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Lecture – 6

Good morning and welcome to the next lecture of this video course on Advanced Hydrology. In the last lecture we looked at an example of momentum equation, application of momentum equation in the form of a 60 degree elbow as a pipe fitting, and we found out how much will be the force that will be acting on the elbow. Then we looked at the example of a one-dimensional unsteady uniform flow, and then we took the example of steady uniform flow one dimensional in the open channel. And we applied all the three basic equations that is continuity, momentum, and energy equations to look at the basic equations that govern the flow of that kind. Then we looked at the energy equation derived from the Reynolds transport theorem in the form of the heat energy. And we briefly looked at the transport processes that are conduction, convection, and radiation. And as part of the reading assignment as I said that you are supposed to look at these transport processes and the associated equations.

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So, we would end this chapter at this point of time, and today we will start the next chapter on the Sub-Surface Water from our PG book. So, it will be the fourth chapter on Sub-Surface Water. We would look at first, I am intentionally skipping the chapter 3 in

the Venti Charles book and we will come back to that after this chapter 4. As the rain falls on a catchment there lots of processes which go on in the catchment in the form of runoff and infiltration and so on. So, if we look at what happens in a catchment in a simple schematic diagram, this is the ground or the ground level, this is some stream or river and this is the water surface elevation in the river, and then associated with that we may have what is called is the Ground Water Table. And within the subsurface zone below the Ground Water Table we have, as you all know a Saturated Zone.

And this is what we call as the Unsaturated Zone of the earth which is above the Ground Water Table all right. On the ground as you know we may have lots of vegetation and trees etcetera and as the rain is falling some of this rain gets trapped on to the vegetation that is called the interception storage. But we will not go into the details of that we will come to that in the surface water chapter.

Some of the rain water that falls on the ground runs off in the form of runoff and some of it goes into the ground as you know as infiltration. So, the arrows here show what we call as the infiltration ok. Now within this Unsaturated Zone, we have a different phenomena or sub-classification of this Unsaturated Zone in the form of what is called a Capillary Zone right. It may be it may have a variable length, all right. So, if these heights you see right, this is called the Capillary Zone. I think all of you understand what is a Capillary Zone, it is basically the zone in which the water moves up due to adhesive forces or due to the suction forces or the capillary action although, the total zone is unsaturated that is it has some moisture content but which is not maximum moisture content but still you have some water which is sucked up by the soil which is called the Capillary Zone.

Now what happens is, during the rain you have the surface runoff taking place on the ground you may have some lateral movement into the stream in the form of what is called the ground water contribution, it is also known as the base flow. So, the base flow is the movement of water towards the streams or the rivers all right and when the ground water conditions are such that the ground water contributes to the stream we call that as the a base flow. And then within the Unsaturated Zone also sometimes we may have the gradients in such manner that it will favor the moment of water like this and this will appear into the stream and this part is known as the interflow or the subsurface flow.

So, we see that in this subsurface zone there is a lot of activity that is taking place, there is the vertical movement of water infiltration, there is lateral movement of water in the Unsaturated Zone, and there is lateral movement of water in the saturated zone and so on. The law of physics which we apply to all of these different types of movements of water in this Sub-Surface Zone is again very complex and we will try to look at some of those equations in this chapter. Before we move on to that, it is important to understand certain basic definitions, and certain fundamental concepts which we are going to use while we derive those equations.

So, let us first look at a sample of soil. If you take a sample of soil from the Unsaturated Zone and we blow it up or we magnify it how does it look like. So, let us say that this is your control volume or sample of a soil we have taken which consists of some solid particles. These are the soil particles or soil solids, I am going to denote this by hatched lines this is of course, a magnified view of what actually is there in the field and then you would see that one can imagine that there is a slight zone around these. Can you think of what is it? Yeah, this is the water which gets stuck around the solid particles due to the adhesive forces.

So, what we are looking at is a cross section through an unsaturated porous medium and if we denote it this hatched portion is the solid particles or the soil solids this is our regular symbol for water within the soil and this is the air field voids. So, you see that this portion are the voids where only air is there some of the voids are filled with water, due to you know the rain fall or whatever and then we have the solid particles, all right. In the Unsaturated Zone, in the Unsaturated Zone the forces that are predominant are gravity, friction and suction. As we just said in the Unsaturated Zone, we have the suction forces also dominating.

However, in the saturated zone suction is not important because soil moisture content is at maximum value. So, in the saturated zone or that is below the Ground Water Table we have only gravity and friction forces taking part. So, this is a blown up view of different components in an unsaturated soil sample. Now, associated with that we will define certain basic soil properties which we are going to use and I am sure all of you have seen this in your under graduate classes. One of them is we will define is called the porosity and we will denote this as 'eta'. And what is eta or porosity? It is defined as the volume of voids divided by the total volume of the soil sample. So, if we write it is volume of voids so it is a volumetric ratio between the volume of the voids and the total volume.

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Associated with that when we move to the next page we will define what is called moisture content. And how we are going to define we denote this by 'theta' in this course. How we are going to define the moisture content which may be slightly different than what you may have studied earlier in this course. We will take theta as the volume of water so; again it is by volume divided by the total volume of your soil sample. If I go back would like to number these equations this is equation number 1 and moving on and I would call this as number 2 and I may be referring to these equations little later. Can you think of what will be the units of these 2 properties which we are just defined that is, eta and theta? They are the ratio of the volumes. So, they do not have any units. So, they are dimension less numbers.

Now, the value of theta will vary between what and what limits? If you think about it, theta is the moisture content, moisture content in the unsaturated soil. What is the minimum value of the moisture content in the soil? Obviously, it is 0. Completely dry sample will have 0 moisture content. And what will be the maximum moisture content in the unsaturated soil? The way we have defined that is volume of water divided by the total volume. If you think about it, it will be porosity. That is, if you keep on adding the

water in the soil sample which is completely dry, the maximum amount of moisture we unsaturated soil sample can hold will be equal to its porosity.

So, the maximum value of theta will be eta. So, you can say that for saturated soil the theta will be equal to eta and for dry soil or completely dry soil the theta will be equal to 0. So, with these 3 basic concepts we will move on and we will look at the derivation of an equation which is very famous and which is the mother of all the empirical equations which you may have seen in your under graduate classes on infiltration. So, what we will do is we are basically trying to derive equation for the one dimensional unsteady unsaturated flow. So, we are trying to derive the governing differential equation or governing equation for one dimensional flow in the Sub-Surface Zone which is unsteady and unsaturated porous medium in the unsaturated porous medium. This will be combination of again 2 laws or 2 basic laws that is, continuity equation plus momentum equation.

So, when we combine these 2 equations we will come up with what is called the famous Richard's equation which governs the movement of water in the Unsaturated Zone and all the infiltration equations. For example, Horton's equation and Phillip's equation and other equations which you may have seen can be derived from this Richard's equation. So, what we will do is first we will look at the continuity equation part of the Richard's equation then we will write the momentum equation and then we combine these 2. Once we combined we will simplify the whole equation and that will become your Richard's equation.

So, let us look at the continuity equation for this type of flow situation. So, then what we do is we take a control volume approach in which we take a sample or a control volume. My hand drawn sketching is not very good but you understand what we are trying to do is here, this is your x direction, this is our y direction and this is the z direction or the vertical direction. We are taking an element which is of length d x in the x direction, d z in the vertical direction and d y in the y direction and we are saying that z is positive upwards and negative downwards.

So, let us say we consider an elemental volume in the soil unsaturated soil of sided d x, d y, d z as shown. And let again let the moisture content in this soil sample be theta. So, what will be the volume of this element, the total volume of this element clearly will be d

x, d y, d z. So, d x, d y, d z that is the volume of any cube or parallelogram. What will be the volume of water in this? At any given point of time remember that this theta is the moisture content which is a function of time.

So, theta at any time will dictate the total amount of water that is available in this soil sample. So what will be the total amount of water? We know the volume is d x, d y, d z and the moisture content is theta. So, the amount of water in this will be equal to we can easily see theta times the volume which is d x, d y and d z from the definition of our moisture content. Now, in this let us say that some amount of water is flowing in the vertical direction. Let us say, this is q and if we use this Taylor series expansion, what is coming out is q plus the rate of change of q in the z direction times the length in the z direction which is dz. Here you are saying what is q, q is called the Darcy's flux which is that is a total Q by A, where this is nothing but the volumetric flow rate per unit area of the soil.

So, what we have done is we have just define the Darcy's flux or small q which is like the discharge intensity, capital Q is the total amount of discharge which is flow through this soil sample at any given time. And small q is nothing but the capital Q divided by the area of cross sectional the soil.

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Now, what we will do is we will derive or we will apply our Reynolds transport theorem to this. Whenever we apply the Reynolds transport theorem we define what is the extensive property to a given situation and in this case we are looking at the continuity equation. So, it would be the mass of the soil water capital B that is and the intensive property corresponding or beta as we have seen earlier would be d B over d m which is going to be equal to 1 and as per the law of conservation of mass that is mass cannot be created or destroyed.

So, this d B over d t should be equal to what the time rate of change of a mass it cannot be created or destroyed should be equal to 0. So, the left hand side of this Reynolds transport theorem will be equal to 0. And now, we write the two components on the right hand side once we do that or try to do that we can see easily that we have 0 is equal to d over d t triple integral of your beta rho d v beta is one in this case rho is the rho of the moving fluid which is water rho w, and then d v plus the extensive is equal to 0 and I am going to call this as equation number 3 and here rho w, we have taken is the density of moving fluid which is floater in this case.

Now, we try to look at these two components one by one. If we look at the first one that is d over d t of this over the whole control volume of your rho w d v is equal to what physically or in words it is the time rate of change of extensive property stored within the control volume or the time rate of change of mass stored within the control volume in our case. This is nothing but as you can see d by d t of your rho w is the density of water and d v, what is the total volume of water in the control volume? We have just written this down if you go back is this volume of water is by theta d x, d y, d z.

So, we write theta d x, d y, d z. So, this will then will be equal to or let me say that here will be density is constant so we come out of that. What about d x, d y, d z, is it varying with respect to time? No, the control volume if fixed so d x, d y, d z is constant so w is also constant so that comes out of v brackets and what we have is the n partial derivative del theta del t and this I say is equation number 4 which is basically this one. So, what we have written is the first part on the right hand side of the Reynolds transport theorem which is rho w volume del theta del t.

Now working on the second term, it is the flux of your extensive property or the net out flux of the extensive property flowing across the control surface. What we have is the two components in flow in the vertical direction is q flowing across what area d x, d y. If you go back to the figure you see q is flowing across this you see this in the z direction it is flowing across what area d x and d y. Similarly, what is coming out is this one q plus del q, del z, d z flowing across same area d x, d y. So, we write this in flux and out flux. So, inflow is v dot d A, v is the Darcy's velocity or Darcy's flux q flowing across the area d x, d y.

Similarly, what is outflow? It is equal to your q plus del q over del z over a distance d z, this is the Darcy's flux multiplied by the area v dot d A and area is d x times d y. So therefore, your net quantity or the whole cross section of your rho w of v dot d A that is what actually we have to write what we have written here is the inflow and outflow but what we have to write is the rho w v dot d A. So, that will be equal to rho w q plus del q over del z d, z times d x, d y that is the outflow minus q over d x, d y, I am sorry at there is a rho w here so let me put it here rho w. So, we have rho w d x, d y. We had seen earlier that when we write this flux term the inflow is always negative and outflow is positive.

So, we have applied the same thing here in front of the inflow we have put a negative sign and in front of the outflow we have put a positive sign. So, if we simply this it should be easy to see that this whole term then will be equal to a certain things will cancel out here it is easy to see that, it will be d x, d y, d z times del q over del z and let me say that this your equation number 5.

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Now what we do is, we just make some algebraic manipulations we use these 2 equations 4 and 5 and we put it back into 3. So, if we do the operation that is putting equation 4 and 5 into 3 what do we get? You do that and divide this result by rho w and the volume which is d x, d y and dz.

In the simplified form what we are going to get is del theta del t plus del q over del z this whole thing is equal to 0, this is my equation number 6. So, what we have done is we have derived the continuity equation for the one dimensional unsteady flow through unsaturated porous media. So, this is only the continuity part of the Richard's equation and going back again because it is important all we have done is we have taken a control volume approach in which we have defined its volume moisture content and we have written the two components of the Reynolds transport theorem and simplified v resulting expression to derive this equation.

So, let me write it down that this is the continuity equation for 1 dimensional unsteady unsaturated flow in the porous medium. Now what we do is we will move on to the momentum equation and then we combine those 2. For the momentum equation, we had seen in the last few lectures we looked at the momentum equation for the ground water flow situation and also for the channel flow. Remember we had done the channel flow it was tau 0 is equal to gamma RS and we applied the similar concept to the ground water and then we had said that this q is equal to K times S f. These equations represent the momentum equation or these 2 different types of situations.

So, what we will do is we will take this equation; the Darcy's law or q is equal to K S f and then apply this to this unsaturated flow situation. So, with that assumption I am going to take this as the momentum equation q is equal to K S f and let us say this is equation number 7 where your q is the Darcy's flux or the velocity S f is the rate of head loss per unit length of the medium and K as you all know is the hydraulic conductivity or permeability.

Now, we have the flow taking place in the vertical direction. For this situation we will take this head loss as del h over del z where, h is the driving force or the head which is causing this water to move and due to our a nomenclature or the way we have defined our a co-ordinates, as the z increases h decreases they are in opposite direction so we

take a negative in front. So, we than have our equation as q is equal to negative K of your del h del z. So, this your Darcy's law or the momentum equation.

Now, what we will work is on this quantity h, which is the driving force or which is causing the water to move vertically downwards.

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Total energy h, it consists of 3 components in the Unsaturated Zone of the subsurface flow. One is the suction head or suction forces which are causing the water to you know suck, suck the water due to the capillary action other is the gravity force which is the datum head and other is the kinetic energy in terms of the velocity head.

So, if we write down these 3 then we have h is equal to psi plus z plus v square over 2g where, psi is the energy due to suction forces or the suction head, z is the datum head or the potential energy and v square by 2g as you know is v velocity head or the kinetic head. In the ground water flow situation the velocities of flows are very small in the, channel flow the velocities cannot be neglected because the velocities are quite significant and v square by 2g will be quite significant as compare to z the potential and other energies.

But here in comparison to v z and psi this quantity, because the velocities are very small v neglected we say negligible. So, we say that this is 0 so the energy which is causing the flow of water in the ground water or the Unsaturated Zone is nothing but the sum of 2

components that is suction head and v datum head. Now, I would like to look at this suction head little bit more closely for those of you who may not have looked at it earlier. In the unsaturated soil, there are or there is electrostatic force which is acting between the water particles or water molecules and the soil particles. This surface adhesion or this electrostatic forces in the form of adhesion draws the water into the soil up or down and where does this water go around the soil particles surfaces leaving the air in the center of the.

As more water is added in the form of rain fall that is what will happen more air will come out of the soil and water will replace that, the air exits upwards and the area of free surfaces diminishes within the medium until the medium is saturated. So, what is happening basically is that we take the soil sample for example, and we put some water into it what will happen is initially due to the adhesive forces the water will place itself around the solid particles. Once that is done and we increase the moisture content or we add more water what happens is more air will be replaced by this water and we keep on adding the water and due to rain fall more water enters into the Unsaturated Zone, what happens is that the Unsaturated Zone becomes completely saturated.

So, what we will do then is we will look at this kind of situation and then how we can write this the momentum equation.

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So, what we have is we have 2 components of this energy which is driving the water in the Unsaturated Zone and we have written this as h is equal to suction head plus the datum head and I am going to call this equation as 9. Now in order to derive this momentum equation for this flow situation what we do is we put this equation 9 in equation 8 and what is equation 8? It is nothing but your Darcy's law I think this is equation 8, I am sorry this is 8.

So, this is q is equal to minus K del h del z. So, if we do that then what we will have is q is equal to minus K del of psi plus z over del z and let us say this is my equation number 10 and this is equal to, we do slight algebraic manipulations here. So, it is the sum of 2 variables we take the derivative so what we do is we have del psi del z and del z del z. So, for that we have K for del psi del z we have using the chain rule we write this as del psi del theta and del theta del z because we have the theta which is the moisture content which is varying as a function of times so we want to involve that plus the second term is K times del z del z which is 1.

Now what we do is we define a soil property D as K times del psi over del theta which is this quantity and this is called "Soil Water Diffusivity" and its units will be L square by t. For example, meter s per second or meter square per day whatever way do you want to define it. So, with this notation we can write q as minus of your D del theta over del z plus K and this is let us say my equation number 11. Now, what I do is I substitute or combine 11 into my continuity equation which is 6. What do we get let me go back and see where is my equation number 6 this is my equation number 6.

So, for q which I have just derived I put that equation here take the derivative of that with respect to z put in this equation six and simplify what I will get I am not going to do that, you can verify that you have del theta over del t is equal to del over del z of d del theta over del z plus and this is my equation number 12. Equation 12 is your famous Richard's equation. So, this is your final form of the governing differential equation for one dimensional flow which is unsteady in the unsaturated porous medium.

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So, let me write this down, this equation 12 which we have just derive is the famous Richard's equation which is the governing differential equation for 1 dimensional unsteady unsaturated flow in the porous medium. It was first represented or derived by Richard's as the name suggests in 1931 and many people or many researchers have a taken this Richard's equation and made certain modifications, made certain simplifications, some assumptions, derived some other types of equations of infiltration in the Unsaturated Zone.

So, now what we will do is we will move on to some you know simple things and look at the infiltration process and some of the equations which you may have seen in your earlier classes. So, first of all let us define infiltration and then we will look at some empirical equations. As you all know infiltration is the process of water penetrating from the ground surface into the soil very simple. What are the factors on which infiltration depends? I am sure you may have seen this earlier. There are 2 main factors on which the infiltration depends on all right the magnitude of the infiltration varies with respect to time.

One is the soil properties themselves all right if your sandy soil we have more infiltration taking place, if you have the clay soil you have less infiltration all right also the surface condition all right. What are the types of land use or land cover look, if it is an industrial area there will be lot of caved areas so there will be hardly any infiltration. But in the rural areas where there is lot of vegetation you will have more infiltration. So, the runoff response from different types of catchment will be different depending upon the infiltration that is why the infiltration process then the standing of the infiltration process is extremely important.

So, writing the factors we have the first 1 is let us say the property of the soil, in which it may be the porosity, the hydraulic conductivity, the texture of the soil and so on. But these 2 are the important factors also the land use, land cover conditions or the surface conditions, that is the vegetal cover and urbanization. There is more urbanization means there will be less infiltration. Ok. Also there is one more important factor on which the infiltration depends let us say you have two different types of soils or rather two different cases in which the soil type is same land use land cover condition is the same all right and the rain occurs on those two catchments. But in one catchment there was some moisture all right and the other one the catchment was completely dry. Do you think that the amount of infiltration will be different? Yes and the reason is because in one of the catchments the initial conditions were only at the higher level of moisture all right.

So, the initial conditions or the current soil moisture conditions also dictate the amount and pattern of infiltration into soil or initial conditions. That is to say if your initial conditions are let us say completely saturated, if you have, if you just had some a rain fall event and the catchment is completely saturated all right. After some time there is some rain fall that occurs what will happen will be hardly an infiltration and most of the rain that falls will become surface runoff. As a case then you had very dry catchment so lot of water will be going as the storage into the infiltration at underground.

So, with this we move on and we will look at what happens when the water goes into the ground. When the water goes into the ground initially the ground is unsaturated or dries. But this moisture content in the Sub-Surface Zone changes as a function of depth. So, if we go vertically down into the ground and if we look at the moisture content how does this look like, that is very important to understand when we are trying to model or develop some equations.

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So, let us look at that which is called the soil moisture profile. This is your ground level and we are looking at v depth and on the this x axis, I am plotting theta or the moisture content basically this is your during the rain fall event very close to the ground what will happen just below the ground level what is the soil moisture content during rain fall event it will be very close to saturation all right. Initially the soil was dry but as the rain fall started just below the ground you have completely saturated conditions.

So if we plot this, you will have something like this, this is your maximum moisture content which is theta is equal to eta very close to the ground. As we come down at any given point of time soil moisture profile will look like this where let me draw different zones, this is your saturated zone very close to the ground, this is the transmission zone and just at the point where the moisture content changes vary sharply this is called the "wetting zone" and this thing is called the "plotting front".

So, what is happening is in the transition zone the moisture content reduces all right slightly all right it is all most constant here. But as we go down because initially the moisture content was very little it was all all most dry the soil was all most dry. So, what we have is a very sharp decline, very sharp decline in the moisture content at the wetting front. So, let me just rephrase it in the saturated zone just below the ground during a rain fall event which is near the surface. We have your theta is equal to maximum which is

equal to the porosity. In the transmission zone in the transmission zone the moisture content in fairly uniform.

And theta is less than eta and in the wetting zone theta decreases with the depth quite rapidly. And what is wetting front? A sun is the zone or the surface rather theta changes or decreases very sharply. So, to understand this basically what we are trying to do is let us say during a rain fall event you just take a snap shot of the soil moisture content at any given time. Then as the moisture is moving downwards into the soil what is happening is it is wetting the soil all right because there is a rain falling from the top initially it was dry but due to the rain fall water has gone in it has made the soil completely saturated for some time and then as you have gone down there is a wetting front.

As the time will axis this wetting front will be completely moving down into the soil and it is length will be increasing and the amount of infiltration will be increasing. Now, what we do is let us look at the relationship between different types of infiltration quantities this was as far as the soil moisture profile is concern. Now, let us say we want to calculate during all this process which we have seen our objective as engineers or hydrologists is to find out the rate of infiltration and the cumulative infiltration. So, let us say the infiltration rate is equal to f (t). 'f' is the rate of infiltration at any given time at what rate the water is infiltrating into this soil that is what we want to calculate for this understanding the process of movement of the our moisture profile is important.

And let us say the cumulative infiltration we define as F (t). So, any infiltration equation we study we try to determine the expression or we try to calculate or estimate small f and capital F that is rate of infiltration and cumulative infiltration at any given point of time. There is a relationship between these 2 as all of you know I suppose this f (t) is nothing but the cumulative that is from 0 to a particular time t of your f tau d tau or in other words we can say your small f is nothing but the slope of your capital F curve.

So, small f is like your intensity curve and capital F is your mass curve. So, if you know the one you can always calculate or find out the other one all right. So, if you have the infiltration rate curve available we just integrate and if you have the cumulative infiltration we can differentiate. I think, I would like to stop at this point of time where we have discuss the soil moisture profile and define what is the rate of infiltration and cumulative infiltration. Now, in the next class we will look at some empirical equations such as the Horton's equation, Philips equation and Green amp equation. How we look at those equations? How we establish those and how we can use them for practical purposes?