

**Advanced Hydrology**  
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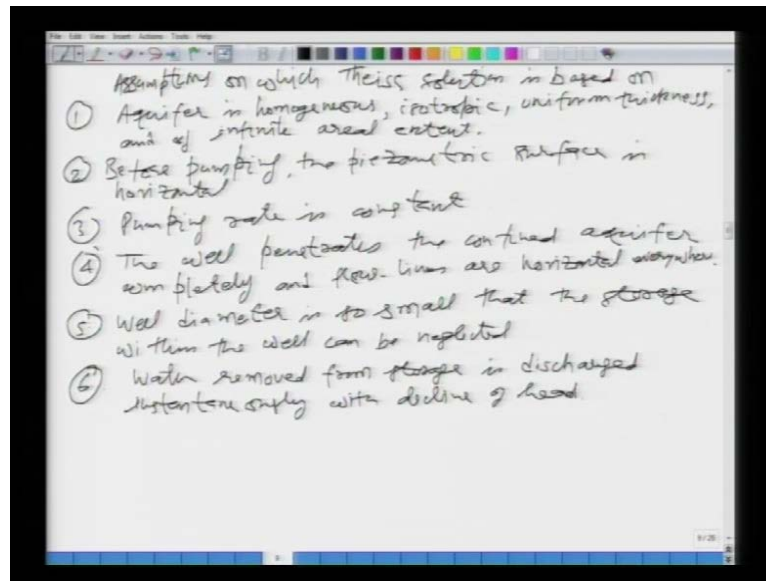
**Lecture – 44**

Good morning and welcome to the last lecture of this video course on advanced hydrology. In the last class, we looked at an example a numerical example of radial steady flow in a confined aquifer. Then we looked at the derivation of the pumping discharge in a unconfined aquifer which is isotropic and homogenous and of course, we looked at the steady state case.

Then we looked up an, at an example for this case that is unconfined aquifer. Then we looked at a very special case in which we analyzed or we looked at the derivation of the water table profile in an unconfined aquifer, which is subjected to a discharge, and also we are pumping some water at the rate  $q$ . We said that the flow at any radial distance  $r$  will be not constant and it will be varying because of the recharge rate that is coming in.

Then towards the end of the last lecture, we looked at the governing differential equation for the unsteady case in a confined aquifer. Then we said that the Theis has proposed a solution to that governing differential equation for the unsteady case. What we would like to do next is we would like to look at the application of this unsteady case for the determination of the aquifer parameters, which is the transmissibility and the storage coefficient. Before we go to that, let us look at the assumptions on which that unsteady state governing differential equation was derived and the solution has been arrived at by Theis.

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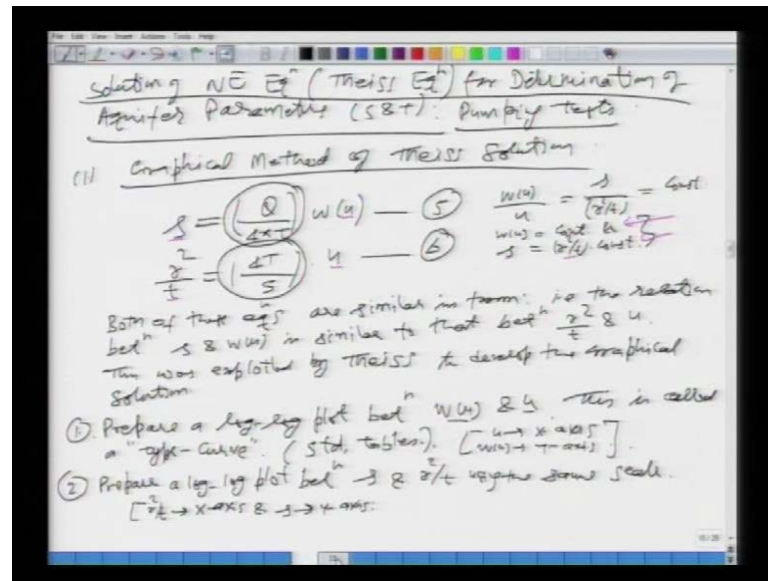


So, the next thing we are going to look at is the assumptions on which the Theis solution is based on and I will list you these assumptions, give you a list of all this one by one. Aquifer is homogenous, isotropic and of uniform thickness is of uniform thickness and of infinite and of infinite areal extent. And these are the basic assumptions we have seen some of them earlier. Second one is that before pumping the piezometric surface, the piezometric surface is horizontal. Number three, the pumping rate is constant. You are not varying the rate at which we are pumping water out of this confined aquifer.

Number four, the well penetrates the confined aquifer completely and flow lines are horizontal everywhere, everywhere means in the aquifer. If the well will not penetrate completely, then the flow lines actually will not be horizontal towards the well or the closed to the well. They will be bending so as to reach the well. The well diameter is so small; the well diameter is so small that the storage within the well can be neglected. So, in deriving the solution by Theis, he has neglected the storage within the well.

The last assumption is that water removed from storage is discharged instantaneously with decline of the head because we are in the sorry, we are in the confined aquifer. That is why we know that the when we are pumping water out then it is not coming from the storage. And if there is any water which is coming out of the storage we are assuming that it is replaced instantaneously, immediately.

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Now, what we do is we look at the solution of the non equilibrium equation or the Theis solution or the Theis equation for determination of the aquifer parameters. We see that the problem of determination of the aquifer parameters is very important in the ground water hydrology. There are two basic aquifer parameters which is the storage constant and the transmissivity as we have said earlier. The values of these parameters are needed in any ground water flow model and these parameters are normally determined through what is called pumping (( )).

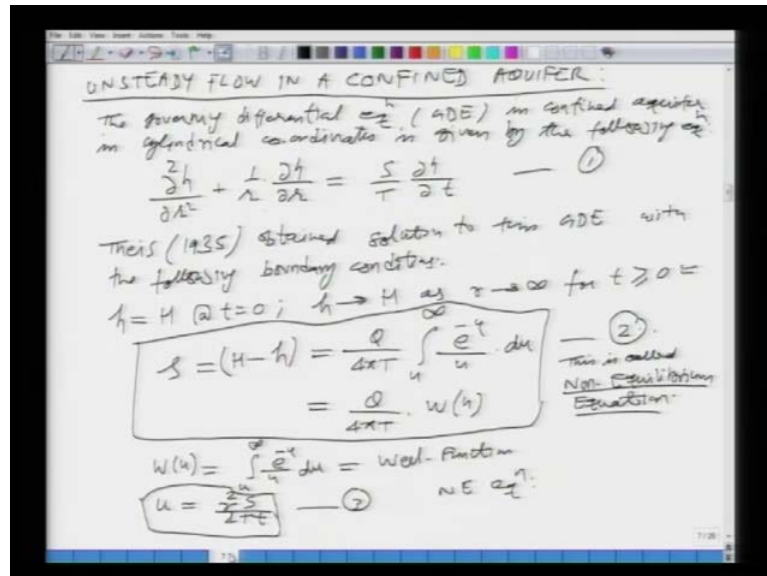
So, what we do is we pump the water and then we make certain observations. We have seen, how it is used in the thyme's equation or the steady state condition. Then we said that steady state conditions are difficult to reach, it has you know lots of problems and it is not economical and so on. So, we use the non equilibrium equation or the Theis equation.

So, what we will do is we will look at a couple of methods of determination of these aquifer parameters as in t and the first one of them is what is called the Theis method of solution using the graphical form. The second one is what is called the Cooper Jacob method. So, let us start looking at these two through the use of pumping test.

So, the first one is the graphical method of Theis solution. Now, if we look at the equations which were proposed by Theis that is what it is?  $s$  is equal to  $\frac{Q}{4\pi T} W(u)$ , let me number this as five and then there is a definition of  $u$  the parameter itself. And

if you slightly readjust it, it can be written as follows  $r^2 t$  is equal to  $4 T$  over  $S$  times  $u$  and let us say this is your equation number six.

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If you go back this is what we are talking about.  $s$  is equal to  $Q$  by  $4 \pi T$   $w$   $u$  that was number one and the other one is this equation three. So, I have just rewritten equation two and equation three here, these are five and six. Now, both of these equations are similar in form. Well, what do we mean by that? If you look at this equation we see that  $s$  is a variable, which needs to be observed or the data needs to be obtained and  $u$  is another variable, which is changing as a function of time.

However, in this equation this  $Q$  over  $4 \pi T$  this is constant in equation five. Similarly, if you look at equation six your  $r^2$  by  $t$  it is variable, it is changing as a function of time and  $u$  is also which is changing as a function of time. However, this thing is constant. So, what we are saying is that the equation number five and six have a similar form and this fact was actually exploited by Theis that is the relation between  $s$  and  $w$   $u$  that is equation number five is similar to that between  $r^2$  by  $t$  and  $u$ .

So, this was exploited or used rather I should say by Theis to develop the graphical solution. So, we see that the, he has written down these two equations and its similarity has being identified between these two equations. Then what he has done is he has proposed this graphical solution in which we prepare two different graphs. I am going to go through the step by step procedure of the solution in a minute. So, what we basically

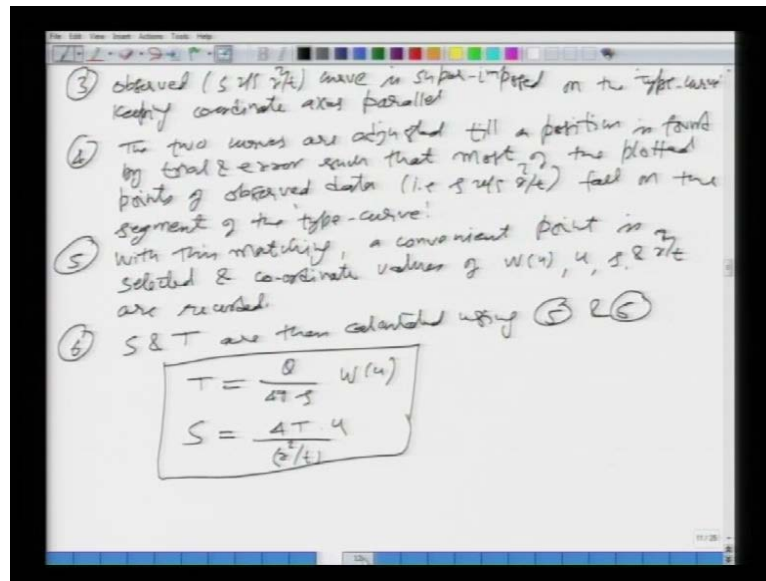
do in this is we prepare one graph, second graph we superimpose them. We find the data point using the observed data, using that observed data point we try to make certain calculations to obtain  $s$  and  $t$ .

So, it is a very simple procedure and I will give you a step by step method of obtaining  $s$  and  $t$  using Theis method of graphical solution. So, let us see the procedure. First of all we prepare a log log plot, a double log plot between what?  $w_u$  and  $u$ . This is nothing but your well function for different values of  $u$ .  $w$  is your well function. This is called a type curve. So, we select different values of  $u$ , find the well function. You just plot it on a log log paper and in fact this well function is available in the standard tables, for different values of  $u$  you can find the values of  $w_u$  or the well function in standard tables and how do we plot?

Well we plot  $u$  on the x axis and  $w_u$  on the y axis that is the type curve. Then number two is we prepare, we prepare a log log plot, another log log plot between your  $s$  and scale or the same or using the same scale. I should say using the same scale, the scale should be same so that we can superimpose them and how do we do it? When we take  $r^2$  on the x axis and the drawdowns on the y axis. Why we are doing this, is basically if you take these two equations. We are saying five and six are similar to each other, so if you take the ratio  $w_u$  over  $u$ .

You divide these two equations let us say will be equal to what? Will be equal to your  $s$  over  $r^2$  by  $t$ . What is all of this is equal to? This is equal to ratio of this quantity and this quantity which is constant. So, you have  $w_u$  is equal to this constant times  $u$  and also your  $s$  is equal to  $r^2$  by  $t$  times that constant. So, this is the relationship you are trying to exploit. So, what you do then is you prepare a log log plot using this data that is  $u$  versus  $w_u$  and the another log log plot using this data. That is  $s$  and  $r^2$  by  $t$ . And what we are assuming is that we have done the pumping test and as a function of time we have the values of drawdowns. So, that was your step number two.

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Let us move on and then we look at the third step in which the observed s versus r square by t curve that is the second curve, which we have prepared on the log log scale which is the same scale is superimposed, is superimposed on the type curve. Both of them are representing the same kind of relationship, we are using the same scale, so we are trying to superimpose one on the other. How? Basically, you are keeping your coordinate axis parallel. Once, you superimpose you try to see that the two curves are then adjusted. How? Using a trial and error basically till a position, till a position is found.

How? By trial and error such that, such that most of the plotted points, most of the plotted points of the observed data, of the observed data, so which curve we are talking? s versus r square by t that is the data, fall on the segment. It will not completely match because of the errors; fall on the segment of the type curve. So, what we are trying to do is we have prepared a type curve and an observed data curve using the same scale. They are on the log log plot, you superimpose that is you put one on top of each other. Then we try to adjust such that most of the data points will follow the type curve or will go through the or pass through the type curve. So, we find that segment.

Once we have done that what is the next step? Number five, with this matching a convenient point is selected, is selected and the coordinate values, coordinate values of all these things x y for both the curves are found which would be what, w u, u, s and r square by t from the two curves, which are matching are recorded. So, it is a manual

method you see and then the last one is the values of your aquifer parameter that is S and T are then calculated using equations five and six, which we have just written.

Well, how do we do that? We will just use those T is equal to it will be Q over 4 pi s, s is just you have found from the matching point. Then w u is also you have found and then capital S is your storage constant that will be 4 T u over r square by t. You see that it is a very simple method in which you are using a trial and error procedure of matching the two different curves. Then noting down the observed values of four different variables and then using these equations to calculate the aquifer parameters.

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(2) Cooper-Jacob Method  $u = \frac{r^2 S}{4 T t}$

For small  $r$  & large  $t \Rightarrow u \rightarrow$  small.  
 For small  $u \leq 0.01 \Rightarrow$  the first two terms in the definition of  $w(u)$  are sufficient.

$$w(u) = \frac{Q}{4 \pi T} \left[ -0.5772 - \ln u + u - \frac{u^2}{2} + \frac{u^3}{3} - \dots \right]$$

$$\Rightarrow w(u) = \frac{Q}{4 \pi T} \left[ -0.5772 - \ln \frac{r^2 S}{4 T t} \right]$$

Rearranging the solution  $\Rightarrow$  a set of equations:

$$S = \frac{Q}{4 \pi T} \left[ -0.5772 - \ln \frac{r^2 S}{4 T t} \right]$$

$$S = \frac{2.30 Q}{4 \pi T} \log \frac{2.25 T t}{r^2 S}$$

So, that was the method number one and the second method we are going to look at is what is called the Cooper Jacob method of solution. Now, what they have done is they have exploited the relationship of u. How is u defined? It is r square S over 4 capital T small t. If you look at this equation closely or carefully for small values of r and large values of t would mean what? Would obviously mean that u is small, is it not? For small values of r then you have r square and then you have the large value of t, for this combination the u will be very small.

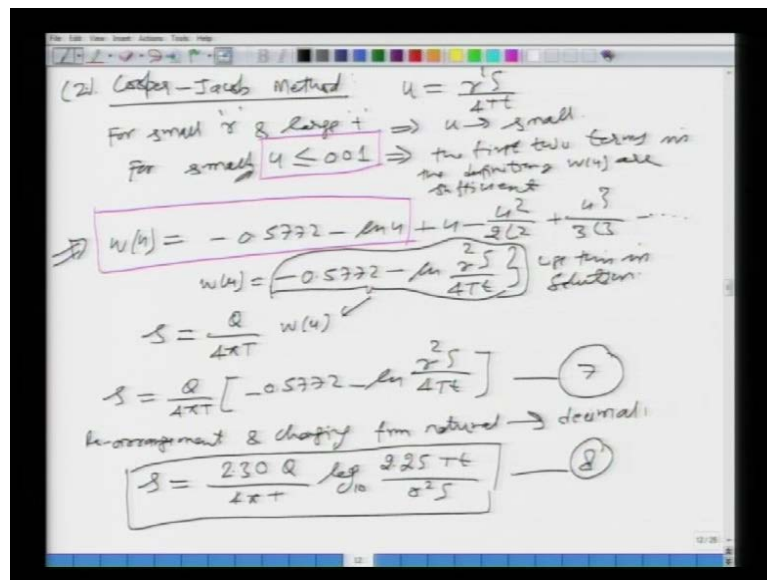
Then for small u which is less than 0.01. What these gentlemen have said is that the first two terms in the Taylor series expansion term the definition of w or the well function are sufficient. So, if your u is small or less than 0.01, then if you look at your w u or the well function is given as Q over 4 pi T times minus of your 0.5772 minus of your l n of your u

plus  $u$  minus  $u$  squared over  $2 \cdot 2$  factorial plus  $u$  cubed over  $3 \cdot 3$  factorial and so on. That is what we had seen is the expression for  $w$   $u$ , but for very small value of  $u$ , what he is saying is that you can just take only the first two and ignore these.

Why ignore? Because you have  $u$   $u$  squared  $u$  cubed and  $u$  is very small anyways. So, if you did that what you have is  $w$   $u$  is equal to this.  $Q$  over  $4 \pi T$  of your minus  $0.5772$  minus  $\log$  of  $u$  which is  $r$  square  $S$  over  $4 T t$ . So, what we then do is then we slightly rearrange the solution and then we can obtain the expression for the draw down, to get the draw down. It will be  $S$  is equal to your  $Q$  over  $4 \pi T$  of minus of  $0.5772$  minus natural log of  $u$  which is  $r$  square  $S$  over  $4 T t$ .

Then if we do the rearrangements the final solution will come out as  $2.30$  capital  $Q$  over  $4 \pi T$  and the log of  $2.25$  capital  $T$  small  $t$  over  $r$  square over  $S$ . So, we take the first two terms in the well function which is given as minus  $5772$  minus  $1 \ln u$  plus  $u$  minus  $u$  squared over  $2$  factorial plus  $u$  cubed over  $3 \cdot 3$  factorial and so on. So, what we do is then we take the first two terms that is these two.

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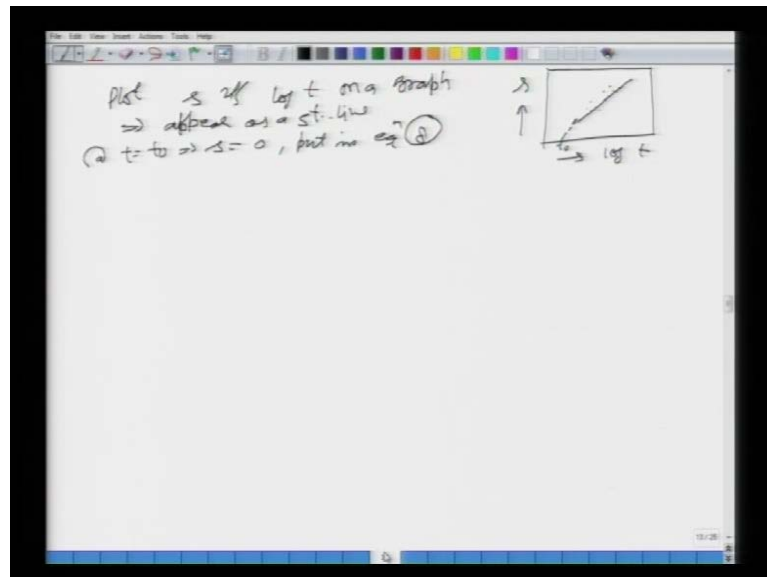
So, that your  $w$   $u$  will be equal to minus of your  $5772$  times or minus natural log of  $u$  and what is  $u$ ? It is  $r$  square  $S$  over  $4 T t$ . Then we use this in your solution the Theis solution. What is that? That is  $S$  is equal to  $Q$  over  $4 \pi T$  of your  $w$   $u$ . So, we put for  $w$   $u$ , all of this thing here. Once we do that what we will have is this. So, I just substituted this



whole quantity which is the approximate one into this solution. And let me say that this is my equation number seven.

Now, what we do is we do some rearrangement of this equation and changing from natural to decimal log. So, we convert this equation into decimal log and we do slight adjustments, what we are going to get is this  $2.30 Q$  over  $4 \pi T$  of your log to the base 10 now of  $2.25 T t$  over  $r$  square s. So, this is the draw down under the simplification and what is that simplification? This one in which we are taking only the first two terms such that  $u$  is smaller than 0.01.

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So, with this what we do is plot your,  $s$  versus  $\log t$  on a graph. So, if you go back and look at this equation you are plotting  $s$  and  $\log$  of  $t$ . How will that appear? Obviously, it will appear as a straight line, is it not? So, if you see here you have  $\log t$  here and  $s$  here and you will have these points. This is thus that is how it will appear something similar to a straight line and few of the data points actually may be falling outside the frame. So, what you do? This then is you extrapolate this curve, so that it cuts the  $x$  axis and let us call the time as  $t$  naught or  $t_0$ . So, at  $t$  is equal to  $t_0$  what you are saying is then, then  $s$  is equal to 0. Then put these values in equation eight, which we have just written.

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(2) Casper-Jacobs Method:  $u = \frac{rS}{4\pi T}$

For small  $r$  & large  $t \Rightarrow u \rightarrow$  small.

For small  $u \leq 0.01 \Rightarrow$  the first two terms in the definition  $w(u)$  are sufficient.

$w(u) = -0.5772 - \ln u + u - \frac{u^2}{2} + \frac{u^3}{3} - \dots$

$w(u) = \left[ -0.5772 - \ln \frac{rS}{4\pi T} \right]$  put this in solution.

$s = \frac{Q}{4\pi T} w(u)$

$s = \frac{Q}{4\pi T} \left[ -0.5772 - \ln \frac{rS}{4\pi T} \right]$  — (7)

re-arrangement & dropping from natural  $\rightarrow$  decimal.

$s = \frac{2.30 Q}{4\pi T} \log_{10} \frac{2.25 T t}{r^2 S}$  — (8)

What is that equation eight? This one. So, you put  $s$  is equal to 0 and  $t$  is equal to  $t_0$  in this.

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Plot  $s$  of  $\log t$  on a graph  $\Rightarrow$  appear as a st. line.

@  $t = t_0 \Rightarrow s = 0$ , put in eq (8)

$\frac{2.30 Q}{4\pi T} \log_{10} \left[ \frac{2.25 T t_0}{r^2 S} \right] = 0$  — (9)

$\Rightarrow \frac{2.25 T t_0}{r^2 S} = 1$  — (10)

$s = \frac{2.25 T t_0}{r^2}$  — (11)

Now, with eq (8) @  $s_1, s_2$  such that  $(t_2/t_1) = 10$ .

$s_1 = \frac{2.30 Q}{4\pi T} \log_{10} \frac{2.25 T t_1}{r^2 S}$

$s_2 = \frac{2.30 Q}{4\pi T} \log_{10} \frac{2.25 T t_2}{r^2 S}$  } Subtract.

If you do that that is what you will get  $2.30 Q$  over  $4 \pi$  capital  $T$  of your log of  $2.25$  capital  $T t_0$  divided by  $r$  square  $S$  is equal to 0 and this is my equation number nine. So, all I have done is I have set this is equal to 0 and I put  $t$  is equal to  $t_0$ . Now, if for this whole expression to be 0, we know that  $Q$  is not 0. So, this whole quantity is not 0.  $\pi$  is not 0,  $t$  is not 0,  $Q$  is not 0. So, what has to be 0 is log of this whole quantity should

be 0. So, this whole thing has to be 0 that means your  $2.25 T t$  naught over  $r$  square over  $S$  should be equal to what? This is log to the base 10, has to be equal to 1.

So, the quantity in these brackets has to be equal to 1 for the log of to the base 10 of this quantity equal to 0. Log of 1 is 0. So, with this relationship we find out capital  $S$  which is going to be your  $2.25 T t_0$  over  $r$  square. Now, what you have done actually is you have derived a relationship between  $S$  and  $T$ . Let us say this is your equation number eleven. Now, what we do is we need to be able to find this  $T$ . How we do that? Like this. Now, we write equation number eight at two drawdowns or we select the data  $s_1$  and  $s_2$  such that how the ratio of the corresponding times  $t_2$  over  $t_1$  is equal to 10.

You see that you have converted everything into decimal log, log to the base 10 and then we are trying to get the ratio as 10, so that the things will be simplified. So, I am talking about this equation eight you select  $s_1$  and  $s_2$  to such that  $t_2$  over  $t_1$  is 10. Then you can apply this equation and then you can write  $s_1$  will be equal to what?  $2.30 Q$  over  $4 \pi T \log$  to the base 10 of  $2.25 T t_1$  divided by  $r$  square  $S$ . Similarly, you have  $s_2$  as  $2.30 Q$  over  $4 \pi t \log$  to the base 10 of  $2.25 T t_2$  over  $r$  square  $S$ .

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The image shows a whiteboard with handwritten mathematical equations. The top equation is  $\Delta s = s_2 - s_1 = \frac{2.30Q}{4\pi T} \cdot \log\left(\frac{t_2}{t_1}\right) = 10$ . Below this, the equation for  $T$  is derived as  $T = \frac{2.30Q}{4\pi \Delta s}$ . A circled number 12 is written next to the denominator. Below the equations, there is a note: "So, first use eq (12) to get T. Then, we use eq (11) to calculate".

All I have done is I have taken equation eight and I am putting  $s_1$   $t_1$   $s_2$   $t_2$ , so we are writing this equation at those two data points. Now, what you do is you subtract this. If you subtract one from the other what will you get? Let us say you have  $\Delta s$  is equal to  $s_2$  minus  $s_1$ , which will be equal to what  $2.30 Q$  over  $4 \pi T$  of your log of what?  $t_2$

over  $t_1$ , if you go back and see everything else will cancel out, once you subtract you will have  $t_2$  over  $t_1$ .

What is  $t_2$  over  $t_1$ ? This is equal to 10, log of 10 is equal to what? 1. So, this equation then essentially gives you the value of  $T$ ; that will be  $2.30 Q$  over  $4 \pi \Delta S$  because this whole quantity is equal to 1. So, you can find the value of  $t$  as this expression and let us say this is your equation number twelve. So, what we have then is  $Q$  is known, pumping discharge and  $\Delta s$  is the difference in the two drawdowns which we have observed, which we have already observed.

So, we can get the value of  $T$ . So, then we say that we first use equation twelve, this equation to get the  $T$ . Once, we have  $T$  then what we do? Then we use equation eleven to calculate  $S$ . What is equation eleven? It is this one. Once, we have found the value of  $T$  we can find the value of  $S$ , where  $t$  naught is obtained from this curve, this one here and  $r$  is a distance to the observation value which is a single value.

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Handwritten notes on a whiteboard:

Plot  $s$  vs  $\log t$  on a graph  
 $\Rightarrow$  appear as a st. line  
 (a)  $t = t_0 \Rightarrow s = 0$ , put in eq. (8)

$$\frac{2.30Q}{4\pi T} \log \left[ \frac{2.25 T t_0}{r^2 S} \right] = 0 \quad (9)$$

$$\Rightarrow \frac{2.25 T t_0}{r^2 S} = 1 \quad (10)$$

$$S = \frac{2.25 T t_0}{r^2} \quad (11)$$

Now, with eq. (8) (a)  $s_1, s_2$  such that  $(t_2/t_1) = 10$

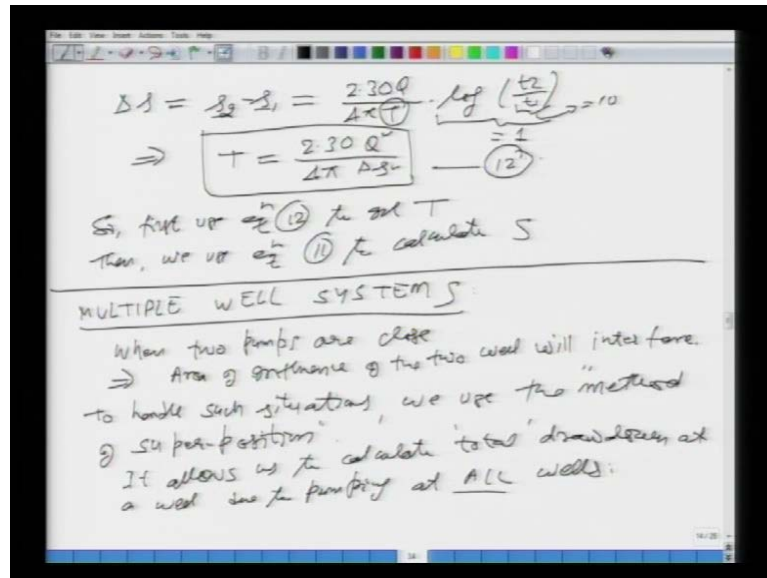
$$s_1 = \frac{2.30Q}{4\pi T} \log \frac{2.25 T t_1}{r^2 S} \quad \text{Subtract}$$

$$s_2 = \frac{2.30Q}{4\pi T} \log \frac{2.25 T t_2}{r^2 S}$$

So, this way we see that we can use the non-equilibrium equation with a single observation well and there are couple of methods we have seen; one is the graphical method proposed by Theis. The other one is called the Cooper Jacob method in which we convert the things into log to the base 10. We exploit the properties of the logarithm to obtain the value of  $T$  and  $S$ .

The last thing which we would do in this course is look at the case of the multiple well system. In the last class I mentioned about interference effects. If you have more than one pumping wells, then what happens is the draw down at one well will affect the draw down at the other. So, let us look at how we analyze this multiple well system.

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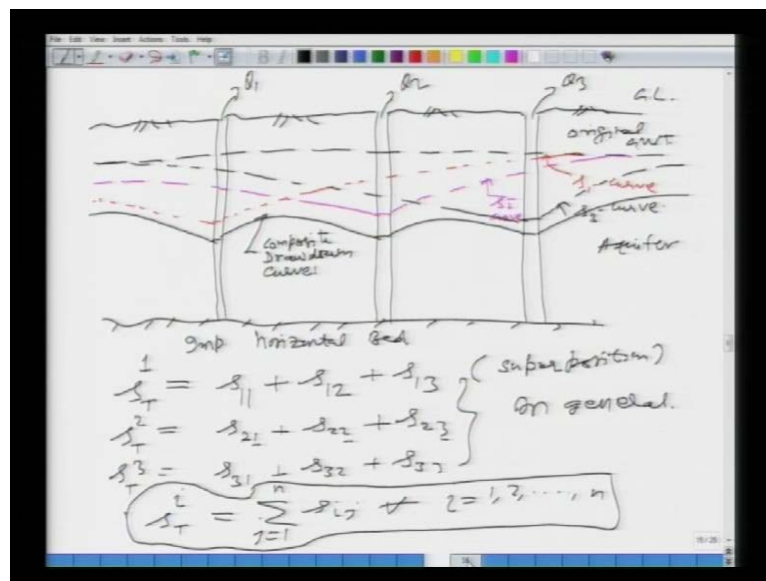
The next thing or the last thing we will look at is multiple well systems. Why do we need to do this? Well, when there are two pumps or two pumping wells are close or located very close to each other then what will happen? Essentially, the area of influence, the area of influence of the two wells will interfere with each other. Do you see that we have seen that when we are pumping water from a confined aquifer or unconfined aquifer. It has a certain area of influence, the ground water table or the piezometric surface will extend up to the area of influence. But let us say you have a, another pumping well within this area of influence.

So, then the two areas of influence will interfere or will impact each other in that case how do you handle the situation, what will be the draw down? So, the draw down in one well will not only be because of the pumping in well one, but it will also be due to the pumping in well two because the water tables are interacting with each other. So, what do we do with that? Then what we do is to handle such situations, to handle such situations what we do is we use what is called the method of super position, we use the

method of super position, to do what? It allows us to calculate or determine total draw down at a well due to pumping at all the wells.

So, what we are saying is that let us say you have three four different wells in a aquifer. So, what will be the draw down in one particular well? It will be the draw down due to the pumping in this well plus the draw down due to the pumping in the second well which is interacting with it, the third well and the fourth well and so on. All we do is we superimpose them. So, what we would like to do is we will look at an example. First we will look at how the total drawdowns will be and then we will look at an example of this interaction of the areas of influence and calculation of the total draw downs. Before we go to the, this method of super position in calculating the total draw down in a multiple well system let us look at the whole system graphically, what is exactly happening.

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So, if you come here let us say you have or let me use more space in fact, this is your impervious horizontal boundary like we normally do and then we have let us say three different wells which are penetrating into this. And then this is your ground level, this is your ground level and then let us say this is your original ground water table and each of these are pumping wells is pumping water at some rate. Let us say this is  $Q_1$ , it is  $Q_2$  and  $Q_3$ . So, you have this aquifer, this is your aquifer in which there are three wells.

They may be in line or they may be in you know, if you look at from the top they may be staggered. So, what we have then is let us say due to the pumping in well one we use

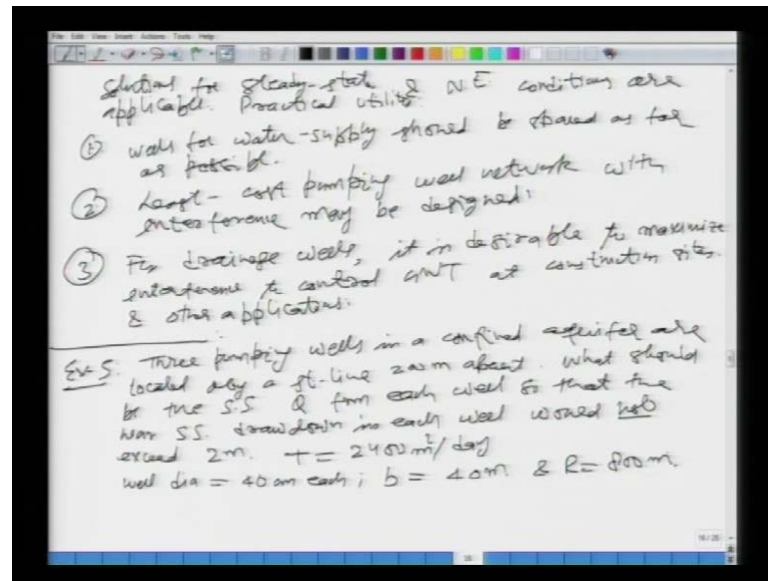
different colors, let us say this is your draw down curve for the well one or you say that this is your  $s_1$  curve. The second one is let us say here in the second well. So, this is your  $s_2$  curve, these are individual you know draw down curves and then the last one let us say is the third one, the draw down curve in the third well and this is your  $s_3$  curve. So, what we are looking at is there are three different cones of you know depression or the draw down curves we are looking at due to pumping in well one, two and three and we have represented them with different colors.

Then we want to find out the total. Let us say, what we will do is we will add the corresponding ordinates or we will see how we can actually do that, but let us say that this is your composite draw down curve. Then it will go like this. So, what is this well this one is you're what is called composite draw down curve, which can be determined using the method of super position. How do we do that? Well, let us say try to write the expression for the total draw down in well one,  $s_{T1}$  is what, total draw down in well one. It will be equal to what?

They are how many wells operating? Three wells, so it will be equal to the draw down at well one due to pumping in well one,  $s_{11}$  plus draw down at well one due to pumping in well two and draw down at well one due to pumping in well three. So, this will be your total draw down. So, all we are doing is we are using the super position here. Then similarly, you can write the expression for the total draw down in well two will be equal to  $s_{21}$  plus  $s_{22}$  plus  $s_{23}$ , which is the draw downs at well two due to pumping in 1, 2, 3. Similarly, you have  $s_{T3}$  going to be equal to  $s_{31}$  plus  $s_{32}$  plus  $s_{33}$ .

Therefore, in general what we can write, we can say that  $s_T$  in any  $i$ th well will be equal to what? Will be the summation of  $s_{ij}$ , you have two subscripts here where  $j$  is going for 1 to  $n$ , there are  $n$  number of wells. This equation is true for all  $i$ 's.  $i$  going from 1 to all the way to  $n$ . So, this is your method of super position which can be used.

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And once we apply this method of super position in a multiple well system then we say that the solutions, which we have just seen. The solutions for the steady state conditions and non-equilibrium that is unsteady state are applicable in this case, for the multiple well system also. Well, what is the practical utility of multiple well system? We will look at a few cases. Number one you have the wells for water supply, in a particular area you may be tapping water using different pumping wells.

Water supply should ideally be spaced as far as possible; when we are pumping water we should try to see that they are very far apart, so that the areas of influence are not interacting with each other. However, it is not possible all the time. The other thing is that the least cost pumping, least cost pumping well network. When you have a network of wells you want to minimize your cost with interference may be design, when it is not possible you need to have lot of pumping wells then we design them in such a way that our cost is minimized.

Number three is for drainage wells. For drainage wells it is desirable, it is desirable to maximize the interference to control the ground water table at construction sites and some other applications. So, what we are saying essentially is that the knowledge of interaction of these areas of influence or the multiple well system and how we analyze that is important in a few situations. As far as possible we should make sure that it does not happen, but if it is not possible then we can design a drainage, not the drainage the



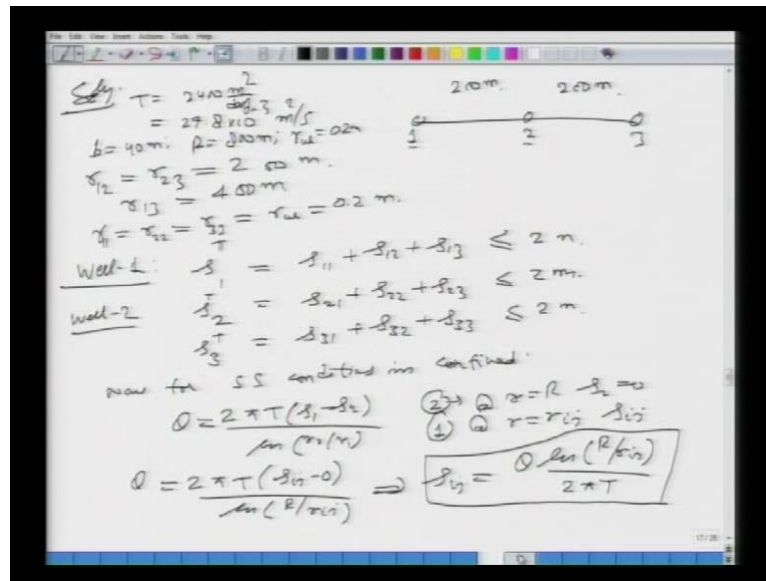
pumping well system in such a way that our cost is minimized. That is number one for the water supply system.

There is another situation in which what we want to do is let us say we want to construct a building in some place. In that case the foundations have to go let us say deep enough. So, what we need to do is we need to pump the water, so that the construction can take place. In that case we want to maximize the interference, so that the draw down is as much as possible. So, there are different types of objectives. For that to be able to achieve these objectives we need to understand this interaction between different wells and draw down curves and so on.

So, what we will do next is we will look up at an example which is based on the superposition. So, let us see we will look at the example which says that the three pumping wells in a confined aquifer are located, are located along the straight line which is 200 meters apart from each other. The question says what should be the steady state discharge from each well, so that so that the nearest, so that the not nearest, but near steady state draw down.

So, things are probably not very steady state, but they can be assumed to be steady state. The nearest steady state draw down in each well, in each well would not exceed, would not exceed 2 meters. The data that is given to you is  $T$  is equal to 2400 meter square per day, the well diameter all of them is 40 centimeter each,  $b$  is equal to 40 meter that is the thickness of your confined aquifer and the radius of influence  $R$  is 800 meters.

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So, if we look at this solution of this problem, you have these three wells one here, two here and the third one here and they are located 200 meters apart in a line. The T is given to you as 2400 meter square per day. You can convert it into s i unit, so it will be 27.8 10 minus 3 of your meter square per second. The other data is b is equal to 40 meters, R is 800 meters t r w is same for all the three wells, 40 centimeter is the dia. So, radius will be 0.2 meters. Now, I notice that what will be r 1 2? It will be equal to r 2 3, it will be equal to what, the radius which we are going to use to get the draw down at pump one due to pumping at well two.

So, one two what is the distance? One, between points one and two it is 200 meters. So, r 1 2 is equal to r 2 3 is equal to 200 meters. What is r 1 3? r 1 3 is 400 meters. So, if you are trying to find out the effect of draw down on one due to pumping at three the distance you have to use is 400 meters. Similarly, it should be easy to see that r 1 1 will be equal to r 2 2 and that will be equal r 3 3, which is equal to r w which is 0.2 meter.

All of these things should be very easy to see. Now, what we are saying is that at well one let us say the draw down at 1 due to 1 is equal to your s 1 1 plus s 1 2 plus s 1 3 all of that should be less than equal to 2 meters. Similarly, the draw down at 2 or your well two for example, will be equal to what? The total at 2, this is total actually, let me say total will be equal to s 2 1 plus s 2 2 plus s 2 3 that should be also this. And similarly, at s

3 the total is  $s_{31}$  plus  $s_{32}$  plus  $s_{33}$ . At each well the total draw down has to be less or equal to 2 meters.

Now, for steady state conditions in a confined aquifer, what is our equation? The equation is  $Q$  is equal to  $2\pi$  of your  $s_1$  minus  $s_2$  over natural log of  $r_2$  over  $r_1$ . We have derived this earlier. So, if we take the cross section two at  $r$  is equal to capital  $R$ . So, that your  $s_2$  will be 0 and if you take one at  $r$  is equal to  $r_{ij}$ , any general one and then it will be  $s_{ij}$ . If you use that, then you can write a general expression as  $2\pi$  of your  $s_{ij}$  minus 0 divided by natural log of your capital  $R$  over  $r_{ij}$ .

So, we have all these different combinations one, one, one, two, two, two, three and so on. This is representing your  $r_{ij}$  and  $s_{ij}$ . So, from this you can find out your  $s_{ij}$  is going to be equal to what,  $Q$  natural log of  $R$  over  $r_{ij}$ . This thing divided by  $2\pi T$ . So, we are going to apply, we are going to use different combinations of  $i$  and  $j$  and write the equation for this draw down in terms of  $Q$  and then we can find the value of  $Q$ .

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Well-1:

$$s_{11} = \frac{Q \ln\left(\frac{R}{r_{11}}\right)}{2\pi T} = \frac{Q \ln\left(\frac{800}{0.2}\right)}{2\pi (27.8 \times 10^{-3})} = 47.48Q$$

$$s_{12} = \frac{Q \ln\left(\frac{R}{r_{12}}\right)}{2\pi T} = \frac{Q \ln\left(\frac{800}{100}\right)}{2\pi (27.8 \times 10^{-3})} = 7.94Q$$

$$s_{13} = \frac{Q \ln\left(\frac{R}{r_{13}}\right)}{2\pi T} = 2.97Q$$

$$s_T^1 = s_{11} + s_{12} + s_{13} = 47.48Q + 7.94Q + 2.97Q = 59.39Q \leq 2m$$

Well-2:

$$s_{21} = s_{12} = 7.94Q$$

$$s_{22} = s_{11} = 47.48Q$$

$$s_{23} = s_{21} = 7.94Q$$

$$s_T^2 = 63.32Q$$

Well-3:

$$s_T^3 = 59.39Q$$

So, let us do this for well one. What will be  $s_{11}$ ?  $s_{11}$  will be what,  $Q$  natural log of  $R$  over  $r_{11}$ ,  $s_{11}$  here,  $s_{11}$  here divided by  $2\pi T$ . So, that will be equal to your  $Q$  natural log of capital  $R$  is what,  $800$   $r_{11}$  is  $0.2$ . So, I am just putting these values now and then  $2\pi T$  is what  $27.8$  times  $10$  minus  $3$  and once you simplify that it will be  $47.48Q$ . So, that is your  $s_{11}$ . Similarly, you can find  $s_{12}$ , which will be  $Q$  natural log of  $R$  divided by  $r_{12}$ ,  $s_{12}$ ,  $s_{12}$  and then it same  $2\pi T$ . You plug the values in, it will be  $Q$  natural log of  $800$

divided by  $r_1^2$ . What was  $r_1^2$ ? It was 200 meters divided by again the same thing  $2\pi$   
 $27.8 \times 10^{-3}$  and that will come out to be 7.94 Q.

Similarly, you can find  $s_1^3$  which will be  $Q \ln$  of capital R over  $r_1^3$  over  $2\pi$   
T and once you put these values it will be 3.97 Q. So, what will be the total draw down at  
well one? It is the sum of these three quantities that is  $s_1^1$  plus  $s_1^2$  plus  $s_1^3$ , which  
will be 47.48 Q plus 7.94 Q plus 3.97 Q. That is to say  $s_{T1}$  will be equal to 59.39 Q.  
So, you get one condition. Now, this should be less than equal to 2 meters. So, we can  
use this condition and find the value of Q basically.

However, we have to see that this draw down should be less than equal to 2 meter at  
other wells also. So, what we can do is we can do a very similar analysis at well two and  
at well three. So, what you will find is for well two you will have  $s_2^1$  will be equal to  $s_1^2$   
is equal to 7.94 Q because of the symmetry. Then you will have  $s_2^2$  is equal to  $s_1^1$   
which will be same, which is 47.48 Q we have already found and  $s_2^3$  will be  
equal to  $s_2^1$  which is 7.94 Q.

That will imply what your  $s$  total at second well will be equal to 63.32 Q. So, let us say  
this is your second relationship. Similarly, you can get this at well three which will give  
you your  $s$  total at the third well will come out as 59.39 Q. So, you have let us say these  
three equations. This is number one, this is number two and this is number three. So, you  
can find three different values of Q, but they, there will be only one equation that will  
give you the limiting condition or the conservative estimate of Q. Which one will be  
that? Let us see, it will be the number two.

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Condition 2 will be conservative.  
 $63.32 Q \leq 2m. \Rightarrow Q = \frac{2}{63.32}$   
 $Q = 3.16 \times 10^{-2} \frac{m^3}{s}$  or  $113 \frac{m^3}{hr.}$  or  $31.6 \text{ cfs}$  Am.

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So, condition two will be conservative; that is what we use. So, that you will have 63.32 Q should be less than equal to 2 meters. Once you do that you can find Q as 2 over 63.32 or your Q will be equal to or less than equal to you can say 3.16 10 to the power minus 2 meter cube per second or 113 meter cube per hour, you can convert it into any other suitable units or 31.6 meters per second. So, you see that what we have done is we have determined the value of Q, which will be needed to pump the water, such that the draw down at each of the wells will be less than or equal to 2 meters. This kind of problem as you saw involved the super position method in a multiple well system and we can also design a drainage network in which we want to maximize this.

So, with this I am afraid we come to the end of this lecture today and also the end of this course. So, what we have done is basically we have looked at the basic you know governing equations and the laws of physics and the statistical hydrology also and we have looked at both surface hydrology and the ground water hydrology. In case you are looking at this video still I am sure you will have lots of questions. If in case you want to clarify them you can always you know come and you know talk to me, you can give me a call or you can contact me through the email. I will give you this detail once again, I had given to you in the beginning, that is my name and you can send me an email at this address.

So, this will be our point of contact from this point on. I have really enjoyed doing these lectures, 40 or 44 different lectures. I hope that after you have gone through carefully each of these lectures you will gain something out of them. I am sure you would have learnt something, which will be useful and you can take this forward and which will motivate you to do further studies. You know advance in your career, with that I would like to close this video course and I would like to wish all of you a very bright future. If you need to contact me, you can contact me at this email address.

Thank you very much.