Advanced Hydrology
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Lecture – 40

Good morning and welcome to the video course on advanced hydrology. We are into the last leg of this course and looking at the groundwater hydrology part. In the last class we looked at the anisotropic aquifer, we looked at how we can deal with the situation when we have a aquifer in which there are different types of soils.

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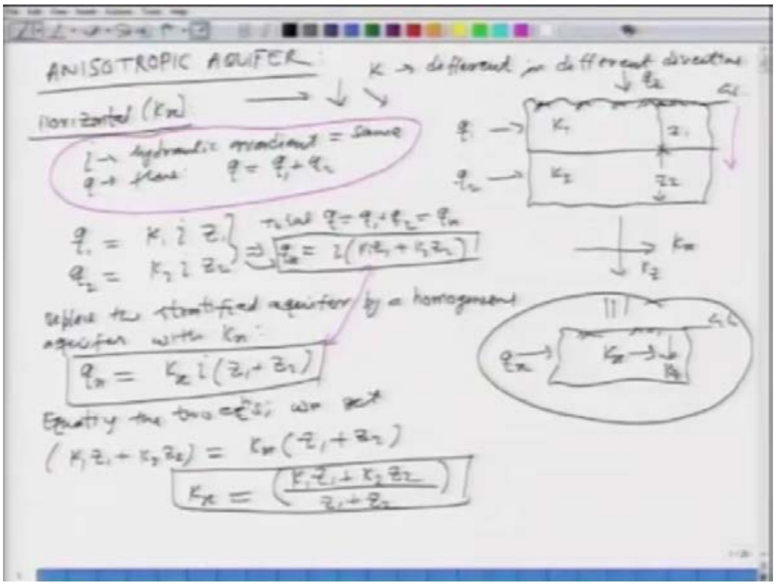

So, if we come here I would like to quickly go over the material which we have you know dealt within the last class. So, we saw the horizontal case, what will be the expression for K x? And that is what it comes out when you have 2 layers of hydraulic conductivity K 1 and K 2. Then we moved on and we looked at the vertical case in which the equivalent expression for hydraulic conductivity comes out to be the harmonic mean and for the earlier case or the horizontal case it was the weighted average. And then we defined the Darcy's law for any general direction beta from horizontal, and then we said that the K beta can be given by this equation number 14. Then we derived what is called the general flow equations, in the saturated zone the steady kind of flow in a confined aquifer.

So, we took a cube and in which we written down the continuity equation which is of this form number 22. And then we wrote this equation in the Z direction and we combined all the mass fluxes in all the 3 directions, and the final continuity equation comes out as in equation number 23 right here this one. And then we embedded the Darcy's law or the momentum equation along with this continuity equation. Once we substitute that and simplify and assume that the flow is incompressible and also isotropic that is to say K x K y K z all are same. Then that results in what is called the Laplace equation which is the governing differential equation for this kind of a situation.

Then we moved ahead and we said that for the unsteady flow in a confined aquifer the compressibility effects become important. And without going through the complete derivation we said that this governing differential equation can be derived like this the equation number 26 which is called actually the diffusion equation. The left hand side is similar to the Laplace equation except that in the Laplace equation, the right hand side is 0, but in the diffusion equation the right hand side contains the variable h with respect to time the derivative. Then we started looking at the two dimensional saturated flow in the unconfined aquifer. We said that the confined aquifer things are very nice flow is a truly horizontal same lines are parallel, but unconfined aquifer posses certain difficulties which ever listed here.

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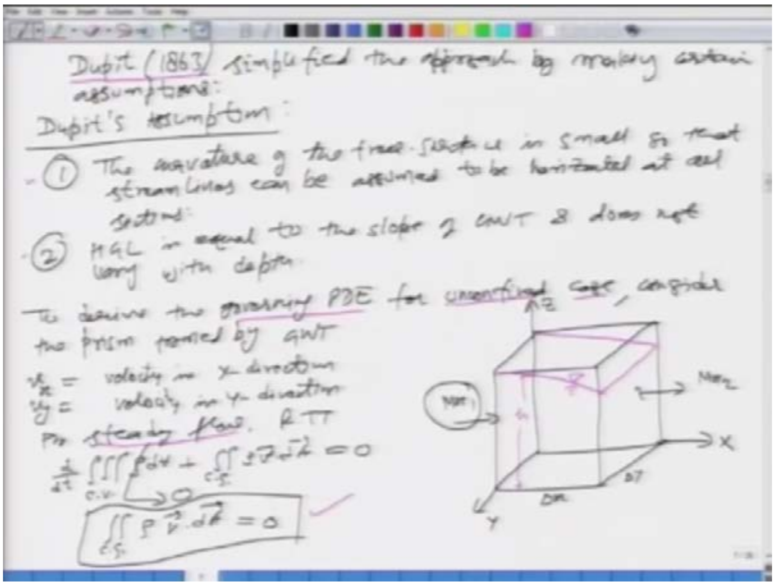

To overcome these difficulties Dupit in 1863, simplified this approach of attaching this problem. And he made certain assumptions and we listed them here the 1 and 2. Under these assumptions then we started to derive the governing partial differential equation for the unconfined case. We considered the cube similarly, like we did earlier. And then we started looking at the continuity equation. We said because of the steady flow your continuity equation will be like this. And then once we look at the cube and the water surface elevation which is at the or the groundwater table which is at the atmospheric pressure in the case of the unconfined aquifer.

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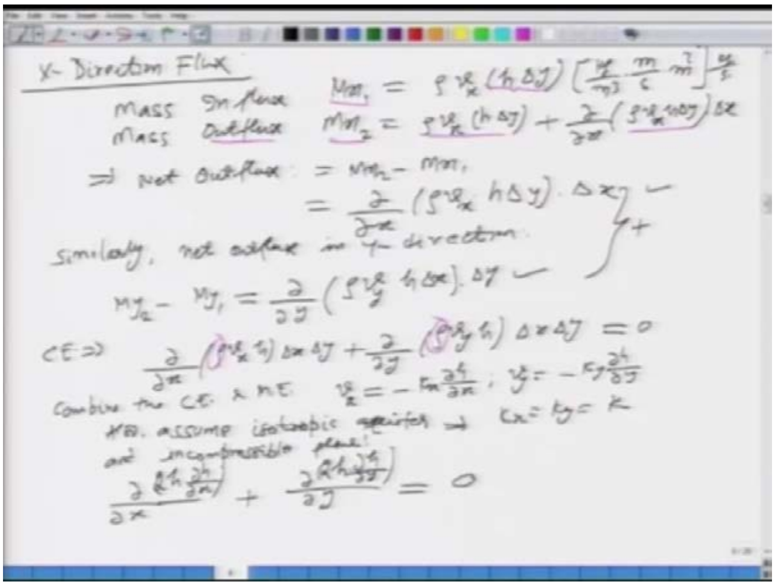

You have slightly different fluxes. This is what we had written that in the x direction you have the mass in flux M x 1 is equal to rho v x h times delta y. And if you look at the units it will come out to be what kg per second. And if I go back so you are looking at this is the cross sectional area h times delta y. So, you are looking at the distance h and delta y is the distance or the side of the cube along the y direction and M x 1 is what is the input going in... So this is the expression for the input or the influx and similarly, the out flux coming out is by Taylor series expansion this term plus the changes in the same thing multiplied by the distance delta x. And then we said that if we subtract these 2 the net out flux will be M x 2 minus M x 1 which comes out like this.

So, if we continue from here what we can do is we can say that similarly, we can write the expression for the net out flux in the y direction. So, if you did that you will have M y 2 minus M y 1 will be equal to what? Well it will be del del y of your rho the velocity in the y direction times h delta x is the area across which this flux is flowing. Then delta y is the direction in which some changes are taking place.

Now, as per the continuity equation, so this is your x direction flux and this is the y direction flux. The continuity equation tells us what, this one controls double integral of the over the control surface of rho v dot d x is equal to 0, so that would basically mean that the sum of these 2 quantities should be equal to 0. So, you have del del x of your rho v x h and del x del y, you can take outside plus del del y of your rho v y h and del x del y outside is equal to 0. So, all I am doing is sum of these 2 equations.

Now, we combine the CE and the momentum equation, what is the momentum equation? Well it is nothing but the Darcy's law which is like this in the x direction. And what will be the y? It will be minus of K by del h del y. Also we assume isotropic aquifer, isotropic aquifer would mean your K x is equal to K y is equal to constant which is let us say K. Assume and you can say incompressible flow, incompressible flow means what the density changes are not important. So, that the density does not change as a function of space or time. So, this rho can come out of the derivative or it will drop off.

So, once we use all these things what I am going to get is this; del del x over h del h del x that is term number 1 plus you have del del y of h del h del y is equal to 0. So, this is the form of the equation I get which is the governing differential equation we are trying to derive for this particular case, the unconfined case. So, what we do is we do slight mathematical or algebra like manipulations here. So, you can always multiply any equation by a constant. So, what I am going to do is I multiply this equation by 2, I can multiply this equation by 2, and right hand side is 0.

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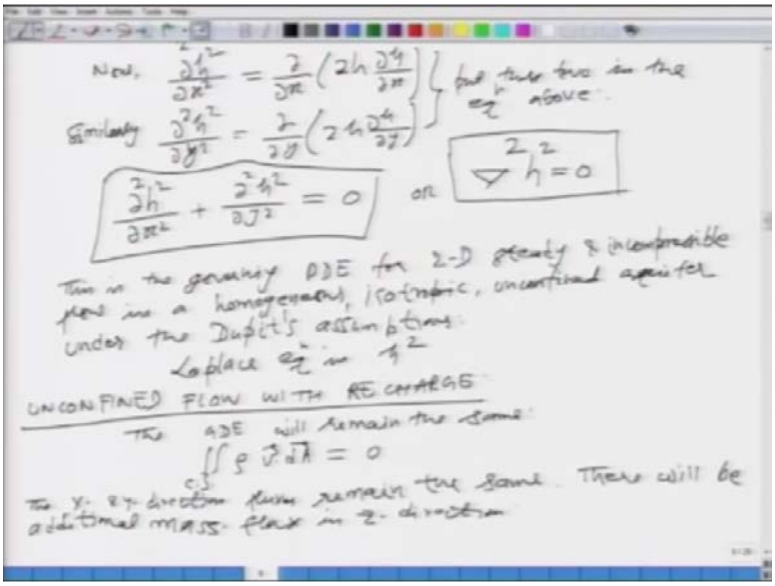

And then I use my knowledge of calculus to say that what is the derivative first derivative of or the second derivative of your this thing? Yes, if we look at the second derivative of this h square of your with respect to x, we can write this as del del x of twice h del h del x. Similarly, we can use del 2 del y 2 will be del over del y of what of your twice of h del h del y. So, it is basically if you take the first derivative of h square that is what it is going to be. So, what I do is I put these 2 in the equation above. Once we do that and simplify what we are going to get is, this or in the short form I can write this as del 2 h 2 is equal to 0.

Now, this equation is called the Laplace equation, this is very similar to the Laplace equation which we had seen which was the governing differential equation for the steady flow in the confined aquifer. And we have just seen that the governing differential equation for the unconfined case is what it is similar to the Laplace equation except that the variable involved is h square rather than h. So it is del 2 h 2 del x square plus del 2 h 2 del y square is equal to 0. So, then we say that this is the governing partial differential equation for what for two-dimensional flow we are looking at which is steady and incompressible density changes but ignored in a homogenous. All these are the assumptions while deriving this equation isotropic, isotropic means the K is same in all the directions and unconfined aquifer under the Dupits assumptions. That is what this is and what is this? This is given as or known as, we can say Laplace equation in h square.

So, this is the equation we have derived which is which we said is the Laplace equation in h square which is the governing partial differential equation for unconfined case steady flow which is incompressible. And the aquifer is considered as homogenous and isotropic; this is the case we looked at in which there is no recharge. Remember in the case of the confined aquifer the recharge areas are very far away very small areas. But in case of an unconfined aquifer we may have recharged due to rainfall due to infiltration and the depercolation. And also due to some you know existence of some lake or pond or some agriculture practices and so on. So, in the real life there are lots of situations in which we have the mass in flux taking place in the vertical direction also right. Now, what we have derived is a governing differential equation by considering the flow only in the x and y directions. However, we may have a case or lot of practical situations in which we have the vertical flow also taking place in the form of a recharge.

So, what we will do is we will try to write the governing differential equation for the case of unconfined aquifer with recharge, so that is our next thing. So, we have unconfined flow with recharge. As you can see what will be the changes? Well the governing differential equation will remain the same as far as the integral continuity equation is concerned that does not change whether you are considering 1 d flow, 2 d flow, 3 d flow, etcetera. So, it will be this equation which we had written earlier from your Reynolds's transport theorem. This will remain the same. That is to say what, that is to say the second component in your Reynolds's transport theorem which is the mass flux or the net out flux of the extensive property flowing across the control surfaces is equal to 0? That is your continuity equation. So that remains the same the only difference would be what you will have to account for the mass flux which is taking place in the vertical direction. So, let us look at that then we can say that the x and y direction fluxes remain the same. And then there will be additional mass flux which we need to consider, additional mass flux in the z direction which needs to be considered.

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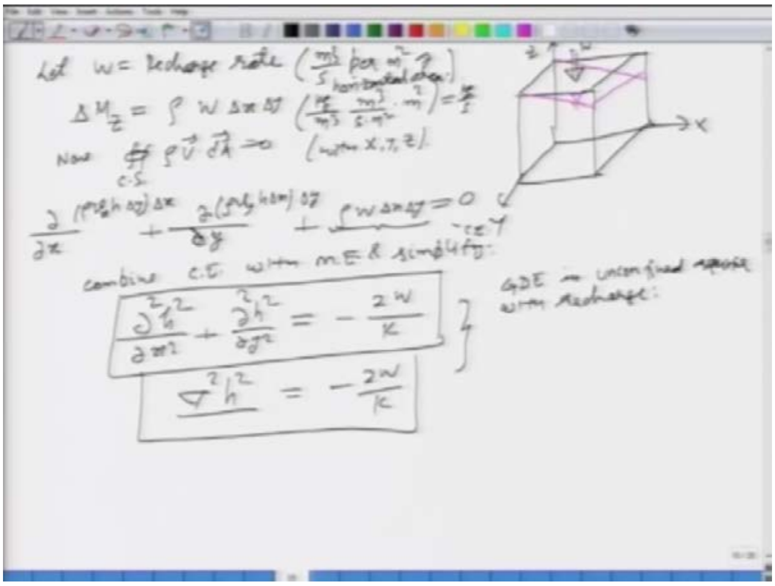

So, let us say that we have a similar situation like we discussed or saw earlier you take a cube it is not drawn very nicely, but you understand what we are trying to do. And then you have the groundwater table like we had considered last time. This is your X direction this is our Y direction and the vertical one is the Z direction that is the Z direction. And what we have is some recharge rate W is applied in the vertical direction. So, what we say is that let W be the recharge rate, what are the units of the recharge rate? Let us say it is meter cube per second per square meter of horizontal area. So, what is this; the essentially this unit is what meters per second or centimeters per hour or millimeters per day or something like that it may be the c pitch you know going into the ground corresponding to that what will be this mass flux in the Z direction? M Z 2 minus M Z 1 or delta M Z. Well it will be rho W and it is occurring across which area del x and del y.

So, if we look at the equations or the units rho is or kg per meter cube right W we said is meter cube per second per meter square. And the area del x del y is what? Meter square; so meter cube will cancel, meter square will cancel, the ultimately the final unit will be what it will be Kg per second? So, this is the mass flux which is taking place in the vertical direction. And now if you apply rho V dot d A is equal to 0 with considering X Y Z all the directions. Then what are we going to get? Well it will be del del x of your rho v x h del y del x that was part 1 plus del del y of your rho v y h delta x times delta y. Now, we have the third term which is rho W del x del y. So, we have added this in the Z direction, all of this should be equal to 0 as per your continuity equation.

Now, we do the same thing and what is that we combine this continuity equation. So, this is your continuity equation right with the momentum equation which is your Darcy's law and simplify. I am not going to do that, because we have seen it already for v x and v y we can write the Darcy's law. And then we can make the assumptions that the conductivity is same in all the directions rho can be taken out from the incompressible flow and so on. Finally, what you are going to get is this del 2 h 2 over del x 2 plus del 2 h 2 over del y square will be equal to minus 2 W over K. So, this is our governing differential equation and in short we can say del 2 h 2 is equal to minus of your 2 W over k where W is the recharge rate.

So, this is your governing differential equation in unconfined aquifer with recharge. And then all the other assumptions or simplifications are valid here, that is to say it is a two dimensional flow. And the aquifer is homogenous it is isotropic and the flow is steady incompressible and so on. So, with this we come to the kind of you know a stop or a brief stop where we have looked at the derivation of the governing differential equations. In general what we are going to do next is that we will look at some situations or we will apply these governing differential equations which we have seen for different kind of situations which are encountered in practice. So, we will look at the confined aquifer case one dimensional flow, we will look at the unconfined aquifer, how the application of these equations can result into the solution of the real life problems encountered in the groundwater aquifers? So, let us move on to that.

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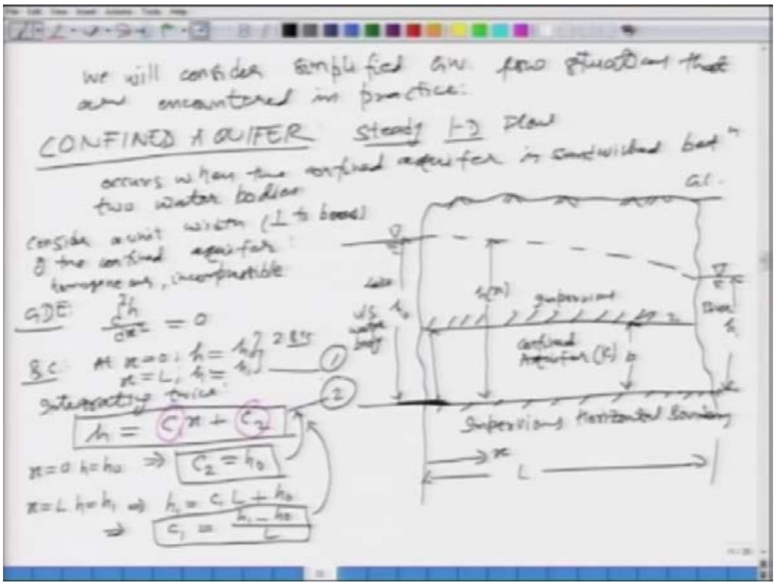

So, now we will consider simplified groundwater flow situations, situations or problems that are encountered in practice. The first one of such cases we will consider actually first we will consider the confined aquifer. And in this what we will look at is steady, steady state one dimensional flow in a confined aquifer. And what we will be talking about basically here is you have a confined aquifer in which a flow will be taking place only in one direction, when will that happen? When a confined aquifer is sandwiched between 2 water bodies, let us say you have these 2 confining layers flow is taking place in only in x direction or in one direction. Then the hydraulic grade line or there has to be head difference or the driving force which will cause the flow to go let us say from left to right. So, there may be a lake on the left hand side or there may be a pond or a river on the right hand side with level difference which will cause the water to flow from left to right or right to left depending upon the relative elevation of these 2 water bodies. So, let us look at this kind of a situation, this kind of situation occurs when the confined aquifer is sandwiched between 2 water bodies.

So, we will look at this in the graphically or in a figure first. And then we will work on this is the impervious horizontal boundary or strata. Then you have another confining layer this is also impervious. And then you have the ground somewhere up here. So, this is your ground level or the surface of the earth and down here, let us say you have some water body where water is up to this and upstream let us say you have another water body. So, this may be let us say a lake this may be a river or a small lake or anything. The thickness of this aquifer is let us say b this is your confined aquifer hydraulic conductivity is K. And let us say that this is the piezometric surface, how will it look like that is what we want to find out. So, what we want to find out is basically this solution h as a function of x, what is x? Well x is the direction of flow which is in this direction from left to right. And let us say that these 2 water bodies are some distance l apart it may be 100 kilometers or you know whatever, but we know that distance. So, this is the situation.

Now what we do is we consider a unit width which is perpendicular to the board we are looking at. So, this is the aquifer this one; this is the aquifer. And if you look perpendicular to this we are looking at unit width, let us say 1 kilometer or may be one meter or whatever across that phase, how much flow will be taking place? What will be the velocities and what will be the head as a function of x? As we go into the aquifer that is our objective of the confined aquifer. And we care saying that the flow is steady it is one dimensional and all the other assumptions are valid that is aquifer is homogenous, incompressible flow is taking place. We will always assume these things and it is isotropic actually the question of isotropic does not arrive here, because we are considering only flow in one direction.

So, what will be the governing differential equation of this? What is the governing differential equation for one dimensional flow in a confined aquifer from left to right in this case which is steady and homogenous and isotropic and incompressible and so on. All those things we just saw that this is a Laplace equation which was a del 2 h over del x square plus del 2 h over del y square and so on. But we have only one basic dimension flow is taking place only in x direction. So, what will be the governing differential equation well Laplace equation in x only one dimension so that is what it is going to be you have del 2 h over del x square is equal to 0.

What will be the boundary conditions from the knowledge of your mathematics? You know that each differential equation actually I can write, because there is only one so I can use a capital D actually. So, let me remove this is the total derivative you can have d 2 h over d x 2 that is equal to 0. So, boundary conditions are what, I think I have not defined them here so let me do that. So, let us say that the height of the water surface in water body upstream is h 0. So, this is your upstream water body upstream means what the h 0 is higher than the water surface elevation here, let us say this is h 1. And we are measuring all the elevations from the bottom layer which is this 1. So, what are the boundary conditions? Well at x is equal to 0 what is h? h is equal to h 0 right. Similarly, the other one is x is equal to l at a distance of l what is h? h is equal to h 1.

So, these are the 2 boundary conditions which we need we have a differential equation of second order. So, we need two boundary conditions to solve it, what will be the solution? Well we integrate this equation twice it is very easy to find out the solution is going to be what C 1 x plus C 2. So, this is your final solution but we do need to find out these coefficients of integration C 1 and C 2. And how can we do that well we apply the boundary conditions. So, if you apply the first boundary condition that is x equal to 0 and h is equal to h 0 will give you what x is 0 means C 1 will drop out. So, what we will have is C 2 is equal to h naught. So, you have the value of C 2 from the first boundary condition, what will be the second one?

Well to obtain C 1 we just use the second boundary condition so at x is equal to l, you say h is equal to h 1. So, if you put these things in this equation or in your solution, what you will have is h 1 is equal to C 1 times l plus C 2, what is C 2? C 2 is h naught we just found out. So that will give you c of one is equal to what h 1 minus h naught divided by l So, we have found both C 1 and C 2. So, we put these 2 values back into your solution what you are going to get is this.

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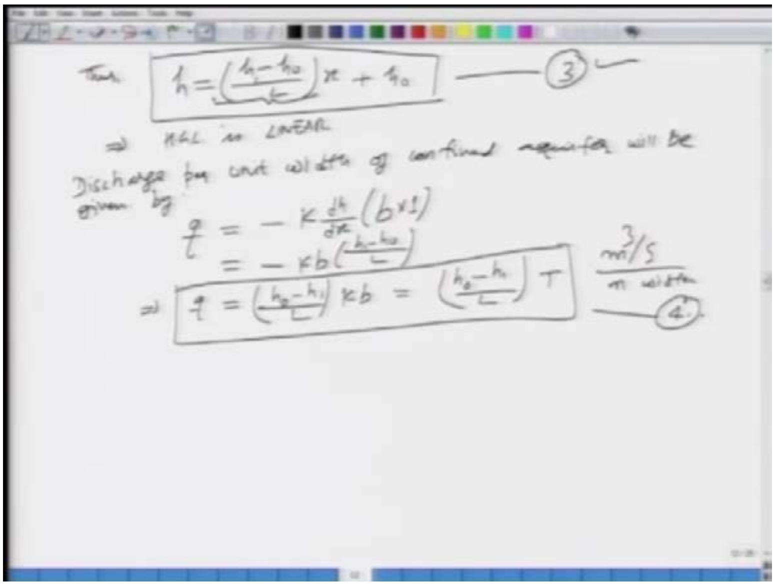

So this is a very straightforward a simple problem actually in which you have this solution h as a function of x is going to be given by this equation plus h naught. Let me see if I can number these equations otherwise we will be running into problems later on. So, the problem which I have defined here is let us say this is equation number 1. The solution which I found this one let us say is 2 and the final solution which I found in the next page with the value rho C 1 and C 2 is let us say 3. So, what does that mean? That means your hydraulic grade line is what is linear?

So, if you go back and see, this is your hydraulic grade line, so this is the solution which is going to be linear. So, if you want to find the height of the piezometer in any of the distance x, you can linearly interpolate that, so it is the simplest of the problem. What will be the discharge? This is the solution once we have the solution, we can find out the discharge per unit width of the confined aquifer will be given by what? Well this is the discharge we can use the Darcy's law, this is going to be equal to what? Minus K d h d x v is equal to K I, but if this is the discharge I can multiply it this by b and this is multiplied by 1 unit width.

So, you are multiplying by the area. So, this is basically your discharge intensity, what will that be? What is that will be minus K b, what is d h d x? You can differentiate this is the solution equation number 3 you differentiate that, that will give you the d h d x which is nothing but equal to this expression. So, it will be d h d x is h 1 minus h 0 right divided by l. So, what will q be the final expression? Then you say is h naught minus h 1 over l times K b. Or you can say that this is going to be h naught minus h 1 over l times T, where T is your transmissive. What will be the units of this? This will be meter cubed per second per meter width of your aquifer. And let us number this equation as 4.

So, this way we see that we have looked at a situation in which we can solve the problem of a confined aquifer in which one dimensional flow is taking place. And the aquifer is sandwiched between 2 water body it is a very simple case in which the solution is linear. So, we can find out the velocity we can find out the hydraulic head. And we can find out how much flow actually will be taking place from left to right or right to let. So, now what we are going to do is we will look at the unconfined case, we just looked at the case of a confined you know a flow or the confined aquifer one dimensional flow. Now, we will look at the similar situation where we have unconfined aquifer.

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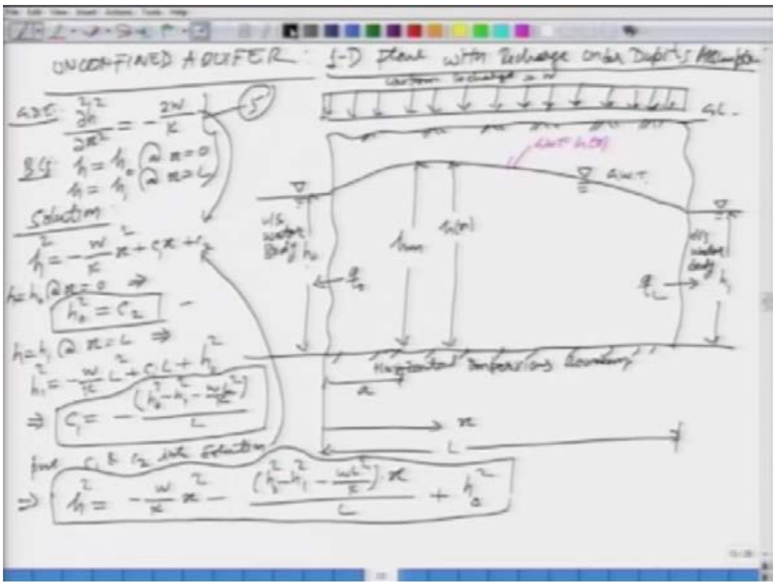

So, let us see you have unconfined case or unconfined aquifer. And what we are going to do again is we will look at the one dimensional flow in the saturated zone we are with the recharge, will recharge under Dupits assumptions. This will be the most general case as far as the one dimensional flow is concerned. And then if the recharge is not there we can just put W is equal to 0 in the resulting equations. So, we would like to derive an expression which is the most general one.

So, what I will do is I will first look at the schematic, we have a similar situation in which this is your ground, and you have an upstream water body. And you have a downstream water body. It may be lake or a river this is your horizontal impervious boundary of your unconfined aquifer. So, there is no confining layer at the top, this is my direction of flow x, the distance between the 2 water bodies is l. And then on top of this I have a uniform recharge which is feeding the ground level, this may be the infiltration due to rainfall or you know any other artificial needs. So, this is your uniform recharge and the rate let us say is W like we had defined earlier. Let us say that your final solution may be something like this that is what we are interested in; this is your ground water table. And we want to find out the equation of this ground water table in the unconfined aquifer under recharge rate of W.

So, let us say at any distance x what is the h of x that is what we want to find out. And all the distance is we are measuring are from the bottom and the upstream water body as an elevation of h naught like before. And this is h 1 let us say the location of this maximum water surface or groundwater table is we denote this as h m. And let us say this occurs at some location a then we would be we would need to find out let us say how much is the groundwater contribution. What is the, this discharge which is taking place into the downstream water body? And then how much is the base flow or the discharge which is taking place at x is equal to 0. So, this is the flow situation, the first thing we have to do is write the governing differential equation. And we had just derived the governing differential equation which is what it is del 2 h 2 over del x 2 is equal to what? Minus 2 W over K. this is the governing differential equation for the unconfined case one dimensional steady flow. And I could have written total derivatives here, but you understand we have flown in one direction. Similarly, what are the boundary conditions well h is equal to h naught at x is equal to 0 the conditions are same in fact, like earlier the differential equation is different this is at x is equal l.

So, at h at x is equal to l the hydraulic is h 1 at 0 it is h naught, what will be the solution? The solution is going to be you just integrate it twice. So, the equation will come out like this you will have h square is equal to minus W by k of your x square plus C 1 x plus C 2 where C 1 and C 2 are the constants of integration which need to be determined using the boundary condition coming from here to here should be very easy. We are just integrating it twice. Now to determine these coefficients, let us use these boundary condition, the first one is h is equal to h naught at x is equal to what? 0, so put x is equal to 0 in this equation and h is equal to h naught. So, what are you going to get is h 0 square is equal to what? x is 0, the 2 terms will drop out that is your C. So, this is the value of your C 2. The second condition says what h is equal to h 1 at what at x is equal to l. So, you just put these things you will have h 1 square is equal to minus W over K of your l squared plus C 1 l plus C 2 and C 2 is what? h naught square we just found out.

So, once you simplified this you will have C 1 is equal to minus of your h 0 square minus h 1 square minus W l squared over K. This whole thing divided by l you can verify that that is going to be your C 1. And let me number this equation this whole governing differential equation with boundary condition is equation number 5. So, we put C 1 and C 2 back into your solution into the solution meaning what into this equation? What you will get is the solution h square is equal to minus W over K x squared minus h 0 square minus h 1 square minus W l squared over K. This whole thing multiplied by x and this whole thing divided by l plus C 2 which is h naught square. So, this is the solution of the groundwater table which we are looking at here; this one this is the groundwater table equation h which is given by this equation. And let me number this, this is as equation number 6, now what is this?

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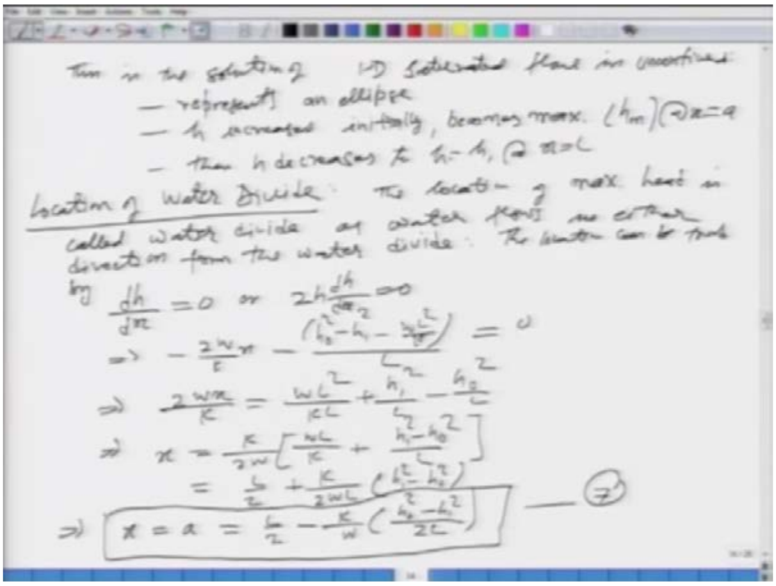

Well this is the solution of one dimensional saturated flow in unconfined aquifer under all the assumptions, Dupit's assumption homogenous aquifer which is incompressible flow isotropic and so on. Now, this equation 6 which we have derived it represents what, what kind of an equation is this? This represents the equation of an ellipse. If you go back to your knowledge of mathematics this looks like the equation of an ellipse and you see this you see that as h increases or actually the h increases initially. In the ellipse, what do you have the, the y variable increases initially becomes maximum at certain distance x. Let us say that value is h m and let us say that this occurs at x is equal to some distance which we do not know we need to find it out. And then h decreases to what value h is equal to h 1 at x are equal to l. So, initially you see that it starts at h is equal to h naught it becomes maximum h m. And then it goes back to the boundary condition the boundary conditions have to met on have to be met on either side.

Now, what is this h m? h m is the maximum groundwater table whenever we have an unconfined aquifer. And there is a recharge taking place; we know that the groundwater table will rise in the middle somewhere and finding out the location of this maximum head or the maximum groundwater table. And what will be the value of this h m? The magnitude and location of this h m is extremely important. For example, in agricultural practices, in the fields we apply water for you know irrigation purposes. Then we need to know that how much water we apply? What should be the recharge rate? So that this h m does not become higher than a certain value, or in other words we may say that the distance between the ground and this h or the maximum h should be some safe a limit. So that there is no water logging and there is no problems.

So, determination of this location of this maximum groundwater table is important. And let us do that how can we do this location of what is called the water divide. The location of the maximum head is called the water divide. Why, as the water flows in either direction from the water divide. Go back and see if this is your water divides this location. Then what is happening is the water will be flowing in this direction, and it will be flowing in the other direction so this is called the water divide, because water will be flowing in either direction. How can we find the location of this? We can use the knowledge of our mathematics. What will be the slope of the groundwater table at this location? It has to be 0, this will be a horizontal line. So that is the property of the groundwater table we will use at the water divide the location can be then found how by setting d h d x is equal to 0 is not it? Or you can set twice of h d h d x is equal to 0 it is 1 and the same thing you have the equation or the solution of this h in terms of h square this one. So, what you do is you take the first derivative of this equation 6. So, it will be twice of h d h d x, so you differentiate on the right hand side equate it to 0 that will give you the location of the water divide.

So, let us do that that will imply your minus 2 W over K x minus h 0 square minus h 1 square minus W l squared over K. You can do that easily over l. All of that equal to 0 would mean what? Let me give you the next step which is going to be your 2 W x over K will be equal to W l squared over K l. You can simplify this, plus your h 1 squared over l minus h 0 squared over l so that will give you your x is equal to K by 2 W outside. And then you will have W l over K plus h 1 square minu S h naught square over l or you can simplify this further it will be l by 2 plus K over 2 W l times h 1 square minus h naught square so that would basically mean that your x is equal to a is equal to what? l by 2 minus K over W of your h 0 square minus h 1 square over twice of l. So, after you simplify this whole thing, this will be the location of your water divide. And then we say that this is your equation number 7. So, you get the location of the water divide. Now, how can you find the value itself, what will be h m?

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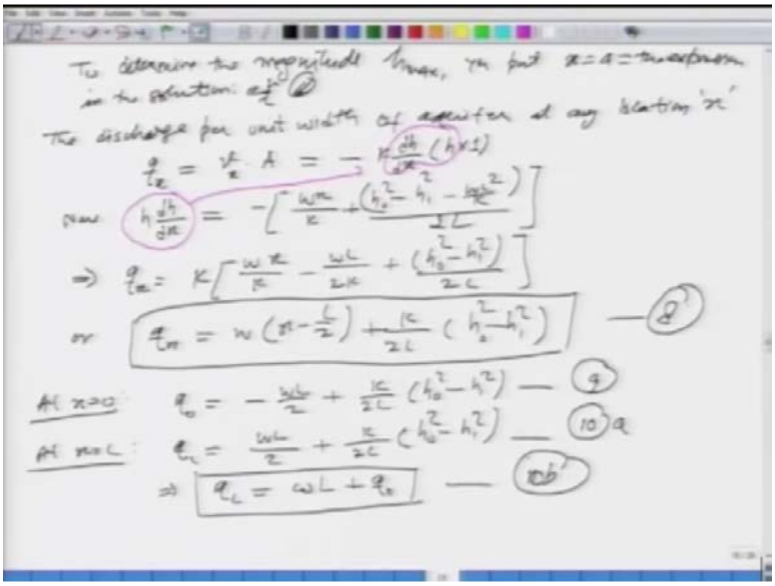

So, to determine the magnitude h max, all you do is you put your x is equal to a is equal to whatever the expression we have found which is this one, you put this in the solution is what I think it is equation 6, let me go back and check equation number 6. So, this is the solution so you put x is equal to whatever we had found for a in this equation. And that will give you the value of h max. We will not do that you can verify that on your own. The next thing which we are going to do is we look at the discharge that will be taking place per unit width per unit width of aquifer at any location; at any location x can be found as follows. Let us say q x is going to be equal to what, it will be v x multiplied by the area the discharge is going to be what, the velocity times the area across which it is taking place, what will that be as per the Darcy's law I can say v x is minus K v h d s right and what is the area? Area is going to be h times 1.

Now I had just derived the equation for d h d x which was minus of W x over K plus h 0 square minus h 1 square minus W L squared by K this whole thing divided by 2 L. So, if I use this then I can say what will be your q x. You see that the h d h d x appears in this equation h times d h d x. And that is what we have just found out after differentiating the solution. So, I put this, this whole expression on the right hand side into this. And then simplify what I am going to get is this is your K of your W x over K minus W L over 2 K plus h 0 square minus h 1 square over 2 L or your q x is equal to W times x minus L by 2 plus K by 2 L of your h 0 squared minus h 1 square. So, this is the equation for finding out the discharge per unit width of the unconfined aquifer at any location x, I am going to number this equation as 8. So, you see that this discharge which, which is taking place; it may be either in the positive direction or it may be in the negative direction depending upon the location you know that at x is equal to a is the water divide.

So, to the left of the water divide flow will be taking place in the negative x direction, after the water divide it will be taking place in the positive x direction. And the other important thing to note here is that this discharge per unit width of the aquifer will be a function of x. Like in the earlier case in the confined case, we had seen that it is constant it is the same discharge taking place from left to right which is not the case here. So, what we can do is we can find out the groundwater flow contribution at x is equal to 0 or the upstream water body and also at x is equal to l. We can find out how much flow actually will be taking place or how much groundwater contribution will be taking place into these water bodies? How do you do that? Well we just put x is equal to 0 and x is equal to l in equation 8.

So, if you do that, you will have q 0 is equal to minus W L by 2 plus your K by 2 L of your h 0 square minus h 1 square, this is my equation number 9. And at x is equal to l the q of l is going to be W L by 2 plus your K over 2 L times h 0 square minus h 1 square. This is your 10. And then you can simplify this further actually you can use q naught you will have q l is equal to W l plus q 0. I am not going to do that you can work on that. And let me say that if this was 10 a, I am going to say that this is our 10 b. So, this way we see that we can find out how much will be the groundwater flow which is taking place in a unconfined aquifer with recharge at the upstream water body and the downstream water body. I am afraid I am running out of time here I would like to stop today. And then look at some other simple groundwater flow situations in the confined and unconfined cases. So, we will stop here and come back tomorrow.

Thank you.