

Advanced Hydrology
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Lecture – 43

Good morning and welcome to the video course on advanced hydrology. We started the well hydraulics in the last class in which we looked at, what goes on inside in aquifer when we pump the water out of the aquifer? And we looked at the two cases of confined and unconfined aquifer the dynamics is slightly different, because in the unconfined aquifer the water comes out of the storage, where as in the case of the confined aquifer the water is coming from the recharge areas.

Then we looked at the derivation of the equation for pumping discharge cube towards a well in the confined aquifer. And we assumed a very simplified case of radial flow which is steady in a confined aquifer which is homogeneous and isotropic and the flow is incompatible and so on. We derive that equation and what we will like to do today is look at an example for the confined case. And then look at the derivation of the similar equation for the unconfined aquifer. And then look at that example for that and then we will move on from there. So, I would like to get started with looking at an example of confined aquifer radial flow into a ramp.

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Ex-3 A 30 cm diameter well completely penetrates a confined aquifer ($K = 45 \text{ m/day}$). Length of the aquifer = 20 m (b). Under steady pumping, the drawdown @ well (S_w) was found to be 30 m. Assume radius of influence as 300 m. $Q = ?$

Soln

$$r_w = 0.15 \text{ m}; \quad R = 300 \text{ m}; \quad S_w = 30 \text{ m}$$

$$b = 20 \text{ m}; \quad K = 45 \frac{\text{m}}{\text{day}} = 45 \frac{\text{m}}{24 \times 60 \times 60 \text{ s}} = 5.208 \times 10^{-5} \frac{\text{m}}{\text{s}}$$

$$\therefore T = Kb = 5.208 \times 10^{-5} \times 20 = 10.416 \times 10^{-7} \text{ m}^2/\text{s}$$

For steady radial flow in confined aquifer:

$$Q = \frac{2\pi T S_w}{\ln(R/r_w)} = \frac{2\pi (10.416 \times 10^{-7}) (30)}{\ln(300/0.15)}$$

$$= 0.02583 \text{ m}^3/\text{s}$$

$$= 0.02583 \times 1000 \text{ lps} = 25.83 \text{ lps}$$

$$= 25.83 \times 60 \text{ lpm} = 1550 \text{ lpm}$$

$Q = 0.02583 \frac{\text{m}^3}{\text{s}} \approx 25.83 \text{ lps} \text{ or } 1550 \text{ lpm}$

So, in this chapter that is example 3 and it goes like this. A 30 centimeter diameter well completely penetrates, completely penetrates or confined aquifers. A confined aquifer with hydraulic conductivity K given to us a 45 meters per day which is quite high. The length of the strainer is given to us as 20 meters, what is a strainer? Well strainer is a, is a casing in the confined aquifer through which the water gets filter. So, so that, no sand or debris is coming in. So basically that what is this length of strainer is representing is the thickness of the aquifer equal to b . Other data other information that is given is that the under steady state pumping, under steady state pumping the draw down at the well at the pumping well that is S_w was found to be, was found to be 3 meters under steady state conditions. Assuming the radius of influence as 300 meters, find out what is the rate of pumping.

So, these are data given it is a very simple problem in fact, so let us see how we can find out this discharge or pumping rate cube. It is a good a healthy practice to write down all the data first, the diameter is given the diameter of the well. So, the radius at the well will be half of that so which is 0.15 meters. The radius of influence capital R is given to you as 300 meters. Now, you know that which data you are going to use well your first cross section will be at the radius of influence and the other one will be at the well itself. So, 1.1 and 0.2 are at the radius of the influence and at the well. The drawdown at the well is 3 meters and drawdown at r obviously is 0.

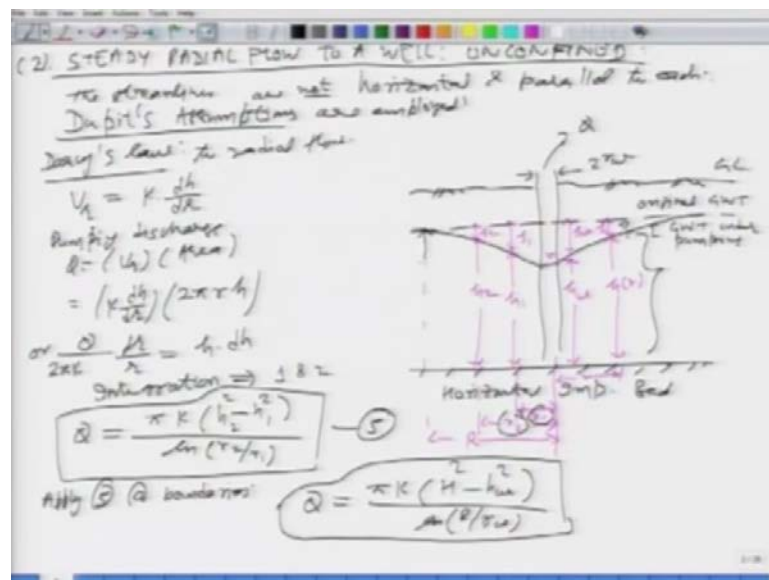
So we are not writing that b is equal to 20 meters, K is 45 meters per day is equal to you can converted into meters per second, we want to find out the things in let us say in s^{-1} units. So, we have 45 times 1 over it will be 24 hours times 60 times 60, number of seconds in a day. So, it is going to be 5.208 times 10 minus 4 of your meters per seconds. And you can calculate the capital T which will be K times b and it is 5.208 10 minus 4 we just found out times 20, b is 20. So, it will be 10 times 416 times 10 minus 3 meter square per second. We can do this competition fairly easily. For the steady radial flow in confined aquifer, we derived that equation in the last class, what is Q ? It is $2\pi T S_w s^{-1} \frac{r_2^2 - r_1^2}{R^2 - r_w^2}$ so s^{-1} is your $S_w s^{-1}$ is 0 divided by natural log of what? r_2^2 over r_1^2 which is capital R over r_w .

So, we have calculated T as w is given, r is given, r_w is given, π is a constant. So, if you put all the values you can find what is the value of Q ? It is going to be 2π times T is 10.416 10 minus 3 meter square per second times drawdown at the well is S_w is 3

meters. This whole thing divided by natural log of capital R is 300 divided by radius at the well is 0.15 everything is in meter square. This will come out to be 0.02583 meter cube per second, everything is in s i units. So, your discharge is going to be in meter cube per second. We can convert this discharge into other suitable units you multiply by 1000, what is that? That is liter per second, because there are 1000 liters in it meter cube, so what is that? 25.83 liters per second, you want you can converted into some other units 25.83 times 60 that will be what? Liters per minute.

So that is approximately 1550 lpm. Therefore, you say that your discharge coming out of that pumping well in the confined aquifer is 0.02583 meter cube per second or 25.83 of your liters per second or 1550 liters per minute or we can express this in any other suitable units we want so that is your concern. So, this way we see that it is a very simplified or very simple numerical example of calculating the discharge or the pumping discharge coming from a well in a confined aquifer under steady state condition. We have not looked at the unsteady case which we will do little later in this course. Now, what we will do is we will move to the unconfined aquifer case. We will apply the similar analysis which we used in deriving the expression for cube.

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So, let us do that first and then will see an example. So, the second case we are looking at is steady radial flow towards a well or to a well and we are looking at unconfined case. As we have seen earlier when we were deriving the governing differential equation for

the unconfined aquifer, the main problem there is that the stream lines are not horizontal and parallel to each other that is a problem in the unconfined aquifer. So, what we do is we simplify our analysis and we say that we are going to use the Dupit's assumption so the where for the case of well hydraulic also what we will do is we will employ or we will use this Dupit's assumption. So, the solution which we are going to get will be under the, these assumptions. So, let us look at the schematic diagram of what is actually happening, you have the horizontal improvise bed as usual.

And then you have the well which is penetrating completely this is your ground surface the discharge that is coming out is Q from this well. This is twice of r_w to start with initially this is let say your original ground water table. And under the steady state conditions it will drop and a cone will get form and you will have this as your, this is your ground water table under pumping and this is under the steady state conditions. So, let us define all the parameters like we had done earlier everything we are going to measure from the center of the well, this is your capital R you are going to take a couple of observation wells. So, let say this is your s_1 and this is your s_2 the corresponding values of the height of the ground water table are going to be h_1 and h_2 from the bottom of the aquifer and these the things are located there at r_1 and at r_2 from the well.

At any general distance let say this is your s and this is your h as a function of r and this is your r . The height in the well is h_w and this is of course, your S_w . So, what we will do is then we will employ exactly the similar analysis. So, what we will do is if you look at from the top you take a cylinder of radius r . And then we will apply the Darcy's law to this radial flow which is taking place towards the well, what would that be? Well it will be V_r is the radial velocity which will be $K \frac{dh}{dr}$ like we have said earlier. Now, what will be the pumping discharge? This Q is going to be equal to you are the velocity times the area, the velocity we have said is $K \frac{dh}{dr}$ as per your Darcy's law. And what is the area of cross section across which this flow is not taking place? Earlier we had the cylinder of height b . And now at any distance r what is the height of that cylinder? It is h this one at any general distance r .

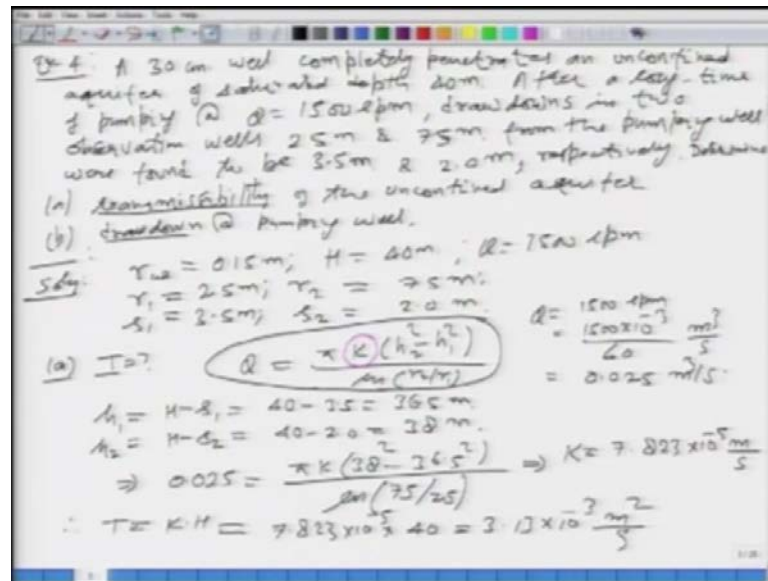
So, the area the cross sectional area cross which this radial flow is accruing is going to be $2\pi r$ times h , $2\pi r$ is the circumference h is the height that will be cross sectional area of your cylinder if you open it out. Once we understand that the remaining thing is simple mathematic so you separate the variables like we had done earlier. Then you will

have $\frac{dr}{r}$ on the left hand side. And then on the right hand side you will have $h \frac{dh}{dr}$. And if you integrate on both the sides integration will give you the equation of Q. And integration we are doing between location 1 and 2 which are corresponding to your r_1 and r_2 . So, once you do that your expression would come out to be $\pi K \frac{h_2^2 - h_1^2}{\ln \frac{r_2}{r_1}}$. So, this is your relationship or corresponding relationship for the unconfined case. And we are going to number this equation as equation number 5. We can apply 5 at the boundaries like we have done earlier that is the radius of influence capital R and the, the valid itself r_w .

Once you do that what you will have is Q is equal to $\pi K \frac{h_2^2 - h_1^2}{\ln \frac{R}{r_w}}$. We have to find all those things earlier I may not have written capital H. So, let me do that this is your capital H. So, this way you see that the expression for the unconfined case involves h^2 . While deriving the ground water you know general flow equation we had seen that the governing differential equation was a Laplace equation for the confined aquifer. For the unconfined case it was Laplace in h^2 for the steady state. And on the right hand side we had the recharge term and the, you know unsteady case we had the $\theta \frac{dh}{dt}$.

So, we see that for the case of the well hydraulic also the discharge expression involves simple h or $h_2 - h_1$ basically in the confined case but for the unconfined case we have a square of that, because of that variation in the ground water table. The next thing we will do is look up at an example based on this unconfined case and then we will move on from there.

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So, the next example is your example number 4 for this chapter basically. And that is how it goes. A 30 centimeter well completely penetrates completely penetrates and unconfined aquifer of saturated depth, saturated depth of 40 meters. So, what is this? This is your capital H. After a long time of pumping, so your pumping water from the unconfined aquifer and after a long time a pumping at the rate of Q is equal to 1500 liters per minute that is the heal from that aquifer, the drawdown's in the two observation well, two observations well which are 25 meters and 75 meters from the pumping well. So, these are your r 1 and r 2, were found to be were found to be 3.5 meters and 2.0 meters. What are these things? These are the drawdown's s 1 and s 2 at r 1 and r 2 which is given to us then we say respectively. Now, what we have to do in this problem is determine or find a, the transmissibility, the aquifer parameter of the unconfined aquifer that is number 1 unconfined aquifer.

And the second one which is b is the drawdown at the pumping well. So, let see how we can do this like earlier let us list all the data that are given to us first r w is 0.15 meters, capital H is the height of the original or the ground water table originally which is given to you as 40 meters. The discharge is 1500 liters per minute, the radial distant from the pumping well of the first observation well is 25 meters. And similarly, your r 2 is 75 meter, the corresponding values of the drawdown s 1 here is 3.5 meters and s 2 is 2 meters. Now, a; you want find out T. Well what we know is this for the case of unconfined aquifer $\pi K h_2^2 \text{ minus } h_1^2 \text{ over natural log of } r_2 \text{ over } r_1$. This

is what we just derive for the unconfined case. And we have to find the transmissibility and transmissibility is what? Hydraulic conductivity times the thickness of the aquifer.

So first we need to find K to able to find the transmissibility. In this equation everything is given to us, so Q is given h_1 h_2 r_1 r_2 these things are given to us. In the sense that the drawdown's are given we can find out what will be the h_1 and h_2 . So, you see that h_1 is going to be what nothing but capital H minus s_1 which is 40 minus 3.5 which is 36.5 meters, h_2 is capital H minus s_2 which will be 40 minus 2.0 which is 38 meters. And also you can say Q you can converted into meter cube per seconds it is 1500 liters per minute. So, you can multiply this by 10 to the power minus 3 to converted in to meter cube and minutes you have 60 seconds. So, this will be in meter cube per second and this will come out to be 0.025 meter cube per second.

So, now you have all the data you can plug into this equation and get the value of K. So, if you did that you have 0.025 is equal to πK of your 38 square minus 36.5 square divided by natural log of this is r_2 over r_1 which is 75 divided by 25 everything is in meters here. This will give you capital K as 7.823 times 10 minus 5 meters per second. Once you calculate that you can verify that, what will be T h, the T? T will be K times H. For the confined case it was K times b so it will be K times capital H for the unconfined case for the original condition so it will be 7.823 10 minus 5 of course, and multiplied by capital H is 40. So, you have 3.13 10 minus 3 the units are going to be meters square per second. So, that is part a in which we find the transmissibility of the aquifer. Next we have to find out what will be the drawdown at the pumping well S w.

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$$(h) \quad r_w = 0.15 \text{ m.}$$

$$Q = \frac{\pi K (h_1^2 - h_w^2)}{\ln(r_1/r_w)}$$

$$0.025 = \frac{\pi (7823 \times 10^{-5} \frac{\text{m}}{\text{s}}) (36.5^2 - h_w^2)}{\ln(25/0.15)}$$

$$\Rightarrow h_w^2 = 811.84 \text{ m}^2 \text{ or } h_w = 28.49 \text{ m.}$$

$$S_w = H - h_w = 40 - 28.49 \text{ m.}$$

$$\Rightarrow S_w = 11.51 \text{ m.} \quad \text{Ans.}$$

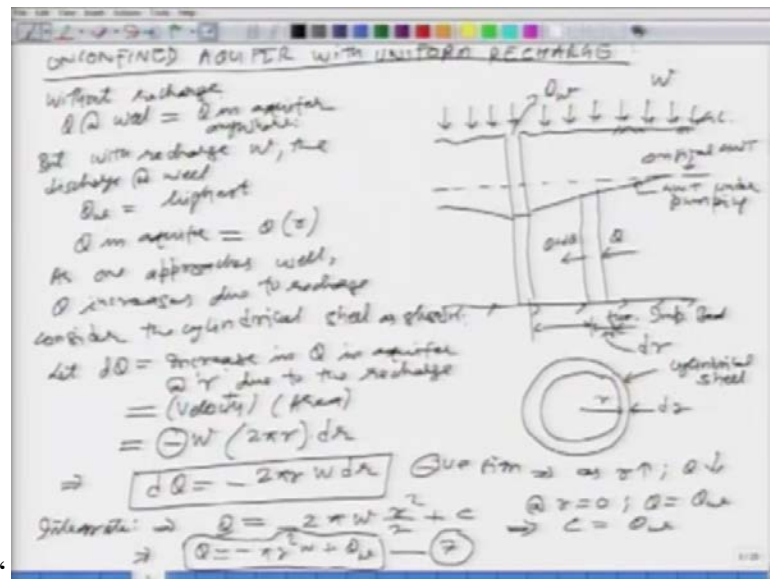
So, how do we do that? Let us move one you have r_w is equal to 0.15 at the well. Now, we use the same equation which is Q is equal to πK of your h_1 square minus h_w square its h_2 minus h_1 basically and h_2 is h_1 here. And the second one is at the well, so do not get confused. So, you have r_1 over r_w by 1, we have all the values here except h_w . If you find h_w from this we can find the drawdown, you have 0.025 is the discharge which is equal to what? π times the K we just found out is 823 times 10 minus 5 this is meters per second. And then you have 36.5 square they said that is h_1 minus h_w that is what you have to determine divided by the natural log of r_1 was 25 r_w is 0.15.

So, from this equation you can calculate this guide here easily. If you did that what you will have is the h_w square is 811.84. Please verify all these things it will be square meters or that will give you h_w as 28.49 of your meter. So, this is the height of the water table in the well, once you have that how do find the draw down what this is capital H minus h_w this was 40 and this is we have just found as 28.49 meters. So that way you see S_w is equal to 11.51 meters that is going to be your answer. So, we see that the case of the unconfined aquifer is slightly different and slightly more complicated, because it involves the square of your ground water table. None the last the computation are very similar and simple. And we can use the derive expression for calculating Q and if you know the Q we can find out the drawdown or the height of the water table, etcetera so that, this will help us in managing our ground water resources.

Upon till now what we have done is we have looked at a single well, you have; you have an aquifer whether it is confined or whether it is unconfined you have single aquifer. Later on in this course we will look at the multiple well case in which you may have more than one wells. And then you need to be able to calculate the drawdown and the ground water table due to pumping from one aquifer from one pumping well what will be is effect on the other pumping well.

So, we are going to look at those things little later, but before we move on to that what I would like to do is I would like to look at the case of unconfined aquifer when there is a recharge. Until now, we looked at the steady state case of pumping the water out when there is no recharges happening. However, we may have a situation in which the rain is also taking place and you pumping water out also. In that case you have certain amount of recharge taking place and the dynamics in the aquifer ground water flow will be slightly different. So let us look at that.

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So, now we are looking at the unconfined aquifer with uniform recharge. So, let me first draw the figure and then we will look at the dynamics, this is your horizontal impervious bed as usual. Then you have this is your pumping well and this is your ground level your pumping at the rate of Q_w , Q_w we are representing as discharge at the well. And let say this was your original ground water table after sometime it will have ground water table will be like this that is your ground water table under pumping. Now, what is also

happening is there is a recharge taking place in to the aquifer from the top. And the rate of recharge is let say W .

Now, it is important to see that if we did not have any recharge, without recharge the Q at the well is equal to Q in the aquifer anywhere that is what we had use. So, what we are saying is that if you have no recharge taking place. Then what discharge we are pumping out the Q from the well that is equal to the discharge that is taking place anywhere in the aquifer. In fact, that the equations we are used Q was velocity time area we use the Darcy's law and found out the cross sectional area so that is the concept we had used. Now, there is an additional mass flux entering in due to the recharge. So, the water which we will pumping from the well actually will not be equal to or will not be same as the discharge that will be taking place at any point in the aquifer. Why is that? Because there is an additional input, additional amount of water that is going in and contributing to the storage of your aquifer un confined aquifer. And and this discharge which is or the, this recharge basically will contributes certain amount of flow towards the well when we are pumping out.

So, as we go away from the pumping well so as a r is increasing the amount of discharge in the unconfined aquifer will be wearing or it will be different. So, this is very important to realize that so once we do that. So, let say but with recharge, then we refrain what we are trying to say the recharge value is W the discharge at the well, what is that? Q w actually is going to be highest and Q in the aquifer at any location will be function of r as I said it will be bearing in the aquifer. As one approaches as one approaches the well or as we go towards the well, this Q increases, because of the recharge that is taking place, because you have additional water going in so that will be contributing through the discharge or the flow due to this recharge.

So, what do we do? How do we handle this? Let me first a say what we had? How we can represent this situation? If you take a small strip, let us consider a small strip of length d r let say this distance is r and this is this thing is d r . And if you look at this whole thing from the top, if you look at this strip from the top it will look like this. So, you have the radius is r and this is d r . And what is this whole thing? This is basically yours cylindrical shell, cylindrical shell. So, if you are looking at from the top it is a cylinder, but it hole of at the bottom or in the middle, so it is a strip.

So, what we do is let us consider the cylindrical shell as shown. If we do that then what is happening is if you look here the discharge at this location is let say Q . And what we have said is as we are approaching towards the well the Q is increasing. So, let say the discharge as we go come out of this scrip is Q plus $d q$. So, $d Q$ is the discharge or flow which is taking place due to the recharge that is coming in from the top. So, we select this $d Q$ is what is the increase in the flow Q in the aquifer? So, $d Q$ is the increase in the flow in the aquifer at some radial distance r , why? Due to the recharge, recharge that is taking place, what will that be equal to? So, what will this $d Q$ be well? It will be nothing but the velocity, velocity water at that point of time at that location rather times area is not it.

So, what will that be? What is the velocity of water? It is the rate at which water is entering into this shell, W is the rate at which water is entering. And what is the area across which it is taking place? Well it is $2 \pi r$ is the $2 \pi r$ is the circumference multiplied by $d r$. So, this is the change and as we are going away from the well or as r is increasing Q will be decreasing or this $d Q$ will be decreasing. So, we then we say that is why we are taken this negative sign, so you say $d Q$ is equal to minus of your $2 \pi r W d r$. And we have taken negative sign which basically indicates that as r increases your Q decreases as your going away from the well Q is decreasing an if you are approaching towards the well or r is decreasing Q is increasing. So, just this the rotation part, so then what we do is we integrate this quantity, integrate this equation rather. Integration will give you what? You just do Q is equal to minus 2π , W is constant the recharged it. And then you have $r d r$ if you will integrate. So, you will have r square by 2 and then there is a constant of integration.

How do you find the constant of integration? Well to use the boundary condition. And what is the boundary condition? At your r is equal to 0 or at the well, what is the Q ? They are saying Q is equal to let say Q_w , so that will imply what? You put Q is equal to Q_w here and r is equal to 0. So, your C will be equal to what? Q_w . So, that basically means then your Q is equal to minus πr square W plus Q_w this we will define as the equation number 7. So, this is the relationship between Q at any distance r and the Q at the well. And how the W is W is constant and how the r is changing. Now, this Q is what? This Q is the flow in the unconfined aquifer.

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For unconfined aquifer, we have:

$$Q = -2\pi r K \left(h \frac{dh}{dr} \right)$$

we put from ②

$$-\pi r^2 W + Q_w = -2\pi r K h \frac{dh}{dr}$$

Separate the variables

$$\left(\frac{1}{r} Q_w - \pi r W \right) dr = -2\pi K h dh$$

Integration \Rightarrow

$$\frac{2}{r^2} h = \frac{W}{2K} (r^2 - R^2) + \frac{Q_w}{\pi K} \left(\ln \frac{R}{r} \right) \quad \text{--- ③}$$

\Rightarrow ③ \Rightarrow at $r=R$; $h=0$

$$0 = -\pi R^2 W + Q_w \Rightarrow Q_w = \pi R^2 W$$

And we have seen that earlier that for the unconfined aquifer, what do we have? What is this Q well? This Q we have taken earlier as minus of your $2\pi r K$ times $h \frac{dh}{dr}$. So, this is basically your velocity $K \frac{dh}{dr}$ you know $K \frac{dh}{dr}$. And then $2\pi r h$ is your area across which it is flowing. Now, what is this Q? This Q we put from 7 which is what minus of your $\pi r^2 W$ plus Q_w . Let see you go back that is what Q is Q is equal to from equation 7 it is Q_w minus $\pi r^2 W$. You just put that value of Q here and then you equated to the right hand side which will be minus of your $2\pi r K$ of your $h \frac{dh}{dr}$ over dr .

Now, we separate the variables like W normally do r on one side and h 1 other side so that we can do the integration. If you did that you will have and slight rearrangements you will have $\frac{1}{r} Q_w$ minus $\pi r W$ times dr that will be on the left hand side. And on the right hand side you will have minus of your $2\pi K$ of your $h \frac{dh}{dr}$; you can verify that. Integrated between the limits what we will get is this I will not do that complete test procedure here integration would heal you this h^2 minus h^2 will be equal to $\frac{W}{2K}$ times what? Time r^2 minus R^2 plus we have the Q_w divided by π times K of your natural log of R over small r . So, you are using capital H and R as one boundary and general small h and small r as the further boundary. So, this is the equation of your water table profile and various other things which involves W with recharge and pumping rate Q_w in a unconfined aquifer.

So, this equation 8 is basically representing what it is representing your profile water table profile. In which case in the unconfined case where radial flow is taking place under steady state conditions. And there is also recharge taking place W from the top due to rainfall or agricultural water or something like that. This is a very important equation which can be used when we have the recharge taking place in the unconfined aquifer. So, if we look at a special case of this we can determine what the discharge at the well would be. So, equation 7 basically all let me say equation 7 will give you at r is equal to capital R , what is Q ? Where Q is 0 at the radius of the influence there is no flow taking place. So, if we use this in 7, what is the equation number 7? This one at r small r is equal to capital R your Q is 0 the left hand side is 0 that will give you the discharge at the well Q_w .

So if you did that you will have 0 is equal to minus of your π capital R square W plus Q_w that means your Q_w is equal to what π capital R square W . That will be the pumping rate or that will be the discharge at the well which can then be use actually in this equation knowing the radius of the influence and knowing the recharge rate this can be put into this equation together. So, you see that we can combine the equation 7 and equation 8 to a simplify our equation and get a composite relationship between profile of the ground water table in unconfined aquifer under the recharge under steady state conditions. I would like to a stop here as for as the steady state analysis is concern. The next thing we will do in this course is on the unsteady flow towards a well. We have seen after a prolonged pumping when the steady state conditions are reached, how we can determine in the drawdown in the ground water table and the pumping discharge and so on.

However, in the many of the aquifers do to the permeability conditions or the characteristic of your aquifer it takes a lot of time for things to become steady state. So, it is very closely let say you are trying to determine the aquifer parameter if the objective of you know mention your water resources is basically you have to first determine the aquifer parameters to be able to use this steady state conditions. You need the observation in terms of the drawdown and if you are going to use the steady state conditions then they will reach after a very long time. So, you have to pump the water for a very long time which is not economical. So, we will use the unsteady state analysis to be able to a manage our water resources the aquifer system.

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UNSTEADY FLOW IN A CONFINED AQUIFER:

The governing differential equation (ADE) in confined aquifer in cylindrical coordinates is given by the following equation:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \quad \text{--- (1)}$$

Theis (1935) obtained solution to this GDE with the following boundary conditions:

$h = H$ at $t = 0$; $h \rightarrow H$ as $r \rightarrow \infty$ for $t \geq 0$

$$s = (H - h) = \frac{Q}{4\pi T} \int_u^{\infty} \frac{e^{-u}}{u} du \quad \text{--- (2)}$$

This is called Well Function Equation.

$$W(u) = \int_u^{\infty} \frac{e^{-u}}{u} du = \text{Well Function}$$

$$u = \frac{r^2 S}{4 T t} \quad \text{--- (3)}$$

NE of t :

So, let us move on and start looking at the unsteady state flow unsteady flow in a confined aquifer. So, we are no longer going to worry about the things to become steady state. This we are actually going to look at very quickly the governing differential equation in a confined aquifer in the cylindrical coordinates is given by the following equation. This, this equation we have actually seen earlier we did the derivation of governing differential equations in the confined and unconfined aquifer, this we have done for the Cartesian coordinate system. And then we have written down the equation for the cylindrical coordinate system.

So, we will just take that the equation as the governing differential equation. And then we will see how it can be solved they will be a couple of if there many method in a solutions or the methodology which has been propose for this however we will be looking at only a couple of them. So, let us look at the governing differential equation and one particular solution which is very famous and important in the ground water hydrology is proposed by Theis so we will look at that. So, first let see the governing differential equation which is described as this $\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t}$ will not go into the details of the derivation of this is equal to S by T of your $\frac{\partial h}{\partial t}$. So, this is this is your governing differential equation and then Theis proposed a solution to this governing differential equation with the following boundary conditions I was point out that this is a slightly complicated equation as for as the solution is concerned.

So, the solution of this equation is extremely difficult using the analytical you know methodology or analytical solution is very difficult. Many numerical methods have been you know applied. However This approach this problem under certain simplified assumptions or simplified boundary conditions. And propose a solution the h is equal to capital H that is your height of your piezometric surface at t is equal to 0 is equal to some capital H and that is number 1. And then what he said is that this height of the piezometric surface approaches towards the original value when as your r stand to infinity or you go very far away from the well when for the times which is larger than 0.

So, under these boundary conditions he has a obtained this solution as follows in which he defined the drawdown s as capital H minus small h is equal to Q over $4\pi t$ of your integral from u to infinity of your e^{-u} minus u over u du . And let say if this is my equation 1; this is your equation 2. We have renumbered the equation system for the unsteady case. This is written as Q over $4\pi t$ of $W(u)$. So, this is the solution which was given by This to the governing differential equation number 1 under these boundary conditions these ones, this equation is known as this is called non equilibrium equation; this equation is known as the non equilibrium equation.

Now, let me first define what is this $W(u)$? $W(u)$ is special mathematical function which is defined like this, integration from u to infinity of your e^{-u} minus u over u du . It is called a well function it is encountered in this kind of situation you know for aquifers or the well that is why it is called the well function in ground water hydrology. And u is an intermediate parameter which involves time, u is something which is representing time.

And also that radial distance are so $r^2 S$ over 4 capital T and small t . And let me say that this is this is actually like this, this your equation 3 you see that the combination of this equations 2 and 3 are these represents the complete solution to equation number 1 and this is called the non equilibrium or any equation. Now this well function W is something which is very difficult to obtain. This W is a very complicated you know mathematical function which involves integration of e^{-u} minus u over u du which is not easy to obtain. And many people have proposed certain approximate expression or the Taylor series expansion of this $r w$. and this is available actually in standard table. So, it is very difficult to find out the analytical value or the final solution however, certain table or the standard tables are available. What we will do is we will look at one simple expression which is the based on the Taylor series expansion.

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The NE eqⁿ is used extensively to determine the aquifer parameters (S & T). The NE eqⁿ is better than the Thiem's eqⁿ (Equilibrium Eqⁿ) for SS conditions because:

- ① S can also be determined.
- ② only one observation well is needed.
- ③ shorter period of pumping is needed.
⇒ more economical (as compared to Thiem's eqⁿ).
- ④ no assumption of steady state conditions is needed.

This $w(u)$ is defined as or can be expressed as this minus 0.577216 it is a constant minus natural log of u plus u minus u square over 2 factorial 2 plus u cube over 3 factorial 3 and so on. So, you see that we can consider you know few terms in this expansion depending upon the accuracy which is desired. So, this is the expression of your W which can be used in a computer program if we have to do this. Now, as I said earlier that this non equilibrium equation is something which is applicable under the unsteady state condition when things are changing as function of time when you start pumping the cone of influence or the cone of depression will be forming which will be changing as a function of time.

So, if we can make certain observations in the observation well we can use this equation the solution which we have just obtained. And we can determine the value of the aquifer parameter which will be much more economical. So, people suggest that the non equilibrium or the use of the non equilibrium equation is much more you know efficient as compare to using the steady state equation for the determination of the aquifer parameters. And we will look at a few reasons for that the NE equation the NE equation is used extensively already popularly I should say to determine the aquifer parameters, which is your S and T .

And then the why is that the NE equation is better than the Thiem's equation we had seen earlier which is the equilibrium also called the equilibrium equation for the steady

state things for steady state conditions why because of the following reasons I am going to give you a few of them. Number 1; S can also be determine you see that if we use the, the equilibrium equation or the equation for the steady state conditions we can only determine the transmissibility. However, if we use the non equilibrium equation it allows as to determine not only the transmissibility T , but also we can determine the storage constant of the aquifer S .

The next one is that only one observation, one observation well is needed. In the case of this non equilibrium equation, we need to use only one observation well will see that little later in the course. Number 3; the shorter period of pumping is needed shorter period of pumping is needed why because we are using under the on steady state conditions which are in the beginning or as you start pumping initially the things are unsteady. So, if you have an observation well you can make the observation you cannot down the time and drawdown, etcetera. And we can apply this equation that means it is more economical as compared to using Thiem's equation, because there you have to wait for the steady state conditions. And number 4 is simple that no assumption of steady state conditions, no assumption of steady state conditions is needed. That is to say we are assuming that the things are steady state when we apply the Thiem's equation to determine the aquifer parameter.

We are assuming that things are steady state, but it is very difficult actually to attain the steady state conditions are in real life. So, because the things are going to be unsteady this will be fluctuating all the time. So, it is difficult to obtain the steady state conditions. So, application of a steady state condition equation is actually will be validating the assumption if the conditions are not such. That is why application of this the non equilibrium equation will give you better results, because there is no assumption involve. We see that there are 3 4 advantages of using the unsteady state equation for aquifer parameters. I think I would like to stop at this point of time. And then will come back and look at these solution of this equation are there couple of solution technique to determine the ground water table profile. So, we will come back and see couple of other methods.

Thank you.