Advanced Hydrology
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Lecture – 28

Good morning and welcome to this a post graduate video course on advance hydrology. In the last class, we looked at the new concept of synthetic unit hydrograph in which we looked at the snider's synthetic unit hydrograph; which is based on the hydrograph characteristics. Today, we will start with a slightly different topic which is very useful in in a practice or in the practical usage. For example, let us say we have a unit hydrograph of certain duration available to us. Let us say a 3 hours duration, but we want to find out the direct run of hydrograph response; from the catchment which is subjected to a rainfall event which is of a different duration. For example, 6 hours or 4 hours or 7 hours. So, in that case how we can get that direct run off hydrograph respond.

What we can do is once we have a unit hydrograph, we can convert its duration or we can find the unit hydrograph of a different duration. So, what we will do today is we will look at couple of methods of finding the unit hydrograph of different duration given a particular duration. So, we will look at 2 cases; in the first case the desire duration is the integer multiple of the given duration alright.

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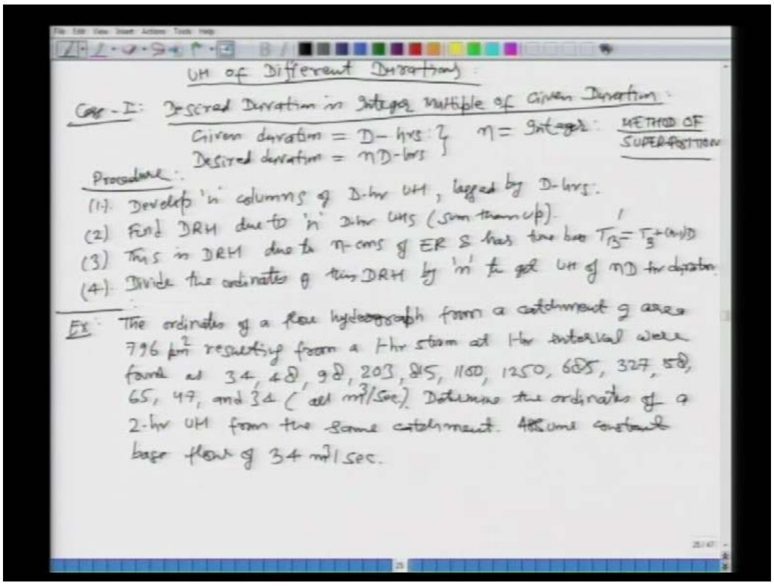

So, let me see here what we are going to study today is UH of different durations. And, in this we will look at or first see the case 1 which corresponds to the case when the desired duration is integer multiple of the given duration. That is to say if you are a duration of the given unit hydrograph or let us say the given duration is D hours. And, then you want to find out the unit hydrograph of a duration which is n times D hours; where n is an integer.

So, this is case 1 in which n is an integer. Later on we will look at case 2 in which n will or can be a fraction it could be a real number. For example, 2.3, 1.2, and so on. So, as always what we will do is we will look at a step by step procedure. And, then we will take up an example which we will demonstrate how we can carry out these steps? So, let us look at the procedure first what we do is we develop and columns of your D hour duration unit hydrograph which is led by D hours. What do we mean by this step is that let us say you we have the unit hydrograph given to us.

So, we have the time base at the certain interval we have the values of the unit hydrograph of the hour duration. Now, we want to find out the n times D hour duration unit hydrograph. And, let us say a for example, the value n is 3. Now, what we are saying in step 1 is we develop n columns of this D hour duration that is n is equals to 3. So, we develop 3 columns. So, we just have time verses the unit hydrograph and then we lag it by the hours; once and lag it by the hours once more. So, that we will have a total of 3 columns, but the lagging will be done n minus 1 times or 2 times for n is equals to 3. So, this step will be more clear when we take up the example. The next step is find the DRH due to n different D hour unit hydrographs.

What are we saying is that we find the composite or compound response in terms of direct runoff hydrograph? That is due to all of this n different unit hydrographs which are generated by back to back rainfalls of 1 centimetre each. How can we do that? Well, we just some these are. So, we can say that sum them up. So, which principle we are using here? This is the method of a principle of superposition. The next step is we say that this D is the DRH due to what, due to how much rainfall? n centimetres of effective rainfall is this clear. We have one unit hydrograph resulting from 1 centimetre of rainfall. Then, we led it by D hours, the second column is also due to 1 centimetres of rainfall, and the third column is also due to the 1 centimetres of rainfall. So, if you are summing these 3 up the resulting director of hydrograph is due to 3 centimetre of rainfall.

So, similarly when value of n is other than 3 or any other value we say that in general the sum of the composite direct run of hydrograph; which we get is due to n centimetre of rainfall. And how about the time base? And, it has the time base of I am going to use prime here to define the modified time base; which will be the time base of your original unit hydrograph plus n minus 1 times D you can verify that. Now, we have a direct run off hydrograph with us which is produced by n centimetre of rainfall. And, this n centimetre of rainfall occurs in how much duration? For n is equals to 3 we have n times D or 3 times D. Let us say D was 3 hours and if n is equals to 3 this rainfall is occurring in 3 plus 3 plus 3 9 hours.

So, we have a DRH which results from 3 centimetres of rainfall which occurs uniformly over the whole catchment in 9 hours. How do we find the unit hydrograph now? We use the principle of proportionality. So, we divide the direct run off hydrograph by the total rainfall which is 3 centimetres in this particular case. So, then we say the final step is divide the ordinates of this DRH by n; which is actually representing the n centimetre of effective rainfall to get your UH of how much n times D hour duration. So, it is a very simple method and actually it is called method of super position; I think I forgot to mention that. So, let me do that here this is called method of super position.

In which all we are doing is we are using the principles of Lenoir curie which are method of super position or the principle of super position. And, also we are using the principles of proportionality both of them. So, what I am going to do next is I will take a pen example which will demonstrate these tabs. So, let us look at that now it goes like this. The ordinates of a flow hydrograph the ordinates of a flow hydrograph from a catchment of area 796 square kilometres resulting from a 1 hour storm at 1 hour interval were obtained to be or were found as these numbers So, these are the data given to us in terms of the flow hydrograph ordinates.

And, the last one is 34 and all of these numbers that are given to us are in the units meter cube per second. What we have to do is determining the ordinates of a 2 hour unit hydrograph from the same catchment. Assume constant base flow of your 34 meter cube per second. So, this is the problem in which we are given that the flow hydrograph ordinates resulting from a 1 hour rainfall; which are given at 1 hour time interval. Otherwise always ordinates are at a 1 hour each and then the duration of the rainfall is also 1 hour.

So, we have 1 hour unit we have to first find out the 1 hour unit hydrograph and then we have to determine the 2 hour unit hydrograph? How do we find the 1 hour unit hydrograph? Well, the first sector in the base flow find the DRH, we find the total rainfall or effective rainfall which produces this? How can we do that? We can do that by estimating the area under the curve. We know the area of catchment a drainage area of the catchment we can find out the total volume divide the volume by the area that will give us the a effective rainfall tab. So, what I am gone a do is then I will give you some steps in a tabular form.

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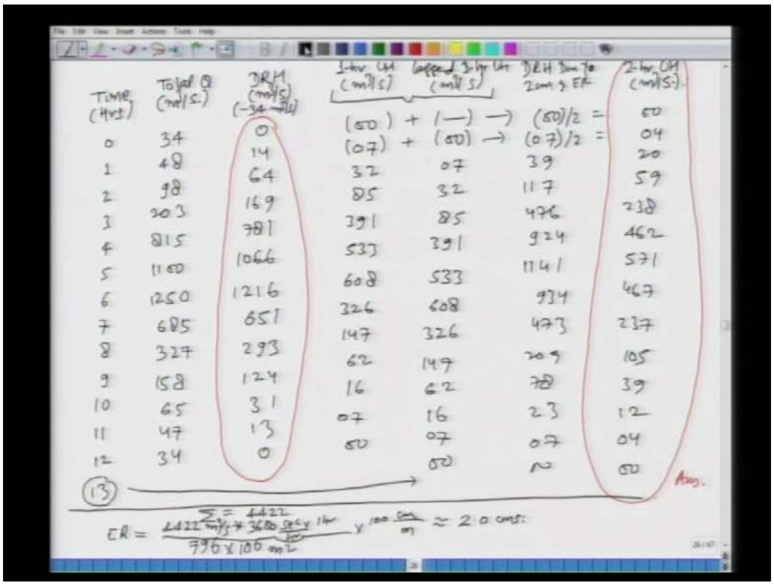

Let us go to the next page and arrange the data in a tabular form. You have the time in hours which will go from 0, 1, 2, 3, 4, 5, 6, 7 all the way to 10. So, these are the times which are given to us as 1 hour interval. The total flow is given to us in meter cube per second, this is the data which I have just written. So, you have 34, 48, 98, 203, 815, 1100 and so on. I am just coping all these data which were given to us 65, 47 and 34 is that the 12 at the end of the twelfth hour. Now, what we do is, we find out the DRH for this 1 hour storm. How do we do that? While, we separate the base flow. And, what is the base flow? It is given to us as 34 meter cube per second. So, all we do is we calculate the next column by subtracting 34 from this second column.

If you did that you will have 0, 14, 64, 169, 781, 1066, 1216. You can verify all these numbers I am writing that quickly we have seen one example which actually does this. I am not spending too much time on this. The last one will be 0 because we are subtracting 34 from this column. So, what we have now is a director of hydrograph which is resulting from certain amount of effective rainfall. Now, we want to find out the volume. How do we do that? We just sum this column up sum up all of these. If you sum them you will get 4422. And, then the effective rainfall depth would be equal to what? Well, it will be 4422 this is meter cube per second. We multiply this by the number of seconds in an hour; which is 3600 seconds times 1 hour divided by hour.

And, then we divide this whole thing by the area of the catchment which is 796 square kilo meters; we need to convert it in to a square meters. So, you multiply by 10 to the power 6 this is square meter. You take care of the units and then you multiply this whole thing by 100 centimetres in a meter. So, this will come out to be approximately 2.0 centimetres. So, this whole a runoff hydrograph ordinates they result from 2 centimetre of rainfall. Now, I have a DRH which results from 2 centimetres of rainfall. How do I find the unit hydrograph? I use the principle of proportionality divide the ordinate by 2.

So, that will give me 1 hour UH in meter cube per second. How do I find that? I divide the previous column by 2 that is all. You will have 0, 07, 32, 85, 391, 533, 608, 326, 147, 62, 16, 07, 00. So, what I have obtained is the 1 hour unit hydrograph given the data and this is the same procedure we had seen in the last class for a similar example. What we have to do is now we have to develop n columns. What is the value of n here? n is equals to 2 because from 1 hour we have to find out 2 hour unit hydrograph. So, we have to develop 2 columns and one of them is this one. So, we need to develop one more column which will be what? Which will be lagged by d hours. And, what is d? D is 1 hour here which is the duration of the effective rainfall. So, you have lagged d hour UH this is also in meter cube per second.

So, when you lag it you can just put a dash at the initial one and then just copy this whole thing, this whole column you just shift it. So, this is called lagging. So, you have 0, 07, 32, 85, 391, 533, 608, 326, 147, 62, 16, 07 and then the last one will be 00; and it will go up to thirteenth hour this guy will be at thirteenth hour. So, if I go back to my a procedure which I had given to you look here. We have to develop the n columns of d hour duration this is the first tab; that is what we have done. The next step is what find the d r s due to n d hour u x. How? By summing them up that; that is the next step.

So, what you do is you have these 2 n that is 2 d hour unit hydrograph. You sum them up corresponding ordinates that will be a total DRH due to it will be due to 2 centimetre of yam which a which occurs back to back. So, this column plus this column. Basically what you have is this plus this that goes here; this plus this goes here and so on. If you did that what you are going to get is 0, 07, 39, 117, 476, 924, 1141, 934, 473, 209, 78, 23, 07 and 00. So, you have obtained a direct runoff hydrograph which results from 2 centimetres of rainfall occurring in 2 hours now. So, how do you find? The unit hydrograph just divide this by n and n is equal to 2. So, these ordinates you divide by 2 that will give you 2 hour unit hydrograph in meter cube per second.

So, if you did that so what I am doing is you just divide by 2 all these numbers divided by 2 and so on. So, you will have 0 it will be 4 do not worry about the rounding of. This is I am taking the whole numbers 238, 462, 571, 467, 237 and so on. So, this is your answer for 2 hour unit hydrograph given 1 hour of unit hydrograph by using the method of superposition; in which we look that the method step by step. And, then you seen how we can implement that method? Let us move on what I am going to do is let us look at the second method which is applicable for the case when n is a fraction.

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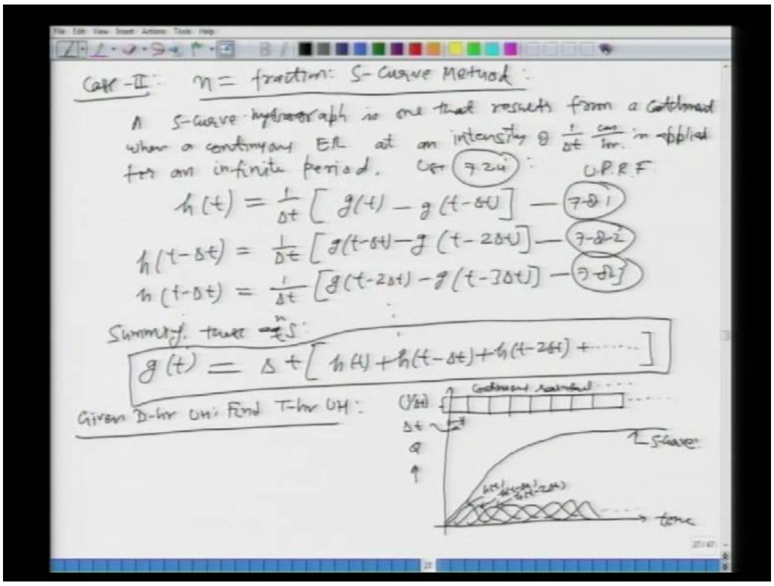

And, we are going to use what is called the s curve method for this? We have seen what is an s curve? We will briefly a revisit or look at a what is the theoretically and how we can actually calculated with the help of an example? And, once we have the s curve; we can find out unit hydrograph or we can convert the duration of any unit hydrograph of any duration to any other duration. So, this is the major advantage of having this s curve. So, let us look at what we are trying to do here? A s curve hydrograph is one that results from a catchment that results from a catchment when a continues effective rainfall at an intensity of 1 by delta t centimetres per hour is applied on that catchment that is for an infinite period, we have seen that earlier. How can we find this? We can use equation 7.2.4 which we had seen earlier which is this actually; which is for your unit pulse response function. What is that? h(t) is nothing but one over delta t of your g(t) minus g of your t minus delta t; remember this relation we have derive this earlier. So, let me number this as 7.8.1. This is a relationship between what and what? h is the unit pulse response function and g is what it is the unit step response function. So, it is a relationship between these 2. Using this we can find out what is called your s hydrograph? As per this definition which we are just a stated here.

Now, you can find out the unit pulse response function due to another rainfall event which starts delta t hours later and which has same type of input 1 by delta t centimetres per hour occurring after delta t hour. What will that be? That will be h of t minus delta t and you can put t minus delta t in case of t in this equation. So, if you do it that you will have g of t minus delta t minus g of t minus 2 delta t this is 7 8 2. Similarly, you can keep on writing the response from another rainfall which occurs 2 times delta t hours later. So, it would be t minus 2 delta t minus g of t minus 3 delta t this is 7 8 3 and So on for infinite number of times.

What is an s curve? s curve summation of all of these that is what we are saying. So, it results from an intensity which is continuously occurring. So, if did that and if you sum these up. What do you get? You will get summing these when we write it here summing these equations. What we are going to get is your g(t) is nothing but delta t of your h(t) plus h t minus delta t plus h t minus 2 delta t and So on; where g is your unit step response function at any time or the s hydrograph. So, theoretically it is given by the summation of the ordinates of your unit hydrograph which are separated by delta t hours or the d hour duration. If you want to look at it graphically it is equivalent to saying this in the same manner. This is your time ordinate and this is your let us say Q or a the hydrograph ordinate; this is your rainfall input, this is occurring an infinite period. So, this is your continuous rainfall.

What is this duration? This is delta t and what is this intensity we have defined already one over delta t. So, the intensity is 1 over delta t which occurs for delta t durations. So, that what is the total amount of rainfall of what is the duration? One by delta t multiplied by delta t, that is one centimetre. That is your definition of unit hydrograph. So, basically what we have is the unit hydrograph resulting from these 1 centimetre of rainfall occurring back to back for delta t durations. That will be let say something like this; these are the unit hydrographs continuing indefinitely. This is your h t, this is next one is your h t minus delta t, next one is h t minus 2 delta t and. So on. So, if you are going to sum these up; these ordinates you will get a response which will look like this. And this is what is called your s curve.

So, this is the concept of the s curve theoretically as well as graphically. Now, what I am going to do is I will give you the procedure for finding out the unit hydrograph of a t hour duration. The given the unit hydrograph of d hour duration and d and t can be a any numbers and we will take the help of the s curve. So, let me give you this procedure given a d hour UH. Find your t hour UH? Let me go to the next page for that.

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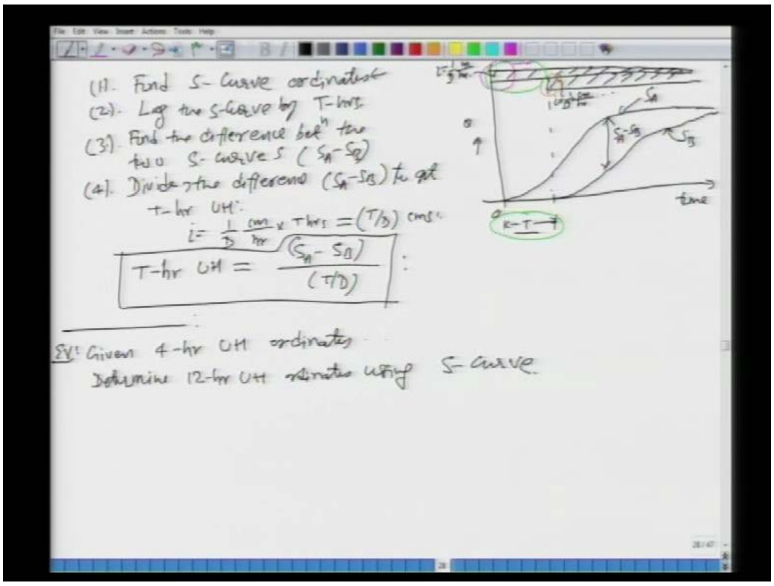

The procedure goes like this given the data first what we do is we find the s curve? And, I just explain you graphically and theoretically how we can do that? And I am going to take an example to demonstrate that. So, we first find the S curve ordinates or S I do graph; that is first step number one. And, in the second step we lag the s curve by by what? Any ideas by T hours by T hours. Let me a try to give to you what we are trying to do here. So, this is you are time domain and this is you have a domain. What you have is first rainfall, this is intensity is equals to 1 by D centimetres per hour; it is a occurring a starting at t is equals to 0.

And, these results in this s curve and let us call this as A. So, what we are doing is we a first step is find the S curve ordinates. So, this is your S curve ordinates or the first S curve; in the second step we are saying lag the S curve by T hours. So, we have this s curve we lag it by T hours that is it this is your T; there is no relation between D and T. So, we lag it by T hours what we are going to get is same response which is dilate by T hours and then ultimately release the equilibrium discharge is going to meet the same discharge. And, I am going to call this as of D let us say. And, what is the input? From which this is resulting well this is starting here. And, this is the rainfall which is s occurring continuously and this is also I is equals to 1 by D centimetre per hour occurring infinitely.

So, you have the first S curve that results from an intensity of 1 by D centimetre per hour which occurs infinitely. The second one is another S curve which is lag by T hours that occurs by the same pattern of input or intensity of rainfall; but that rainfall pattern occurs T hours later. So, in the next step what you do is find the difference between the 2 S curves that is S A minus S B? So, what we do is you separate S B from S 1. In a tabular from you will separate the data like we have done in the previous example. So, this is your S A minus S B. So, we can find this S A minus S B. What is this S A minus S B? This represents what? The S A minus S B represents a direct run of hydrograph which results from what kind of input?

If you look here what we are saying is the first curve S A results from this whole input which is continuing indefinitely whereas the second one is resulting from this input. And, when you subtract these 2 what is the resulting a direct run of hydrograph? That is resulting from which input? It is basically this input do you see that. It is resulting from an input of intensity 1 by D centimetres per hour occurring for how much duration T hour. What is this? T hour do you see that. You have the 2 curves we are using the method of superposition we are subtracting the 2. We know the input patterns of the 2. So, we know the input which causes this S A minus S B which is the direct run of hydrograph resulting from this rainfall. Now, if we have understood this then the last step would be very easy divide the difference; that is your S A minus S B to get to get T hour UH; divide the difference by what?

What is D amount of rainfall which is causing this S A minus S B? The amount of rainfall is what, intensity is what? Intensity is 1 by D centimetres per hour. And, it is occurring for how much? This T hour duration, so multiplied by T hour. So, how much is the rainfall then? It is T by D centimetres you see that. So, the S A minus S B is the direct runoff hydrograph response from the catchment that results from T by D centimetre of rainfall. Once I have a known that I can find out by proportionality your T hour UH is going to be equal to what it is S A minus S B those DRH ordinates divided by T minus D; this is as simple as that. So, this way we see that how we can determine the S curve ordinate number one? And, then once we have the s curve we can use the method of superposition and the method of proportionality because they are all linear system. To determine or to convert the duration of one hydrograph from to the different one; which is attraction of the other.

So, this is going to be given this is an example; I am looking at given a 4 hour unit hydrograph ordinates these are given to us. And, we have to determine a 12 hour unit hydrograph ordinate using S curve. You see that in this case n is equals to actually integer, but we are going to demonstrate the use of S curve method for the conversion of the duration of this unit hydrograph. I am not coping the data here I will do that directly on the next page.

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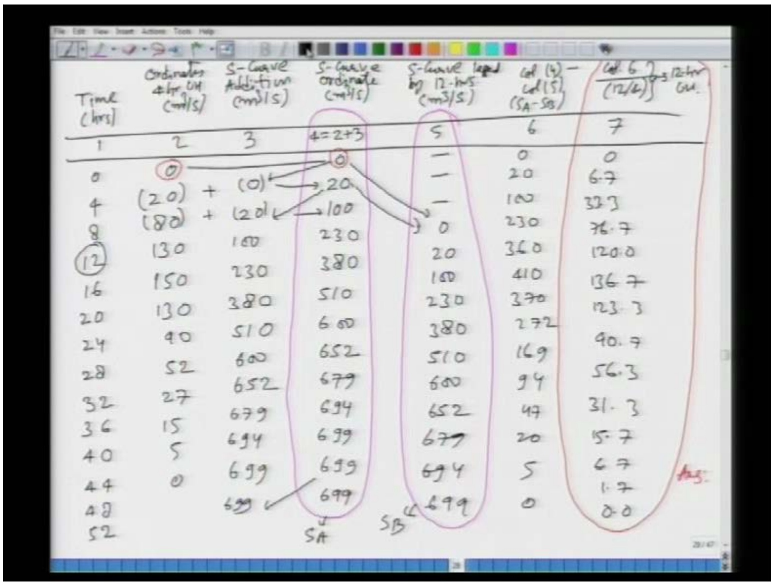

Time (hrs)	Ordinates 4 hr UH (m3/s)	S-Curve Addition (m3/s)	S-Curve ordinate (m3/s)	S-Curve lagged by 12 hrs (m3/s)	Col (4) − Col (5) (SA−SB)	Col 6 / (12/4) = 12 hr UH
1	2	3	4=2+3	5	6	7
0	0		0	—	0	0
4	(20)	(0)	20	—	20	6.7
8	(80)	(20)	100	—	100	33.3
12	130	100	230	0	230	76.7
16	150	230	380	20	360	120.0
20	130	380	510	100	410	136.7
24	90	510	600	230	370	123.3
28	52	600	652	380	272	90.7
32	27	652	679	510	169	56.3
36	15	679	694	600	94	31.3
40	5	694	699	652	47	15.7
44	0	699	699	679	20	6.7
48		699	699	694	5	1.7
52				699	0	0.0

So, what we have is we are given the data of time verses what ordinates of a 4 hour unit hydrograph. So, that is what is given to me this is in meter cube per second. So, let us say this your column 1, this is your column 2. And, these are given at 4 hour intervals does not have to be they could be at any other interval 40, 44, 48, 52 etcetera. As the ordinates that are given to us are 0, 20, 80, 130, 150, 130, 90, 52, 27, 15, 5 and 0. So, these are the data that are given to us. So, first what we have to do actually is we have to first calculate the s hydrograph or s curve. Theoretically, we have seen what it is and what is the equation? But in a tabular form we will use a slight trick. So, that our computations are simplified. So, I am going to give that method how we can calculate this s hydrograph quickly using only 2 column? Otherwise you have to write these unit a hydrograph ordinates infinite many number of times. And, then keep summing them up then we do not want to do that. Of course if we did that we can get the same answer, but we will use a very compact manner.

So, what we do is we develop 2 columns and in developing only these 2 columns we will get the s curve. So, the next one is what is called we s curve addition column and this is also in meter cube per second. Let us call it column number 3 and the next column is going to be your s curve ordinate directly. And, this is going to be in meter per second; and this is your column 4 which we say is equal to column 2 plus column 3. So, what we do is the first entry we just throw here. So, this is your ordinate what has doing is you are transferring it here that we can do that easily. And, then what we do is we transfer this ordinate to the next column or the previous column in the next entry. So, we can transfer this 0 in third column just after 20. Then, what you do is you do the column 2 plus 3 that is this 1 plus this 1 and this will go here.

So, what is it? This is going to be 20 is it clear? Let me go over it again. The first ordinate of your unit hydrograph which is what a 0 let say it is 0 we just transfer it to the s curve ordinate. So, the first data in the s curve ordinate is same as the first one in the unit hydrograph. Then, what you do is we transfer this number let me see this arrow actually at should reflect here. You are transferring it here.

To the previous column in the as the next entry. You transfer this number here and then the next entries at 4 hour will be the sum of next ordinate plus the s curve additional column. So, then we have 20 and then this 20 we are going to transfer to the next which is at the end of 8 hour which is let me say 20 here. What is going to be the next entry? Here, it is 80 plus 20 which is how much 100, and we keep on repeating this process this way using only this 2 columns; 1 is the s curve addition column other is your directly s curve ordinate. It will this procedure which we are doing essentially amounts to the same thing as if you are a lagging a 4 hour unit hydrograph infinitely; that is you are doing so many numbers. And, then keep on adding them it has the same effect. So, let me quickly then do this you will have a 100. And, then let me put 100 here and then you will have 230 here, then you have 230 here, and you will have 380 here. Then, 380 comes down here then you sum these you will have 510 here.

If 510 here then this will be 600 and then 600 and plus 52 is going to be 652 and so on 699 plus 0 is going to be 699. And, we can keep on doing this actually and then you will get the same number 699. That is to say the s curve hydrograph will saturate at what is called the equilibrium discharge or a constant discharge which is 699 in this case. So, these are your s curve ordinates this whole column. How do we calculate? It should be very clear. Now, we are going to apply these steps basically this one. To calculate a t hour unit hydrograph and t is 12 hour in this particular case.

So, what are these find the s curve ordinates which we have done lag this s curve by capital T hours and T is equals to 12. So, we lag it by 12 hours then we will find the difference s a minus s b and we will divide that by t by d. So, that is as simple as that let me show you that. So, the next column is going to be your s curve lag by how much t hours which is 12 hours in this case. It could be 11, it could be 10, it could be anything; 12 hours means what? Let me first say this is a column number 5. So, you will have dash in the first 3 entry 0, 4 and 8. So, the first entry is going to be at what at 12 hour. So, because we are lagging it by 12 hours. How do we do that? Well, we just transfer all these ordinates here we are just this lagging it.

So, you can cut and paste all these if you are doing it in excel. Let me do that you will have at 12 hour, it will be 0. Then, you have 20, 100, 230 and so on 380, 510, 600, 652, these may not correspond to the times I have put here. But you understand which entry corresponds to which time there should not be any confusion 694 and 699. So, now, I have got this is your s a. And, what is this the other one? This column is s b. Then, the next one is going to be column 4 minus column 5 which is nothing, but your s a minus s b if you did that what you are going? To get is 0, 20, 100, 230, 360, 410, 372, 272. You can verify these numbers you know these steps. And, how I am doing it? I am just a subtracting these previous 2 columns, 5 and 0.

So, now we have got column number 6 which is the deferens of 4 and 5. Finally, you will have column 6 divided by what divided by t by d t is 12; d is 4. So, we divide this column by 3. To obtain what this is going to be your 12 hour unit hydrograph which is a desire. And this particular case it happens to be an integer multiple, but it does not have to be for s curve. So, the next step is simply dividing the last column by 3 that will give you the desired answer. So, let me say that this your column number 7; the last column which is 12 hour unit hydrograph ordinates. So, this going to be these numbers we can verify these very easily, and so on 1.7 and 0.0 is the last one. So, this is your final answer which is the 12 hour unit hydrograph.

So, what we have a done today is we have look that 2 methods of determining the unit hydrograph of different duration; given a d hour UH we are finding n times d hour unit hydrograph; where n was an integer. In the first case we use what is called the method of superposition. And if the n is not an integer or say fraction then we can use the s curve and we have seen a very compact method of how to determine and how to find the s curve? And, then how we can use this s curve technique by lagging and by using the principle of proportionality and superposition; to find out the unit hydrograph of a different duration.

So, with that actually we come to the end of this particular chapter on unit hydrograph; in which we have seen comprehensively the Lenoir systems floury, the definition of the unit hydrograph, it is basic consumptions. Then, we have seen this 3 basic a functions which are needed in applying these linier system theory. These work unit impulse response function which we said is equivalent to the instantaneous unit hydrograph. The second one we said was the unit step response function which was nothing, but the s curve. And the third function we define it was the unit pulse response function which is nothing, but the unit hydrograph.

And, then we look that 4 different methods of derivation of the unit hydrograph for a simple storm and the complex storm. And, you also look that the multi storm analysis in which we can combine the data from different storms and then get a composite unit hydrograph. And, today as I summarise we look that the conversion of the duration of the unit hydrograph. So, with that we come to the completion of this what I should say is the deterministic hydrology or the conceptual methods at least partly. And, then in the next lecture we are going to start a new chapter; in which we will move on to these stochastic hydrology where in we will look at some probability and statistical concept. And, how they are implemented in hydrology?

Thank you.