Advanced Hydrology
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Lecture – 4

Hello and good morning. And welcome to the lecture number 4 of this video course on advanced hydrology. In the last class, we looked at the derivation of the Reynolds transport theorem. What I would like to do today is go over it again, and look at some of the steps are more closely just in case the some of your not able to follow it. I am doing it, because this is extremely important to understand various steps involved in the derivation, and understand different terms in this Reynolds transport theorem.

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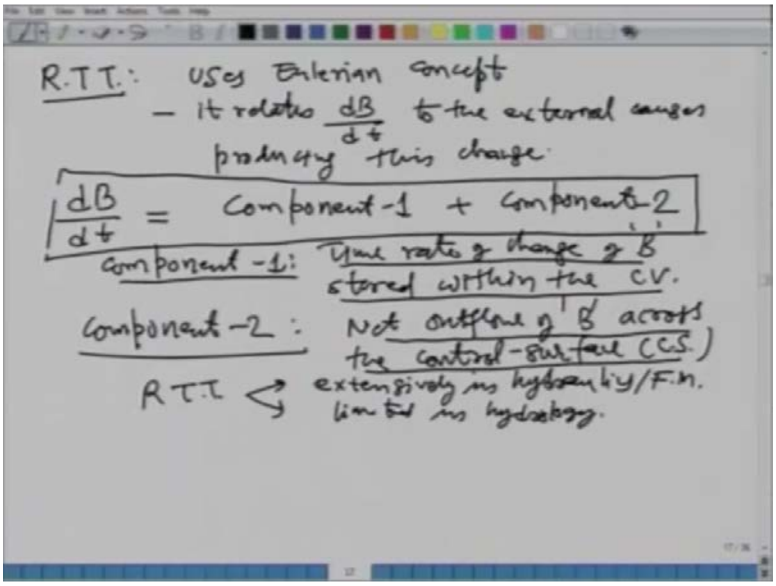

So, what I like to do is come back to this board and define that the Reynolds transport theorem basically gives us this. The time rate of change of extensive property is equal to the sub of 2 components; that is component 1 and component 2 as shown here. What is component 1? Component 1 is nothing but the time rate of change of extensive property sold within the control volume that is this; this is your component 1 that is defined as the time rate of change of the stored within the control volume. And component 2 is the outflows of the extensive property flowing across the control surface, that is to say component 2 is the net out plus or net outflow of the extensive property across the control surfaces. So, what we will do is will just write these two quantities.

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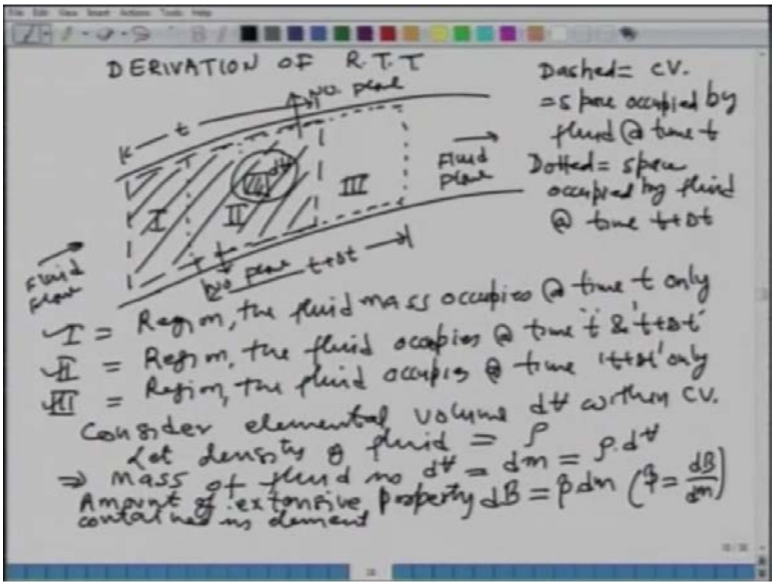

And what we did is we defined the control volume by using this schematic diagram in which we defined regions 1, 2, and 3 where region 1 was the region at time p only; region 2 was the region the fluid occupies at the time t and t plus delta t and region 3 is the region the fluid occupies at time t plus delta t only. Then what we did is we took a very small elemental volume d V this one within the control volume. And then what we did is we wrote the expression for the amount of extensive property within this d B. Then we said that find the total we integrated over the whole area or whole volume.

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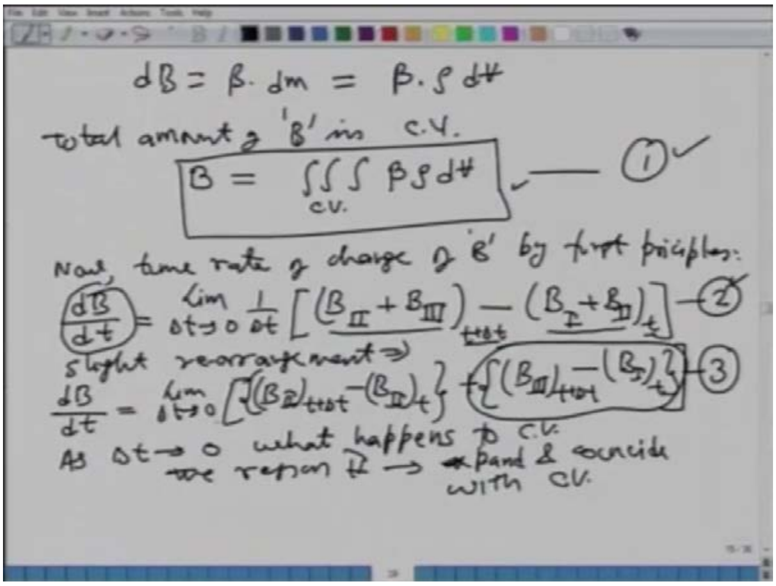

So, I will just jump to the next step. This is the expression B stat equation number 1 which comes out to be the extensive property stored within the control volume. Then we moved ahead and we said that we will write the expression of d B by d t that is the left hand side of our Reynolds transport theorem using the first principles that is how we defined the first derivative using our knowledge of calculus in which we said that d B by d t is nothing but the under the limits of your extensive property at time t plus delta t minus extensive property at time E. And using the definitions decoration of our different regions we have extensive property at time t plus delta t in regions 2 and 3 and the extensive property at time t in regions 1 and 2. So, we wrote this equation and the, we did some slide rearrangement. And then we will deal with the first term is this and the second term this one by one.

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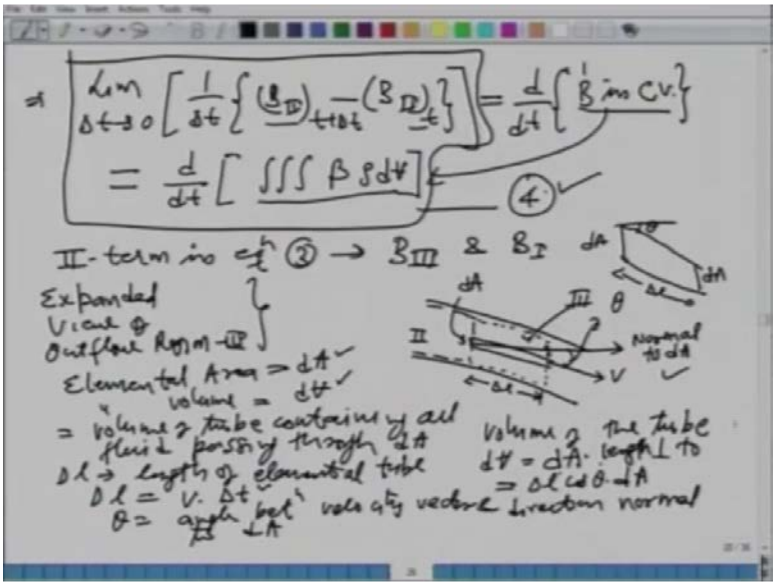

Moving then further it write to write this first term as under the limits of 1 over delta t of your B 2 at time t plus delta t minus B 2 at t, this we say is nothing but the first derivative of extensive property in the C V. And we have just derived the expression for B in C V as this quantity. So, we say that this whole thing; this whole expression we derived and we said this is equation number 4 so that was the first component which is the time rate of change of extensive property stored within the control volume that is component number one. In the component 2 of the Reynolds transport theorem we have two terms; one is at the outflow region and other is at the inflow region, what we did is we look at the magnified or we took the magnified view or elastration of what is happening at the outflow region.

So, we look that this region which we have drawn here in which at in the outflow region we took a very small elemental cube of volume d V and cross sectional area d A and where in we defined the various terms where d A is the cross sectional area the volume is d V. And theta is the angle this one between the velocity factor and the normal vector normal to the d A. Using this we get a different terms that is length of this tube is V time delta t and d V was defined as area multiplied by the length of that particular element so it will come out to be delta l cos theta d A. Then what we did is we try to write the expression for the total amount of B in region 3, why because region 3 is the outflow region, and we want to write down the expression across the whole cross sectional area d A at the outflow region.

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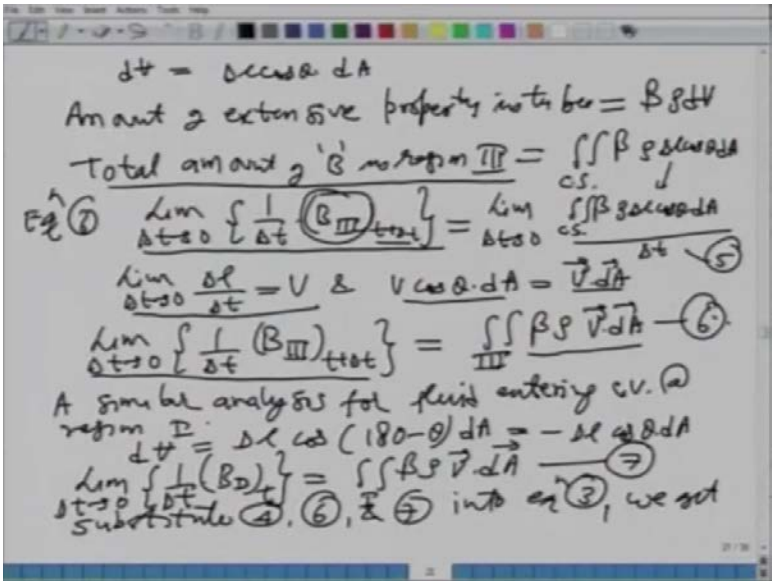

This we said under the limits of your B 3 at time t plus delta t is nothing but under the limits of this expression which we derived here. Then we use the simple definition of the, this is how the velocity is defined. This is the dot product of 2 vectors V and d A that is V dot d A. We put all these things in this expression and this came out to be the total integral or double integral over the whole cross sectional area or the control surface times beta row V dot d A. Then what we said is we can carry out a similar analysis at the inflow region.

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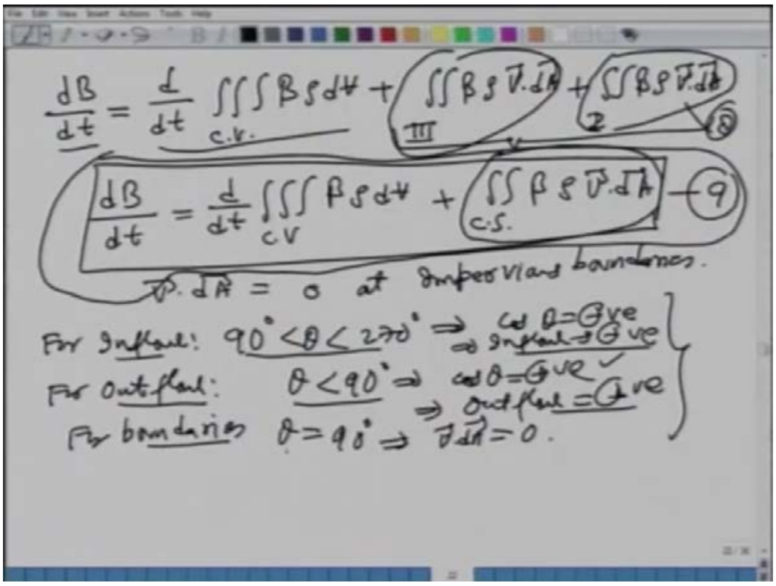

And we will come up with a similar equation which was this, this was the similar analysis for the inflow region; this is what we had just derived. And this was the component 1 and this is the left hand side. So, if we combine the last two terms into a single term and we say that across the whole cross sectional area, what is the quantity beta row V dot B A flowing across the whole control surface? So, this is your final expression for the Reynolds transport theorem say which consists of these two components. First one is the time rate of change of extensive property stored between the control volume. And the second one is the extensive property flowing across the control surface. Then we look that some simple concepts of various possibility of value of theta in which we said that at the inflow region the theta will be in this range that is why the inflow if it take negative, we do not have to worry about the dot product. At the outflow region your theta will always be less than 90 degrees that is why cos theta is positive.

So, we take the outflow plug as always positive. And across the boundaries there is no flow. So, theta is 90 degree and V dot d A will always be 0. So, this is what we had done in the last class, the derivation of the Reynolds transport theorem. What we will like to do today is to take these concepts of the Reynolds transport theorem and look at the continuity equation. So, first we will start with the continuity equation and then we will take up the momentum and energy equation also. How we can derive or how we can reduce these basic laws of physics which we apply routinely to all our fruitful problems using that Reynolds transport theorem.

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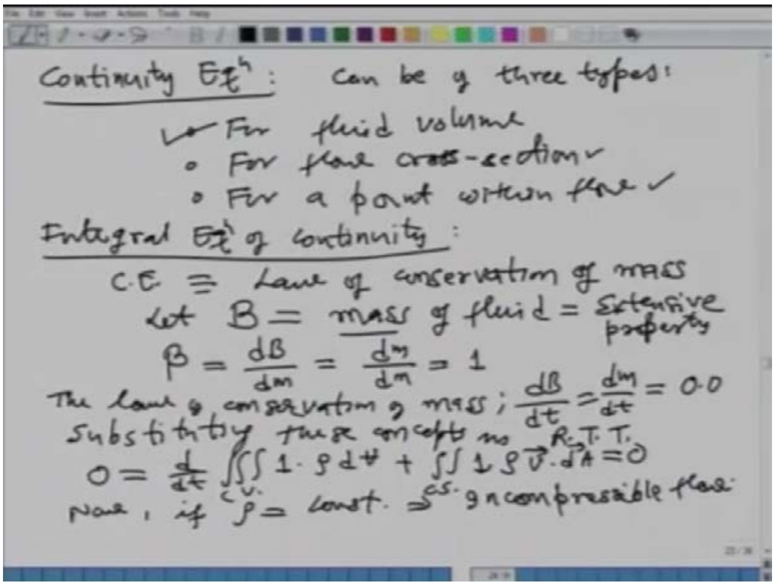

First we look at the continuity equation, the continuity equation can be written or can be of 3 types, it could be for the fluid Volume; it could be for flow cross section or for a point within the flow or flow region. I am sure you have seen some of the examples of these continuity equation. Can you think of an example of continuity equation for a point within the flow region I am sure you may have done the problems on the pipe fluid analysis or the water distribution system designed in which what we do at each junction is all the flow coming into the junction is equal to all of the flow going out of that junction. So, that is the example of a continuity equation at a point that is this one. And flow across the cross section we all are familiar the amount of you know what the volumetric flow rate flowing across a particular cross section is equal to the same at the next cross section if the flow is steady.

However, what we are going to do is we will look at the integral continuity equation in the fluid Volume. So, let us look at this integral equation of continuity as we know what is a continuity equation? It is nothing but the expression for or representation of law of conservation of mass. So, the extensive property in our, this Reynolds transport theorem will be the mass of the fluid. As we said that we derived this a general control volume theorem or that Reynolds transport theorem for any general extensive property.

When we are trying to derive the continuity equation continuity equation is written for mass we will take capital B is equal to the mass of the fluid that is knowing. So, mass is the extensive property what will be beta then? I will like to do think about it for a second and that means yourself what would it be? What is the relation between beta and B? That is intensive and extensive property. It is the d B over d m so what will be the d m over d m will be nothing but 1. So, for the law conservation mass your extensive property is the mass and intensive property will be 1 constant always. Also as per the law of conservation of mass, what do we have? It states that the matter cannot be created or destroyed it has to be conserved.

So, what does that mean in terms of d B over d t that is d m over d t that has to be equal to what? If the mass has to be conserved over time 0 so d B by d t will be 0. So, this is the left hand side of your Reynolds transport theorem. So, we substitute all these things substituting these concepts in your Reynolds transport theorem, what we will get is the left hand side is 0 and then the first term is d over d t of your triple integral over the control volume of your beta row d V and beta is 1. So, you have 1 row d V plus the extensive property that is mass flowing across the control surface that is double integral over the cross section or the control surface beta is 1 row V dot d A is equal to 0. Now, say that your density of the fluid when the fluid row is taking place is constant, you make that assumption that is to say that we are saying that the flow is in compressible that is there are no density differences during the flow motion we consider under the assumption of incompressible flow. What will happen is row will drop out this equation we got row becomes the constant and we can take it out of the integral.

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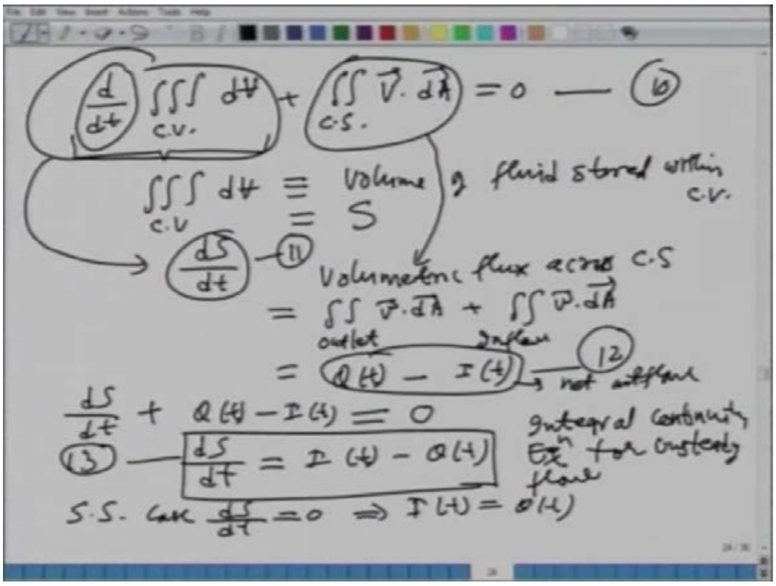

So, what we have than is d over d t of triple integral over the whole control volume row is gone. So, you have d V plus over the cross section or control surface you have V dot d A is equal to 0. And I am going to name this number this equation as 10. Now, what we do is we look at this 2 expressions individually. Just think about physically, what is this quantity represent? That is to say or let us leave the d by d t side for a second. And try to think what is C V or the integral of the whole volume within the control volume is what if you put row here it is basically the total mass whish is there in the control volume. But row is out so is the total volume, what we say is then this is the volume of fluid stored within the C V.

And let us represent that the total volume that is stored within the control volume as capital S, S is the storage that is it then what is this whole quantity? This is nothing but d S over d t; this here, try to convince yourself. The first quantity for the first expression in equation 10 is nothing but d S over d t. Now, let us look at the second quantity, what is this represent or what does it represent basically? Basically it represents the extensive property flowing across the control surface. And the extensive property is mass, and because we have taken the row out we have the volumetric flux flowing across the control surface. So, let us say that your this second quantity represents the volumetric flux across the control surface. This we say is equal to we can break it up into 2 parts, let us say at the outlet region or the outflow region what is V dot d A?

And at the inflow region, what is V dot d A? And this we say is equal to what is the low at the outlet region? Let say it is Q volumetric flow rate and this is I or the inflow in the volumetric flow units. And let say this is your equation number if this was 11, I say this is 12; we put 11 and 12 into 10. So, what we will get is d S over d t plus your Q t minus I t. Remember what is Q t minus I t it is the net outflow and all of this should equal to what? Should be equal to 0 or we take the inflow and outflow quantity on the other side you have this is equal to I minus Q that this sound familiar. I am sure it does, this is nothing but your famous continuity equation for the unsteady flow.

So, this is the integral or volumetric continuity equation for unsteady flow. What happens when the flow is steady? There will be no change in storage or any quantity which depends on time will be constant or the change in the respective time will be 0. So, for the study case or for study state case your d S by d t term will be 0 that would mean your I t is equal to Q t inflow is equal to outflow. If you think of an example of a channel flow in which the same amount of flow is passing through a particular reach of the channel. Then we know that the discharge is constant same discharge is flowing. So, inflow is equal to out flow within and it is. And I will number this equation as let say 13 and this is further steady flow. So, moving further I would like to just caution you.

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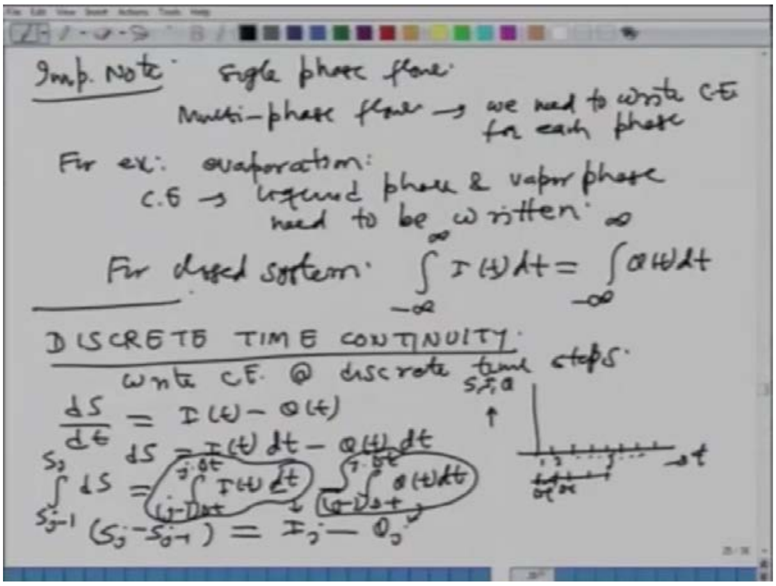

One important point about the use of this continuity equation, the continuity equation which we are just written above applies to a single phase flow that is the fluid is in a single phase either liquid or gas. However, if you have the multi phase flow that is if you have a physical process in which the fluid is occurring in more than one phases that is there is gases state involved; there is liquid state involve or any other solid phase may also be involve. If that is the case, then we need to write the continuity equation for each phase. For example, evaporation and the water is evaporating from the water bottle let say rivers and lakes and dams and reservoirs. So, you have a multi phase flow kind of a equation there, there is condensation taking place there is evaporation taking place in the net escape of water is called the evaporation.

So, in that case we need to write the continuity equation for liquid phase and vapor phase separately need to be written. We will come back to this concept little later in this course when we will look at the process of evaporation. The other thing I would like to mention that for a closed system we should have the integral of the quantity of the inflow should be equal to the total quantity of the your or in other words if we total inflow volume is equal to the total out flow volume, then it is a closed system moving further. The next thing we are going to look at this what is called the discrete time continuity. As we know most of the hydrological data are continuous in nature or most of the hydrological variables I should say are continuous in nature. For example, rainfall or flow in a river the steam flow in a river is changing continuously with with respective time. However, when we measure these variables for example, when we measure the flow in a river the the data which we have available are in discrete time step. For example, we will probably measure them on an hourly bases or on in daily bases similarly, for the rainfall. Rainfall also we will measure at discrete time step.

So, all the equations which we have written we need to write them at discrete time steps, that is why understanding the discrete time continuity becomes important. What we do is we write the C E at discrete times steps; that is called the discrete time continuity. So, if we look at the concept this is your time domain and time may be in hours or days or a any suitable unit. And the quantity if you have is either S or I or Q it may be wearing I do not want to draw the graph. The important thing here is that we discretize the time domain like this where in we say this is 1 2 3 and so on. Then let say this is your j eth time interval and so on.

And each of these interval let say is delta t delta E which can be either constant or or variable, but in most cases we keep it as constant for the easing implementation for example every day we measure the rainfall. So, if we apply the, our continuity equation which we have just derived d S by d t is equal to I t minus Q t or we say d S is I t d t minus Q d t. Now what I will do is I will integrate this equation on both side that is to say if I integrate this under the limits the storage at at say being general at S j minus 1 we are writing this at the j eth time interval.

So, S j minus 1 to S j will be equal to you are the integral of your i t d t where the variable is d t. And the time is wearing during that j eth interval from j minus 1 delta t to j delta t. Similarly, integral of your Q t d t of j minus 1 delta t to j delta t or we say that this is after you integrate it will be S j minus S j minus 1, and I represent this as let say I j minus Q j there, what is I j? I j is this whole quantity; this is I j and this whole quantity is Q j.

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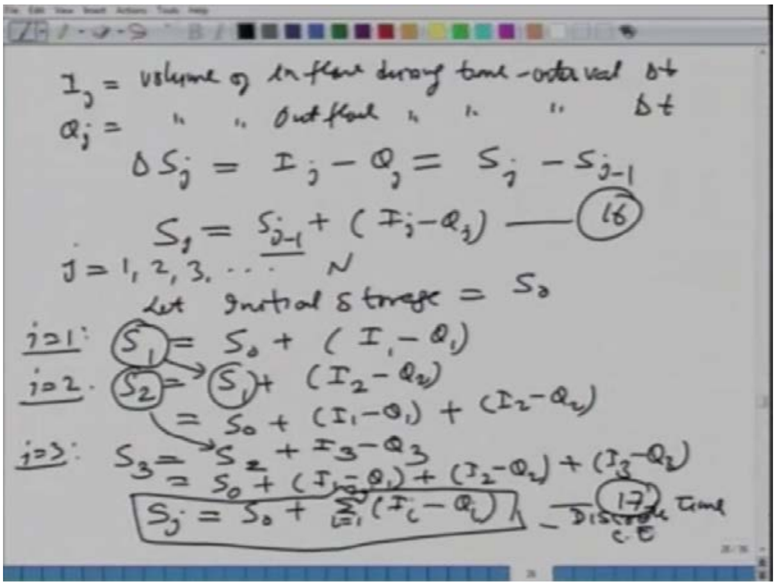

And how I define them is basically your I j is the volume of inflow during time interval delta t and similarly, your Q j is the volume of out flow during time interval delta t. So, in other words I can write the equation delta S j is equal to I j minus Q j which is nothing but S j minus S j minus 1 or you can write the expression for S j as S j minus 1 plus I j minus Q j and let say I am going to name this as a equation number 16. So, this is my discrete time continuity equation at least in the partial sense.

Now, what we do is we can apply this equation at each time interval. So, that we will have a general expression each time interval means let say we will apply this at j is equal to 1 j is equal to 2 j is equal to 3 and so on if we have capital N number of time intervals. So, if we do that j is equal to 1 then we will have S 0 and S 0 is something the initial condition of the initial storage. So, if we say that the initially the storage is let say S 0 then what will be S 1 as per equation number 16 it will be nothing but S 0 plus j is 1 so I 1 minus Q 1. Now, we write the same equation at let say this per as j is equal to 1. Now, we write this as j is equal to 2 that will be S 2 is equal to S 1 plus, what I 2 minus Q 2. Now, what is S 1? You put S 1 from this equation into this so it would be nothing but S 0 plus I 1 minus Q 1 plus I 2 minus Q 2.

Similarly, for j is equal to 3 you will have S 3 is equal to S 2 plus I 3 minus Q 3 and the value of S 2 will put from here into this. And that will give you this as S 0 plus I 1 minus Q 1 plus I 2 minus Q 2 plus I 3 minus Q 3 you see that there is a pattern here once we recognize the pattern we can just capture it. So, you say that in general at any j eth times that your S j is going to be equal to S 0 plus summation where summation will be wearing from one to j let say this is I it will be I I minus Q I. So, this summation is I actually let me write it again I am going to write it again. So, it will be summation I running from 1 to j. And let say this is your equation number 17 which is called the discrete time continuity or in other words it is the continuity equation indiscreet time domain which we can use to analyze the data which are measured at discrete time. So, let us move ahead and look at the different ways of data representation methods.

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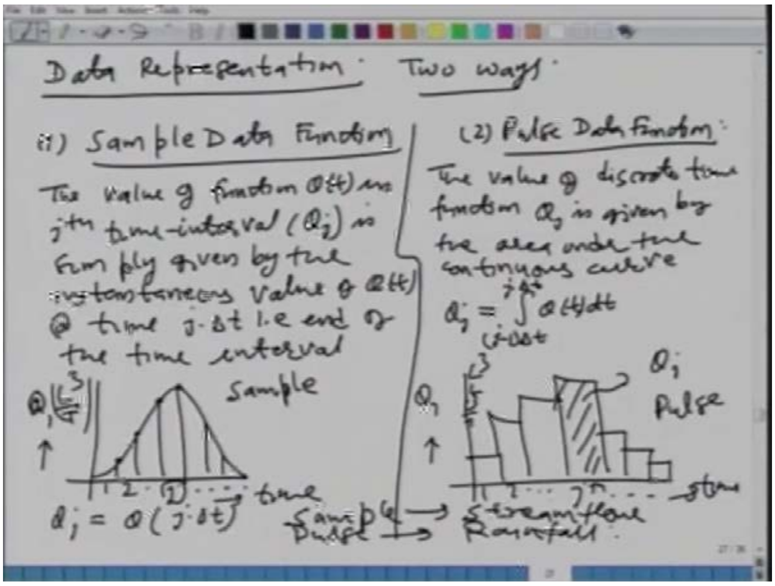

The various variables which are measured over time, but even at discrete time intervals at which we are measured we represented the store the data or the method of representation can be different. So, there are basically 2 different types of method or 2 different ways of data representation these are called sample data representation of function. And the other one is the pulse data representation or function. In the sample data representation what we do is we just simply measure the data at the end of the timing interval period and we say that the value of the physical variable during a time interval is its value at the end of the time interval. And in the pulse data representation we say that the cumulative value of a particular variable which we measure, we say represents the total value for that time interval which will be represented by the area under the curve for that time interval.

So, Let us look at these definitions and the graphically how we represented. So, here we stay that the value of a function at say the function is Q in the jth time interval jth time interval and we will say that it is Q j is simply given by the instantaneous value of your Q t the function at time j delta t that is at the end of the time interval. If you, you look at this graphically, what does it mean? If you have a inflow it look like this, then we have discreties the time domain. So, this is your first interval, second interval and so on and the j eth interval so during the j eth interval this one what we are saying is that we take the value at the end of the time interval. Similarly, j minus 1 is this; this one is this and so on. We say that Q at j is nothing but value of Q at time j delta t.

So, this is called the sample data representation. Let us look at the second one will draw a line here in this one the value of discrete time function Q j is given by the area under the continuous curve that is to say your Q j is equal to the area under the curve and that curve is your that function is Q t and j eth, j is the time interval. So, we run the integration between j minus one delta t and j dash. If you want to look at the graphically how we represent this as let say it is in the form of bar chart this is 1 2; this is your j eth time interval and so on.

We take the area under the, this represents your Q j for the pulse data function. The x axis is time and y axis is Q j, you see that in this first one that is the sample data function this Q is normally an L cube by T unit. And in the second case this could be an either L cube unit or L unit or L cube by T unit depend upon depends upon how we are defining our particular variable. Can you think of an example of these two data representation, sample data representation and pulse data representation? Yes these sample data representation is the instantaneous value and that is how we represent the stream flow and for the pulse as you can see from this bar chart it is the rainfall in the form of rainfall density for unit time we did. So, we looked at the derivation of the continuity equation from the Reynolds transport theorem. Then we look at the volumetric form of the continuity equation or the integral continuity equation. And then we also look that the continuity equation at the discrete times steps.

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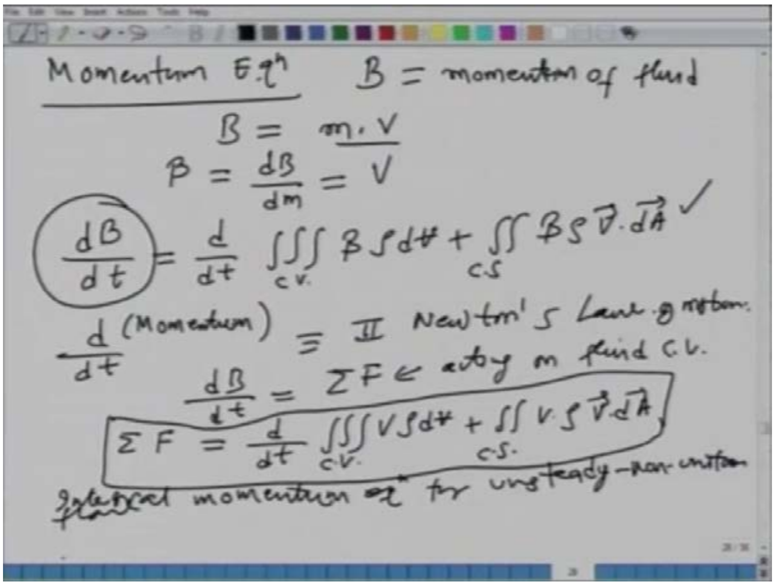

So, now, what we will do is will, we will move on to the momentum equation or derivation of momentum equation from our knowledge of this Reynolds transport theorem. So, then we are writing the momentum equation what is the extensive property B? Obviously, it has to be the momentum or the fluid moment of fluid or the B is equal to what is the momentum? Mass times velocity that is how the momentum is defined. Now, can you tell me what will be the intensive property beta? This is defined as your d B over d m. So, if you take the derivative of this quantity m times V with respect to m what we will have is the velocity V. So, for the momentum has be extensive property, velocity is the intensive property, keep that in mind. Now, what we do is we write our Reynolds transport theorem, you have to remember this equation by your heart it is like Mannes equation; you cannot, you never forget. So, the Reynolds transport theorem, you should understand in such a way that it just comes out. So, d B by d t is your d over d t triple integral beta row d V plus double integral control surface beta row V dot d A.

Remember in the continuity equation we said that d B by d t is equal to 0, what was that? that was the law of conservation of mass. So, we write the left hand side using an external knowledge. Similarly, what is the d over d t of momentum? What is the time rate of change of momentum? You have to go back to your knowledge of physics, twelfth standard or may be your in a first few years of engineering degree. And tell me, what is the basic law that gives you the time rates of change of momentum? Yes it is the, it is given by the second law of second Newton's law for the Newton's second law of motion. And as per that, what is the time rate of change of momentum? It is nothing but the net force acting on the body, earlier we have done in terms of the body. Now, it is in terms of the control volume on the flowing fluid.

So, this is you can write your d B d B over d t is equal to the net force summation F which is acting on the fluid control volume. So, with this knowledge what we will do is we will put all these equation in our Reynolds transport theorem. So that you will have summation F that is the left hand side is equal to d over d t of triple integral beta is V, we have seen that row d V plus double integral over the control surface V row V dot d A. This is your most general momentum equation for any kind of fluid flow problem. And this we will say is the integral momentum equation for unsteady where things are wearing as a function of time non uniform flow, non uniform flow, what is a non uniform flow or a uniform flow?

You understand the difference between uniform flow and non uniform flow. Yes the uniform flow is something in which things are not wearing with the respective space. And in a non uniform flow, the velocity and other parameters will be wearing this respective space. So, let us move on further.

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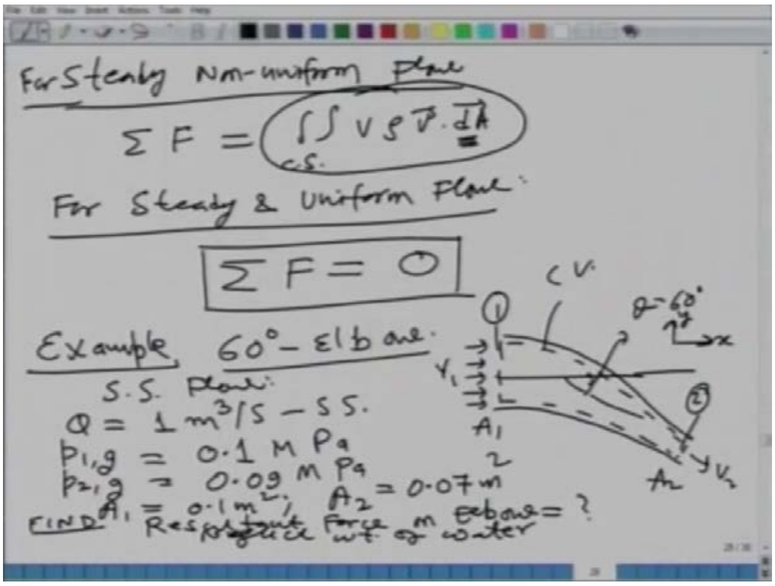

And we want to write out this equation for steady non uniform flow if you want to write out the momentum equation for steady flow, but it may be non uniform what will happened is the things are not changing as a function of time. So, the first term on the right hand side will vanish. So, we can write that the net force acting the first term is 0. So, it is only the second term V row V dot d A so this is for the steady non uniform flow. How about for steady and uniform? When the flow is uniform what does it mean? It means that the velocity and other hydraulic parameters are not changing with respective space.

If the things are not changing with the respective space and in this term we are taking the integral with respect to the area. If with respect to different areas or different points in your in a control volume if the velocity is constant than this whole quantity will also be equal to 0. If you take the V dot d A term, because the velocity is not changing then this will be 0, because of the uniform flow. So, for this momentum equation then will become summation F is equal to 0 so this is the momentum equation for the steady uniform flow. Now, what we will do next is will try to apply some of this different types of some momentum equations to some flows equation.

I would like to take up a example problem and this kind of a example problem I am sure you may have done earlier in your classes. So, this is the example of a 60 degree elbow. In the pipe distribution network system as we know there many kinds of pipe fitting, there will be t junction; there will be elbow; there will be 90 degree elbows and valves and so on. What we are doing here is let us say we have a situation. So, this is a pipe fitting where in all the data are given; this is the 60 degree means this angle theta is 60 degrees. This is your x direction; this is your y direction the velocity of flow for the direction is in this horizontal direction.

Let say this is V 1 here and the water comes out whether velocity V 2 here, and the cross sectional area of this elbow here is A 1. And let me defined it that this is cross section 1; this is cross section 2, and this is A 2, so flow is taking place under steady state condition, you have a case in which water is flowing through this 60 degree elbow constant quantity Q is flowing under steady state condition. So, things are not changing or flow is not changing with respect to time. And the data that are given are Q is equal to 1 meter cube per second which is static state other thing that is given to you is the p 1 g which is the pressure force or the pressure that is acting at cross section 1 there will be some pressure acting is 0.1 million Pascal’s. Also given to you is the pressure at the cross section 2, g is the gate pressure that is above atmosphere that is I am sure all of you understand this terminology.

So, this is 0.09 million Pascal's. So, these two pressure are given also given to you is let say the area of cross section at the inflow region is 0.1 square meters or you may be given the pipe diameter to at the inflow region where this elbow is being attached. So, you can find out that area A 1 and similarly, your A 2 is given as 0.07 square meters. What we have to find this the resultant force on the elbow is equal to what? Another data that is given to you is neglect the weight of water. So, these are the data that is given to us, the flow is given cross sectional area are given the pressure are given; the inflow and out flow region. Neglect the weight and find out how much force will be acting on this elbow? All this analysis is important to design all this fittings. So, that they are able to visitant the force that will be acting on them in the real life situation. So, what I have drawn here is that this is the control volume.

So, what we will do is we will apply our steady state momentum equation to this control volume. And find out what are the forces which will be acting on this in the x and y direction? Remember the momentum is a vector quantity. So, we will write the x direction momentum and y direction momentum equation.

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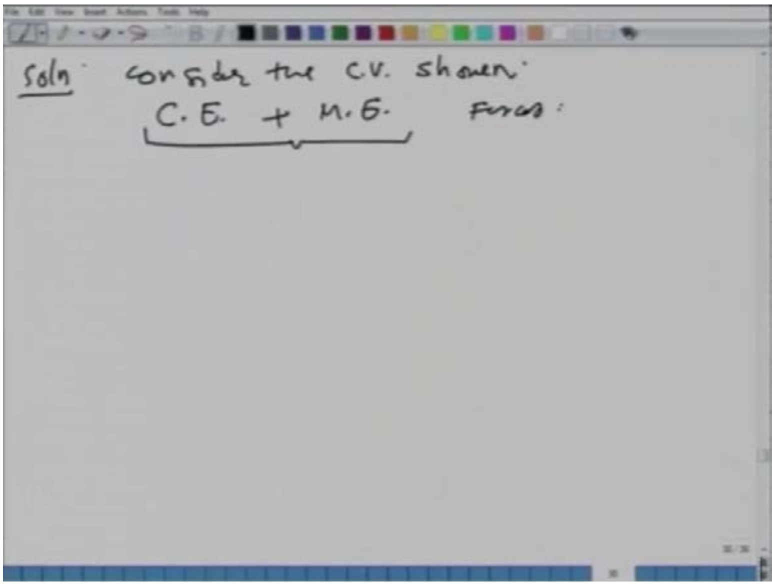

Let us do that, so for the solution you say consider the C V shown. So, in fact, what we will do is we will solve this equation by using continuity equation and momentum equation. It will be a combination of these two things. And once we apply this we will be able to calculate the forces at what I would like to do is I would like you to think about this problem. And in the next class, you come back and look at how we are going to solve this particular problem. So, at these two cross section, we have seen the continuity equation, we have seen the momentum equation to apply this at this two in a cross section or this whole control volume. And see if you can come up with the answer of what is the force that will acting on this elbow? We will look at the solution in the next lecture.