Advanced Hydrology
Prof. Dr Ashu Jain
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 29

Good morning and welcome to this post graduate video course on advanced hydrology. In the last few lectures, we looked at the chapter on unit hydrograph. And in just concluded class, yesterday we looked at the methods of determination of unit a hydrograph of a different duration, then the one which is given to us. For that we looked at 2 methods. First one was what is what is called method of super position in which the desired duration is the integer multiple of the given duration.

The second method involves the use of esker. And then we also used or looked at 2 different examples, numerical examples of these 2 methods. So, with that we will move on and today we are going to start a new topic or a new module in this course which is on stochastic hydrology. In this module we will basically looked at we will study we will look at 2 chapters which are given in the Ven Te Chow's book. One is the chapter 11 which is on the hydrology statistic and the next is the chapter 12.

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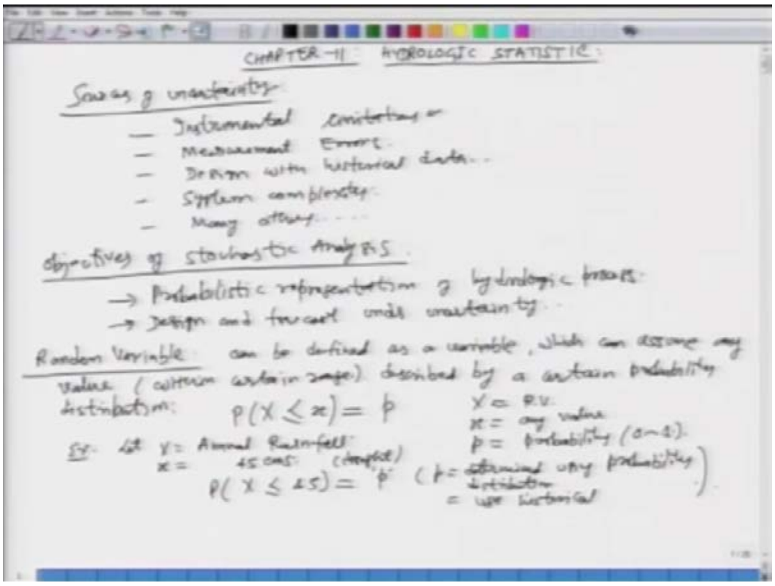

So, what we will do is we will get started with the chapter 11 today from your Ven Te Chow's book and the title is hydrologic statistic. If I ask you what is going to be rainfall tomorrow what will be your answer? Well you can say that the rainfall tomorrow may be 1.2 centimetres or 0.2 you know centimetres or few millimetres or you can say that there may not be any rain tomorrow. But can you say that can you make that state statement with 100 percent confidence. No, we can never make any such statement about hydrologic variable with 100 percent confidence or 100 percent probability. Why because most of the hydrologic variables are uncertain in nature. There are lots of a sources of error while we are analysing we are measuring the data hydrologic data. That is that is why we need to consider the uncertainty or we need to incorporate the uncertainty into our analyses.

So, what we are going to do in this chapter is we will look at some of the methods which are used to analyse or incorporate this uncertainty into our analyses. And how we can fit a probability distribution? How we can use the loss of probability and statistic to make judies decision under uncertainty? So, if we get started there are many sources of uncertainty, but we will not look at all of them just mention a few. There maybe some instrumental limitations. There are some instrumental errors maybe involve while measuring the values of rainfall and follow etcetera. Another is the measurement error themselves, first one was the errors involves with the instrument. The second one is the measurement error that is different people may read a different reading are depending upon you know different times or subjectivity maybe involve with that. Then we are doing a design hydrologic design with histological racers or histological data which maybe ironies.

So, the design is based on the data which may contain error or uncertainty so over the design as to be a reliable. There maybe errors in your analyses due to system complexity what do you mean by that our system maybe extremely complex? But we may be substituting that or assuming a simpler approach. So, that may be a source error in your analyses. And then you know as I said I am not going to list the remaining ones, many others. Whatever is the reason of the sources of errors or uncertainty in our data hydrological variables, our objective is what our objective is to somehow incorporate this uncertainty in the hydrological process which we are modelling or representing.

So, what are the objectives? The first one as I said is, let me look at that the objectives of stochastic analyses, the mainly 2 objectives. And the first one is the probabilistic representation of hydrologic process. For example, rainfall is a hydrologic process. It is not a deterministic system. There are lots of complexity there may be a errors in the data and so on. So, what we do is we try to represent this process or any hydrological process using a probabilistic approach. When we say probabilistic approach or probabilistic representation of your system what do me mean by that? As I have just said at the beginning of this a lecture today that what will be the rainfall tomorrow. The answer cannot be 2.3 centimetres with 100 percent accuracy. What we may be able to say is that there is a 50 percent chance that the rainfall tomorrow will be less than or more than certain number. Or you can say that there is the 60 percent chance that it will rain tomorrow, or there is a 90 percent chance that it will not rain tomorrow. So, you can make this kind of statements in which you are associating certain probability some factor between 0 and 1 or 0 to 100 percent.

So, this is called the probabilistic representation of your hydrological process. So, this is 1 of the objectives and there are various ways of doing that we cannot just put the number you know to the rainfall tomorrow. It has to be properly analysed, it has to be properly deduce using the laws of probability and the historical data. So, we will come to that a later in this chapter, what is the second objective? What is the purpose of carrying out these hydrological analyses of your variables under uncertainty? Well all this analyses is going to lead us to either some kind design we want to design. Let us say a dam or a bridge for which we want to know what is the magnitude of the flood or the discharge that is accepted in the river. Because your design as to be safe for the worst case in scenario. If you make some error in the estimation of the magnitude of the discharge or the peak follow for which you are designing your hydraulic structure, then the as you can accept as a as you can you know understand the results can be or the consequence can be catastrophic. If you make you know 10 percent 50 percent.

You know 20 percent error in the estimation of the peak discharge you know you cannot be actually effort that. So, design is 1 important thing hydrological design or hydraulic designs are in which the knowledge of probability or stochastic analyses is required so that we can develop what is called a relievable design. What is the other one? Well we have many this water resources system projects in which we need to operate the system or the water system under uncertainty. For example, flood management and rout management. You want to go through the process of this you know this extremes or your floods and routs. But you want to be able to forecast these variables that are your discharge rainfall etcetera well in advance. If you can say that next month this is going to be rainfall and you can be accurate then you can conserve the water today. And then you can go through the rout situation in a better manner. So, being able to forecast accurately is also required in which the knowledge of statistic is essential. So, these are some of the reason. So, for the second 1 I will say design and forecast under uncertainty and there can be many other application of stochastic analyses or stochastic hydrology we will not go into the detail of all of them.

So, with this little bit of background what I would like to do is start defining certain basic principles and concepts which we are going to need when we do this stochastic analyses or statistical analyses. Some of this concept or definitions are very simple you may have seen them earlier, but none the less they are very important. And we are going to just review them quickly. So, I am going to start with the definition of what is called a random variable. What is a random variable? As the name suggest is some physical variable which as uncertainty involved in it which is which is random in nature. That is you know as simple as that, but that is for a limit as hydrolyses what how we are going to define a random variable is that, it can defined as a variable how which can assume which can assume any value. And that value is within certain range. The random variable can assume any value or take up any value. But that as to be within certain range described by described by a certain probability distribution.

So, this is the definition of a random variable what is it can defined as a variable which can assume any value it can take up any value. For example, let us say we have rainfall, rainfall can take up any value tomorrow or next year depending upon the time frame between let us say 0 minimum value is 0 maximum theoretically infinity, but maximum can be 100 centimetres or something. So it is a variable which is uncertain in nature and it can take any value within the specified range. But the important condition is that, the value which it will take is described by the certain probability distribution the beauty or the good factor or the advantage about defending the random variable is that; that the random variable assumes a value, however it is described by certain law although it is random but it follows certain probability distribution. And there are many kinds of probability distribution we will see in this chapter or in this module different variables will follow different hydrologic probability distribution.

So, this is the definition and the mathematical representation of a random variable is like this. You have probability of your random variable x having a value less than or equal 2, small x is equal to small p where your x is the random variable small x is any value. For example, if capital X is your rainfall or the annual rainfall and small x can be let us say 100 centimetres. Then let me first say that your P is equal to the probability which will be described by certain distribution and this is between 0 and 1. So, for example, if you take the case of the annual rainfall for example let you capital X be annual rainfall.

So your random variable is annual rainfall what is going to be the annual rainfall next year or what is going to be peck discharge next year? So, that will relate to what kind of you know droughts situation maybe in what kind of flood situation maybe in? So, these are the kind of decision we can take. So, if you say that x is equal to annual rainfall and let us say x you want to define if you are let us say in the if you want to analyse the drought condition And then you are looking at let us say if the rainfall annual rainfall is less than or equal to 45 centimetres then you say we are in trouble as for as the drought is constant.

So, we can define this kind of thing or these kinds of event are depending upon our need. So, for this case it is going to be the random variable will be or can be define as your x less than equal to 45 is equal to some value of P. And this P can be determine using some probability distribution which the annual rainfall follows in that area it can be determined using that for which we are we use the historical data. So, you take the historical data you fit a certain probability distribution. So, this is the definition of what is called the random variable.

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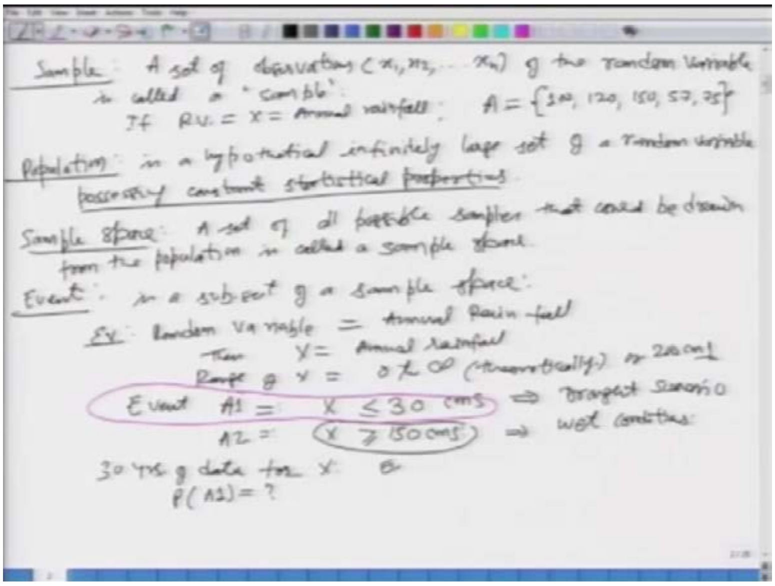

Moving on we defined what is called a sample, what is a sample? Well it is nothing but a set of observations and let us say that this x 1 x 2 up to x n of the random variable of the random variable is called a sample. So, what is a sample if your random variable as I said in the previous page is your annual rainfall? Then let us say see your set A is the values of the annual rainfall in the past 3 4 5 years 10 years 20 years. Let say we have 20 years data of rainfall data for example, here if I say that the annual rainfall for the last 5 year was 1 100 120 150 57. And let us say 75, what is this? This is a set of observation this is the annual rainfall observed last 5 year.

So a sample is nothing but you know collection of the data for that particular type of event or the random variable which we have define. So, this is a sample most of our probabilistic analyses will be carried out on the sample that we need to understand what we have is what is called population. Population is a hypostatical concept infinitely large set of a random variable possessing constant statistical properties, this is important possessing constant statistical properties as for as the population is concerned.

So, what is the different between the sample and the population sample? We said it is the just collection of data points of from your universal satellites 5 years of data or 10 years of data. But population is something high piratical the data set is extremely large infinite. But the condition is that it possesses what constant statistical properties. And the statistical proprieties we are talking about what is what is lived you know mean standard division quince. Those kind of things we will come to these later to in this chapter, but if you have a population the mean of the population is always going to be constant which is not a case for a sample.

If you take a sample of 20 yard you know 20 data points, it will have certain mean standard division and so on. You take another sample of the same hydrology variable let us say annual rainfall in the same area you have a 20 years data here another 20 data years there means and standard divisions and other statistical properties will not necessarily be the same. So, sample is the very small set and population is infinitely large but the statistical properties are constant. Let us move on then we have what is called as sample space, what is the sample space? It is a set of all possible samples, set of all possible samples that could be drawn, that could be drawn from the population is called the sample space. What is the sample space? It is a set of all possible samples that can be drawn a sample is the collection of the observation of the data.

So, it is the set and sample space is what let us say you can draw 10 different sets or 10 different samples. So, that whole space actually be from which you are drawing this samples is called the sample space. Next thing is what is called and event, what is an event? Event is a subset of a sample space; an event is a subset of a sample space. This event is something which we can define whenever we are dealing with the hydrological variables of data. And event is something which we can define depending upon our need or the practical use. This is very important to understand.

How I will take up you know maybe one example which will demonstrate we can define an event. For example, let us say you are analysing the flood situation when you want to manage your flood situation. You want to define an event in such manner that you are able to define the flood a you know in a certain manner. For example, you have the annual peck discharge you know data that is what we want to analyse. And then you say that the flood is defined such a manner that or an event is define event is equal to flood for example. So, the event is suppose to have accrued when the peak discharge in a year is greater than or equals to certain the magnitude.

Similarly, how would you like to define an event when you are dealing in a drought situation, you would say that the annual rainfall is less then this particular value if that is case we are into a drought situation. It can be a teared system also for the drought for the drought case for example, you can say event A is what if the annual rainfall is between this and this limit than you have moderate drought situation. If the annual rainfall is between this and this limit then you have severe droughts situation and so on. So, an event is something we can define based on our need. So, for example, if you are random variable is annual rainfall. Then you say x is equal to annual rainfall. That is your random variable.

What is it is range? Well range of x let us say is between 0 to some maximum value theoretically it is infinity, but practically it would be the maximum value which has been observed which maybe annual value of 2100 centimetre or something or something like that. Then you may want to define an event A is such that this annual rainfall x is less then equal to 30 centimetres. Means what are what is our emprises? We are trying to look at drought situation then you say let me say that this is A 1, then you define another event A 2 such that your x is greater than equal to 1 100 50 centimetre means what you are dealing with the wet conditions or maybe flooding. So, this x which is the annual rainfall you may have do rainfall runoff modelling. And determine what will be the value of follow and the peak discharge. And then put the limits on the or define the event in terms of the peak discharge.

So, this way we see that we can define an event the way we want depending upon our need. And then we can analyses the data the historical data we have and determine all kinds of probabilities. For example, let us say you have let us say 30 years of data with you for x means annual rainfall. And your event you have defined as this one. Let us say this one how can you find the probability of the event A 1, occurring probability of A 1 equal to what how will you find this out? If you have 30 years of data for annual rainfall think about it I will give the answer little later. It is the simple concept of probability you can just count the number of years in which the rainfall was less than 30.

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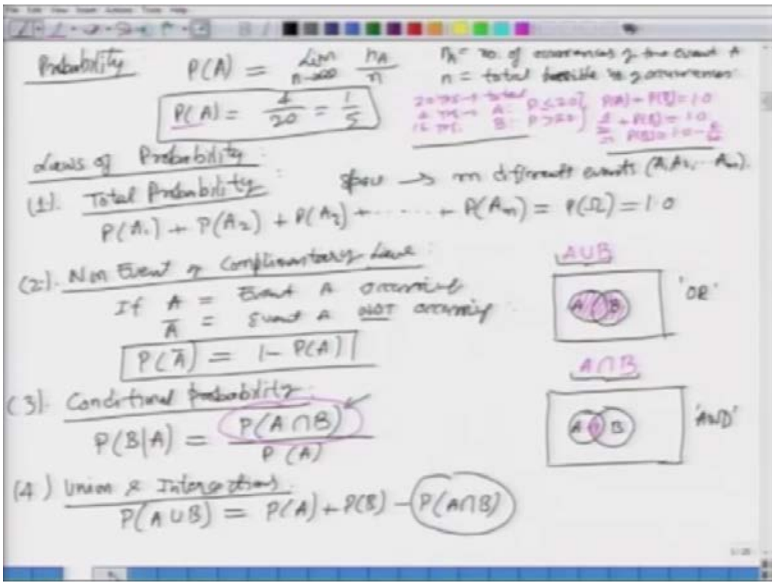

So, let us move on and now we defined what is the called the probability. What is a probability of occurrence of a event A. So, we have defined you see the sample the population, the sample space, the event. Now, we are defining probability of occurrence of an event that is P of A is defined as under the limits as n stands to infinity of what n A over n let me define this things what is N subscript A is what? It is number of occurrences of the event A and this N is what with total possible number of occurrences of your event A of course.

So, in the previous example let us say we had 20 years of data, and out of those 20 years of data. In 4 of them the annual rainfall was less than 30 centimetres. So, what will be the probability of event A occurring that is what is your probability that the annual rainfall is going to be less than or equal to be 30. So, it will be nothing but 4 divided by 20 or it is 1 over 5. So, you can get that using your data very easily. So, you see that we can analyse our historical data and then we can define the event Based on our needs. And then we can find out various probabilities by looking at the data.

Now, a simple definition of probability will give you this kind of information which we have just looked at. However there may be many events which may be interacting with each other. For example, you may have to define an event A event B event C. And then event A B C maybe interacting with each other or they may be independent, you may want to find out the relationship in between those events that is for that we need many different types of analyses. And there are what are called laws of probabilities. I am sure you have see this laws of probability, but we will look at some of this.

So, the next thing we are going to do is what is called the laws of probability there are many of them. And the first one which we will look at is called the law of total probability law of total probability. Let us say that the space the total space you can divided into m different events or you can draw m different events. And let us say these are A 1 A 2 A 3 all the way to A m. So, let us say you have a sample space in which you can draw m number of samples or m number of events can be defined. Then the law total probability states that we some of the probability of all this events A 1 A 2 A 3 is equal to what will be equal to 1 that is all. So, mathematical what we can say is that probability of the event A 1 occurring plus probability of the event A 2 occurring plus probability of the event A 3 occurring and so on. All the way to what plus probability of your event n A n occurring should be equal to what probability of your total sample space occurring that as to be equal to 1.0.

For example, in your earlier case let me go back up here you had 20 years of data, total 4 years you said what was the event A event A was your annual rainfall was less than equal to 20. So, if you could say that your event B was your rainfall value was more than 20. If that was your event B then how many years are there 16 years of data? If the 4 years of data had rainfall value less than 20 the remaining had to have a rainfall value more than 20 so that the total sample space which is your 20 could be divided into how many 2 so m value is here 2. So, you see that your P 1 P of your a plus P of your B has to be equal to be what has to be equal to 1 right as per the law of probability. Now, we have seen that P of A is what 1 by 5 or 4 by 20 so of this is 4 by 20 plus P of your B is equal to 1.

So, that will give you P of your B is equal to 1.0 minus 4 by 20 or you could say that it will be sixteen by 20 you have you would have know that already. So, you could verify either the law of total probability or knowing the law of probability total of law probability is true you can find out something that is unknown. So, that is where some of these law are useful to us number 2 is what is called the non event or law of complimentary or complementary law or complimentary probability. As it is clear if A is your event A occurring. And if you define another event A bar as event A not occurring.

Then your probability of complimentary event which is a bar is given as what 1 minus of your P A. This is what is called your law of non event or complimentary law whatever, we call it or law of compliment tarity. So, you have an event A occurring if you can find out the probability for event A occurring you always know what is the probability of the that event not occurring. For example, flood if you can say that the probability of occurrence of flood next year is 0.5. What is the probability of non occurrence of the flood 1 minus 0.5 that is also 50 percent? Another example is we just looked at if you come here in the pink this is also you can find out by as non occurrence, because occurrence on an event was annual rainfall value being less than equal to 20, and the non occurrence is what greater than 20.So, it is similar thing.

Now, the third one we are going to look at is what is called conditional probability or the law of what it is called conditional probability. As the name suggests when a when do we need to use the conditional probability. Well, when there is a condition about another event which has already occurred or not occurred or something like that. So, conditional probabilities are useful when you are dealing with more than one event such that one event will depend on the other. So, that we are defining a conditional probability of occurrence of a event given another event As a accrued. So, let me first define it then will maybe discuss about the physical nature of this.

So, this is denoted as P or the probability of event B given A, what is the condition probability? Probability of an event B occurring given A as occurred is equal to what as per the law of this condition probability? It is given as the probability of A intersection B divided by the probability of A. This is what is called the law of conditional probability. Now, this involves a something which we are not discussing. What is this P or probability of A intersection B. This we are going to define next which is 4 your union and intersections. So, what are the unions and intersection we will look at this concept? Let me first give you the relationship about this. So, if you have events A and B. See if you look here in the conditional probability you see this thing in the numerator we have A intersection B. So, you have 2 events A and B and we want to find out there unions and intersections. So, the union is defined as A union B is what as per the laws of this a unions and intersection it is defined has probability of A plus probability of B minus your probability of A intersection B. I am sure you know all this concepts.

You may have seen in your earlier classes a set theory and union intersection, but let me quickly give you this let us say this is your over all sample space in which you have 2 events. this is A and this is B. So, what is the A union B so you have event A occurring you have A event B occurring when can you say that the both the events A and B are occurring and when can you say either A or B is occurring depending upon that we define A union B and A intersection B. So, this is A union B means this whole thing.

So this whole area is how you defined as the A union B and which probability is given by this. How about A intersection B. Intersection is basically end in which things are common to both the events. So, this is your let us say this where your A and this is your B, what it is your intersection B? And this is your A union B. So, when you have this 2 events the 2 events are related with and or operator. So, when you say A union B you are saying the probability of occurrence of this or that right event A or event B that is A union B. And when you say intersection what you saying is that it is the space is common to both the events A and B. So, the 2 events are connected within and operator. So, we are dealing with you know mathematical operators. So, coming back here what is your A union B is given by this equation in which then disappears, how do we define a intersection mathematical knowing the probability of individual events?

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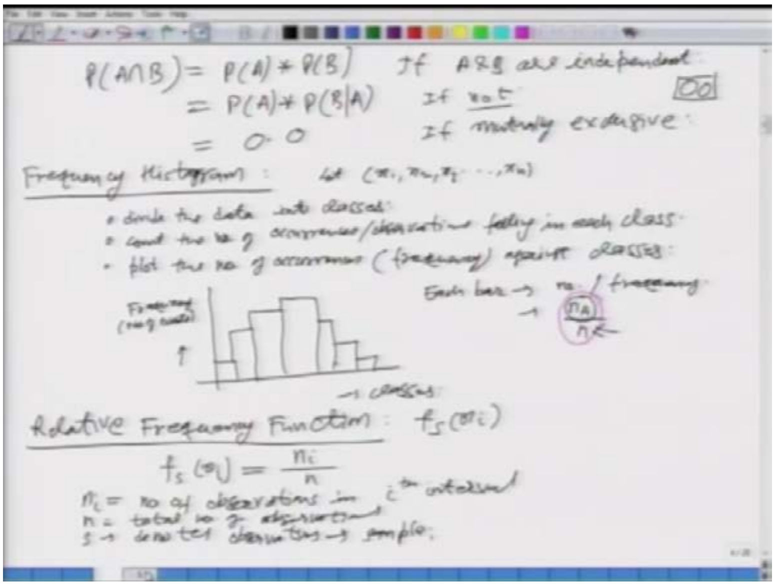

This is define like this is equal to probability of event A multiplied by the probability of event B when or if A and B are independent. So, when the 2 events A and B or the occurrence of events a dose not depend on the occurrence of event B. Then your A intersection B is given by the product of the individual probabilities P A times P B. That is what we are saying. And this is equal to your P of your A multiplied by what P of your B given A that is your conditional probability if not. You see that your this situation. The second one is same as the law of your conditional probability you see this P B given A is P A intersection B divided by P of A. It is the same equation we have written there. So, P A intersection B is equal to what it is P A multiplied by P of your B given A if there are dependent on each other or if the occurrence will dependent on the other one. However, if they are independent then you just say A intersection B is just the just multiplication of the 2 probabilities. What about if they are mutually exclusive they are mutually exclusive means what they are just separate means you have this space they are separate completely separate they have nothing common.

So, when you have 2 events in which there is nothing common they are called mutually exclusive. So, A intersection B is going to be equal to what A intersection B is the actually defining the common space and the common space here is what 0. So, it is going to be your 0. So, this way we see that there are basic laws of probability, we have seen the total law the conditional and the intersection and the unions which are quite useful in analysing certain situation in which we have more that 1 event. And there are interrelationships between them. So, from here we will move on and then look at the data representation and data analyses. And then we will start defining certain basic concepts and then we start analysing our hydrological data. What do you do with your data first thing when you are given in your statistic chapter in your earlier classes? The first thing we do is it what is called the frequency histogram or we analyse the data in terms of frequency of occurrence for each class or of is for each magnitude.

So, the next thing we are going to look at is what is called frequency histogram? What is done in this is you have the data let us say that that we have data set x 1, x 2, x 3 all the way to some x n. What we do is we divided the data into classes. Let us say you have a rainfall, annual rainfall minimum value is 0 or may be 10 centimetres maximum is let us say 200 centimetre. So, this is a range, this range we can divided into different classes that is 10 to 30, 30 to 50, 50 to 70, and so on. So, this are called different classes. So, first thing what we do is divided the data in to different classes then what do you do then you count the number of occurrences or observations right remember this following in each class.

So, what you do is you have the data you are taken the minimum or the maximum data you have divided that range into different classes. And then you manually just scan through the data and put them into different classes so that you will have the number of occurrence of your event or observation in each class. And then what you do is you just plot the number of occurrence which is called frequency actually against the classes. So, it will look like this, this is our classes which will actually represent the magnitude of your random variable. And this is your frequency or you know it could be in number in terms number of events or occurrences in that class. So, you would see that this is normally done in the form of a bar chat. I am sure you have done this in your earlier classes your each bar represents what either the number or the frequency or a fraction.

How do we define the frequency or a fraction it is basically the number of occurrences divided by the total number of occurrences. So, either you can plot this number n A as the length of this bar or you can plot this whole thing the fraction n A over it. So, it dependents, how you are representing your frequency histogram? So, this is very simple it basically a simple data analyses in which we get an idea of which class is occurring most frequently, what is the average value we get a lot of good information you know once we just plot this data or this frequency histogram.

Let us move on the next thing we will defined is what is called relative frequency function. This is f s of your x i where the f s of your x i is defined has what it is n i over n as I said earlier already there n i is what n i is the number of observations in ith interval or ith class. And is the total number of observations in your data said that is and this S is actually denotes what that these observations are from a sample after all we are working with the sample only. And then we can plot this. So, it basically are nothing but this one here. So, if you are plotting n a over n or n i over n it is called the relative frequency function, but if your potting plotting n A or the number occurrence it is called the frequency histogram. So there is a minute difference here.

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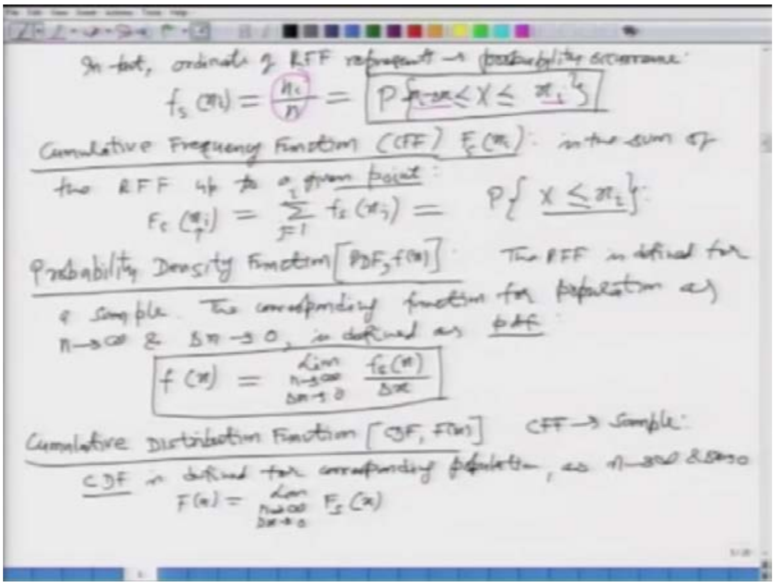

In fact, this the ordinate the ordinate of your relative frequency function which you just define represents what? It basically represent the probability of occurrence do you see that we had already defined this ratio which is n i over n. What is this under the limits when your a sample size is to large? This is the probability of occurrence of that event A what is that event A? It is the class i. So, we say that the probability of occurrence of your random variable such that it is value is between the lower class limit to the upper class limit that class value minimum or maximum of that class or the ith class is equal to what is n i over n.

So, this is actually than nothing but the mathematically we can define it as what it is the probability of your random variable having a value what between certain range and what is that range the upper limit is let us say x i. And if the interval the class interval is delta x the lower limit is going to be x i a minus delta x. Do you see that what is this probability this expression is saying what this expression is saying that? The probability of occurrence of your random variable x between this and this range which and which the lower limit this and the higher limit this is given by this guy here or this frication. So, this related frequency function actually gives you the information about the probability of occurrence of your random variable within a certain range which is a very useful and important information.

So, the class interval can be defined in a manner which is a useful for our water resource management. So, related to this relative frequency function we will define another a term which is called cumulative frequency function. And let us say this is CFF and this we denote has capital F subscript s of your x i as the name suggest it what it is the cumulative type of function? What is a cumulative function which is a cumulated if you integrate another function in this case that another function your RFF or the relative frequency function. So, the cumulative frequency function is nothing but the cumulative of you relative frequency functions. So, if you define it, it is nothing but is the sum of the relative frequency function I am not going to write it I will use this a abbreviation up to a given point, any given point or any given magnitude of your random variable.

So, how is this than define your F s of you x i is equal to the simulation of your F s of your x j. So, we are using another index here such that your j varies from where to where from 1 to i you see that the index here is i. So, this defined up to that point that point is the ith class up to that what is it? It is a simulation of everything before it. So, again this will represent what? This will represent the probability of what of your random variable having a value less than or equal to x i? So, what you are saying is that the relative frequency function gives you the probability of occurrence of the random variable within a certain range. However, the cumulative frequency function or CFF is the accumulative value of your RFF up to that point, that is to say it will give you what the probability of occurrence of your random variable? Before that just earlier that is why we are having P x less than equal to x i that particular value.

So, let us continue and then we define what is called the probability density function. This is a term which will be encountered very frequently PDF or it is simply written as F of x it is like any function. Well what is it the relative frequency function which we are just seen is defined for a sample? The relative frequency function was defined for a sample. What is a PDF? The corresponding function for the population functions for population, what is a population? A sample of size infinite or infinity as n stands to infinity and delta x stands to 0. So, we are putting 2 conditions what are those 2 conditions? First one is that the sample size becomes infinity or you have very large sample also that delta x stands to 0. So, that you are talking about from discreet to continues. So, delta x extremely small under those conditions at RFF is defined as the PDF. Mathematically we say your F of x or the probably density function is nothing but under the limits as n stands to infinity and delta x tends to 0 of what of your F s x i or x divided by delta x. We are dividing delta by x and we are taking the limits.

So, this is the definition of your PDF. So, essentially what we are saying is that the RFF is for your sample and the PDF is for the population similar to that we will define what is called a cumulative distribution function. What is a cumulative distribution function? It is denoted as capital F x. The CDF is defined for corresponding population it is similar concept actually let me first say that your cumulative frequency function is defined for a a sample right I should have said that earlier. And then the CDF or the cumulative distribution function is defined for a population or it is the corresponding function which is defined for a population as same thing n tends to infinity and delta x tends to 0.

So, mathematically what we are going to say is that your capital F of x is equal to what is equal to the function under the limits of what an n tends to infinity and delta x tends to 0 of your F s of your x. So, we have seen that there are 2 functions; one is the PDF and other one is the CDF it should be very clear that 1 is the cumulative of the other. So, if you have PDF with you, you can always find the CDF by integrating that on the other hand. If you have the CDF the cumulative function you can differentiate that to get the PDF. So, what is the relationship between the two?

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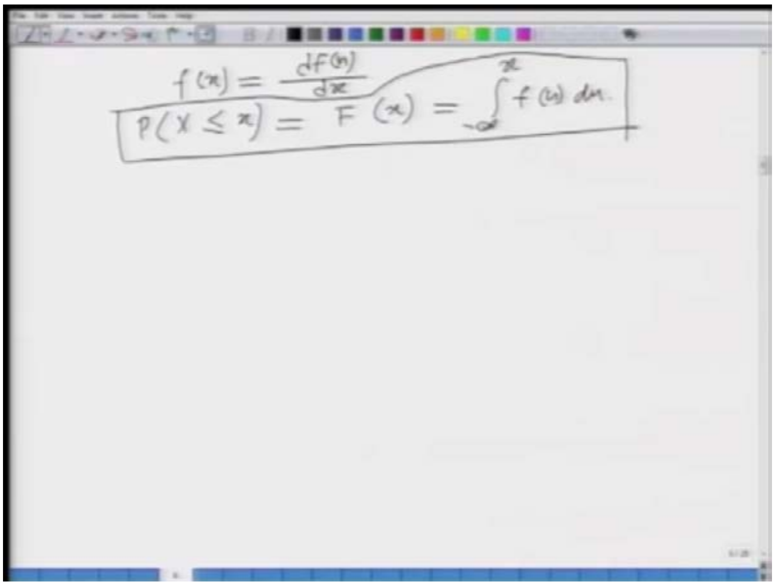

Well let me go to the next page and say that your PDF is nothing but the derivative of your CDF with respect to x. And also what does this represent? This represents your x less than equal to x, what is this probability of random variable having a value less than or equal to a particular value? This is given by your CDF which is given by minus infinity to x of your f u d u. So, this is a very important result which will through which we can find out the probabilities of different kinds. So, I am afraid I am running out of time today. I will come back and revisit some of these relations more closely. So, basically what we have seen today is we have started the probability chapter, we have looked at some basic concepts what is how is the probability defined? Why the statistical analysis is needed? And then you know different laws of probability and then towards the end we defined the concept of relative frequency function, cumulative function and PDF and CDF. So, I would like to stop here today. And then we will pick it up from here tomorrow.

Thank you.