

**Advanced Hydrology**  
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**Lecture – 41**

Good morning and welcome to the next class of this video course on advanced hydrology. In the last class, we looked at or we rather completed the derivation of the governing partial differential equation for the unconfined aquifer right which is homogenous isotropic, and we looked at the simple study case. And we looked at this for 2 cases, in first one it was with recharge and the second one it was without the recharge. With that we completed the derivations of various types of governing differential equations for different situations. And then what we did is, we looked at couple of you know real life situations in which we applied these governing differential equations.

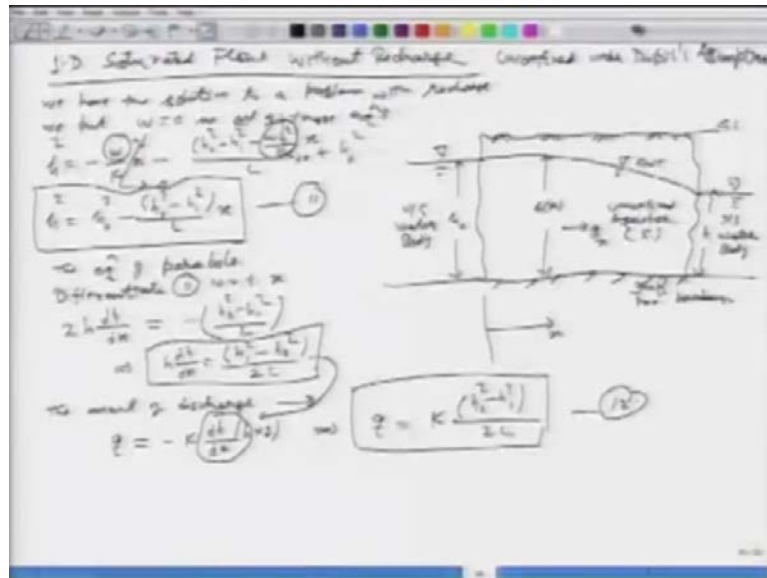
First case was for the confined aquifer in which we had 2 water bodies; one on the left, one on the right and due to the level difference in these two water bodies, the flow or the ground water flow will take place from left to right which will be one-dimensional. And we said that the governing differential equation will be the Laplace equation, and then using the boundary conditions we can find out what will be the complete form of the solution.

Then we look that the unconfined aquifer, we looked at the most general case of a applying the governing differential equation with recharge. For the one-dimensional case, we looked at what will be the solution and we said that the ground water table will be in the form of an ellipse. From the upstream water body if we the water surface or the ground water table will start to rise as we go along the  $x$  or along the direction of the flow. Then it will it will attain a maximum and start to go down and meet the boundary condition at the downstream lake or river or whatever is the water body.

Then we said that finding out this location of this water divide or the maximum ground water table is important in agricultural, and other kind of applications. We said it can be a determined the location and the magnitude of this water divide can be determined by equating the slope of the hydraulic grid line at that location equal to 0. Once we do that we have the solution in terms of  $h$ , we differentiate it with respect to  $x$  and we equate it to 0 that will give you the value of  $x$  at which the water divide will occur or we will have

the maximum water surface elevation. That is where I think we stopped, and what we would like to do today is we would look at a special case of this last case which we looked at.

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So, let us get started today, and we will look at the one dimensional saturated flow without recharge, without recharge. And we are looking at unconfined aquifer of course, because we can talk about recharge and no recharge only in the unconfined aquifer under Dupit's assumptions. This is all important, there are still valid under the Dupit's assumptions. So, if you look at the semantic it will be quite similar to what we had seen last time. So, this is your impervious horizontal boundary cos theta, you have an upstream water body; you have a downstream water body and their heights as are same. And they are measured from this day term this is  $h$  naught and this is  $h_1$ . That is how we had defined it then is or let us say the ground water table which we are actually interested in finding out the equation of this is the ground surface elevation or this will be your ground level. This we are looking at unconfined aquifer of hydraulic conductivity  $K$ , this is your origin. So, you are measuring  $x$  along this direction and at any  $x$  we are interested in finding out what will be the ground water table.

So, this is a equation which is similar to what we have seen of the case we have already seen. So, we have the solution to a problem or to a more general problem with recharge. So, what we do basically is that to find the solution for this case without recharge, we put

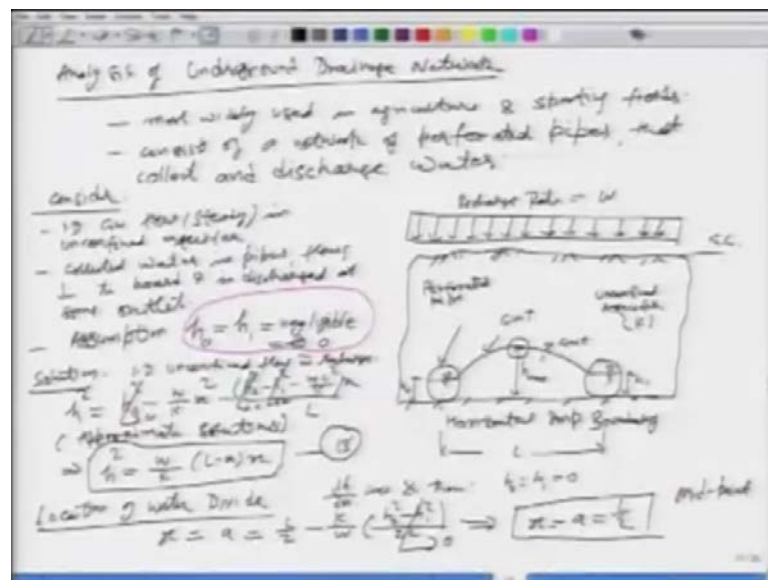
W is equal to 0 in all of those equations. What was the solution for the case we have recharged? Well it was given by equation number 6 I think if you go back and let me reproduce it here. So we have  $W$  by  $Kx^2 - h_0^2 - h_1^2 - \frac{WL^2}{K}$ , all of these multiplied by  $x$  divided by  $L$  plus that constant at  $0$  square. So, to find this solution we say  $W$  is equal to  $0$  means what this term will actually drop out  $W$  appearing here. Anything else, well this term also will drop out that would be  $0$ . And then what will be your solution then? You have the solution will be in this form of  $\frac{h_0^2 - h_1^2}{L}$  of your  $x$ . So, this is your solution and let me number this as 11.

What kind of an equation is this? If you look at this equation, the earlier solution with recharge we had said that it is an equation of an ellipse. You have the  $h^2$  is equal to  $x^2$  term is also there. Now that  $x^2$  term has been dropped or that no longer is there in the solution. So, what is this kind of a mathematical solution is this? This is the equation of a parabola; we say that this solution for 1 dimensional saturated flow without recharge in an unconfined aquifer under Dupit's assumption is represented by the equation of a parabola given above. So, equation 11 is the equation of the parabola. Now, in order to find out in order to find out the discharge let us say, that will be taking place is let us say  $q$  at any distance  $x$ , what we do is we differentiate this 11, equation 11 with respect to  $x$ . So, what will that be? So, if you differentiate this equation 11, you will have  $2h \frac{dh}{dx}$ , that is the left hand side;  $h_0^2$  is a constant.

So, that term will drop out and what you will have is minus of  $h_0^2 - h_1^2$  over  $L$ , that is what we will have or you have  $h \frac{dh}{dx}$  will be equal to what? It will be you change the or you take the minus sign inside you will have  $h_1^2 - h_0^2$  over twice of  $L$ . So,  $2$  you have brought on the other side. So, that means the amount of discharge or flow which is taking place in the  $x$  direction will be equal to what? Well we have  $q$  is equal to what, is your  $K \frac{dh}{dx}$  times  $h$ ? Because that is the height, so  $h$  multiplied by  $1$  is the area actually we are looking at. Instead of  $b$  in case of  $v$  the unconfined aquifer. Now, what is this  $h \frac{dh}{dx}$  we put from here if you did that what you will have is  $q$  is equal to  $K$  times. Again we change the sequence of  $h_0$  and  $h_1$  and you will have this divided by twice of  $L$ , this is my equation number 12.

So, you see that the discharge that will know from left to right or in the favorable radiant direction we see that it comes out independent of  $x$  in the earlier case when we had the recharge in the confined aquifer it was a function of  $x$ . So, depending upon the location, the magnitude of the  $q$  was different when we had the recharge taking place in the unconfined aquifer. However, we see that this is not the case for the case when we do not have any recharge. So, if you have an unconfined aquifer sandwiched between 2 water bodies, the flow that will be taking place will be constant from, from one water body to another. And that is basically your base flow we can find out or determine what will be the contribution from the ground water to a lake or to a river. So let us move on.

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And then what we are going to look at is the practical problem of analysis of underground drainage network. See what we have looked at is the 2 cases of unconfined aquifer in which the flow is taking place due to difference in the water surface elevation on either side. One is the upstream water body and one is the downstream water body. So, flow is taking place in  $x$  direction or in a single direction, the 2 cases are with recharge and without recharge. An important application of one of these cases is that is with recharge is in the design of the underground drainage system in the agricultural fields and in many sporting fields. What we have is when we put a certain amount of recharge, what we want or what is desirable is that we do not want the water to stand for a very long time. For example, in a sporting field, if it drains we want the water to drain

very quickly, but if you do not have any under drainage system that will take a lot of time for water to sweep through or percolate.

So, what is done is an under drainage system is designed in which we lay a network of pipes, a perforated pipes underground, what will that do due to the perforations the water will get collected into those pipes and these pipes are laid in such a manner that the gravity due to gravity the water will flow at some outlet location. So, the purpose of this underground drainage network is basically to collect the water which is sweeping or infiltrating into the ground and then take it away from the field. So, how do we analyze that system? We will look at that and we will apply one of the cases which we have just seen.

So, let us first basically define that the underground drainage network system is most widely used in agricultural or in agricultural and sporting fields. And it essentially consists of a network of perforated pipes, perforated pipes that collect and discharge the water. Now, this is going to be a slight difference between what we have learnt and what we are going to apply. It will not be a straight forward application. So, let us look at the situation what is actually happening.

So, what you have is some horizontal in pervious boundary we have to locate it or maybe we can lay straight arc which is like that. And then what we do is we put these pipes in. And as you know the pipes are normally a half full or  $\frac{3}{4}$  full. That is how they are designed then this is your ground level; this is your ground water table and there is some recharge which is being applied on this which is uniform with respect to space and time also. Let us say this is your recharge rate is equal to  $W$  and what these are, are this is your perforated pipe? This is your  $h_M$  or  $h_{max}$ , and let us says that these 2 pipes are laid such that the distance between them is  $l$ . And this is the unconfined aquifer with hydraulic conductivity  $K$  we are looking at. So, what we are going to analyze it or consider it that is as follows. So, it will be a slight simplification of the overall situation we have looked at. So, we will have one dimensional ground water flow which is steady in nature in this unconfined aquifer.

What is happening is the collected water in pipes or the perforated pipes flows how, flows perpendicular to the board we are looking at. So, the direction of fluid is just perpendicular to this diagram which we have drawn and is discharged at some outlet.

Now, the main basic assumption is that your  $h_0$  is equal to  $h_1$  is equal to negligible. That is the assumption. And let us say that is equal to 0, what this  $h_1$  and  $h_0$ ? Well this was your  $h_0$  this was your  $h_1$  and what we are saying is that these 2 are negligible as compared to other dimensions, so that we can take them equal to 0. However, the flow is still 1 dimensional towards the pipe, because we have the gradient which is driving the flow of water, because water is being discharged at the outlet.

So, what is the solution, what is the solution for this type of situation 1 d and unconfined flow with recharge? We have just seen that, that this is nothing but your  $h$  square is equal to  $h_0$  squared minus  $W$  over  $K$   $x$  square. This is an ellipse, we have the recharge as taking place so it is an equation of ellipse minus you have  $h$  naught square minus  $h_1$  square minus  $W L$  square over  $K$   $x$  over  $L$ .

Remember that this is an approximate solution in our case or this case of the underground drainage network, why because we are making certain assumptions. It will not be exact however; it is good enough design or analyzes this system. Now, what we do is we apply this assumption, this one in the solution. So, we say that this is 0; this is 0 and this quantity is also 0. So, that would be basically mean that your  $h$  square or this profile the ground water table profile is given by this equation you will have  $W$   $y$   $K$  times  $L$  minus  $x$  into  $x$ . Then you can see that easily from that easily from above equation and we say that this equation number 13 with the numbering notation we have been following. So, you see that this will be the equation which is again equation of a parabola. How do we locate this or basically this one? Our objective in the under drainage design or analysis of this underground drainage network is what is to make sure that the maximum ground water table the  $h_M$  is not very close to the ground.

So that water or the rain which is falling or the water which is applied for agricultural practices it maintains a certain distance let us say may be 1 meter or one and half meters or 2 meters. So, we do not want to we do not want the ground water table to exceed by a certain amount, so that we can design our system accordingly. So, it is important to determine what will be the value of this achievements and given this a solution which we are determined we can find out how what will be the location. So, let us do that it will be slightly different than what we had found earlier. So, if you want to find out the location of the water divide right what would that be one we can do the same thing your  $d h$  by  $d x$  is equal to 0. And then we put  $h_0$  is equal to  $h_1$  is equal to 0 in that, we already have

that. So, let me write the solution first  $x$  is equal to  $a$ , we had found out as  $L$  minus  $2$  minus  $K$  over  $W$ . If you go back to your notes that are what it was  $h_0$  square minus  $h_1$  squared divided by  $2L$ , means what we have said that  $h_0$  is equal to  $h_1$  both of them are  $0$ s. This whole term will actually drop out. That means your  $x$  is equal to  $a$  is equal to  $L$  by  $2$ . So, that means the maximum ground water table will occur at the midpoint between the or along the distance between the perforated pipes. That was number 1 that is we have found out the location.

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Handwritten mathematical derivation on a whiteboard:

magnitude of  $(h_{max})$  we put  $x = a = \frac{L}{2}$  in the solution

$$h^2 = \frac{w}{K} (L - x)x \quad x = \frac{L}{2}$$

$$\Rightarrow h_{max} = \frac{w}{K} (L - \frac{L}{2}) \frac{L}{2} = \frac{w}{K} \frac{L^2}{4}$$

$$\Rightarrow \boxed{h_{max} = \frac{L}{2} \sqrt{\frac{w}{K}}} \quad \text{--- (15)}$$

Discharge Between Pipes

$$q_x = w(x - \frac{L}{2}) + \frac{x}{2L} (h_0^2 - h_1^2) = 0$$

$$\Rightarrow \boxed{q_x = w(x - \frac{L}{2})}$$

$x = 0 \Rightarrow q_x = -\frac{wL}{2}$  (Dir of  $Q$  is  $x$  direction)

$x = L \Rightarrow q_x = +\frac{wL}{2}$  (Dir of  $Q$  is  $x$  direction)

- total discharge entering one pipe per unit length

$$q_{in} = 2(q_x = \frac{wL}{2})$$

$$\Rightarrow \boxed{q = wL}$$

Now, how about the magnitude of  $h_{max}$ , what is it? Well in order to determine this  $h_{max}$ , what we do is we put your  $x$  is equal to  $a$  is equal to  $L$  by  $2$  in the solution. What is the solution we have just seen for this case? It is  $W$  over  $K$  times  $L$  minus  $x$  times  $x$  that was the solution, so we put  $x$  is equal to  $L$  by  $2$  in this. So, what will be  $h_{max}$  square will be  $W$  over  $K$  of your  $L$  minus  $L$  by  $2$  times  $L$  by  $2$ . So, this is  $W$  by  $K$  of your  $L$  squared by  $4$ . So, that would mean your  $h_{max}$ , you take the square root of this whole thing will be  $L$  by  $2$  and  $W$  by  $K$  in the square root. So, that is your equation to find out the magnitude of the ground water table from the bottom of those pipes. This is equation number 15. So, you see that this way we can determine or we can actually apply the case of 1 dimensional ground water flow in the saturated zone in. And unconfined aquifer with recharge to analyze the underground drainage system under certain simplified assumptions which actually gives you know results which are practically utilizable you can apply those results to design the system.

So, we have seen what is  $h_{max}$  and what is the location its midpoint and the equation for  $h_{max}$ ? Now, we want to know how much water will actually get collected in these pipes given the recharge rate given the hydraulic conductivity. What will be the amount of flow that will be entering into a perforated pipe? That is nothing but you can find out  $q$ . So, if we did that that is the next thing, so that we can design these pipes accordingly to handle that kind of discharge. So discharge entering the pipes. We have seen that for the general case your  $q_x$  is given as this  $x$  minus  $L$  by  $2$  plus your  $K$  over  $2$ , well of what  $h_0$  squared minus  $h_1$  square. This was the equation for the discharge  $q$  along the direction of flow at any location  $x$ .

Now, for the case of underground drainage system my this term is  $0$ , because  $h_0$  is equal to  $h_1$ . So, that essentially means that discharge at any location at  $x$  is going to be  $W$  times  $x$  minus  $L$  by  $2$ . Now, I am interested in finding out the discharge were I am interested to finding out. Let me go back if I go back here I am interested in finding out the discharge at your  $x$  is equal to  $0$  and at  $x$  is equal to  $L$  that is where I am. So, that I know how much water is entering into this pipe, let us say from this direction and from this direction. How can I do that, well I just put  $x$  is equal to  $0$  and  $x$  is equal to  $L$  in this equation.

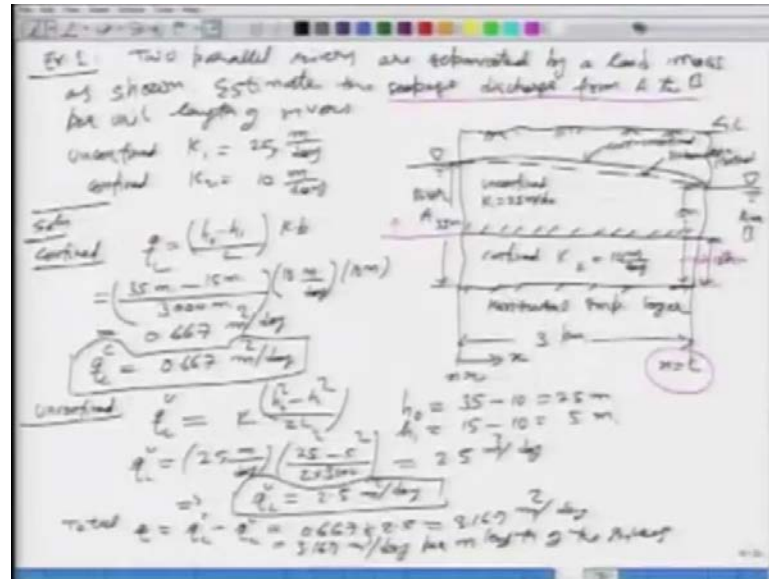
So, at  $x$  is equal to  $0$  you will have  $q_0$  equal to minus  $W L$  by  $2$  you see that and that  $x$  is equal to  $L$  your  $q_L$  will be equal to plus  $W L$  by  $2$ . What does this negative mean? Negative basically means that it is in negative  $x$  direction or from right to left and this is positive means in the positive  $x$  direction. So, if you go back this is the discharge which is taking place in the, at  $x$  is equal to  $0$  in the negative  $x$  direction. So, it is going into the left pipe and similarly, at  $x$  is equal to  $L$  is taking place towards the right side. So, if we assume that there is a network of such pipes what will be the total discharge entering into any pipe? Let us say what will be the total charge in this whole pipe? We have some contribution coming from the left there will be some contribution coming from the other side.

So, what will be the total discharge entering one pipe? That is per unit length of the pipe. Remember we are considering the unit width of the aquifer. And the aquifer is basically is a representing this unit length of this pipe.  $q$  in the pipe will be twice of your either  $q_0$  or  $q_L$  you see that half of from the left and half of from the other side. So that means the total  $q$  will be  $W L$ . So, we want to design these pipes you want to design these pipes to



handle what, W L. So, it essentially depends upon the recharge rate at which the aquifer is being subjected to it. So, this way we see that we have made certain simplified assumptions and in which we can apply or analyze and design in our underground drainage network system. Now, what I would like to do is I would like to look at a couple of examples, a numerical example on some of these cases which we have seen.

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So, the first one so let us look at this example we have 2 parallel rivers which are separated by a land mass as shown. And I am going to draw the picture for you. What we have to do estimate the seepage right discharge or the base flow or the ground water flow discharge from A to B there are 2 rivers A and B actually. So, we have to find the discharge from river A to river B per unit length of the rivers. And what is given is that for the unconfined case, you have K 1 is equal to 25 meters per day it is 25. And for the confined case this, you have K 2 as 10 meters per day. So, let me first draw the picture actually that will make things more clearly.

So, this is your bottom or your horizontal imperials here, then this is your aquifer that is your ground, there is a river upstream. Let us say this is our A and there is river downstream which is we are saying river B. What we have a confined aquifer at the bottom. This is your confined aquifer with K 2 is equal to 10 meters per day. This is given to us. On top of that what is given to us is that there is an unconfined aquifer with K 1 as 25 meters per day.

And in this unconfined aquifer this will result into some kind of ground water table distribution like that. And that in the confined case you know that the solution is actually linear through the, this is your ground water table for the unconfined case. And what is this? This is your phreatic surface for the confined case. The distances that are given let me mark those out the thickness of the confined aquifer is 10 meters. And the height of the water the height of the water at the upstream water body this much this is given to us as 35 meters which is constant over time, so because we are looking at a steady case. So, this is 25 meters and the height of the water body at the downstream is let me put it inside this is 15 meters. And everything is measured from the bottom in pervious layer. Also given is that the distance between the 2 rivers is 3 kilometers.

So, this is your  $x$  is in this direction here you have  $x$  is equal to 0. This is your  $x$  is equal to  $L$ . This is the situation in which you have an upstream water body, you have a downstream water body it is sandwiched or it is you know an encounter passing a land mass in between in which we have 2 aquifers; one is the confined at the bottom other is the unconfined. What we have to find is that how much will be the, or what will be the total flow? That will be taking place from left to right into river B from A to B. So, let us start looking at the solution of the problem in which we have both the aquifers confined and unconfined. And we have the equations or the solution of both such cases and there is a no recharge taking place here. So, let us start with this the solution. Let us first look at the confined case which is at the bottom and actually it is a simpler one also, what we have to do is we have to estimate the seepage discharge from A to B.

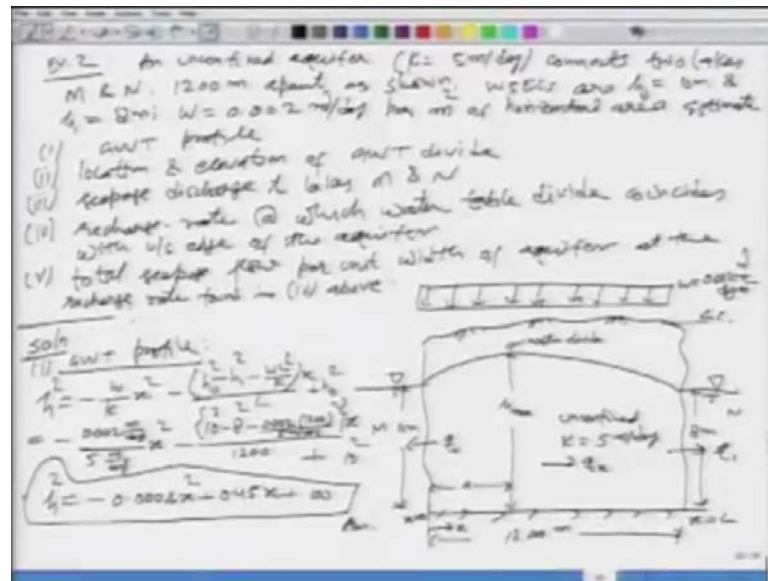
We are going to concentrate on that, what is the seepage discharge in a confined aquifer? Well we had seen that this  $q$  is actually equal to  $h_0$  minus  $h_1$  divided by  $L$  multiplied by  $t$  or  $K$  times  $B$  which is uniform or it does not depend upon the distance  $x$ . So, you can say that  $q$  at  $L$ ,  $q$  at  $L$  means what from A to B means that is what we want to find out how much is this  $q$   $L$  is taking place from A to B? So, we have the values of  $h_0$   $h_1$   $L$   $K$   $B$  all those things given. So, this will be equal to what is  $h_0$ ? It is 35 meters what is  $h_1$ ? The height of the water table on the right side is 15 meters and it is over a distance of 3 kilometers means 3000 meters all the meters will cancel out. What is the value of  $K$  for the confined case? It is 10 meters per day. And  $B$  is the thickness of we have to prefer is 10 meters, what will be the final units of this? It will be meters per square day. So, if

you calculate this you should be able to get as 6.67 meter square per day or you say  $q_c L$  confined is equal to 0.667 meter square per day.

You can convert this into some other suitable units if you would like, but we are going to keep it like this always. So, let us now, next look at the unconfined case, for the unconfined case, we are going to look at only that is a  $q_u L$  at unconfined. I am going to write and this equation is going to be equal to what? This as you know  $h_0^2$  square minus  $h_1^2$  square over  $2L$ . We have derived this just now little earlier now. One thing we have to keep in mind is for unconfined aquifer it will be  $h_0$  can somebody tell me this, this  $h_0$  is what  $h_0$  is the height of the water table from where, from the bottom of the confined aquifer? No this is from this location. So, for unconfined case this is your datum that is where the unconfined aquifer actually starts. So, what will be at 0 well it is 35 minus 10. So, it will be 25 meters. Similarly, we can find out what will be  $h_1$ , well it will be 15 minus 10 which is 5 meters. All we can do is we put these values into this equation.

And we can find  $q_u L$  the unconfined case  $K$  for the unconfined case is 25 meters per day right  $h_0^2$  square is 25 square minus 5 square over 2 times 2000. And that will come out to be approximately 2.5 square meters per day. You see that the conductivity of the unconfined aquifer is much higher more than twice of the confined aquifer. So, this is this is a contributing a major fraction, the overall discharge. So, you have  $q_u L$  is equal to 2.5 square meter per day or you can say meter cube per day per unit width of the  $q_u L$ . So, what is the total  $q$  will be?  $q_c L$  plus  $q_u L$ , it will be 0.687 plus 2.5 which is equal to 3.167 square root as per day. That is to say it will be in another word you can say 3.167 meter cube per day per meter length of the rivers.

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So, this way we see that we can determine the ground water flow contribution, we can find out the ground water flow contribution from one water body to the other by using some of these equations of which we have derived. So, this was the example 1 and what we are going to do next is look at another example which will be the most general. So, I am going to look at the unconfined aquifer with recharge in which we will find out the discharges we will find out the water table profile.

And we will also find out the location of the water divide and its magnitude and are the most of things we have seen. Let us do that. I am looking at example number 2. It goes like this an unconfined aquifer, if  $K$  is equal to 5 meters per day. This conductivity is quite low as compared to the last example as we have seen connects 2 Lakes, 2 Lakes in this case. And let us name them M and N, which are 1200 meters apart the distance between 2 Lake is this much as shown, do not worry about it I am going to show it in a minute the water surface elevations are  $h_0$  as 10 meters and  $h_1$  as 8 meters.

The recharge rate is given to you as 0.002 meter cube per day, per square meter of horizontal area. What we have to do is estimate the column number one; ground water table profile, find out the equation of the ground water table, number 2; Location and elevation of the magnitude of the water table divide of ground water table divide, number 3; is the seepage discharge and on the seepage discharge 2 Lakes M and N. So, how much is the discharge taking place towards M and towards N both of them, 4th is

recharge rate, this is important at which at which water table divide the water table divide coincides with coincides with the upstream edge the upstream edge of the aquifer. And this will be an interesting thing to do actually, because we want to may be managing our ground water table. The last thing we have do is the find or estimate the total seepage flow, total seepage flow per unit width of aquifer at the recharge rate found in all 4 above.

So, this is the problem it is a rather long statement of the problem, but you understand that we have an unconfined aquifer there is a recharge taking place the flow will be taking place from the left to right or right to left. We have to find out the ground equation of the ground water table profile. We have to find out the location in an elevation of the water table divide. Then we have to find out how much flow will be taking in place towards the lake M and towards the lake N. And then we have to find an interesting thing that is we have to find the charge rate such that the location of the water table divide is at the upstream edge. So, we do not want the sometimes it may be a useful exercise why because you do not want the maximum ground water table to be occurring between somewhere, because of the agricultural practices and so on.

So, we can manage it so that we can increase or decrease or manage our recharge rate in such a manner that we can shift the location of the water table divide. So, we can do that and the last thing is we have to find out the total seepage flow per unit width of the aquifer at the recharge rate which is the modified M. So, let us start looking at this quickly. First I am going to draw the figure this is the bottom then you have this is your ground level you have lake m upstream. And we have lake N then the downstream the distances that are given to us are this is 8 meters, this is I think 10 meters, this is 10 meter. The ground water table is something like this, this is the maximum. So, we have to find this  $h_{max}$ , you will also have to find this is your  $x$  is equal to 0  $x$  is in this direction.

Let us say this is A, you have to find this A and the distance between the 2 is 1200 meters; this is given of the water bodies. Here you have  $x$  is equal to 0. This is your  $x$  is equal to 0 L and L is 1200 let us say this is  $q_1$  or  $q_L$  you can say and this  $q_0$  taking place this is your unconfined aquifer with  $K$  is equal to 5 meters per day at any location  $x$  you have  $q_x$  taking place. You can find it out, this is your water divide for ground water table which we have to find and there is a recharge on top of this. So, this is the complete

semantic of this problem. And this W is given to us as 0.0002 what meter cube per day per square meter of the horizontal area start looking at the solution. The first one is the ground water table profile. So, first let us look at the ground water table profile. It is equation is given by this elliptical equation  $W$  by  $K \times$  square minus you have  $h_0$  square  $h_1$  square  $W L$  square by  $K \times$  over  $L$  plus  $h_{naught}$  square. This is the equation you have derived.

Now, all we are going to do is will put we know  $W$  we know  $K$  right  $h_0$   $h_1$   $L$  everything is known to us. So, all you do is put all these values the minus 0.20s and 2, this is meters per day divided by  $K$  is 5 meters per day. So, this co efficient  $W$  by  $K$  actually is dimensionless times of your  $x$  square. Then you have minus 10 square minus 8 square right minus 0.002 times 1200 square. Let me use these square brackets here or the middle brackets here and this whole thing divided by 1200 is for  $L$ . And this quantity is divided by 5 meters per day which is  $K W L$  square by  $K$  0.002 is  $W L$  is 1200 square and  $K$  is 5 meters per day. This is multiplied by  $x$ . And the last thing is  $h_{naught}$  square which is going to be the 10 square. So, put all the values and then you can find out the equation of the ellipse. And once you simplify this it will come out as minus 0.0004  $x$  square plus 0.45 of your  $x$  plus 100. So, you can put any value of  $x$  it will give you the square of the height of your ground water table. So, that is your answer of the first part.

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(i) Location of GWT Divide

$$Q = \frac{L}{2} - \frac{K}{W} \left( \frac{h_0^2 - h_1^2}{2L} \right)$$

$$= \frac{1200}{2} - \frac{5 \text{ m/day}}{0.002 \text{ day}} \left[ \frac{10^2 - 8^2}{2(1200)} \right] = 562.5 \text{ m}$$

$\Rightarrow Q = 562.5 \text{ m}$

Also  $h_0^2 = -0.0004 (562.5)^2 + 0.45(562.5) + 100$

$$\Rightarrow h_{\text{max}} = \sqrt{226.56} = h_{\text{max}} = 15.05 \text{ m}$$

(ii) Seepage Discharge at M & N

$$q_s = -\frac{Wx}{2} + \frac{K}{2L} (h_0^2 - h_1^2)$$

$$= -\left( \frac{0.002 \times 1200}{2} \right) + \frac{5}{2(1200)} (10^2 - 8^2)$$

$\Rightarrow q_s = -1.125 \text{ m/day}$  Ans to (ii)

The second thing you have to do in this example is the location of your ground water table divide. We have seen that this is given as follows; you have  $L$  by 2 minus  $K$  by  $W$  of  $h_0$  square minus  $h_1$  square over 12. This is the location of the ground water divide. We put all these values  $L$  is 1200 by 2 minus this is 5 meters per day  $K$  divided by  $W$  is 0.20s and 2 meters per day. And then this whole thing multiplied by  $10$  square minus  $8$  square over 2 of 1200. So, this is going to be equal to what, we should calculate will be 562.5 meters it is slightly towards the left or the upstream water body. So,  $A$  is 562.5 meters, also we have to find the elevation, what is that you just put this  $A$  is equal to this in your solution. What is the solution we just found out  $3$  Os and  $4$  x square x is 562.5, whole square plus  $0.45$  x is 562.5, like we just found out plus 100 that would mean your  $h_{max}$  will be square root of 226.56 that would mean the height of your maximum ground water table will be 15.05 meters.

That is your answer to the second part. If I go back it is 15 meters from where it is from down of the bottom. So, the height of the water is 10 meters in the upstream water body. So, maximum will be about 15 point something. So,  $h_{max}$  is 15 point let me see how much it is 15 meters I can say it is 15.05 meters that is your answer. So, the third part is you have to find out the seepage discharge which is taking place in these 2 places. We have the equation for 20 it is going to be equal to minus of your  $W$   $L$  by 2 plus  $K$  by 12 times  $h_0$  square minus  $h_1$  square. Once we put the value of  $x$  is equal to 0 in you  $q$  x and then you have minus 0.002 times 1200 over 2  $h$  to the power 1 which is this plus  $K$  is 5 over 21200. And then  $10$  square minus  $8$  square that will give you  $q$  naught as minus of your 1.125 with a square bracket do not worry about the minus, minus here only means that it is from right to left. That is your answer 2 M. So, this is the discharge which is taking place towards the left or the upstream water body.





here. So, you will have minus of 1 2 3 1 2 5. So, you are using the revised value of W other things are same plus K is 5 2 times 1200 and other things are same. So this is going to be the seepage flow towards the upstream water body you put all these values and then.

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The image shows a whiteboard with handwritten mathematical work. At the top, it says  $q_x = 0 \Rightarrow$  No flow to lake. Below that, it says  $\frac{dh}{dx} = 0$ . The main calculation is  $q_L = q_0 + WL = 0 + (0.000125) 1200 = 0.15$ . The final result is boxed as  $q_L = 0.15 \frac{m^3}{d \cdot m^2}$ .

You can find it out as  $q_0$  is equal to 0.0 that means no flow to lake this is important. This can also actually we can find out we can find this out using what, because your  $\frac{dh}{dx}$  by  $\frac{dh}{dx}$  is equal to 0 at the edge. If  $\frac{dh}{dz}$  is 0 the flow is 0. So, what is going to be  $q_L$  for this revised value of W? Well it is 2 0 plus W L this is 0 plus revised value of W is 1 2 3 1 2 5 times 1200 which is 0.5. You see that this will be 0.15 by meter cube per day per square meter. That is the answer to your last part of this problem. Well this way we see that we can utilize these equations for the analysis of the ground water flow in the unconfined case, confined case and so on. So that we can solve certain problems and we can also manage the ground water resources and the ground water table, water logging and all those problems. I am afraid I am running out of time and I would like to stop here. And what we will do tomorrow or in the next class is look at the well hydraulics.

Thank you very much.