Advanced Hydrology
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Lecture – 21

Good morning and welcome to this post graduate video course on advanced hydrology. Today we will start the 21 lecture in this course before that we would like to as usual go back and look at what we did in the last lecture. In the previous class we completed the development of an algorithm for estimating the abstractions from precipitation using green empty equation. We said that would develop this algorithm for green empty equation, but any other infiltration equation can also be used while developing this algorithm. We matched in time domain in which we analyze we said that we can analyze each and every time step compare the relative magnitudes of the infiltration rate, and the intensity of rainfall alright at the beginning and at the end of a time step alright.

Then what we said is we can characterize each and every time step into one of the three possible ponding cases, and depending upon that we can use the appropriate equations to calculate the infiltration at the end of the time interval, and once we have that we can calculate the effective rainfall hyetograph. Then we moved on to a new method which is called the SCS or the soil conservation service method, which is applicable for storm rainfall. We can calculate the effective rainfall using this SCS method in terms of a parameter which we need to estimate which is called the curve number, and the SCS as provided tables depending upon the soil type and the land use land cover conditions, and it gives us the curved number. And this curved number is given for the normal antecedent moisture conditions or the initial conditions.

They have also given us the equations to estimate the curved number, if the initial conditions are on the dryer side or on the wetter side. After that we started developing the model the rainfall runoff model for the surface flow or the overland flow. We looked at the schematic diagram, and we defined the various variables, and we said that our objective is to calculate the flow depth and velocity at the end of the over land flow. What I would like to able to do next is take the continuity equation where we stopped in the last class.

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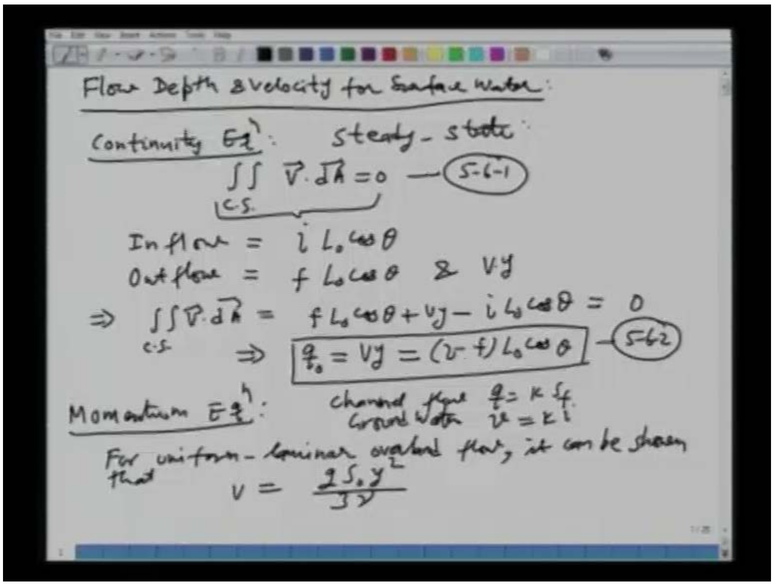

So, what we are looking is at the expression for flow depth and velocity for surface water, we will analyze the surface water first as the overland flow and then later on as the channel flow. We have already written down various variables that are involved and the control volume, and then we said that we would write the continuity and momentum equations for we control volume which we have considered in this particular case.

What are the inputs and outputs well the rainfall is falling from the top there is infiltration taking place, and there is overland flow the flow depth is y and the velocity is v, our objective is to find this v and y at the end of this control volume. And then we write first the continuity equation, and we state conditions we reynolds transport theorem for the for the mass comes out as this beta row v dot d A beta is one row will come out you have v dot d A is equal to 0 this is our 5 6 1.

Now with just with reference to the figure which we have drawn yesterday so basically what is this whole thing this is representing. If you look at physically what it represents is the is the second term on the right hand sides of your Reynolds transport theorem, which represents the net out flux of the extensive property flowing across the control surface and the extensive property. Here is the mass you have taken the density of the water outside which has cancelled out so, instead of mass we can consider the volume. So, net volume metric flux flowing across the control surface r is equal to 0 so, if we apply that then all we need to do is we look at the control surfaces is what is the movement of water across that control surface and then we just take inflows as negative outflows as positive sum them up and equate it to 0.

What is the inflow the only inflow is in the form of I and if we take the inflow perpendicular to your cross section or the control surface it will be l 0 cos theta with reference to the figure we have and then multiplied by one which I am not writing here. Similarly, your outflow would be what there is infiltration going out of v control volume across the same length l 0 is the length of the overland flow which we have defined is there any other outflow yes the other outflow is your water flowing as the surface flow across the control surface at the outlet of depth y.

And the volume metric flux would be v times y all right so it will be v discharge intensity per unit of length of the overland. Therefore, your this term we can write as outflows as positive so, we add them up and inflow as negative so, we put a negative sign in front and then we equate it 2 0 then we say that the discharge intensity coming out is nothing but, v times y would be equal to after you simplify this i minus f l 0 cos theta.

So, this we have continuity equation and I am to number this as 5. 6. 2 so, this gives you this equation gives you the q 0 which is the discharge intensity coming out of that sheet flow of the overland flow. Again our objective is to find v and y this equation gives you v y which is the multiplication of these two quantities so we need one more equation to calculate both v and y all right so we take help from the momentum equation so, let us look at the momentum equation.

We have seen earlier that whenever we are applying the momentum equation either it is the channel flow or it is the ground water flow. It is some form of a equation the famous equation we have seen for channel flow it was q is equal to c s or let me write it for channel flow initially we had written q is equal to k s f. Most of you may remember that for the ground water flow we had said that the Darcy’s law v is equal to k I represents the momentum equation.

Now in this particular case we have the sheet flow or the overland flow which is uniform in nature so, for the uniform laminar flow. Let us assume that the sheet flow is laminar for t now and for the turbulent case we will consider it later. So, for the uniform laminar overland flow what we will do is we will take the momentum equation from the knowledge of our fluid mechanics we will not go into the derivation of that it can be shown. So, we are not going to show it here, but we will take it that the velocity is given by this expression which is g time's s 0 y square divided by 3 times of your mu so, this represents the momentum equation for we flow conditions which we are considering.

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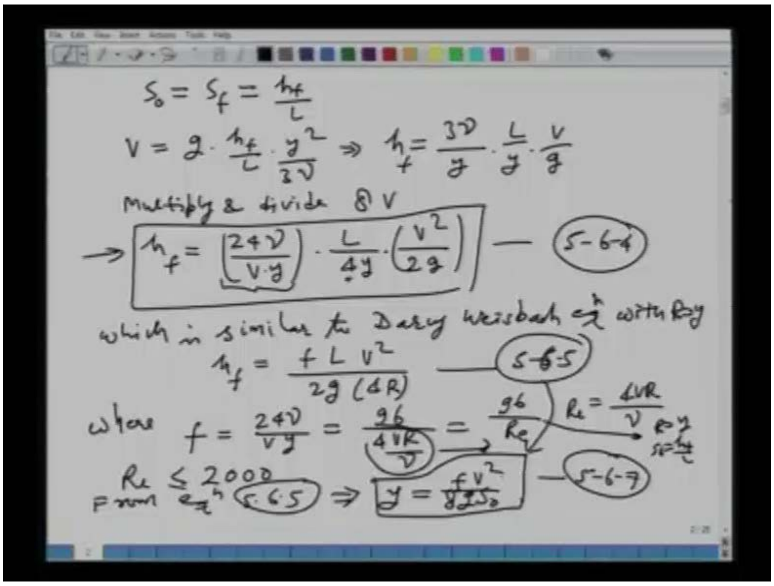

What do you as not as not is the bed slope which is equal to the friction slope for the uniform flow as you know and this is equal to h f by L we have seen that earlier in the first 2 chapters.

Now what we will do is we will try to reduce this momentum equation in a form, which is similar to some other equation we have seen earlier in our undergraduate class. So, that we are trying to analyze this flow condition with another flow situation which we have analyzed already. So, that we can write that equation easily so, it will involve some mathematical or algebraic manipulations essentially. So, with this we say that the velocity then will be equal to g times h f by l if you go back all we are doing is I am trying to write down this equation in which s naught is h f by L that is what we are writing. So, we are slightly rearranging this so you have g times h f by L times y squared over mu. That would imply your h f which is we head loss quantity is nothing but, 3 mu over y times l over y times v over g you can see that we can come from here to here very easily.

Now what I do? Is I multiply and divide I can multiply by a constant or one quantity in both numerator and denominator of an equation. So, what I do is I multiply and divide this equation by 8 v if I did that we would see that h f would be equal to it will be 24 mu over v times y this I collect together then l over 4 y you have multiplied by 8 in the numerator you are multiplying by 8 in the denominator so, 4 is here and then there is a so, it is v squared over 2 g let us say this is 564 this equation 5.6.4 does this sound familiar to you, have you seen an equation.

Similarly, to this earlier somewhere in your undergraduate classes by flow yes this is what is called the famous Darcy Weisbach equation in which let me first write it which is similar to or in the form of what is called the Darcy Weisbach's flow resistance formula in the pipe flow equation. With r is equal to y what is that? That is nothing but, your h f is equal to f l v square over 2 g d n d here is 4 r or 4 y so, if you compare this equation and let me number this as 5 6 5 6 5.

If you compare 565 with 564 they have similar forms h f is equal to this whole thing. Let us say f l v square over 2 g and d is 4 y or 4 r where your f is the friction factor or the friction resistive forces which oppose the motion of the fluid is 24 mu over v times y which we can say is you again do some mathematical manipulations multiplied by 4 divided by 4 we are over mu slight modifications then this is as you know what is that term in the denominator? You recognize this is a non-dimensional number Reynolds number all.

So, this is 96 over r e where your r e is equal to 4 v r over mu with r is equal to y we have only side so, this equation which we see here is something like the flow resistance formula. In which the head loss can be calculated using a Darcy Weisbach equation which is f l v square over 2 g v. This flow conditions will be in the laminar region there is a certain condition in terms of the Reynolds number we all know that so, this will be laminar flow if the Reynolds number is less than or equal to 2000.

So, now moving on what I am going to write is, from equation 5.6.5 what I can do is, I can write the equation for y as this f v square over 8 g as naught. I would like you to be able to see this is 5 6 and 7. I will skip one equation which is not very important at this. Of time so from 5. 6. 5 we are trying to find out what will be the y? Where r is equal to y so, this is what we are doing is we are using r is equal to y is one then s naught is equal to h f by l 1. You are using these two you know basic relations to come from 5652567 so, that is your flow depth at the end in terms of the velocity.

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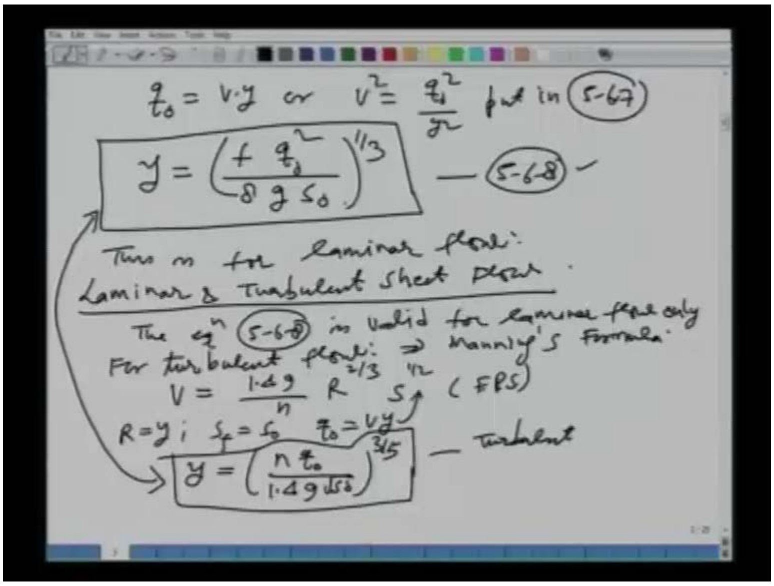

Now what you do? Is from your original equation you know that 2 0 is equal to v times y v squared is equal to 2 0 square over y square put this in your 5 6 7. So, that what we are do is we are eliminating v velocity v so, that we can get the expression for flow depth and once we have depth we can find out whether one we are using q is equal to v y so, it should be easy to see that your y would be equal to f 20 squared over 8 g s naught this whole raise to the power one third.

So, we see that we calculate the flow depth using this equation 5. 6. 8 in terms of f is the friction factor. We can find it out using our standard knowledge about the surface condition of the overland flow q naught is something which we have already written the continuity equation in terms of I minus f l 0 cos theta that expression and all other things are known the bed slope and g n and so on so, we can calculate y once we have the flow depth we can calculate velocity from 2 0 is equal to v times y.

Now this is for laminar flow so, this is assuming the sheet flow or the overland flow during the rainfall runoff processes laminar what happens? when the velocity is increased the overland flow or the steepness of the overland topography maybe very steep so, if that is the case when the velocities will be high and the laminar conditions will change into turbulent. So, in that case we will have to analyze it slightly differently so, what we will do is, we will develop an expression which is most general in which we can utilize this, under certain condition as the laminar flow condition and some other condition as the turbulent flow condition.

So, the next thing we are going to look at is the expression for laminar and turbulent same expression the general expression sheet flow the equation. Which we have derived so, far or let me write it down it is 5. 6. 8 is valid for laminar flow only when the Reynolds number is less than 2000 for turbulent flow which equation is the most important we saw that in the laminar flow we used an expression which is similar to the Darcy Weisbach equation, From the pipe flow for the turbulent flow in and the channel what is the most important equation? For turbulent flow we used the Manning's equation so, in this case when the velocity is in the overland flow becomes significantly high the Manning's equation becomes applicable.

So, what we will do is we will analyze this flow resistance equation or the velocity we can find using Manning's formula what is Manning's formula? This is v is equal to 1.4 9 over n, n is the reference co-efficient as you know r is the hydraulic depth a by p times s to the power half and this I am writing in f p s system as per the book and then we can generalize this for si or the f p s system later so, if you put r is equal to y in this equation and s f is equal to s o then what you will have is q 0 is equal to v terms y. So, if you use all of these equations into your Manning's equation you can derive this like this its again I am not going to do this, but you can easily verify once you utilize all these equations or relation what you will have is the flow depth like this whole raise to the power 3 by 5.

So, we see that this is what turbulent is there a similarity between this equation for the turbulent flow we had just derived and this equation they have very similar forms you see that y is equal to in the laminar case. You have f which is the friction factor here you have n which is the reference co-efficient? And then you have q 0 here it is square it is you know linear then you have some other quantities in the denominator so what we can do is we can write this expression in a very general form.

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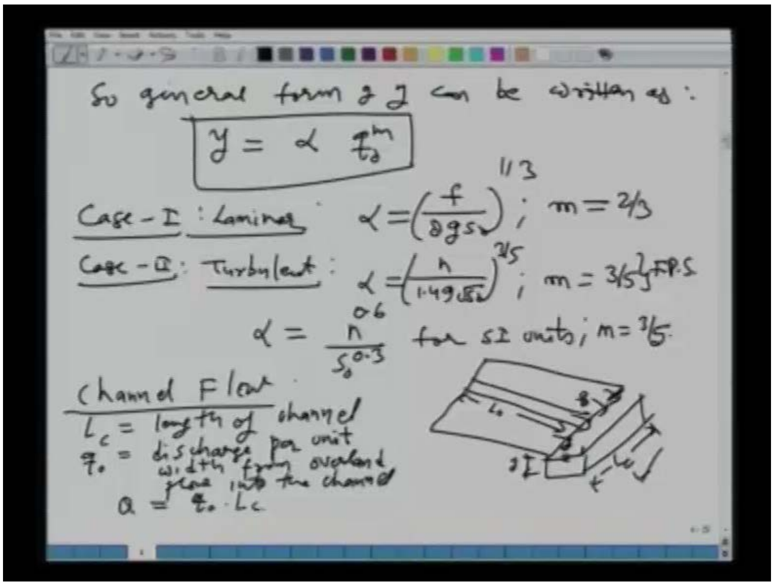

As y is equal to alpha 2 0 to the power n so, this is the general equation we can write for the flow depth where alpha and m will have different value for laminar and turbulent flow components. So, let me give you this without actually deriving all this it should be easy to see so for case 1 which is the laminar flow. We would see that we have already derived this alpha will be equal to f over 8 g s naught all of this raise to the power n by 2 m is the exponent which is equal to 2 by 3.

Let me go back and see this is your laminar flow so, alpha is f over 8 g s 0 raise to the power one by 3 and q 0 raise to the power 2 by 3 so, m is 2 by 3 and case 2 for the turbulent overland flow. You would have alpha is equal to n over 1. 4 9 under root s all of this raise to the power 3 by 5 and m is equal to also 3 by 5 this is true for the f p s system remember in the f p s system in the Manning's equation you have 1 . 4 9 in that factor which converts your units in the si system we do not have that factor of 1. 4 9 so, we can write the alpha n m for si system as well and I will give it you directly so, you will have alpha is equal to n to the power 0. 6 divided by s 0 to the power 0. 3 for the s I units that is alpha and of course,, and m remains the same which is the exponent which is 3 by 5.

So, you see that this way what we have done? Is we have taken a control volume approach of the overland flow in which the rain is falling infiltration is taking place and the overland flow or the surface flow is taking place with that we applied the continuity equation and the momentum equation then we combine the 2 to find out what will be the flow depth and velocity? This we have done for the both laminar and the turbulent cases. So, what we can do is, we can apply these equations and determine what the flow depth is? And velocity and then we can check whether the flow is laminar or turbulent and then we can modify our complications when we are applying these things.

So, this is as per the modeling of the overland flow is concerned whenever and when we are doing the modeling for the rainfall runoff process that is from the rainfall hyetograph we want to find out the direct runoff hydrograph. So, in that process we have to first calculate the effective rainfall hyetograph we have looked at that already in the last lecture today. We have looked at the first part of the rainfall runoff or the transformation of this effective rainfall hyetograph to direct runoff hydrograph how, by doing the modeling on the overland flow.

Now what happens? After that is on that overland flow these small channels would form and after that this q naught that intensity will come into some bigger channels. So, what we will do? Next are we will do the channel flow modeling in order to develop the whole model for the rainfall runoff process?

So, then we have the channel flow so, if we look at the expression not the expression the schematic here my sketching is not very good, but I would like to try none the less I am drawing very simple case. In which we are drawing a channel and this is your overland flow in it is side and there is some water in it so, what we have is? Let us say this unit width of the overland flow that is what we had? Considered remember this was l 0 that is your overland flow and what comes into this channel is your q 0 ok.

So, what you can do is, you can divide your overland flow into different strips of different either the unit widths or any other widths and then you can calculate this q 0 the equations we have already seen this q 0 will be coming into the channel. And then the channel flow will be going into the topographic direction or the favorable slopes so, let us say in this case your flow direction is let say this. So, you will have this q coming in here at different lengths and then you have the depth of flow this y here in the channel. And the velocity is v now our objective is to determine what will be the flow depth? In velocity in the channel given the discharge in this channel how can we do? That how can we determine the? Flow depth and velocity in a channel well we use the famous

Manning's equation we would see that this problem will not be very straight forward it will involve a little mathematical jugglery.

So, let us look at that first define various variable involved so, you have l c it is the length of the channel. This is the length of the channel and let us says that q naught of q 0 is the discharge per unit width from the overland flow into the channel. So, that what will be the total q in the channel the total q in the channel will be q 0 multiplied by the total length of the channel which is l c so, it would be q 0 times l c.

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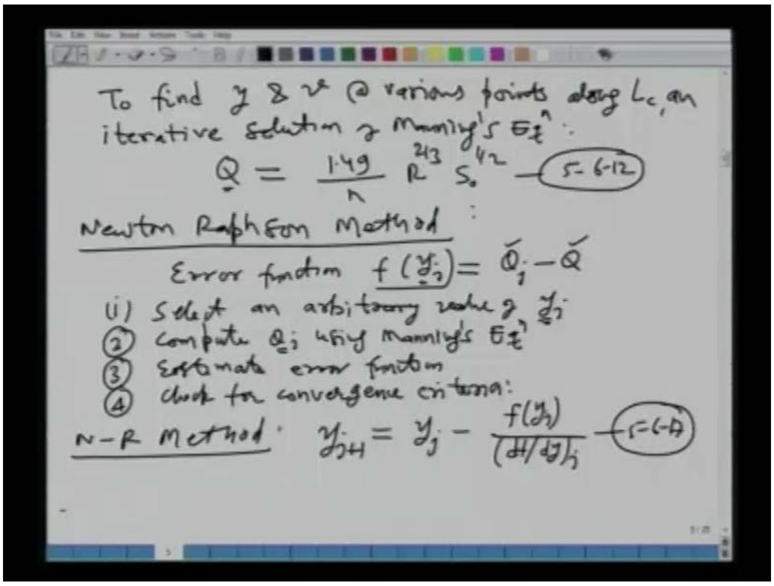

Then we move on then what we are doing? Is our objective is to find y that is the depth of flow and the velocity at various points along l c along the channel length. We need an iterative solution of the Manning's equation well what is the Manning's equation? You know that q is equal to 1. 4 9 we will do this for f p s system which can be easily extended to the s I system r 2 by 3 as half this is s 0 is equal to let us say s naught s 0 and this is my equation number 5. 6. 12

Now, were r is the hydraulic depth which is a by p which will depend upon the shape of the channel as we are talking about the natural in a flow in the natural environment so, it is not a very noise you know shape. It will not be a rectangle or a trapezoidal like we are the used to, but it can be any natural shape in which the both r and a and p will be function of the depth of flow. That is why this equation? Will be implicit in nature and it is non-linear so, we will have to use some iterative category.

When we talk of solving an implicit non-linear equation there is a very famous method which we can use or which is most popular or very efficient method which is called the Newton Raphson method. So, what we will do? Is we will apply the Newton Raphson method to solve for this equation. In the Newton Raphson method? We need to formulate a function for which you have to find out the root in this equation.

The function will be the error what is the error? In any iterative process you start with an initial gas corresponding to that initial gas you can calculate the error and then you try to minimize that error in this successive alterations so, our error function is which we try to minimize let us call this f as a function of y j y j is the depth of flow which we are trying to find out is equal to let us say q j q j is the value or the gas of your q at the end of the jth iteration minus q this q we already know we have already determined using the overland flow and this q is an iterative q which we are gassing based upon this value of by j so, in ideal conditions your y should be such that your q j is equal to q so, that your error is 0 .

So, that is our objective is to determine f of y using the Manning’s equation how do we do? That well the Newton Raphson method it involves the steps where what we do? Is you initially select an arbitrary value of your variable which you are trying to find out y j all right initially you get started and then once we know y j we can calculate q j from this equation from the Manning’s equation.

So, your second step is compute or estimate your q j using Manning’s equation once we have calculated q j the third step would be compute or estimate the error function what is the error function it is f of y j which is nothing but, q j as you have calculated and value of q you already know from our overland flow so, this will be able to give you the difference which is the error and then you just check for the convergence criteria that is to say you calculate the error at the end of the iteration if the error is acceptable you stop if error is not acceptable you start iterating again.

So, this is the step by step procedure of Newton Raphson method and what we will do? Is we will try to apply this in this particular case before we go to that I would like to look at the bases of the Newton Raphson method I am sure all of you have seen what is the Newton Raphson method? How it is applied? Then how it is actually developed all right what I will do? Is I will first give you the equation Newton Raphson methods equation and then we will see how it is actually derived on the basis on which is or the concept on which it is based as you know it gives you the gas for the next equation j plus one as what as the value of the variable at the previous iteration y j minus you are the value of the function at that iteration divided by what divided by the derivative the first derivative of that function at that iteration which is d f over d y in this case all right at the jth iteration and this is your 5 6 17 so, what I would like to do is I would like to derive this expression based on the concept of the Newton Raphson method.

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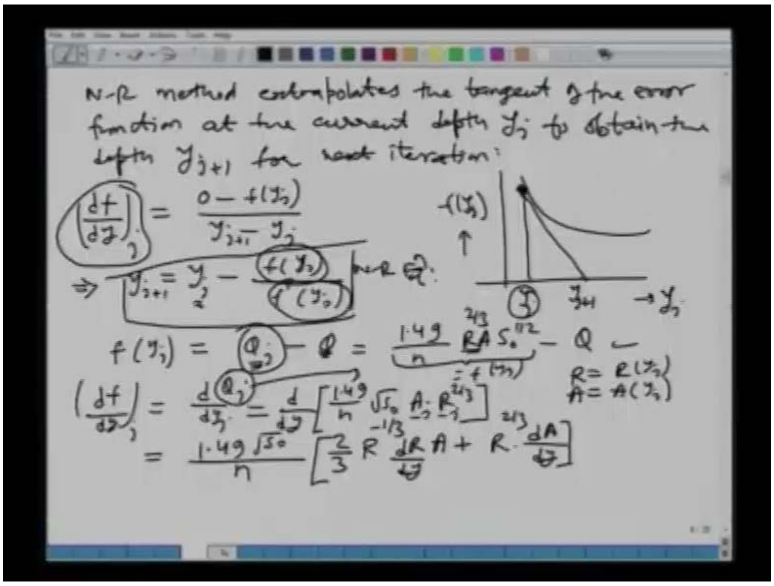

The bases of the Newton Raphson method is that it is or a Newton Raphson method what does it do? It extrapolates the tangent. We have a function an non-linear function you have a starting. And you take a tangent on that. of the error function in this case at the current depth y j current or current root of your equation. To obtain the depth at the next iteration which is y j plus one for next iteration so, if you look at it graphically what it means is that we have depth y j and there is some function non-linear function f of y j what is this f of y j we have already defined it is q j minus q in which q is a constant and q j depends upon the depth of flow y j. So, we are gassing this depth flow y j initially and then we are iterating until some convergence is reach so at any iteration we want to find out what is the next? Depth of flow or the next value of the root of this equation.

So, let us say that you have error function is something like this then let us say you your initial gas is this which is your y j. Then you take the tangent on it what is the? Basis of the Newton Raphson method at the current. At the current. You take the tangent and you see where it cuts the x axis let us say that is where it cuts and that is what will be? To the next .how does it come well let us look at that what we are doing? Is we are saying that the Newton Raphson method extrapolates the tangent of the error what is the equation? Of the tangent at that particular current. Well it is nothing but, the first derivation so, you should write the first derivative d f over d y at your jth iteration means at this. This is yours. On the curve and you want to write the first derivative equation of this line.

What would that be the first derivative of a line is nothing, but y 2 minus y one over x 2 minus x one so, what is? The y 2 minus y one what is the? Value of your f of y at the second. And the first. Well it is 0 minus f at y you see that y 2 minus y one that is the function value at the second. And the first. Divided by x 2 minus x one x is y here so, you have y j plus 1 minus y j. So, all you have done is, you have taken the slope at the current or at the current value of the root of the non-linear equation.

So, if you solve this equation this will essentially give you the method or the y j plus 1 is equal to y j minus f of y j over f prime of y j that f prime of y j is nothing but, this quantity this is your n r equation. You see that this Newton Raphson method is based on a very simple concept if the next gas or the value of your y at the next iteration is calculated using or by drawing the tangent at the current. To the curve and then finding out wherever it cuts your x axis.

So, with this basis this is our equation so, what we can do? Is we can start with any arbitrary value of y j calculate your function and the derivative of the function and that will give you the depth of flow at the next iteration and we keep on doing it. However we need to find out what is going? To be this function n f prime of the function because the function which we are dealing with this extremely complicated. So, let us look at this function which will be in the form of the Manning’s equation.

So, we had said already that what is? F of y j what is our function it is your q j minus q which is 1. 4 9 over n r 2 by 3 s one half minus q where capital q is the discharge coming from the overland flow which is constant and we have calculated that so, that is some number let us say 23 . 4 meter cube per second so, that does not depend upon depth of flow what depends is your gas so, this quantity is equal to function of y j.

So, what would? Then your d f over d y at j be from this equation what we need? To do is you need to differentiate this equation with respect to y and that will give you the f time so, if you did that what you have? Is d over d y j of your q j the derivative of this quantity with respect to y the derivative of this quantity with respect to y will vanish because this is a number so, all you have is this is a function of y so, it will be d q j over d y j is it clear.

So, now q j is given by this expression 1. 4 9 by n and so, on and that is where your r? Is a function of depth of flow so, if you did that then you have let me go back this is actually your velocity. So, I think I need to multiply this by area somewhere so, that is going to be your area. So, in this equation your r is function of y and also the area is you are some function of y j now you have a product of 2 things or a 2 functions which depend upon depth of flow.

So, what will be d q over d y? This will be equal to d over d y of maybe write it down 1. 4 9 over n under root s and then you have a j r j to the power 2 by 3 I have done is d over d y of q j and for q j we have put this value here that is your d j now what would that be? Well you can take the constants outside it will be 1. 4 9 slope is constant it is not changing as a function of depth of flow so, that comes out n is the Manning's coefficient that is also not changing.

So, that also comes out what remains is a j r j 2 by 3 so, it is a product of 2 functions you differentiating with respect to y so, let us do that it will be 2 by 3 please follow it carefully and if you cannot understand it just try to do it independently so, you have product of 2 functions the first derivative would be first 1 multiplied by the derivative of the second.

So, we are differentiating the r term first so, the differential of this quantity would be 2 by 3 r j to the power 2 by 3 minus 1 which is this times d r by d j is also there and then you already have the a here then plus you would have r to the power 2 by 3 times d a over d y see that carefully you have a j and r j to the power 2 by 3 it is a product of 2 functions and then you adjust differentiating it.

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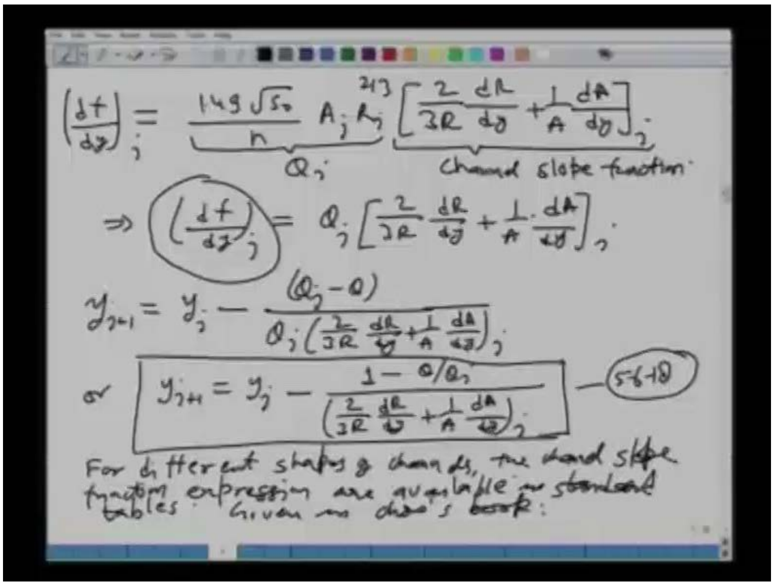

So, let us simplify this then let me move to the next page it would be equal to 1. 4 9 under root s 0 over n and then what you can do is? You can take this thing outside a j r j to the power 2 by 3 s common what will remain? Inside is 2 over 3 r of your d r over d y plus 1 over a because you have taken a and r outside. So, you will have to divide by these things so, you have d a over d y evaluate at j and this is equal to your d f over d y at the jth time step of this whole expression this is function of the depth of flow and it is a complicated function.

This is called the channel slope function. And we will come to that little later what is this quantity? This whole thing this is nothing, but q j so, then your equation for f prime is d f over d y and the jth iteration is equal to what q j times your channel slope function which is 2 over 3 r d r d y plus one over a d a d y at the jth iteration .

So, now we can do what we can do is, we can substitute to this quantity in your Newton Raphson method. We already have the function now we have the derivative of the function. So, we can use the Newton Raphson equation so, if you did that you will have y at j plus one then is equal to what y at j minus function f y j which is what you have? To define as q j minus q that is the function which is the error divided by f prime what is f prime? It is nothing but, q j we have just derived it here multiplied by the channel slope function which is 2 by 3 r d r d y plus 1 over a of your d a d y evaluated at jth iteration.

We can do some slight rearrangements here and simplify this to say that you have y at j plus one is equal to y at j minus you divide the numerator and the denominator by q j so, that you would have one minus q over q j this whole thing divided by 2 over 3 r d r d y plus 1 by a of your d a d y evaluated at the jth times step.

So, this I am going to number as 5 6 18 so, you see that this is a complicated expression we have derived for finding out the depth of flow in the channel using Newton Raphson method which is an iterative procedure at each iteration we need to compute this channel slope function this is the complicated expression we have said and this is a function of what this will depend up on the shape of the channel because it involves what the area of cross section? And also the hydraulic depth.

So, the hydraulic depth and the area of cross section will depend upon the shape. The value of this channel slope functions are given in a table in the book, but different shapes of the channel. So, far different shapes of the channels. The channel slope function expression or the equation actually is available in standard tables and this is given in your in chow's book in this chapter 5 so, once we have calculated this depth of flow we can easily calculate the velocity of flow in the channel by applying the Manning's equation.

So, with that I think? I would like to stop at this moment of time because I am at a good stopping. So, what we have done today? Is we have looked at the modeling of the overland flow and the channel flow and the objective is to find out both flow depth and velocity in the overland flow and in the channel flow and in the process.

We have carried out the complete modeling of the rainfall runoff process in tomorrow's class what we would do? Is we would look at another method of carrying out the rainfall runoff the model which is using, what is called the time area concept? using the time area concept that is how we can develop the iso-crones in a catchment and how we can calculate the direct runoff hydrograph from a time area diagram that will be one thing and then we will look would at an example of the channel slope function not the channel slope function the example of the application of the time area diagram. Once we have done that we would also look at a few topographic characteristics of a catchment so, these are the few things remaining in this chapter and we would try to finish it tomorrow.

Thank you.