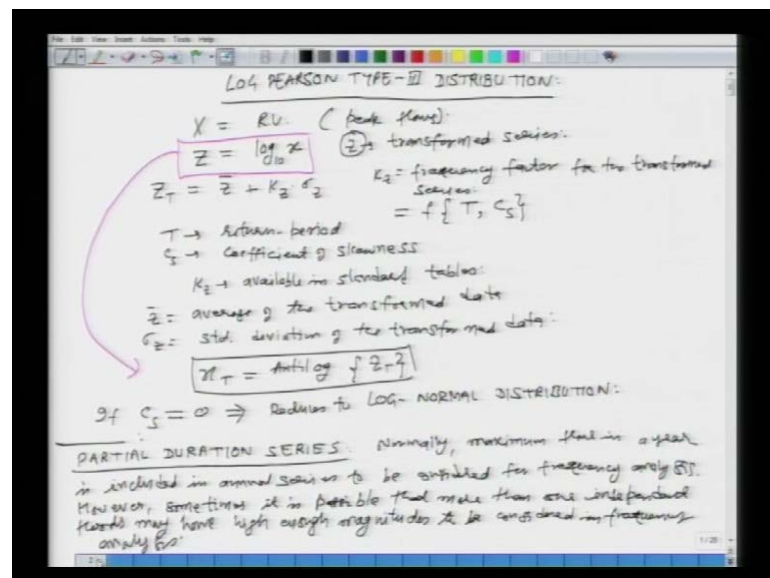


Advanced Hydrology
Prof. Dr. Ashu Jain
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 35

Good morning and welcome to this video course on advance hydrology, we are into the stochastic hydrology module and in this we are looking at chapter 12 on hydrologic frequency analysis. In the last class we saw the Gumbel's distribution method which we said is useful for extreme values or certain variables, such as you know the flood peak or the maximum wind speed or the maximum rainfall in a certain duration, etcetera.

(Refer Slide Time: 00:52)



Today, we will look at another distribution which is called the log Pearson type 3 distribution. So, this is log Pearson type 3 distribution which is used quite extensively in hydrology, as the name suggests we take the data and we transform the data by taking log. So if you have random variable X as a any hydrologic random variable for example, a peak flows or something else then what we do is we transform this into another series let us say Z by taking log.

And normally we take log to the base 10. So Z is what is called the transform series or the log series, and then we carry out all our computation on this transform series of the z series. The general hydrologic frequency analysis equation or relationship remains still

the same that is we find out the value of Z corresponding to a certain return period T sorry T will be equal to what? Will be $\bar{Z} + K Z$ is the frequency factor times sigma Z , where we say $K Z$ is what? It is the frequency factor for the transformed series.

Now this for the log Pearson type 3 distribution is function of two things. One is the return period T itself and other is what is called the $C S$ all right. What is $C S$? It is nothing but, the coefficient of skew. So obviously, T is your return period or recurrence interval and $C S$ is what? It is the coefficient of Skewness in the data all right and this value of $K Z$ is available in standard tables.

So this log Pearson type 3 distribution as we have seen that it takes care of the Skewness in the data all right. What we are doing is we are taking the log when we have a probability distribution or data which are you know skewed you know in terms of the symmetry which can be measured in terms of the coefficient of Skewness all right. So depending upon the magnitude of Skewness this value of $K Z$ will be determine depending upon that particular value of T all right and as I said this is available in the standard tables. So you have a table it is a two dimensional table in which on one side we have the return periods, other side you have the coefficient of Skewness all right. So corresponding to those values you can just read out the value of $K Z$.

And once you have the $K Z$ you can determine $Z T$ what is $Z T$? It is $\bar{Z} + K Z$ times sigma Z all right. \bar{Z} is the average of the transformed series and sigma Z is nothing but, the standard deviation of the transform series.

So let me write it down \bar{Z} is the average or mean of the transformed series or transformed data and sigma Z is the standard deviation of the transformed data. So it is very easy to see that we can apply this method very quickly and easily how? If you have the data in terms of the peak floods annual peak flows what do you do? Well you just take the log of all of them you find out their mean of the transformed series, find out the standard deviation and also find out the coefficient of Skewness and depending upon the return period desired for which you want to find out the magnitude, and the coefficient of skew we can find out $K Z$ from the table and we can use this equation to find $Z T$.

So once we have $Z T$ how can we find x ? Because x is the actual value of your random variable, all we do is just we take the antilog we use this relationship in the reverse direction all right this one all right. So all you do is your x of t will be what your antilog

of Z T which we just found out. I will come back to this in the form an example little later today I would like to take up an example in which will have a set of data set or set of data points to which we will fit Gambel distribution as well as the log Pearson type 3 distribution. So I will come back to this later. Now what happens when the coefficient of Skewness is 0 all right you have a data which is symmetrically distributed that is to say if your C S is 0; that means, what? That means, this log Pearson types 3 distribution reduces to log normal distribution that is what is called a log normal distribution.

Why log normal because working we are working on the log series right we have taken the or we have transformed the data by taking log and it is normal because the coefficient of Skewness is 0 because the normal distribution we know is symmetrical. So that is why if we find that coefficient of Skewness is 0 or very closed to 0 then we can use a log normal distribution all right. So in that table of K Z you have a row which corresponds to C S is equals to 0 you can just read those of all right.

Now, we will look at a some very interesting concept which is called the partial duration series. So let me write it down and then will come back this. What is a partial duration series all right and actually before I go to that what is an annual series most of this hydrologic frequency analysis which we have been studying this is actually applied to the annual data most in most cases it does not have to be I am I am just you know being little bit more pragmatic here, because in most cases we deal with the annual flood flows or annual maximum rainfalls or some variable similar to that. So to collect such data what we do is we look at the whole year all right.

One whole year and you find out what is the maximum amount of flow that occurred in that whole year all right. So let us say you have the peak value of the flow in that particular year similarly, you go back in the past and you find out the peak flood for different years. Let us say you have 50 years of data, so what you will have is 50 different values of peak flows corresponding to 50 years all right that is called annual series. So the series or the data all right 50 data points will form what is called the annual series.

Now in that annual series we may have certain years which are very wet and certain years which may be very dry all right. Now let us say we are going to use this data for some flood management we want to find out the you know 500 year flood magnitude

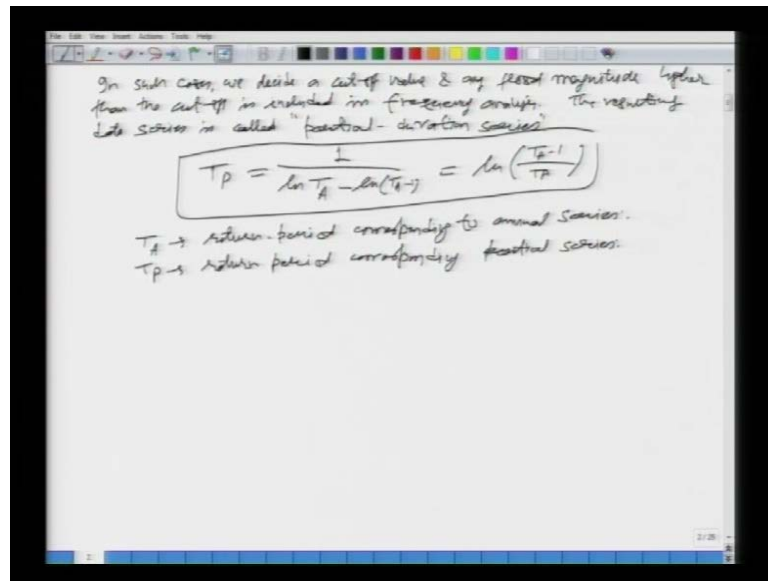
what will be the magnitude of flood which will occur once in 500 years. So when we are carrying out that kind of analysis then which data is more important to us? It is the data which corresponds to the wet years not the dry years all right for example, you may have one particular year in which there were 2 or 3 different magnitudes or different flood events occurred all right which may be higher than the highest magnitude of some different year all right.

So then what we do is we carry out what is called a partial duration analysis or which is called the partial duration series? In which we will say that I am going to put a cut off i will say any flow more than 50,000 meter cube per second constitutes flood. So i will let us say if I have the 50 years of data then what I will do with that cut off limit of 50,000 meter cube per second I will see how many years have flood magnitudes or the peak magnitudes higher than that all right. And I will take all those magnitudes all the peak flows so in from one particular wet year you may have you know 2 or 3 different flood events in which the flow magnitude was 50,000 and more.

So you may have more than one data point from a single year in a partial duration series similarly, there may be a you know few years in which you will not have any data in the partial duration series, because the peak flow in that particular year was probably let us say 40,000 meter cube per second or may be less. So what we do in the partial duration series is basically we modify our data to suit our needs. If we are carrying out a flood analysis we are going to say let us say this is my cut off and anything above that I am going to consider as flood all right. So in a particular year if there is more than one data point I am going to consider that in my analysis and in a particular year if there is you know the peak flow is less than that I am going to discard it all right. So that is the objective of what is called the partial duration series.

So let me just look at that write it down. Normally, the maximum flow in a year is included in annual series that is what we said initially to be considered for frequency analysis. However, sometimes it is possible that more than one that more than one independent floods of flow value that is may have high enough magnitudes to be considered in the frequency analysis. So that gives us the concept of what is called the partial duration series.

(Refer Slide Time: 13:56)



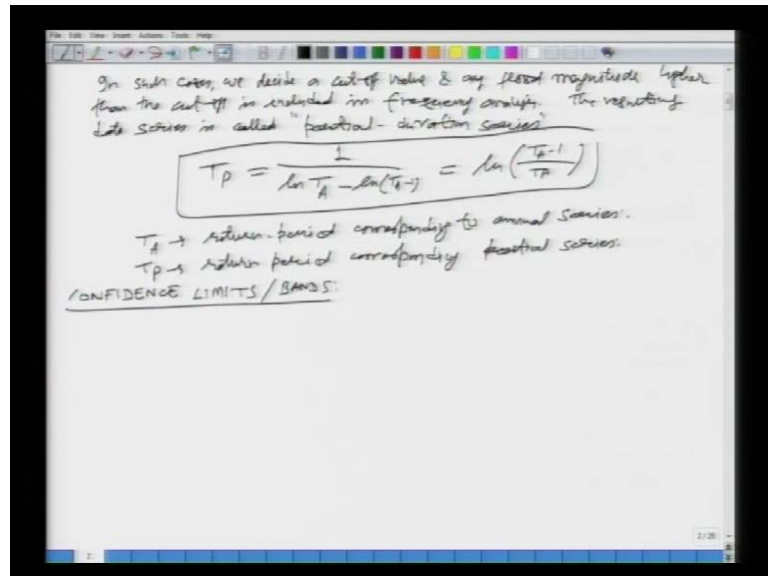
In such cases we select or we decide a cut off value and any flood magnitude any flood magnitude higher than that or higher than the cut off is included is included in the frequency analysis obviously, such a frequency analysis will be more meaningful the resulting data series is then called the partial duration series that is all.

So this is the concept of what is called the partial duration series. The there is a relationship between the return period corresponding to a partial duration series and the annual series which is given like this T_P is equal to $1 / (\ln T_A - \ln(T_A - 1))$ and this you can see easily can be written as natural log of $(T_A - 1) / T_P$.

So will not go into the derivation of this, but this is what people have determined. So obviously, what is T_A ? T_A is your return period corresponding to the annual series A means annual and then T_P is what the return period corresponding to the partial series. So obviously, when we are working with the partial series we need to establish a relationship between the return period for the annual series and the partial series all right. So we can carry out the analysis on the annual series and then we can convert it to the corresponding exceedance probabilities for the partial duration series. So sometimes this partial duration series analysis is important for floods as well as droughts which is carried out for more meaning full frequency analysis otherwise the lower magnitudes or

the dry years will impact the flood analysis which is actually not accurate and that will cause a certain decision to be taken which is not appropriate so that is why we do this.

(Refer Slide Time: 17:13)



The next concept or the next thing we are going to look at is what is called the confidence limits or the confidence band. whenever we are dealing with the prediction or forecasting or estimation under uncertain t that is you know we are not sure but, we are trying to use the statistical principles and we are going to say here in the frequency analysis what are we going to say basically, we are saying is that the magnitude of the flood or whatever the hydrologic variable corresponding to a 500 year return period is going to be this much.

Let us say after you a fit Gambel distribution or a log Pearson type 3 distribution or any other distribution to a given data set and you are asked to predict a 500 year flood for which a structure has to be design. So basically what you will get the output from your module will be some number let us say 50253.8 meter cube per second.

(Refer Slide Time: 18:37)

In such cases, we decide a cut-off value & any flood magnitude higher than the cut-off is included in frequency analysis. The resulting data series is called "partial duration series".

$$T_p = \frac{1}{\ln T_A - \ln(T_p)} = \ln\left(\frac{T_A - 1}{T_p}\right)$$

T_A → return period corresponding to annual series.
 T_p → return period corresponding to partial series. (continues)

CONFIDENCE LIMITS/BANDS:

The diagram shows a horizontal axis labeled 'T' and a vertical axis labeled 'Q'. A horizontal line represents the predicted value. A vertical line is drawn at a specific return period 'T'. A horizontal band is drawn above and below the predicted value, representing the confidence limits. The word 'Band' is written next to the upper limit.

What is the guarantee or what is the confidence in that number. Are we sure? Can we say that with 100 percent accuracy that yes the magnitude of the flood will be 50236 meter cube per second for 500 year return period no. So what we do is this let us say this is your return period and this is your x of T. For 500 years you want to predict some value and let us say your analysis gives you this value whatever that is. Now what we do is instead of saying that the value will be this much all right without giving any probability or without having much confidence in it what we say is or what we do is we say that we give this band which is called the confidence band.

So what are we saying is that instead of pointing out or giving one point as the prediction we are going to say that my value is going to lie between this and this or I am going to say that the lower limit will be this and the upper limit will be this. This is called the confidence limit the lower limit and the upper limit and you are prescribing a band or width all right which will depend upon certain things and we are going to look at that but, we will be able to say there is a 90 percent chance that the magnitude of 500 year flood all right based upon this distribution will be between this and this value that will be a more accurate assessment or prediction or estimation.

(Refer Slide Time: 20:09)

In such cases, we decide a cut-off value & any flood magnitude higher than the cut-off is included in frequency analysis. The resulting data series is called "partial-duration series".

$$T_p = \frac{1}{\ln T_A - \ln(T_p)} = \ln\left(\frac{T_A - 1}{T_p}\right)$$

T_A → return period corresponding to annual series.
 T_p → return period corresponding to partial series. (continues)

CONFIDENCE LIMITS/BANDS:
 are defined as the upper & lower bounds of a statistical estimate for a given confidence level. The bounds can be described as follows:

$$U_{T,\alpha} = \bar{y} + S_y K_{T,\alpha}$$

$$L_{T,\alpha} = \bar{y} - S_y K_{T,\alpha}$$

where $K_{T,\alpha} = \frac{K_T + \sqrt{K_T^2 - ab}}{a}$
 K_T → freq. factor

$$a = \frac{Z_\alpha}{2(n-1)} \quad \& \quad b = \frac{K_T^2 - Z_\alpha^2}{n}$$

Z_α = standard normal variable with cumulative probability α

So this is called the concept of confidence band or prescribing confidence limits. So let us look at that the confidence limits are defined as the upper as I said and lower bounds of a statistical estimate for a given confidence level. We have already seen what we mean by the confidence level and the significance level all right. One is the complement of the other so we say the confidence level is 90 percent or 95 percent the corresponding significance level will be 100 minus that. So we will be able to say that the flood magnitude for the 500 year return period will be between this lower bound and that lower bound with that much confidence with 90 percent confidence.

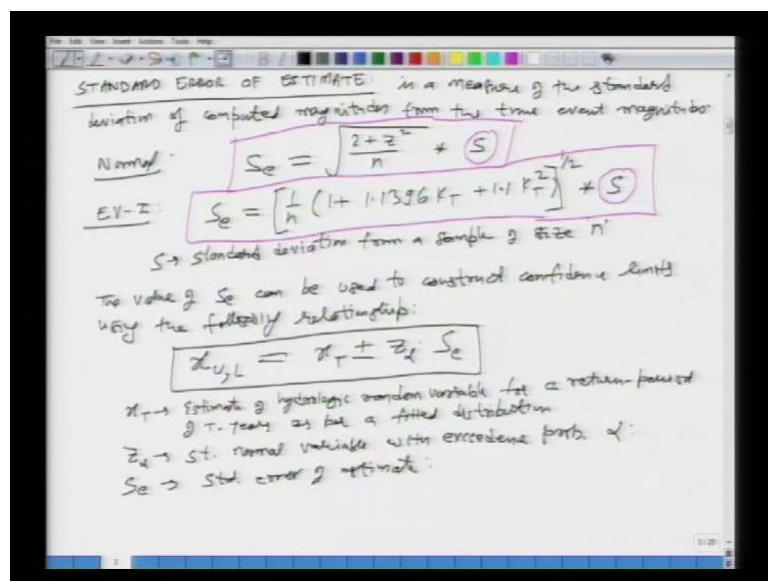
How do we do that? How do we actually determine those limits? That depends upon the distribution the statistical properties of the data and significance level and so on. So let us look at that the upper and lower limits of the confidence interval are defined as follows. The bounds can be described or calculated as follows or using following equations. So this is your upper bound U for upper bound, T is the return period corresponding to alpha is your let us say significance level all right or confidence level this will be equal to what we say \bar{y} plus S of y times what? Times $K_{T,\alpha}$ corresponding to upper all right. So what is this? This number is your frequency factor now this frequency factor is now depending upon the return period the significance level and U is just for saying that this is the upper bound distinguishing.

Similarly, the lower bound will be equal to what? Will be \bar{Y} minus all right same thing S_y times K_T alpha times L where, here I can say where your this bounds are given as follows K_T alpha is equal to K_T all right which we have already seen plus square root of K_T square minus a divided by a and what I am going to do is I am going to put a minus here it is plus and minus. So it will be corresponding to what let me say let me put the upper and lower both of them. So for lower you will take minus, for upper you will take plus all right. I do not want to repeat the equation twice.

Now again we are throwing two parameters here a and b which are given as follows a is equals to Z of alpha squared over 2 times n minus 1 . where n is the number of data points of course and b is another parameter which is K_T square minus Z of alpha squared over L . It should be clear that this K_T is what is nothing but, your frequency factor all right which we have seen already it will depend upon the kind of distribution we are fitting all right. So a we have seen that already and this another important parameter we have thrown here which is called Z of alpha any ideas what is it? This is nothing but, a standard normal variable.

This is a standard normal variable with exceedance probability exceedance probability of what? Of alpha. So Z is nothing but, your standard normal variable with significance level or this exceedance property of alpha.

(Refer Slide Time: 25:45)



So this way you see you using these equations we can find out the upper limit and the lower limit for any type of distribution we know the $K T$, we can find out a and b , we can find out $Z T$ the standard normal variable for any value and then for that we can find out what will be the upper and lower bounds. This another way of expressing this will come back to this a little later like to define another statistical error or some way of quantifying how good or bad our estimate is remember for any mathematical module whether it is statistical or deterministic. We want to find out how good or bad the module is right. So we define what is called the standard error of estimate all right it is a statistical parameter we always calculate.

So let me give you that standard error of estimate any estimate we make using Gumbel distribution or log Pearson or any different distribution it will have certain error associated with that. So we are just trying to quantify that what is it? It is a measure of the standard deviation it is a measure of the standard deviation of what computed magnitudes from the or the from the true value or the true event magnitude. What is a standard error of estimate? It is nothing but, it is a measure of the standard deviation all right. What is standard deviation of the difference between basically the observed value and the computed value all right you take that difference you calculate it for the whole data set and then you calculate the standard deviation of that. It is a measure of that kind of variation in the data in the residual series how is it defined? It is like this for the normal distribution or normally this is what we have S_e is square root of twice plus of your Z square over n all right times your S .

For extreme value type one distribution which we have seen this standard error of estimate S_e is defined as this particular expression $\frac{1}{n} \times (1 + 1.1396 \text{ of your } K T \text{ plus } 1.1 \text{ of your } K T \text{ squared})$ this whole raise to the power half that is everything is in square root actually including $\frac{1}{n}$. So let me put another bracket here $\frac{1}{n}$ is also in the square root and then this thing is multiplied by S where is what is S ? S is nothing but, the standard deviation we have said standard deviation from a sample of your data on which we are working of size n .

So you see that we can calculate the standard deviation all right of the sample from the sample like this and then we can find out S_e depending upon which distribution it is and then we have said that there are two distributions for which we have written this equation all right. So if it normal distribution we are fitting this is the value of standard error of

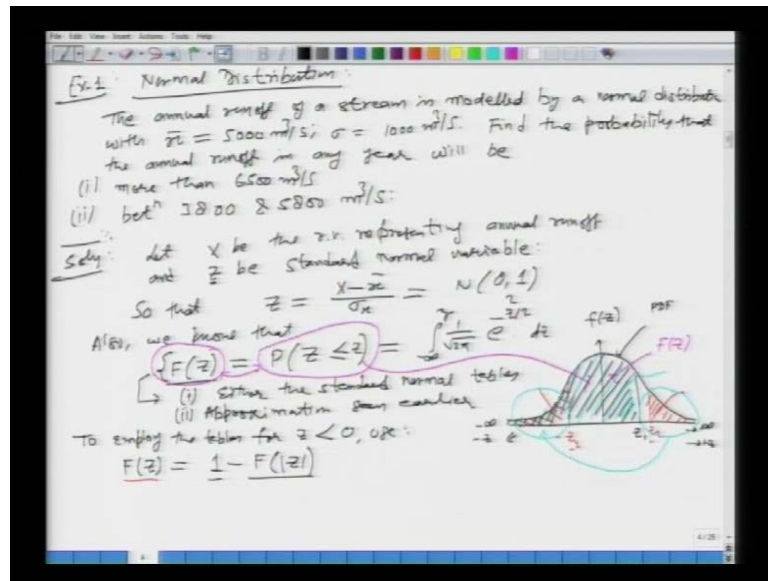
estimate and for the log this $e v t$. So we can calculate this and then there is as I said another way of expressing the confidence limits in terms of the standard error of estimate. So basically what you have is you are making or you are predicting an estimate all right and then you are saying that I know how much is the error associated with that estimate or that you know prediction. So what you do is you define a band around it using the standard normal variable and this is how it is done.

The value of $S e$ can be used to do what, to construct confidence limits, how? Using the following equation what is it? It is the your v value or the upper or the lower limits I am going to put either U or L . U would be corresponding to the upper limit of your confidence band and L will correspond to the lower limit is equal to what? Is equals to $x T$ which we have predicted using some distribution plus minus Z of alpha times $S e$.

And where $S e$ is what we have just seen how we can determine that all right. So $x T$ is the estimate of the hydrologic random variable for a return period of T years how as per a fitted distribution. It may be log Pearson type 3, it may be Gambel or normal or anything all right. So this we know how to determine all right once we have that we can use this equation and Z of alpha we have already defined what is it? It is your standard normal variant where exceedent probability alpha right all right. So we are specifying this confidence limit corresponding to this certain significance level or the exceedance probability alpha $S e$ we have already defined is your standard error of estimate. I have given you two equations corresponding to two different distributions but, for others we can find them in the books.

So we see that we have seen in this chapter the concepts basically associated with the hydrologic frequency analysis all right and we have looked at a couple of distributions mainly the Gambel distribution and the log Pearson type 3 distribution which is very commonly used in hydrology, in addition we also saw the binomial distribution and in the early part of this chapter and where in we looked at that this is a very special type of descript probability distribution that gives a very special type of probability.

(Refer Slide Time: 33:50)



Now what I would like to do is I would like to take up a couple of examples all right numerical examples. One is on the normal distribution and the other is on the application of this hydrologic flood frequency analysis. So let us take this example one on the normal distribution. So what I would like to do is I will state this problem first like we have been doing and then we will come back and look at the solution the annual runoff. The annual runoff of a stream is modelled by a normal distribution it is given to us with the mean and standard deviation given to us. I will just give it to you directly with \bar{X} of 5000 meter cube per second and standard deviation of 1000 meter cube per second. So these data are given to us if they are not given we can calculate them. What we have to find in this example is find the probability find the probability that the annual runoff in any year will be number 1 more than 6500 meter cube per second that is part one and the second part is will be between 3800 and 5800 meter cube per second. So you see that actually this example is not on the flood frequency or the frequency analysis. This is basically on the probability distribution normal distribution I have taken and we will see that how the normal distribution can be applied in the hydrologic analysis.

So let us see how we can solve this problem in which we have to find this probability of two different kinds in which one is you have to find the probability that the annual runoff will be more than a certain value which is given as a 6500 meter cube per second and in the other case we have to find the probability that the annual runoff will be between two

different values and it is given that it follows the normal distribution. So let us look at that.

So we say that let capital X be the random variable representing, representing what? The annual runoff in this case all right and let Z be the standard what? Normal variable remember whenever we are applying the normal distribution we are dealing in terms of this Z which is the standard normal variable. So that your Z is equal to what? Z is equals to nothing but, your all the X values minus the mean divided by the standard deviation of your X series and this we say is normally distributed with the mean of 0 and a variance of 1. So when we transform any data series using the standard normal variable this will get transformed such that its mean will be 0 and the variance will be equal to 1.

Right also in this case when we apply the normal distribution we know that what? That the cumulative distribution function all right capital F corresponding to any value of Z or the random variable capital Z actually I should say is what from the basic laws of probability or the definition of your p d f and c d f we had said that this is the probability what of your random variable lying or being less than or equal to what? This is a small z all right I want to write this is given as your integral of minus infinity to this z of what? Of your equation of your normal p d f right all right. What we do is this whole thing all right is available in two different forms right one is what? Either these standard tables how can we determine this integral or the cumulative distribution function value corresponding to certain values at we can read it of from the standard normal tables that is number one.

What is the other one we have seen in this course how can we determine the cumulative distribution function value of your normal distribution we have seen some approximate equations some you know which had lot of this constants all right. So either we can use the standard normal tables or we can use those equations in a computer program right which will give you a pretty accurate value of the capital F of z all right.

So the second one is the approximation. Approximation seen earlier or given in the book in this course in this particular chapter. So if we look at the standard normal variable or standard normal curve we know that this is symmetrical and the this is your random variable let us say z this is your plus z this side and this is minus z . So this is standard means it is applied on z the transformed series this is symmetrical so it will have the

same shape on both the side it may not look very symmetrical the way I have drawn but, then you understand what it is.

What is the vertical X is representing this is obviously representing your small f of z small f is your $p d f$. So this is what? This is nothing but, your $p d f$ that is how actually it looks I am sure you may have seen all of that. This goes all the way to plus infinity and this one goes to minus infinity all right so this normal distribution is defined from minus infinity to plus infinity.

Then let us say you have any value small z so for that small z what is this whole area, area under the curve such that your value of the random variable is less than or equal to z right which is equal to what? This thing all right or which is equal to this thing. So this is equals to that and all of this is equal to what is equal to this area under this curve all right which is nothing but, you can say that it is your capital F of z all right. So graphically that is what the these probabilities represent that is capital F of z is what is nothing but, the area under the normal $p d f$ up to an including that value small z .

Now, normally when you look at the standard normal tables they are defined only between for some positive values of z if you have notice its only for between 0 to infinity or 0 to some large value of x . It is not defined for the negative values of z why? Because it is symmetrical so what we do is we exploit the property of symmetry of the normal distribution to do what to calculate the cumulative distribution or capital F of z corresponding to z values which are negative and so this is how we do it to employ employ the tables for z values which are less than 0 that is which are negative what we do is we use this concept that is all.

Your F of z is equals to what 1 minus of capital F of this absolute of z do you see that? If you come to this figure what we are saying is that lets say this is what this is some value of z which is minus right. What is F of z corresponding to this value which is negative all right is this area is not it and what we are saying is that this area is nothing but, 1 minus of that particular area all right corresponding to that so if you take plus value of Z let us say that plus value of let me say that this was Z_1 and let me say that this is Z_2 .

So you take a Z_2 here which is equal to this all right and then you take the area on the other tail all right. So this area will be equal to this area all right and now what you are doing is you are saying this F of z is one minus f for the positive value of z all right. So

the one under the whole curve is 1. So if you take 1 minus of this it will be everything which is this one all right. 1 minus of that is basically let me use another colour here it will be this area. So if you take 1 minus of this whole area in the blue will give you equal to this area and this is equal to this area all right. So you see that we exploit this property of normal distribution being negative to find the value of the cumulative distribution when it is negative.

(Refer Slide Time: 45:49)

$$1) P\{X \geq 6500 \text{ m}^3/\text{s}\}; z = \frac{6500 - 5000}{1000} = 1.5$$

$$\therefore P(X \geq 6500) = P(Z \geq 1.5) = 1 - P(Z \leq 1.5) = 1 - F(z)$$

$$= 1 - F(z=1.5) \rightarrow \text{Read out from std. normal table.}$$

$$= 1 - 0.9332 = 0.0668$$

$$\Rightarrow P\{X \geq 6500 \text{ m}^3/\text{s}\} = 0.0668 \approx 6.7\%$$

$$2) P\{5000 \leq X \leq 5800\}; z_1 = -1.2; z_2 = 0.8$$

$$= P\{z_1 \leq Z \leq z_2\} = F(z_2) - F(z_1) = F(0.8) - [1 - F(z=1.2)]$$

$$= 0.7891 - [1 - 0.8849]$$

$$= 0.673$$

$$\Rightarrow P\{5000 \leq X \leq 5800\} = 0.673 \quad \text{Ans.}$$

So now let us come to this particular example problem in which number one we have to find one probability which is your x greater than or equal to 6500 of meter cube per second. So what we can do is we can find out the value of z corresponding to this, this will be equal to what? The value of x minus the x bar is 5000 divided by the standard deviation which is given to us as 1000 right. So what is the standard normal variable it is 1.5.

So that now we can say that P of your X greater than equal to 6500 is equal to the probability of your Z being greater than equal to 1.5 it is the same thing right and what would that be equal to that will be 1 minus of P of Z less than equal to 1.5. What is this? Based upon our definition of the probabilities what is the probability of z being less than equal to z it is nothing but, cumulative distribution function all right.

So this you say is 1 minus F of z . So this is equal to 1 minus F of z is equal to 1.5. Now what we do is this value we read out from the standard normal tables which give us

what? It gives you the value of capital F corresponding to different values of positive values of Z.

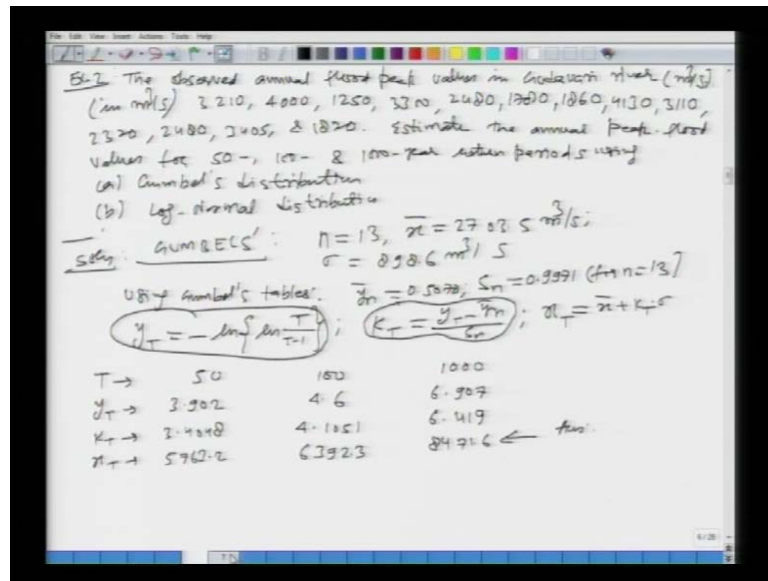
Right so you just do that or you can use the approximate equations I am not going to do that. So if you read it of it will be 1 minus 0.9332 and which will be equal to 0.0668 that is to say your answer for this first part is the probability of your annual runoff being greater than or equal to 6500 meter cube per second is equal to what as per the normal distribution it will be 0. of your 0668 or which is approximately 6.7 percent.

Similarly, we can work on the second part in which what we have to do is we have to find the probability that the value of x will be between two values. One is 3800 other is 5800. So the first thing we have to do is we have to find out the values of corresponding z's for two different values of x's right. So if you did that you will have z 1 corresponding to 3800 will be minus 1.2 because it is less and similarly, your z 2 if you determine it will be 0.8.

So what is this probability equal to this is nothing but, the probability of your z in what range minus 1.2 and 0.8. So these two are equal all right what is this equal to? Well it is the difference in the cumulative distribution function values the higher one which is 0.8 minus F of z is equal to what? minus 1.2 remember this is what we have defined in the beginning of this chapter where the probability of a random variable falling between two different values is given by this difference the upper value and the minus value for the c d f. Now this is negative here so we use that particular property for the negative value of z so it will be F corresponding to 0.8 minus in the brackets what? 1 minus of for z is equal to what? plus 1.2 absolute of z means what 1.2.

So if you read those values out from the standard normal tables it would be equal to what? It will be equal to 0.7881 what is 7.8 this one this is actually equal to this one all right and then you have minus of 1 minus what 0.8849 where in this number is nothing but, this one and this is equal to this one and this one you can read out from standard normal table. So please do that exercise on your own and if you simplified that it will be 0. of your 673. So that you can say that the probability of your x being between 3800 and 5800 this will be equal to what? It will be 0.673 or 67.3 percent.

(Refer Slide Time: 52:12)



So let us move on what I would like to do is I will look at a another example which is on the hydrologic frequency analysis in which the data in terms of the annual flood peaks are given us and we have to fit the two distributions. So let us look at the second one which says that the observed annual peak flood values annual flood peak values in Godavari river.

These are just the data actually I have just made up so may not be necessarily for that river but, this is for demonstration purposes I am saying that are these numbers which are given in meter cube per second to us are 3210, 4000, 1250, 3300, 2480, 1780, 1860, 4130, 3110, 2320, 2480, 3405 and 1820. What we have to do is estimate the annual peak flood estimate the annual peak flood values for different return periods which is 5100 and 1000 year return periods using a Gumbel's distribution and b using log normal distribution. So let us see how we are going to work on that first we will take up the Gumbel's case.

So given the data what is the first step basically you find out we have n is equal to 13 that is 13 different values are given to us we find out the average of this data which is going to be 2703.5 meter cube per second you can verify that and also the sigma which is 898.6 meter cube per second you can calculate that easily.

Now using the Gumbel's tables we find out what is called the y_n and s_n , for n is equal to 13. So there is a table in which for different values of n these numbers are given

these are the Gumbel's parameters. So you will see that it will be 0.5070 and s_n would be what 0.9971. So you can just read it out from the table now we write all these equations which we are going to use what is Y_T ? Y_T is the minus of your natural log of T over T minus 1 this is our one equation.

We need to calculate K_T for Gumbel which will be what Y_T minus Y_n bar over s_n . Y_n bar and s_n for this we have already determine or estimate a and then x_T is equal to x bar plus K_T time sigma. So what I have actually done is I have written down all these equations which we can use to calculate the magnitude of the annual flood peak corresponding to a particular return period all right.

So we calculate the Y_T depending upon Y_T we find out K_T for using that K_T we find out x_T which is the magnitude and we can organise our calculations in a tabular form like this you have T which is 5100 and 1000. So you can calculate given the T using this equation just put value of T three different values you can calculate Y_T right.

So it will be 3.902 it will be 4.6 400 and 6.907 for 1000 similarly, your K_T is going to be what? From this equation once you know Y_T Y_n bar and s_n is known to us we just use this equation. So it will be 3.4048, it will be 4.1051 and it will be 6.419 you can verify some of these numbers and then what is going to be x_T which is your final answer actually it will be 5763.2 these are in meter cube per second right 6392.3 and this is going to be 8471.6 so this row is your answer for the Gumbel's distribution.

(Refer Slide Time: 58:23)

Handwritten notes on a whiteboard showing the transformation of data to a normal distribution and a table of calculated values for different return periods (T).

Notes:

- for NORMAL: Transform the data
- $Z = \frac{Y - \bar{Y}}{\sigma}$
- $\bar{Z} = 2.4078$; $\sigma_Z = 0.1556$; $\bar{Y} = 0.0$
- $Z_T = \bar{Z} + K_T \sigma_Z$; $x_T \rightarrow$ Antilog (Z_T)

T	50	100	1000
K_T	2.054	2.326	3.090
Z_T	3.7223	3.7696	3.8084
x_T	5337	5003	7735 ←

If you see for the log normal what we have to do actually is transform the data right that is Z is equal to log to the base 10 of your x and for that you find out Z bar which will be 3.4078. So all the data 13 data that are given to the to you, you just take the log and find out Z bar which will be this sigma Z will be 0.1556 and c_s actually we are not going to calculate we will just say because we are fitting log normal will take this as 0. So what is your hydrologic frequency equation?

It is same Z bar plus K_T times sigma Z and your x_T will be what antilog of Z_T same thing this is to be read out from the table depending upon the return period all right then all you do is you have the log Pearson type 3 distribution table in that table you will read out values corresponding to c_s is equal to 0 all right. So same thing you do you arrange your calculations in table like this.

You read out K_T for c_s is equal to 0 it will be these values and Z_T will be from this equation you can find out it will be 3.7273, 3.7696 and it will be 3.88 of your 84 take the antilog of these numbers that will be your final answer which will be 5337, 5883 and 7735 so this is your answer.

So you see that implementing these probability distributions is a fairly easy task one we understand the equations. So we have seen in this example you know I have rather hurried up because I am running out of time but, looking at these numbers you can you know calculate all this things on your own and verify how you get these answers. So with this we come to the close of this chapter in fact and this module on stochastic hydrology. What we will do in the next class we is we are going to start the ground water hydrology module all right. So I would like to stop at this point of time and we will take up the ground water hydrology in the next class.

Thank you.