Advanced Hydrology
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Lecture – 42

Good morning and welcome to the video course on advanced hydrology. In the last class we looked at a few examples of unconfined flow and confined flow or rather one dimensional flow in a confined aquifer and unconfined aquifer and for case of unconfined aquifer we had the recharge. So, we looked at the general case of this unconfined flow with recharge. Today, we would look like to get started with the last topic in this course which is on the well hydraulics, because all of you know that majority of our country depends upon the ground water, and how do we extract the ground water is by digging the pumping wells and pumping out the water from it. So, when we pump out the water or the ground water through a pumping well what happens in a aquifer or what kind of flow takes place and how we can find out the aquifer parameters or how we can determine the yield etcetera? So, understanding this well hydraulics is very important. So, this is what we will start with today.

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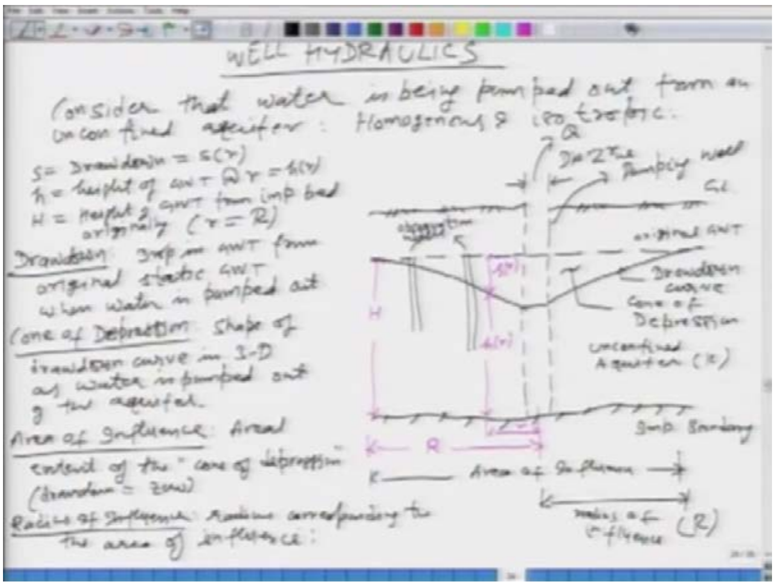

So, we will start with well hydraulics. So, what we will do is let us consider that water is being pumped out from an unconfined aquifer. And the aquifer is homogenous and isotropic. We are going to assume these types of conditions almost always. So, first we are going to look at the case of the unconfined aquifer the well hydraulics, how the radial flow will be taking place towards a pumping well. We will derive the equation for the discharge which can be pumped out from the unconfined aquifer. However, before we go to that it is important to understand certain terminology the definitions and some basic concepts about the well hydraulics.

So, we will get started with that. So, we will look at the schematic here first like here we are doing. This is your impervious boundary which is at the bottom of the aquifer. And then on the top we have the ground level, and then let us say we have we are pumping water through this well. So, this your well and let us say that the well has a diameter of 2 r w. So, r w is essentially the radius at the well that is what we are saying, let us say originally the ground water table is like this.

So, this is your original ground water table. And once you start pumping the water you are going to have drop in the water table like this. Let us say the total height; total height is h originally. And then let us assume that we have some observation wells which are dug into the unconfined aquifer at certain distances, right these are what is called the observation wells, these 2. So, we are now pumping water, but just looking at the water table what we are just measuring were the water is the depth of that this curve is what is called the drawdown curve the lowering of the water table. And this whole area we are looking at is called a cone of depression; we are going to look at all these things in a minute. And then we are going to measure everything from the center of the well. So, let us say this is capital R and we will define all these things. And at any distance R from the well let us say this distance is S and the height of the ground water table is let us say h from the bottom of the aquifer. And this is at some distance R. So, it will be a function of r, and r is this distance.

This is your unconfined aquifer and it has certain conductivity let us say K. So, what we are saying here is that your S is actually called the drawdown, drawdown is what it is the lowering of the water table as a result of the pumping originally it was horizontal. And then after sometime the water table has gone down in the aquifer. This we are saying will be function of the radial distance as you go away from the pumping well this drawdown will reduce. Similarly, h is we are going to define as the height of the ground water table at a distance r. So, this h is function of r. Then we have defined a capital h which is the height of your ground water table from the impervious bed. Of course, all these things originally that is before the pumping or also we can say during pumping at r is equal to capital R. Now, what we will do is, let us look at some of the definitions, what do we mean by drawdown?

It is essentially the drop in the ground water table from original static or stationary rather ground water table when water is pumped out. So, what is the drawdown, drawdown is basically you have the ground water table originally this horizontal. So, you have dug a pumping well you are pumping the water out. As a result of that water is coming from the storage. So, what will happen to that ground water table? It will reduce. So, this reduction in the ground water table is basically will be different as you go away from the radial distance or the center of the pumping well. So, this value of this S, S is the distance between the original ground water table and the current water table that distance is called the drawdown or the lowering of the water table.

The next thing is what is called the cone of depression. So, if we look at the in the 3 dimension it will look like a cone. So, what we are saying is that the shape of the drawdown curve shape of your drawdown curve in three dimensions is called the cone of depression as water is pumped out as a result of water being pumped out of the aquifer. So, what is the cone of depression? You have the original ground water table it is getting lower. So, if we are looking from the side or from any angle you will see that the ground water table is going down, but if you look from the top on a three dimensional picture then you will have a cone forming. So, it will be conical shape of your drawdown curve which is called the cone of depression the whole ground water table is getting depressed and it is forming a cone.

The next concept is we call what is called the area of influence. As the name suggests it is the area aerial extent of the cone of depression. So, it is nothing but the aerial extent, aerial extent of the cone of depression. And here it is easy to see what will be the drawdown at the area of influence or beyond the area influence? Well drawdown will be 0. Because as you go away from the well the drawdown is decreasing and the area of influence it will meet the original ground water table. So, the drawdown is 0 at the area of influence and the beyond. So, you have drawdown is equal to 0. So, if you want to find out the area of influence what you do is on your map you find out the area were the drawdown becomes 0. And then you can delineate or find out the area of the influence.

So, here let me say on this, this is your pumping well; you are pumping certain amount of water q out of this. And we are interested in finding that out. And then this area of influence will be this whole thing area of influence. So, we look at this aquifer from the top a cross section you will have you will see a circular area, that is called the area of influence. And then if you take the radius of that that is called radius of influence, and this we are going to say is capital R. So, the next thing is radius of influence, what is that? It is essentially the radius corresponding to the area of influence.

So, this is the these are the basic terms which actually we are going to be using when we derive the expressions or whenever we talk about you know pumping from a unconfined aquifer or even for the confined aquifer. Now, what we will do is, we will look at what actually goes on inside a well or a aquifer as we start to pump the water out. So, just try to think you know what will happen in the aquifer, it is more of a common sense. As we do that what kind of an activity will take place and what will be the changes in venue of these terms which we have just defined. For example, the cone of depression what will happen to that? What will happen to the drawdown? What will happen to the height of the ground water table? Ultimately our objective is to understand the behavior of all these characteristic in a aquifer.

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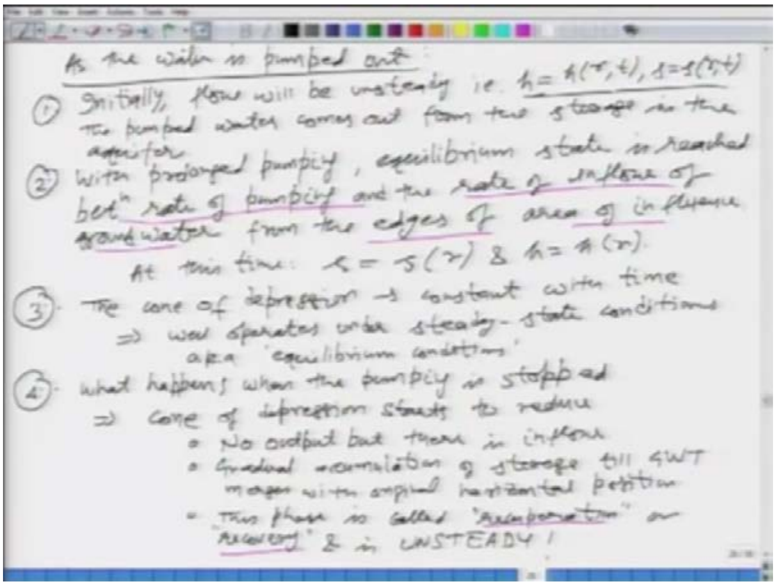

So, let us start looking at what happens as the water is pumped out. So, we are going to look at a few things. First of all as you pump the water out from a unconfined aquifer here. So, initially the flow will be steady or unsteady as you as you pump the water out you just started pumping water out at certain rate which is constant let us say. So, the drawdown’s will be changing, right the height of ground water will be changing ground water table will be changing the area of your cone of depression and all of those things will be changing. However, this process will be steady or unsteady initially; obviously, initially things will be unsteady, because they are starting from 0 and slowly things are changing or improving or increasing or reducing depending upon what you are looking at.

So, initially the flow will be unsteady. That is to say your h will be function of what both r and t. And also your s will be function both r and t. So, during the initial phases of the pumping what we are saying is that the flow is unsteady that is to say if you take any radial distance r for example, you have an observation well you have a pumping well. And then you have an observation well at a distance of r you started pumping water from the pumping well. And then you observe what the drawdown is or what is the surface elevation in the observation? Well initially it will start go down. So, as a function of time it will be decreasing so initially it will be an unsteady process and that will be true for any radial distance r. So, if you have many observation wells in each of them at every hour things will be unsteady initially.

So, ground water table will be changing and the drawdown’s will also be changing area of your cone of depression will be increasing at certain rate So, initially the flow will be unsteady that is to say your both h and S will be function of both r and t. And other important thing to know it is that the pumped water comes out from the storage in the aquifer, unconfined we are talking about. So, initially when the things are unsteady where is the water coming from it is coming from the storage which is there in the aquifer unconfined aquifer. And later on, we will see that after the things have become steady state or equilibrium state has been reached the ground water table will go back to the area of influence at that point of time you will have the ground water flow actually taking place from all the directions all the radial directions.

So, the water will be coming out not only from the storage, but also from the you know flow or the input which is coming from the larger distances we will come to that little later. So, this is the first point we want to emphasize in which we are said that initially things are unsteady as we start to pump the water out number 2 with prolonged pumping.

If you continue the pumping at the same rate q equilibrium state is reached equilibrium state is reached. Now, this is important between the rate of pumping the rate of pumping and the rate of inflow of ground water, inflow of ground water from the edges of your area of influence. So, what we are saying is that, if we continue the pumping with the prolonged pumping what will happen is equilibrium state will be reached or things will become steady state.

That is to say what do we mean by the by saying that the things will be or the flow will be steady state that is to say if you make an observation in the observation well, the drawdown will be constant as a function of time you take any distance r away from the pumping well your drawdown or the water table will not be going down. You keep on pumping the water at the same rate you keep on doing that indefinitely the water level will remain the same once the steady state conditions have been reached. And in other words the height of the water table will also remain constant that is what we mean by that why is it happening? It is happening, because the reason we have said is this let us look at this. Then I will come back to this why because a steady state is reached between the rates of pumping. And the rate of inflow of the ground water from edges of the area of influence as we said during the unsteady case the water was coming out or the pumped water was coming out from the storage mainly.

But when the steady state conditions are reached water is coming from storage or actually it stops coming from storage. And then it is basically resulting from the ground water flow which is coming from the edges of the area of influence. So, you look at this aquifer or the cone of depression as a control volume you have some inflow taking place from the further distances. And then outflow is the q which is being pumped out and there is an equilibrium between this output and the input, same input goes in same output goes out storage is same things are steady state. So, if you think about the Reynolds's transport theorem which we have seen earlier that is no accumulation. If you have this control volume there is no accumulation of mass that is why things are steady state so that is number 2. So, at this time under steady state conditions we can say that your drawdown will be function of r only. This is s and your h also will be function of r only. This it will be no longer function of the time. But it will depend upon the radial distance as you go away the drawdown will be decreasing and height of the ground water table will be increasing. The next point as we pumped the water out is about the cone of depression.

The cone of depression will be constant with time. now when the things have become steady state that means the well is operating or well operates under these steady state conditions. And these steady state conditions are also called as or also known as equilibrium conditions, equilibrium conditions. So, what happens to the cone of depression? Initially when the things were unsteady the ground water table was horizontal. The cone of depression starts to form initially it is size is very small as the pumping is continued. The size or the volume of this cone of depression keeps on increasing after some time prolonged pumping it reaches a constant volume or a constant state. This is the called equilibrium state or equilibrium condition. At that point time both s and h become constant. Now, what happens when we stop the pumping? Until now, we said we start the pumping and we keep on continuing the water from the aquifer as long as we pump the water out we have reached the steady state conditions things will be constant. But if we stop the pumping then what happens? Well obviously, once you stop the pumping your output has stopped, but there is inflow taking place.

So, the water is flowing in the well towards the well because there is lower energy there. So, what will happen to the cone of depression? Well the cone of depression will starts to, will start to reduce until the it will disappear completely and our ground water table will merge with the original ground water table. So, let us look at the next point which is number 4 about what happens, what happens when the pumping is stopped, what happens when the pumping is stopped? You just said that one thing that will happen is that the cone of depressions, cone of depression will start to or starts to reduce in size. That is to say why is it happening, because certain observations we want to make that it is no output from your aquifer. Well system or output has stopped, but there is inflow taking place from the edges of the area of influence. So, inflow is continuously coming into the cone of depression so that the cone of depression starts to reduce.

So, inflow is there, there is no outflow. So, we will have the unsteady conditions what is the other thing? There will be gradual accumulation, gradual accumulation of storage. As a result of all this, till the ground water table merges with the original horizontal position. Now this, this phase which phase after we have stopped the pumping this phase of the activity in the aquifer well system is called recuperation. It is called the recuperation or recovery. And I am going to underline these 2 terms. This process of changes that are taking place in the aquifer well system after you have stopped the pumping as we have said that ground water table will merge with the original position right.

So, all of this process is called the recuperation or the recovery, initially the cone of depression formed due to the pumping once it is stopped it is recovering the aquifer system is recovering to it is original position. So, obviously, you know it is called the recuperation or recovery. Now, this recuperation process this will be steady or unsteady; obviously, it will be unsteady. As we said initially the formation of the cone of depression is unsteady process, but if you continue to pump the water it will remain steady. But if you stop the pumping after let us say 2 days 3 days or 30 days or whatever you have pumped the water. And once you stopped then it will be unsteady process in which recovery of everything will be taking place. So, this recovery process is unsteady. So, if you want to find out the changes in the ground water table, the drawdown, etcetera. During the formation of the cone of depression or initially or during recovery you will have to use the unsteady flow conditions.

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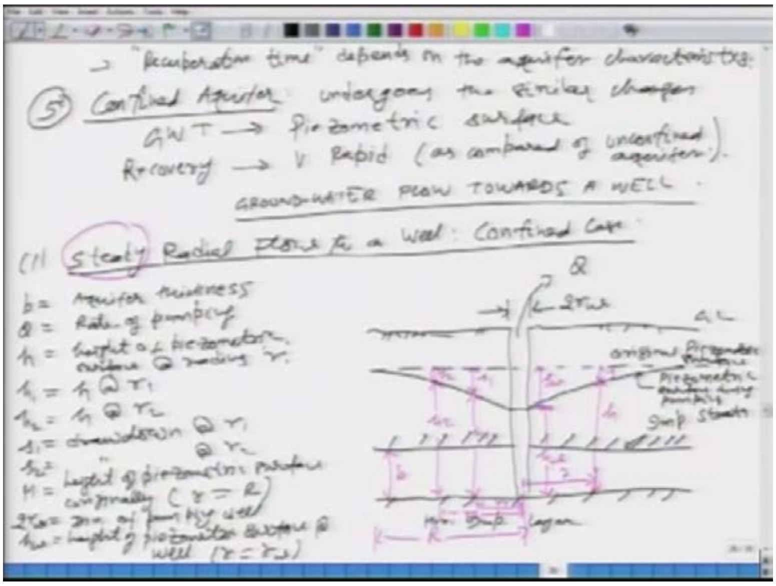

The last thing we will say is that the recuperation, the recuperation time. The recuperation time depends on the aquifer characteristics; obviously, so what is the recuperation time or recovery time? Basically the time which is taken by the aquifer well system to come back to it is original position since the pumping has stopped. So, the time elapsed between stoppage of the pumping and full recovery is called the recovery time this recovery time or the recuperation time will depend upon what the aquifer characteristics, the transmissibility, the capability of the aquifer to transmit the water or the hydraulic conductivity. So, we if you have a highly permeable strata it will take less time to recuperate otherwise it will take very long time.

The next thing or the last point I would like to make about all this process of pumping formation of the cone of depression recovery all of this. What happens in the case of a confined aquifer, we talked about all those things for an unconfined aquifer. Let us say you are pumping deep into a confined aquifer, same thing happens there also. The confined aquifer also undergoes the similar changes. Everything we have looked at it happens in the confined aquifer also except that we do not have a ground water table in the confined aquifer we saw earlier. What do we have? We have what is called the piezometric surface. So, in a confined aquifer if you are pumping water from a very deep well which is confined or sandwiched between a 2 rock strata then what will happen? Exactly the same things will happen the cone of depression will be formed.

Initially things will be unsteady after prolonged pumping things will become steady once you stopped the pumping recovery will be taking place. However, the ground water table is replaced by the piezometric surface, and what is the piezometric surface? Well if you dig an observation well the water will rise up to the piezometric point or energy whatever is available. So, if you dig lots of observation wells you can see that the ground water or the piezometric surface will form that cone of depression which will become constant after prolonged pumping. And after you have stopped the pumping things will start to recuperate back again in the observation wells. So, exactly the same thing is going to happen the last note is about the recovery in a confined aquifer is very rapid, very rapid, very quick as compared to we are saying the unconfined aquifer and why is that?

It is because we the flow is taking place in a confined aquifer due to the pressure differences. So, this is this is about the process of pumping in a well hydraulics we have seen what happens in the aquifer. This is as per as various definitions is concerned you know what is going on inside a aquifer. Now, what we want to do is we want to do some mathematical modeling so that or develop some equations you know apply Darcy’s law and whatever we have learnt so that we are able to find out how much water can be pumped. And what will be the changes in the ground water table as a function of time and as a function of distance radial distance r and so on. So, we want to analyze all these situations. So, what we will do is we will take the cases of confined as well as unconfined case. So, first I am going to take the well hydraulics for the confined case ground water flow towards a well.

So, we are kind of starting a new topic in which we want to model all these things. And within this topic your first case is going to be the steady radial flow to a well. And we are looking at well is in confined aquifer or case one which is your confined case. So, let us first define everything. This is your horizontal impervious layer at the bottom. And we are looking at the confined case and in which this is your ground level. You have a pumping well which completely penetrates the confined aquifer. And this is let us say your confining layer. Original piezometric surface was let us say this much this is your original piezometric surface under the steady state conditions, let us say it has curve like this. So, the thickness of this aquifer is let us say b we will define all the parameters which we are going to be needing. And this is your radius of influence or with distance you are measuring from the center of the pumping well.

And if you take any 2 observation wells let us say this is your s 1 and this is your h 1. We are measuring things from bottom and this is your r 1, so at any radial distance r 1 your height of the piezometric surface is h 1 and the drawdown is s 1. And similarly, at any other distance if these things are s 2 and h 2 and this is at a distance of r 2 and at any general distance let us say at any r these things are h and s. This is obviously, your impermeable strata the upper one this is your piezometric surface during the pumping, during the pumping means what this is actually your steady state thing, because we are looking at what only the steady radial flow. So, keep that in mind it is not unsteady. So, we have defined b h 1 as 2 r the other thing is let us say the height of your water in the well itself. Let us say this is h w this is the height of the water in the well pumping well that is h w. And this let us say is your s w. So, the quantity at the well are s w n h w.

So, let me just write down all these things quickly and then we will go to the derivation. So, b is your aquifer thickness this is confined case we are looking at capital Q is the rate of pumping, it is rate of pumping h is a general height of the piezometric surface where at radius r any general radius r. And then h 1 is what is the height of your piezometric surface h at r 1 h 2 is equal to h at radial distance r 2 the way we have defined in the figure . Similarly, what is s 1? s 1 is your drawdown at radial distance r 1, s 2 is the drawdown at r 2, capital H is the height of the piezometric surface originally initially whatever you want to call it or you say at r is equal to capital R which is your radius of influence. Define 2 r w is what is the dia or the diameter of your pumping well? And put it here that is what we are talking about this is 2 r w. And the water which is pumping out is let us say Q h w is height of piezometric surface in the well or at the well. That is r is equal to r w.

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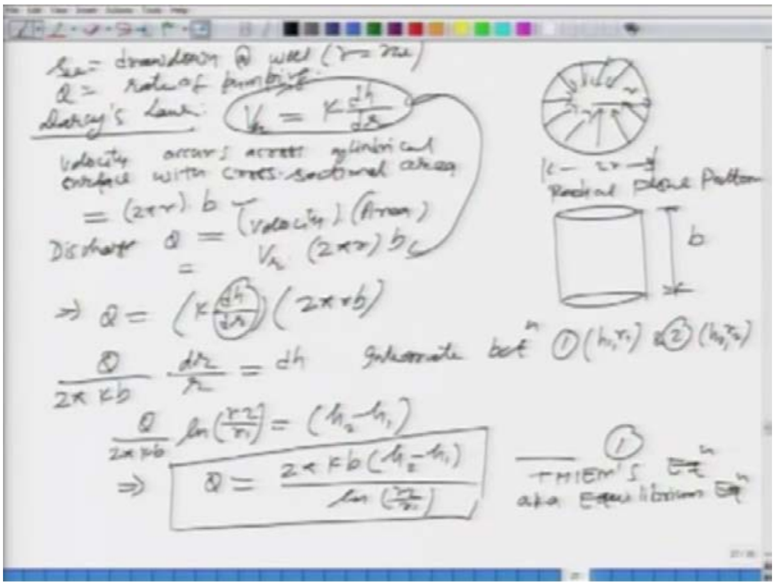

Let me move on and say s w is the draw down at the well r is equal to r w and Q is the rate of pumping which is constant. So, these are all the parameters we have defined. Now what we do is let, let us look at what is actually happening in the aquifer if you look at it from the top. So, if you look at if you take a cross section in the confined aquifer cutting the well and look at from the top, how the water is flowing towards the well? What will be the pattern of the flow? Everything is flowing towards the well let us say the well is a point in a space. So, what you will see is that there is a radial flow taking place from all the direction towards the center of that circle or a radius of influence. So, if you see that from the top, what you will have is if you take any distance r, what you will notice is that the flow is taking place like this towards the well.

So, this is what is this is 2 r of course, at any distance r from the pumping well. So, this is you say is the radial flow pattern. Now, what we will do is we can apply the, the famous Darcy’s law to this radial flow pattern. What will be the radius? Everything is steady aquifer is homogeneous isotropic K is constant in all the directions. So, you have let us say V r is the radial velocity towards the well in any direction that will be equal to what you have K d h d r is not it as you are going towards the well. This is the velocity which is constant. So, velocity basically occurs across a cylindrical surface. If you are looking from the top, you are actually looking at a cylindrical surface with a cross sectional area of what? So, if you are looking at it from the side. Now, this was the top this is your cylinder what is it is height? This is your aquifer thickness b.

So, what we are looking at is radial flow is taking place towards the well. And then if you are taking the cylinder of the complete depth your aquifer the confined aquifer. Then you are looking at a cylinder whose length is b and radius is r. This r is let us say anything it may be the radius of influence or any general radial distance r. So, you are looking at a cylinder of radius r and height a b. So, if that is your control volume then velocity or the radial velocity V r is entering into this cylinder. What is the cross sectional area of this radial or this cylindrical element well we can write it down. So, you did that this cross sectional area will be equal to what the circumference is 2 pi r right or pi d times b. So, you can extend this cylinder and that is what you will find.

So, this is the cross sectional area, this is the velocity. How can you find the discharge q, discharge q which is being pumped out from the pumping well, well from the continuity, what is that? It is nothing but velocity into area is not it Q is equal to V a, what is that? This is your V r times 2 pi r b. What is V r? V r is this. So, I am just going to put it here in this equation. So, if you do that you will have Q is equal to what? K d h over d r is the velocity times 2 pi r b. Now, what you can do is in order to find this Q, you can do some algebraic manipulations, we do not know all this quantity. So, we need to do some integration here, so if you do this Q over 2 pi K b here over r is equal to d h. What you have done is you have separated the variables h on the right hand side and r on the left hand side. That is all we have done here.

And once we have done that what you say is you integrate on both sides between 2 locations; one, one represents what? h 1 comma r 1 and 2; 2 represents h 2 comma r 2. Let me go back and show you what I mean. Remember I showed you this r 1 h 1 and r 2 h 2. So, what you are doing is basically you are integrating between let us say this is 1, and this is your 2, if you do that to this equation what you will obtain is this q over 2 pi K b is constant. This will be log of r integrated between r 1 and r 2 which will be r 2 over r

1 integration of one times d h is h. So, this will be h 2 minus h 1 that means your q is going to be equal to 2 pi K b times h 2 minus h 1 over natural log of your r 2 minus r 2 over r 1. This is the expression for your discharge.

So, what does it mean? So, you have the observation wells 2 observation wells at r 1 and r 2. All you do is you measure the drawdown's or the height of the water table in that once you have that you can find out what the Q is. And if you know the Q you can find out any of these things here let me number this equation as 1. This is the famous equation which is known as Thiem's equation, this is also known as equilibrium equation, why because this is derived for the steady state conditions when everything is in equilibrium state.

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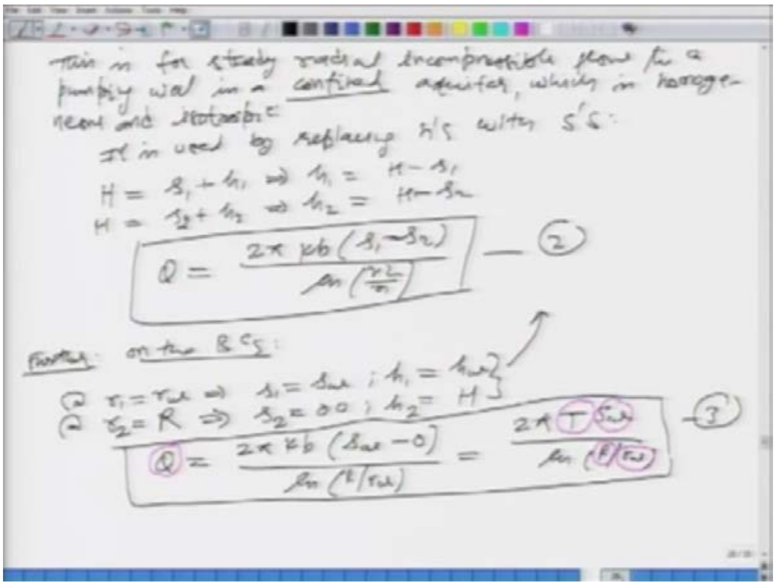

So, move on and say that this is or equation 1 is for the steady radial incompressible flow to a pumping well or towards a pumping well in a confined aquifer. Keep that in mind. We have considered a confined case confined aquifer what kind of a confined aquifer? Well which is homogeneous and isotropic? So, this is this is what is and this equation or Thiem's equation is used basically by replacing H with s s. The equation if you go back you see you have the H involved here so what is normally done or normally we say that we actually exploit this condition what is the total h is equal to s 1 plus h 1 you go back to the definition sketch what is your total H? This is your capital H, what will that be that will be nothing but your h 1 plus s 1 and it will also be equal to h 2 plus s 2.

So, instead of h 1 and h 2 actually we use s 1 and s 2 in that Thiem's equation. So, would mean your h 1 will be equal to what capital h minus s 1 and similarly, your capital H is also equal to s 2 plus h 2. So, that will give you your h 2 as capital H minus s 2. So, you put these things in your equilibrium equation what you will get is this Q is equal to 2 pi K b of your s 1 minus s 2. So, earlier we had h 2 minus h 1 it will be s 1 minus s 2 and then natural log of r 2 over r 1. And this is your equation number 2 let me put it in a box. Now, how can you find out this s 1 and s 2 well basically you have the observation wells. And then you find out the distance between the ground water table and the current ground water table and the steady state condition. That is your drawdown.

Now further we can use this equation on the boundaries or the boundary conditions if you do that at r 1 is equal to r w right at the well right at the well what do you have s 1 as s 1 we have defined as s w. And what is h 1? And we have said that this is h w so the quantities at the well itself are s w and H w. Similarly, what is the other extreme r 2 is equal to let us say capital R the radius of influence at the radius of influence. What is s 2? Can anybody tell me; well we have already said that at the radius of influence the drawdown is 0. So, this is going to be 0 and what will be h 2? h 2 will be nothing but capital H right. So, you put these 2 things in equation 2, you will have a simplified equation which is Q is equal to 2 pi K b times s w minus 0. s 1 minus s 2 s 1 is s w s 2 is 0 divided by natural log of r 2 over r 1 which is going to be equal to capital R over r w. And then you say that this is also equal to 2 pi t s w divided by natural log of capital R over r w.

Well T is equal to K times p which is the transmissibility of your confined aquifer this your equation number 3. So, the advantage of this equation 3 which we have just derived is what this is we can find out the aquifer property T. This Q is actually we would know, because we are pumping certain amount of water. So, we know the rate of pumping at the pumping well. So, if we want to find out the aquifer properties. So, then you can get T from this equation how well all you need to do is you just need to observe the value of drawdown at the well. And you already know the radius at the well and you also need to know the radius of influence.

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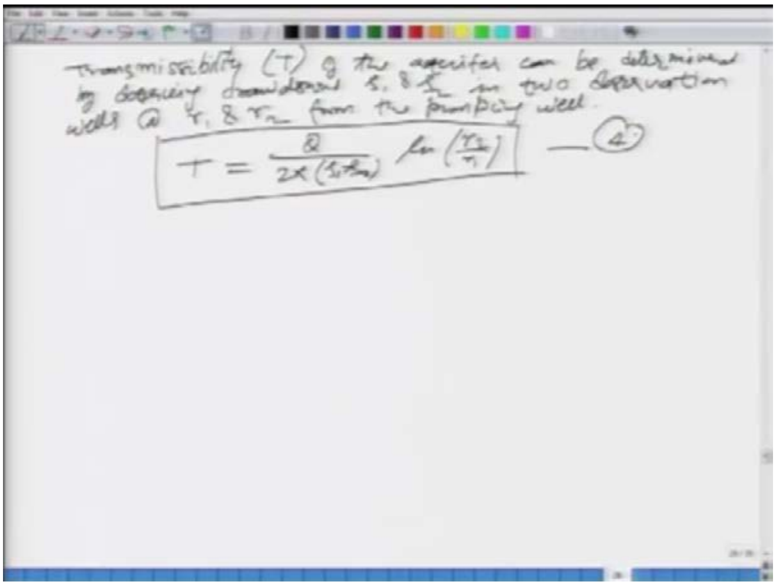

So, knowing all these things you can find out the transmissibility of the confined aquifer. So, transmissibility T of the aquifer can be determined by observing the drawdown. Let us say s 1 and s 2 I am going to be general here and we can apply it to the boundary conditions of course, in observation wells right which is adjacent to the pumping wells at distances r 1 and r 2. Let us say from the pumping well, so that your expression for T then would come out to be Q over 2 pi of your s 1 minus s 2 this is from equation 2 actually. And then natural log of r 2 over r 1 and you can use the well and the radius of influence also and you have put the things accordingly so this is your equation number 4. So, you see that the case of confined aquifer with steady radial flow towards the well can be used to determine or estimate the aquifer property. So, once we have the transmissibility by observing the drawdown's into different wells. Then we can also find out the hydraulic conductivity if we need to... Or in other words if we know these drawdown's we can find out the discharge.

And we can do lots of combinations of these things when we have multiple wells and we can manage the overall ground water resources. So, I think I would like to stop at this point of time. So, basically we have started the well hydraulics, you know topic the last topic in this course. And we looked at certain basic definitions and then we saw the ground water movement towards the well for a confined case. And what kind of flow it was? We said we are we are going to look at only the steady case yet. So, in the next class we will look at the unconfined aquifer. And then we will look at the unsteady case. So, we will stop here today and come back tomorrow.

Thank you.