Advanced Hydrology
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Lecture – 2

Good morning and welcome to the second lecture of this video course on advanced hydrology. First of all I would like to go over what we did yesterday in the first lecture. First thing we did was we looked at the overview of the course in which we looked at the syllabus or the contents. And then we also looked at the class wise or the hour wise distribution of the topics. After that we started with defining what is hydrology in which we said hydrology is a science of water or it is the system in which the circulation distribution and occurrence of water is studied in various parts of the earth. Then we looked at the hydrologic cycle. It is a concept in which the water is constantly moving in various parts of the earth there many components of hydrologic cycle as we saw in filtration, evaporation, surface flow, ground water flow, condensation, precipitation in various forms and so on. And all of them are interconnected with each other. The interconnection and dependence on various variables is highly complex that is why it is very important to understand and model this hydrologic cycle.

Then we looked at the continuity equation or the water balance equation. This can be done either for a catchment or for a lake or for any other small component of the hydrologic cycle. Then we went and looked at the world water Quantities in which we saw there is how much water on the earth how much it is useful and so on. Then we moved on and we defined the concept of what is called as residence time. We also looked at an example of calculating residence time for global reverse, for doing this we use the data or the estimations based on the United Nations approximations from our textbooks. Towards the end of the first lecture we looked at the lake water budget or an example of the simple application of continuity equation to see how we can calculate one of the different variables that is involved in the lake water budget. What I would like to do today is take up that example and extend it.

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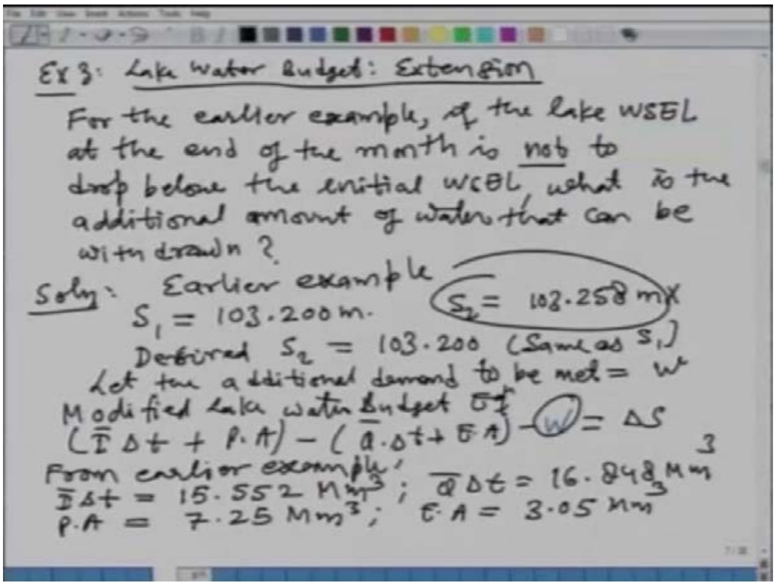

So, what we will do is we will look at the extended example, and I will call it 3 lake water budget. So, the variation which we are going to use for this example is as follows; for the earlier example, if the lake water surface elevation at the end of the month is not to drop below the initial level or water surface elevation in the lake, then what is the additional amount of water that can be withdrawn from the lake? So, problem statement is that in the previous example we found out what should be the water surface elevation at the end of the month in the lake. Now we have modified this problem slightly and we are saying that if the water surface elevation at the end of the month is not to drop below the initial level. Then how much more water or how much additional water can be withdrawn which can be used for other purposes.

For example, there may be some demand in the city or additional demand in that month which needs to be met. So, looking at the solution of this, we take a few numbers from the earlier example, if you remember the initial lake water surface elevation was 103.2 meters. And in that example the S 2 was 103.258 I think meters. However in this example it desired water surface elevation S 2 we are saying is not to drop. And we want to maximize the utilization or maximize the water that can be additional water that can be withdrawn. So, the conservative estimate for S 2 is we take the initial level which is 103.200 to discuss for the earlier example; we are not considering this which is same as S 1. So, we say that let the additional demand to be met the, we denote this by a variable called W. So, in the last example we looked at the water budget equation for the lake.

Now, we have an additional variable W or additional water demand that needs to be met what you think that the water budget equation will be, will it get modified or it will be same.

As we would see that it will get slightly modified. So, we first need to write the modified lake water budget equation in which this W should somehow appear that is the earlier equation was like this in which we have some inflows in the form of upstream flow and the precipitation times the area minus they where some outflows in the form of Q delta t plus the evaporation times the area and then minus I am going to write W is equal to delta S. You see that there is an additional variable which has appeared in this modified water budget equation in the form of the additional water that can be released from the lake. Now, we take the data from the earlier example in terms of I bar delta t P A n and all those things that is the inflow was if we go back to your notes it was 15.552 million meter cube, the Q bar delta t was 16.848 million meter cube. The precipitation input was 7.25 million meter cube, and the evaporation losses from the lake were calculated as 3.05 million meter cube.

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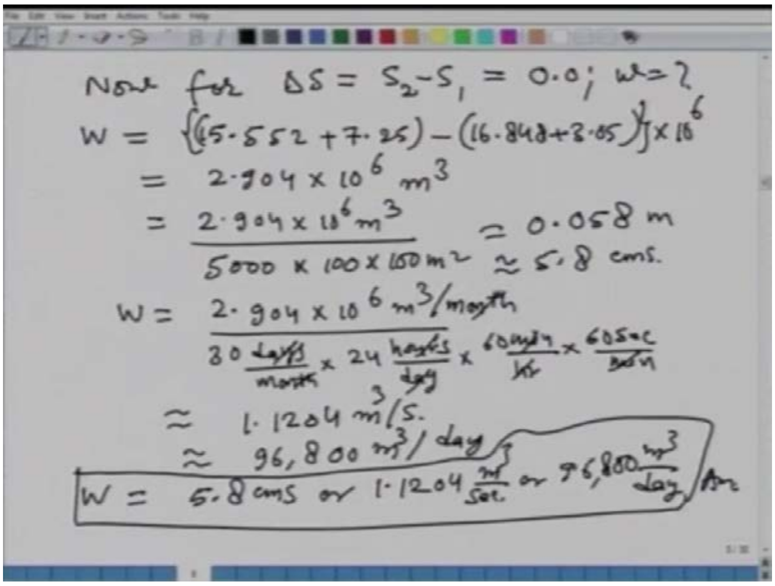

So moving on, now for this particular example for delta S is equal to S 2 minus S 1 is equal to 0, what is the value of W is that is what we have to find. How we do that? We just use that simple continuity equation and find W as put the values 15.552 plus 7.25 minus 16.848 plus 3.05 times 10 to the power 6 delta S disappears, because it is value is

0. So, this will come out to be 2.904 times 10 to the power 6 meter cube. You can convert it into some other appropriate units divided by 5000 hectares was the area of the lake. So, we divide by the area of the lake in square meters. So, the answer we would get is approximately 0.058 meters or it would be about 5.8 centimeters. This number which we have got is over among. So, this is a linear value or the length scale value spread over the whole area of the lake in the whole month. You can convert it into a way or withdrawal rate average rate from the lake as follows we divide it by. So, this is meter cube per month and you have 30 days in a month multiplied by 24 hours in a day then 60 minutes in an hour and 60 seconds in a minute.

So, this month will cancel out, the days will cancel out, the hours and minutes will cancel out. So, what we will have is the meter cube per second. So, this answer will come out to be approximately 1.1204 meter cube per second. And we can convert it into meter cube per day and it would be therefore, the answer is the additional water that can be withdrawn is 5.8 centimeters or 1.1204 meter cube per second or 96800 meter cube per day or we can convert this into any other suitable unit which is desired like liters per minute or liters per second or anything like that. So, this way we see that using a very simple application of the continuity equation or the water budget equation we can play around with different variables, and find out what are the or how we can meet the requirements or additional requirements. This water balance equation is very useful in managing the existing water resources like.

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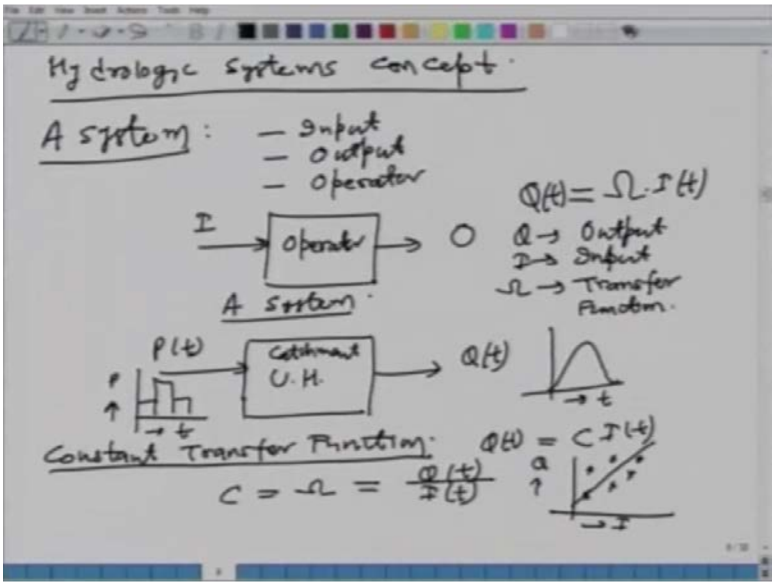

Now, what we will do is we will move away slightly from this. And we will look at what is a hydrologic systems concept. Before we go to any hydrologic system it is important to understand what is a system? First let us look at a system. How do we define a system or how what do we understand by a system? We have various physical natural processes that occur in the nature. It may be related to civil engineering, it may be related to chemical engineering, electrical engineering science or any other field.

So, there is something happening in the nature which may have certain inputs the system will work upon those inputs and produce some outputs. So, the essential components of the system are; that there is some input there needs to be some output and the system is operating upon those input and they produce the output by certain operator or a certain processes which are ongoing in that system. So, we will call it some operator. So, this is what is actually happening. So, if we represent this, these sub components in a schematic diagram, then you have the input to the system this is your, that is a operator or the system operator and it will produce some output.

Now, this I and O, this can be either be a scalar or vector there can be a single input there can be more than one input, there can be only one output and there can be more than one output. So, the system can be very simple or it can be very complex also. And it is up to us how we want to define that particular system. So, this is the schematic of any system in the very simplified or broad terms. What is a hydrologic system? It is nothing but a system in which the input and output variables involved are hydrologic variables. Can you think of any example of hydrologic system? Yes what we can have is rainfall as the input. And this rainfall can be varying with respective time as we all know this is time versus rainfall. And then the system is your catchment, this your catchment on which the rain is falling. And then what comes out of the catchment at the outlet is your Q or the discharge or flow as a function of time which as we know looks like this in which the x axis represents the time and the y axis we have Q.

We can model this system using many approaches and one such approach I am sure all of you have looked at is what is called a unit hydrograph. We will come to this little later in this course as you will know what is a unit hydrograph? It is an operator it is it is a process or it is something which operates on the rainfall, and in this case it is effective rainfall. So, if we dump certain amount of effective rainfall into a catchment, then it will give you the direction of hydrograph. So, unit hydrograph can be considered as some operator or some hydrologic system model. So, if we represent the general equation of a system it would be something like let us say Q. And it can be function of time I am being general here is some operator times I of t where your Q is the output or O is the output I is the input and this omega is called the transfer function. So, you see what we do is when we work with the systems approaches you have a natural physical process that is occurring in the nature. We try to simulate that physical process using mathematical equations or some other type of you now system in which we consider the input we consider the laws of physics and we calculate the output so this is called a systems concept.

In which the main or important task is to determine that operator omega depending upon the utility or the practical usage of that particular system model this operator or the transfer function operator can have different forms. Now, we will look at some simple forms of this operator. First one we will look at is what is called a constant transfer function. It is the most simple form of a transfer function or an operator on any system as the name suggests it is constant does not change with time, does not change with space quick easy to determine or easy to calibrate that kind of model. So, you have a cube or output is equal to some constant times your input. So, once we determine the value of C we have determined the transfer function operator for that system. So, that is to say if the transfer function is constant and we say that this C is nothing but the omega which would be Q t over this I.

Can you think any example of a transfer function operator of this kind? In your undergraduate hydrology course you may studied many empirical equations of rainfall and of models like first class model or English formula. There are many formulas in which the output or the runoff is a linear function of the input or the rainfall. So, that is the example of some of the constant transfer function operators which operate on a catchment So, it can be simply represented as I and Q and since the transfer function operator is constant it would be a linear in nature in this case. So, all we need to is recollect the data of input and output and then we try to just fit a straight line. And we would be able to get the equation of this line. And in this case it will be passing through the origin. So, we will have constant transfer function.

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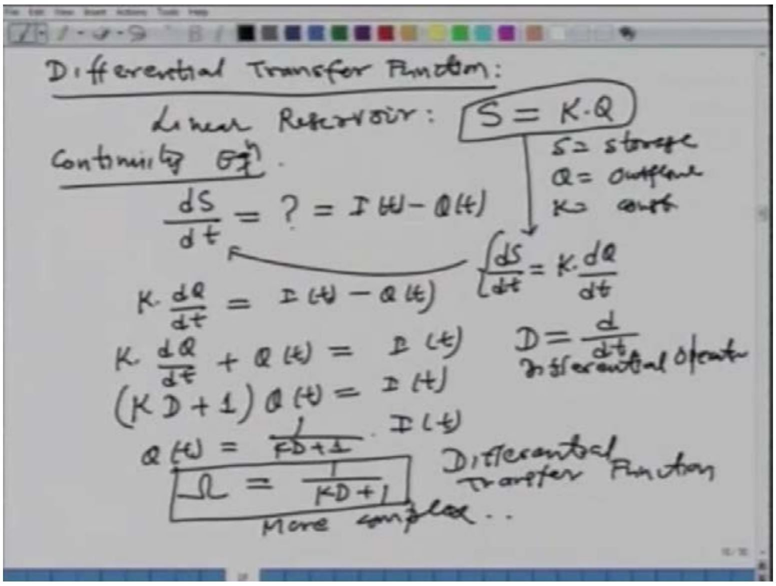

Moving on next we are going to look at what is called a differential operator or differential transfer function. Now, this differential transfer function is slightly more complex than what we had just looked at what we will do is we will try derive the expression of the transfer function operator in the differential form for the simplest cases. I am not sure if you know the concept of what is called a linear reservoir. Does anyone of you know, what is a linear reservoir in hydrology? Linear reservoir is something in which we assume that the storage from in the catchment is a linear function of the output. In other terms or in mathematical terms your storage in the catchment can be defined as linearly related in the output where S is the storage and Q is the output and K is some constant for that particular catchment.

As we know the catchment system is highly nonlinear complex in nature. However, it is difficult to obtain the transfer function operator for that considering this as non linear and more complex. So, what we can do is to get started, we can assume the catchment behavior to be linear using the concept of what is called a linear reservoir. And then let us see how the system will behave and the kind of results we get using the linear reservoir are extremely useful for most practical purposes. So, what we will do is we will use the basic continuity equation to find out what will be the form of the transfer function operator for this case? What is the continuity equation? As we had seen earlier the change in storages what it is nothing but the input minus output? Now, what we do is using our assumption of linear reservoir we find out what is d S by d t. in this your k is constant. So, it would be rate of change of their outflow respective time. And what we do is then we put the value of d S by d t from here in to this equation. Once we do that what we will have is K d Q over d t is equal to I t minus Q t. We can simply this further taking the Q on the other side and I on the right hand side.

Now I define d as a differential operator d over d t. I am sure most of you would be familiar with this mathematical operator, and it is called the differential operator and corresponding to that we are trying to find the differential transfer function. So, what I can do is then you can take the Q outside and then say it is K d plus 1 acting on Q is equal to I, is it clear? What we are doing is we are taking the Q out from here and applying what is remaining that is k d times Q t plus 1 time Q t that is the equation we had above and all that will be equal to the input in the system. From this it is very easy to see that your Q t is nothing but one over k d plus times I t. So, that gives me the format form of this omega or the transfer function. So, this is your differential transfer function and this is for the linear reservoir for which we had assumed this s is equal to k times D.

So, we see that we can use the continuity equation momentum equation and all those basic equations and combine this with our assumptions or simplification of a system and determine the format or equation of the transfer function operator. In this particular case it was very simple. But as you can imagine we can have many more transfer functions which can be more complex in. So, this was about the systems concept in which we looked at the hydrologic systems concept. The next thing we are going to do is we will look at the classification of hydrologic models with this systems concept. As you can see we can have many kinds of models and researchers. And people in the past have developed many kinds of models or hydrologic models using the data.

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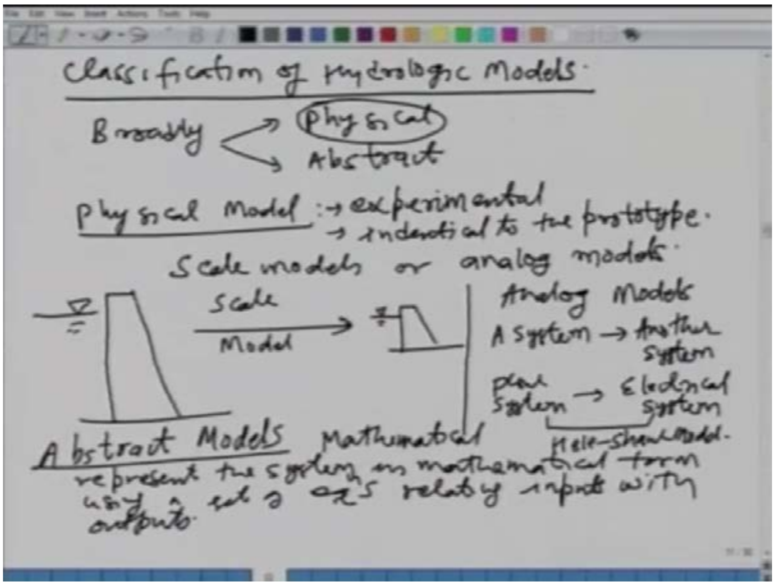

Now, the next step we are going to look at is the classification of these models that is classification of hydrologic models. When we classify any concept, it can be done in many ways depending upon the angle which we look at that particular concept. So, the classification of hydrologic models also we can do in many ways, we will look at 2 different ways. Broadly, first of all the hydrologic models can be classified or grouped into two kinds; one is a physical model and other is an abstract model. You understand what are the physical models and the abstract models. In a physical model what do we do; a physical model is something in which we actually simulate the natural physical process in the lab or in the field or somehow. So, you have a prototype or a full scale model. So, the physical model can either be the full scale or it can be scale model or the small model in the laboratory in which we have the same inputs we have the same outputs, we have the same forces acting, we are just simulating the physical behavior of that particular system.

So, as per as the physical models are concerned they are experimental in nature and when we carry out these experiments what we want to make sure is that their identical as per their behavior is concerned to the prototype or the actual systems the physical systems can be either a scale models or the analog models. I think I am sure almost that all of you would know what is a scale model? A scale model is something in which we either try to reduce the scale of the overall dimensions of the actual system or we try to magnify depending upon the kind of system. For example, if you want to study the dam breaks flow analysis. So, you have a big dam what happens if the dam breaks? So, lots of water will gush towards the downstream we cannot study this kind of thing in the field, because we cannot afford to break a dam and then see what is going to happen downstream, because there will be lot of people there'll be fields and lot of economic value involved. So, what we do is we try to simulate this dam break flow model by constructing a very small dam in a lab it is called as scale model. And then we try to ensure that certain non dimensional numbers depending upon which forces are important are same. So, I will not go into the too much details of the actual scaling of the modeling, but this is something called your scale model.

In this case we are reducing the scale however, we may be studying the micro organism not related to hydrology, but some from scale we want to magnify the overall scale of the system so that we can study it at a large. So, this about a scale models, what are analog models? By the way analog models are not very popular these days. The experimental models are still popular or used rather to understand new phenomena and you know various types of processes. The analog models have not become very popular. However, it is important understand what they are? What is done in the analog models is certain kind of analogy is maintained. That is to say what we do is you have your system or a prototype which behaves in a similar manner to another system. So, what we do is we try to simulate the properties of a particular system by using a completely different or another system. For example, in hydrology we have a flow system is replaced by that is a electrical system. And a famous example of an analog model is Hele Shaw model.

Again as I say it is not going to the details of these things, but it is important to understand that in an analog model. We try to use certain kind of simile or analogy to another system. And then we measure various values of the physical variables involved and then we try to understand how the things would behave. Now, this was about the physical models, 2 types of physical models. Now, coming to the abstract models, what is the abstract model? As the name suggests in this type of models we are not actually simulating the behavior of the system in the lab or using the actual system. What we are trying to do is we try to simulate the behavior of the system using mathematical or the abstract form it can be either on a computer or it can be you know done manually. So, what we do is it is a mathematical form of a model or abstract. So, what we are doing is we represent the system in mathematical form using a set of equations etcetera, which relates the inputs with the output of the system. Here the objective in any kind of hydrologic model is to establish the relationship between various inputs and outputs alright. Either we do it experimentally or we do some computational experiment or we use the knowledge of our mathematics in the laws of physics. So, this was one angle with which we will look and we classify the hydrologic models.

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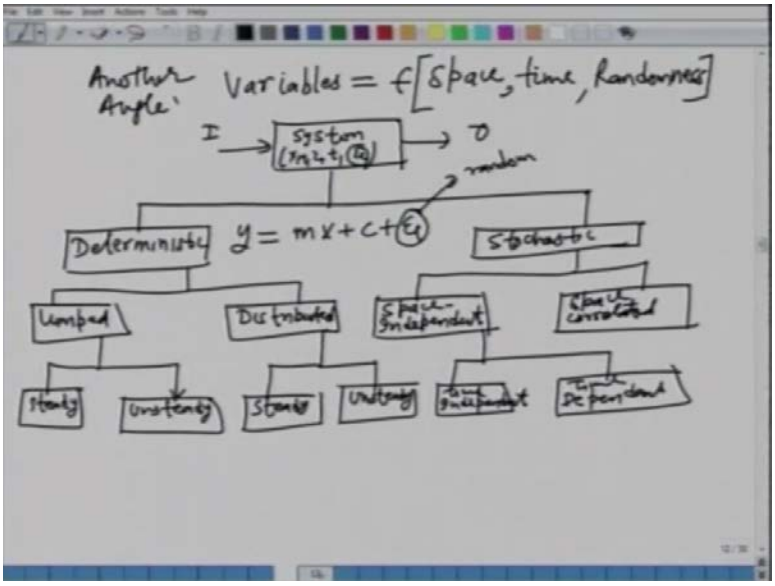

Another way to do that is or another point of view is to realize the fact that all the hydrologic variables or variables for that matter are function of certain basic connections. That is to say the hydrologic variables are function of space time and randomness. The space can be either one dimension or it can have a one dimensional flow or it can be in two dimensions, 2 dimensional flow or we can have a 3 dimensional flow. We can consider the space as a 3 d then time is another basic dimension and the randomness. So, what we do is we will do it in a schematic then we go our each and every type of model. We have a system which is again function of your x y z or time or randomness alright. In this system you have input it can be a vector and you have output it can also be a vector. And what we will do is we will care of each and every dimension and then classify the hydrologic models.

So, if we take care of this randomness then with respect to randomness, a hydrologic model can be defined as or classified as deterministic or stochastic. As we said earlier that the hydrologic variables that is rainfall flow soil moisture profile etcetera they are function of space they vary from one place to other they also vary with respect to time and they are random cause. There are lots of uncertainties involved in them that is to say their values may be different at the same place for a same condition for different time. So, if take care this randomness dimension then the models can be classified as either deterministic or stochastic. So, what is a deterministic model? A deterministic model is one in which we use the laws of physics. And we will ignore the uncertainty or randomness involved in the hydrologic variables.

Another way to look at the deterministic model is that let us say you have a model like y is equal to m x plus C. So, x is an input, it may be a vector and y is the output. So, once you calibrate the model calibration means finding out the values of this and m and C, once you have done that the output calculated from a deterministic model will always be same as long as the input is same. That is to say if the value of x is some particular value let us say 2.3, the output will always be same. However, in the case of stochastic model, because there are uncertainties involved the output will never same for the same input you may have different output. So, what we have is we add another component epsilon and we say that this your random component. So, for the same value of x that is a 2.3 centimeters of rainfall every time. You use this model this stochastic model, because of this random component you will have different output or different y.

And this epsilon or random component can be model using some random number generator or various schemes which I do not want to go to at this point of time. So, I hope the difference between a deterministic model and a stochastic model is clear. Then we go further and we further classify these deterministic models as again 2 types which is either lumped or distributed. What is the difference between a lump model and a distributed model? Well in a lump model we ignore the spatial variation involved in the variables. And in a distributed model we account for the special variations. So, when we are defining or classifying the models as lumped and distributed we are taking care of the special variation right. As we know that the rainfall varies from place to place. We may have seen many times that it rains in a particular area at the same time. But you go you know 5 kilometers away it does not rain there or it may be raining very heavily at some other places. So, there are lots of you know, spatial variations.

But in a lump model, we ignore all those and what we do is we take the average rainfall, and we lump it at particular point in the catchment, and we say that the rainfall is uniform overall the catchment. And then we find out the output. However in a distributed model what we do is we scatter the rain gauges we find out the values of rainfalls at different places in a catchment and then we account for those variations and we divide the overall basin into sub basins and we do the flood routing and we calculate the overall response from the catchment using a distributed approach. So, the difference between lump and distributed is that we take care of the space dimension in a distributed model and we ignore it in case of a lump model. Similarly, when we talk of stochastic models they can also be space co related or a space independent. Space independent or space co related. If the same concept in which if we have a stochastic model in which the variables are independent with respect to space.

Then it is a space independent model, but if we have stochastic model in which the variables are co related with respect to space, then we have space co related model. Now, only thing that remains is the time dimension and with respect to that we have as you all know either a steady model or unsteady model. Now, sure all of you know what is the difference between a steady model and an unsteady model steady model? We ignore the variation of the variables with respect to time and in an unsteady model we try to account for the variations with respect to time. So, we can have steady lump model or we can have unsteady lump model. Similarly, as per as the distributed models are concerned we can have the same classifications. So, we can have steady and unsteady distributed models and similarly, in the stochastic side we can have time independent model or time dependent model.

So, you can see we have various combinations here we can have an unsteady lumped deterministic model or you can have a steady distributed deterministic model, you can have an unsteady distributed model and so on. So, you see that we can have stochastic models which can we space independent which where you know classifieds as time independent or time co related models. Similarly, on right hand side we have space co related models which can also be classified as either time independent or time co-related models. So, this way we can have any kind of combination for example, time co related, space co-related, stochastic model or unsteady distributed deterministic models and so on. So, this way we have this tree in which all kinds of combinations can be considered. So, what we will do next is we will look at some of these examples.

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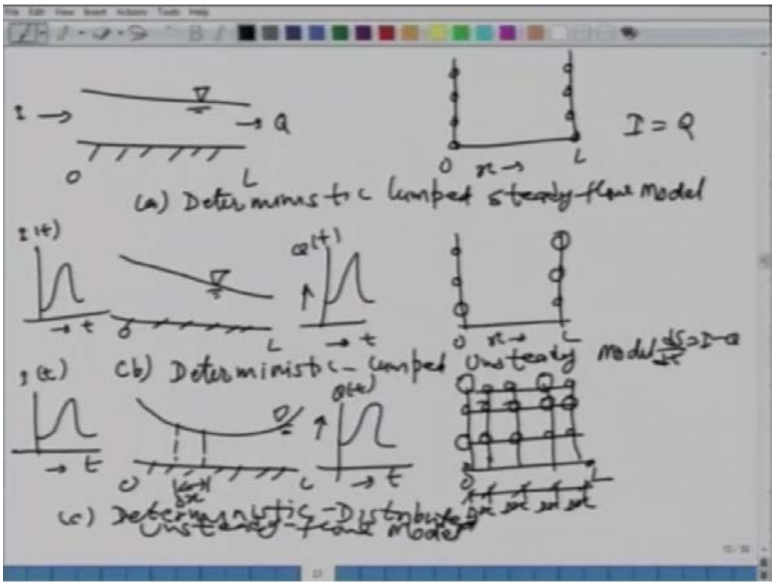

This is a case of a channel flow, this is a back slope from 0 to l. there is some flow I going in and Some Q going out. And if we represent this on a scale from 0 to l in the x direction or this is distance x and on the vertical axis we represent time the circles which we looking at represent the magnitude of the discharge or magnitude of a hydrologic variable. So, we say that the magnitude of the hydrologic variable is constant; this is in case of a deterministic lumped Steady flow model in which I will be actually equal to Q. So, things are lumped or they are not allowed to vary with respect to space and things are steady. That is to say that the discharges of the hydrologic variable values are not changing with respect to time also. We can have another model to see the channel again in which we have the water surface elevation like this, in which your I can be varying with respect to time. This is your 2 l and the output and also be varying with respect to time.

And if you look at the output from the system between 0 and l you have things changing as a function of time the diameters of the circle represent the magnitude. So, it is changing with respect to time that is to say then this case of a deterministic lump on steady model in which as we know the d S by d t is equal to I minus Q. So things are allowed to change with respect to time, but not space. So, it is still a lump model, but it is unsteady in nature. However, we can have the most general type of model in which the input can be varying with respect to time that is a inflow hydrograph, and output also is a function of time, outflow hydrograph. And water surface elevation maybe varying with respect to space also. So, the system delta x so what we do is we (( )) the space dimension and we compute the variations in your depth. And then if you look at the output from the system, you would have between 0 to l there allowing the variables to change with respect to space also here.

So, you have delta x, delta x, delta x and this is the time dimension. So, you see that things are changing with respect to space the diameters of the circle I am putting here are varying with respect space and time as well. This may be different; this may be different and so on. So, this a model which is the most general or we will say that this is a deterministic, distributed, distributed unsteady flow model. In which we are modeling the changes with respect to time as well as the space. However, we are not taken care of the randomness dimension here. So, if we look at the stochastic models they can also be classified in a similar manner.

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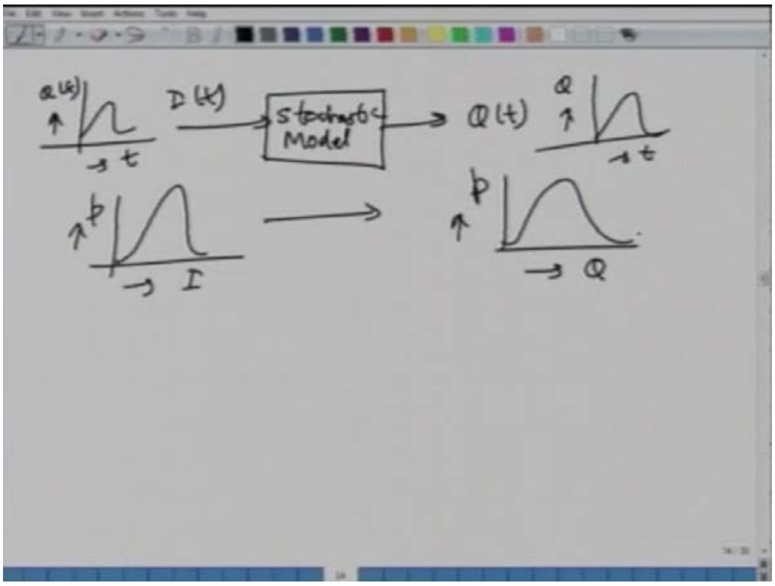

We will not go into the details, but I will just give you a concept. So, if you have a stochastic model in which you have some input and some output as we know it has this curve and this input is also like this. Now, what we do in stochastic model is we say that we are not very sure about the input. That is to say if we have a rainfall on a model you say what is going to be the value of rainfall. Tomorrow we use some modeling and find out that it is 2.3 centimeters however, we are not sure that it is going to be 2.3 centimeters with 100 percent accuracy. We may be able to say that we are 90 percent sure that rainfall will be less than 2.3 or we may be able to say that we are 50 percent sure that the value of rainfall will be that much. So, what we do is we assign a probability to the magnitude of your inputs.

And depending upon the different magnitudes and probabilities we generate a probabilistic distribution. That is to say you have a value of this is p is probability and I is your input. So, for each value of I, you have some or you associate certain probability and similarly, what will be able to this stochastic model is generate the output Q with certain probability distribution. So, you do not have a single output, but you have multiple outputs or a probability distribution of your outputs in a stochastic model. That way this type of information is very helpful in analyzing many water resource systems in which the uncertainty or the reliability or the probabilistic is very necessary. So, with this I think I would like to stop at this point of time the second lecture. And in the next lecture, we will start with the derivation of the Reynolds's transport theorem which is the mother of all the basic 3 equations.