

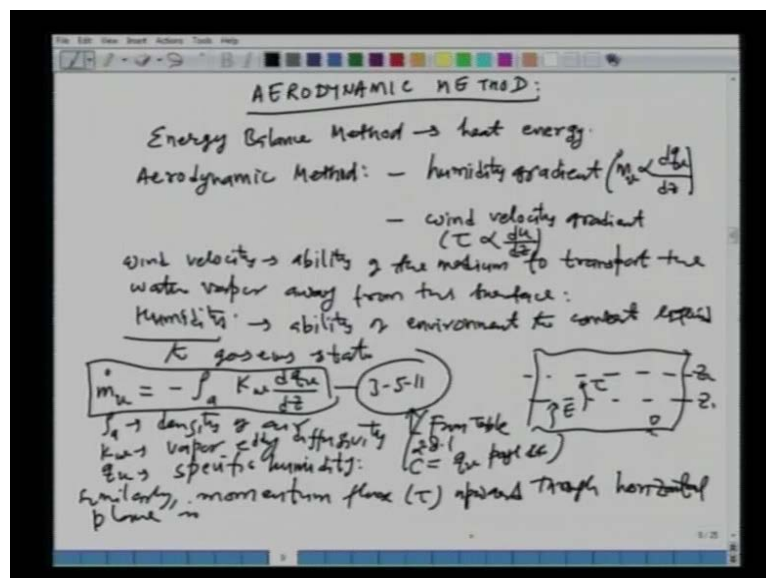
Advanced Hydrology
Prof. Dr. Ashu Jain
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 17

Good morning friends and welcome to the next class of this video course on Advanced Hydrology. We have been looking at the chapter on atmospheric hydrology in which we have already looked at the formation of rainfall and you know various types of models in which we try to find out what is the rainfall intensity coming out of a thunderstorm strength and what is the average rainfall special averaging and so on.

In the last class we had looked at estimation of evaporation using energy balance method, which is one of the analytical methods, which we had said that we would look at in this course. Towards the end of the last class we looked at couple of empirical methods, Rowher's method and Meyer's method and then we said that these two methods are based on the aerodynamic method, which we would look at in the next class. So, today we would start the lecture by looking at what is called the aerodynamic method.

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So, if we come here, aerodynamic method, we had seen earlier, that the energy balance method uses two equations, two basic laws, that is, continuity equation and the energy

equation. And we saw, that the energy balance method for estimation of evaporation accounts for one major factor, which affects the evaporation and that, must be heat energy. However, there are many other factors or physical factors or hydrologic variables that affect the evaporation, alright, from a water body.

The aerodynamic method accounts for two such other factors, what are those two, well we would look at them. Let me first say, that the energy balance method uses the heat energy and the aerodynamic method, which we will see today, is based on two factors, one of them is the humidity gradient, alright. What is humidity gradient? We had seen earlier, that there is a deficit of vapor pressure in the atmosphere, alright. So, depending upon the relative humidity in the atmosphere you have certain vapor pressure and corresponding to the existing climatic conditions, that is, temperature, you have saturation vapor pressure. So, whatever is the difference, alright, that is what drives the movement of water from the water body into the atmosphere.

So, that is one factor, which is the humidity gradient. As we go up how the humidity changes as a function of height that is number one, and the other one is the wind velocity gradient, either the wind velocity or we can say wind velocity gradient, how the wind is changing as a function of depth. So, these are the two basic driving forces for the formation of evaporation and then removal of the vapor, alright, which has, you know, evaporated away from the water body. So, this, this wind velocity actually is what? It is the ability of the medium; it is the ability of the medium to transport the vapor or the water vapor away from the, away from the surface or the water surface, and how about the humidity?

Humidity alright dictates the ability of the environment, I can say, or the climate to convert liquid to gaseous state. We assume, that the enough heat energy is available given the condition, that you have the heat beating up, then how much will be the conversion from liquid to gas will depend upon the humidity, which is existing in the environment or in the atmosphere. So, these are the two factors on which we would base our aerodynamic method.

Now, what is this humidity gradient, alright, which will dictate what is called your m_v dot? m_v dot is what is that we call is the vapor flux, it is directly proportional to d of your q_v over dz , where m_v dot is a vapor flux going upwards and humidity gradient is

represented by dq_v over dz . Similarly, the wind velocity will dictate what is called the momentum flux τ and it will be directly proportional to d over dz . So, these are the two basic, you know, principles or laws of physics, which we are going to use in developing the aerodynamic method equations for evaporation.

Now, before we go to that let me just try to explain you schematically what is happening. Let us say, that this is your water body or the water surface close to the ground, then what we do is, we take any two cross-sections, let us say z_1 and z_2 , just above the water surface body. Then what is actually happening is water is getting evaporated, alright. This is evaporation, alright, due to the heat energy and the humidity gradient, the water gets evaporated into the gaseous ways and then because of the ability of the wind or the medium to transport that evaporated water away from the water body, you have some τ or the momentum flux taking place and that is removed from the effect of the wind. So, what we will do is we will write, try to write this expression, alright.

So, the first one is we will say $m v \dot{}$ is equal to negative $\rho_a K_w dq_v$ or dq_v over dz and I am going to number this equation as 3.5.11. Then what is ρ_a ? It is the density of the air; K_w is what is called vapor eddy diffusivity. It is a constant, which depends upon the environment, which dictates how much vapor flux would be there. And then q_v as you all know is your nothing but the specific humidity.

Now, this equation, alright, I am taking from, from table 2.8.1 from chapter 2, alright, and this was part of your reading assignment in which you were supposed to look at some transport processes. Remember the convection, conduction and radiation, I said that the remaining things you go through on your own and there is a table, please look at. So, what we are doing is, we are drawing our knowledge from some other things we have seen earlier. So, this is the equation, which I have taken from table 2.8.1 and in that equation the concentration c has been replaced by q_v , alright, and it is given on page 44 or those of you who would like to verify this can go to page 44 of the (()) book. So, this is the basic equation, which will govern the vapor flow rate depending upon the humidity gradient. Similarly, we will write or we say, similarly the momentum flux τ upward through the plane or through any horizontal plane is given by...

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$$\tau = \rho K_m \frac{du}{dz} \quad \text{--- 3-5-12}$$

$$u \rightarrow \text{wind velocity @ height } z$$

$$K_m \rightarrow \text{momentum diffusivity or eddy diffusivity } (L^2/t)$$

Now, let us consider two horizontal planes @ heights z_1 & z_2 such that they are sufficiently close to each other.

$$\frac{dq_x}{dz} = \frac{q_{x2} - q_{x1}}{z_2 - z_1}, \quad \frac{du}{dz} = \frac{u_2 - u_1}{z_2 - z_1}$$

$$\Rightarrow \frac{\mu_x}{\tau} = - \frac{K_m (q_{x2} - q_{x1})}{K_m (u_2 - u_1)}$$

$$\mu_x = \tau \frac{K_m (q_{x2} - q_{x1})}{K_m (u_2 - u_1)} \quad \text{--- 3-5-13}$$

Tau is equal to rho a K m and du over dZ. We always said, that tau is directly proportional to du dZ, alright and let us say, this is 3.5.12, alright, and in which your u is the wind velocity at height Z. And what we will take is, we will take two heights, Z 1 and Z 2, and we will take the derivative of this wind velocity and humidity gradient and then try to derive this expression. K m is a constant in this equation, which is called the momentum diffusivity. It is the property of the medium or eddy diffusivity and as earlier, its units are L square over t.

Now, what we do is now let us consider two horizontal planes at heights Z 1 and Z 2 such that they are sufficiently close to each other; sufficiently close to each other. Why do we make that assumption? Well, the advantage is that whenever you have a non-linear relationship and you consider two points, alright, which are very close to each other, within those two regions or two points you can assume the relationship to be linear.

So, what we are doing is, we are (())-wise linearizing the problem, alright. So, we can consider the things to be linear within those two, so Z 1 and Z 2 are close to each other. So, we can write the expression for the gradients using the linear assumption, so that what you can do with n is d of q v over dZ is equal to q v2 q v1 over Z 2 minus Z 1. And similarly, your velocity gradient will be equal to u 2 minus u 1 over Z 2 minus Z 1.

Therefore, if I did, I use these two expressions in 3.5.11 and I divide that by 3.5.12. So, what I do is, I use this definitions of the gradient into the equations 3.5.11 and then 12

and divide the two, that will give me $m \dot{v}$ over τ . These are the two equations, is equal to, ρa would cancel out, you have $K w$ over $K m$ times, Z_2 minus Z_1 will also cancel out, so you would have $q v_2$ minus $q v_1$ divided by u_2 minus u_1 , or you can simplify this and find out the expression for $m \dot{v}$, that is where the evaporation is hidden, is equal to τ times $K w$ over $K m$. Take care of the minus sign and then you will have $q v_1$ minus $q v_2$, alright, divided by u_2 minus u_1 , slightly arrangement of the above equation this we will call 3.5.13.

So, this is the equation, which gives us the $m \dot{v}$ in terms of the $q v$, that is the humidity at cross-section 1 and 2 and also the wind velocity at cross-sections 1 and 2, and $K w$ and $K m$ are constants whose values can be calibrated or estimated somehow. However, there is one quantity on the right hand side of this equation, which is still unknown, alright, which is this τ . How do we find this τ ? So, we still have to do something, you know, some jugglery here, mathematical manipulations to determine or estimate this value of this momentum flux τ .

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For τ we will use wind velocity in boundary layer (50m):

$$\frac{u}{u_*} = \frac{1}{k} \ln\left(\frac{z}{z_0}\right) \quad \text{--- 3.5.14 or (8.2)}$$

$u_* = \text{shear velocity} = \sqrt{\tau/\rho}$
 $k = \text{von Karman constant} (\approx 0.4)$
 $z_0 = \text{roughness height of surface (Table 2.8.2)}$

3.5.14 $\Rightarrow (u_2 - u_1) = \frac{u_*}{k} \left[\ln\left(\frac{z_2}{z_0}\right) - \ln\left(\frac{z_1}{z_0}\right) \right]$
 $= \frac{u_*}{k} \ln\left(\frac{z_2}{z_1}\right)$

$\Rightarrow u_* = \frac{k(u_2 - u_1)}{\ln(z_2/z_1)} \quad \tau = \rho u_*^2$

$\Rightarrow \tau = \rho \left[\frac{k(u_2 - u_1)}{\ln(z_2/z_1)} \right]^2$ Substitute into 3.5.13

So, what we are going to do next is estimation of this τ . What we will do is, we would use or we will use the concept of wind velocity in the boundary layer, in the boundary layer, over the, in the atmosphere, which is approximately 50 meters above the ground, alright, and how is that equation, the boundary layer or the velocity profile described in the boundary layer? This we will take from our knowledge of fluid mechanics, alright,

and this is described by $u/u_* = 1/k \log(Z/Z_0)$ and this we would name a number as 3.5.14.

I am sure you may have seen this equation earlier and this is the velocity profile or the logarithmic velocity profile in the boundary layer where u_* is the shear velocity, which is given by square root of $\tau/\rho a$, k is what is called the von Karman constant, von Karman's constant whose value is taken as 0.4 most of the times and z_0 is what is called the roughness height of the surface, which is the earth or ground in this case for which one can refer to table 2.8.2. You can find the roughness heights for different types of (()) patterns.

So, we use this equation 3.5.14 for 1 and 2, cross-section 1 and cross-section 2 or horizontal plane 1 and horizontal plane 2. We apply this 3.5.14 at these two horizontal planes, so that it will give you the velocity u_1 and u_2 . You do that and we, and then you subtract the two equations, what you would get? So, I am going to say 3.5.14 writing at 1 and 2 will give you $u_2 - u_1$ is equal to, after certain simplifications you have u_* / k times natural log of your z_2 / z_0 minus log of z_1 / z_0 . Should be very easy to see, I have directly applied that equation, you can simplify that and it will be u_* / K times natural log of Z_2 / Z_1 , Z_0 will cancel out.

So, what it gives you? It gives you u_* , remember, or we are doing all this, why? We want to calculate or estimate tau momentum flux and that tau is hidden in the u_* . u_* is equal to square root of $\tau/\rho a$, alright. So, this u_* then is equal to K times $u_2 - u_1$ divided by natural log of Z_2 / Z_1 . I have just come from here to here, but u_* is what it is equal to, square root of $\tau/\rho a$, alright. That means, you just simplify for tau, that will be your ρa times $K u_2 - u_1$ divided by natural log of Z_2 / Z_1 , all of that raise to the power what? Because there is square root here, square.

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$$\dot{m}_v = \frac{k_w \rho S_q (q_1 - q_2) (u_2 - u_1)}{K_m [\ln(Z_2/Z_1)]^2} \quad (3.5.15)$$

This is the famous Thornthwaite-Holzman Eq (1939). q_1, u_1, z_1, S_q at z_1
 Simplifications of Thornthwaite-Holzman Eq

- ① $(k_w/k_m) \approx \text{constant} \approx 1.0$
- ② $u_1 = \text{ZERO}$ @ $z_1 = z_0$
- ③ Air is saturated w moisture @ $z = z_0$
- ④ Use $q_2 = 0.622 \frac{e}{p}$ $e \rightarrow$ vapor e^r
 $p \rightarrow$ atm. p.

$\hookrightarrow r/h = \frac{e_a}{e_{sat}} : q_1 = 0.622 \frac{e_{sat}}{p_2}$
 $e_2 = p_2 \cdot e_{sat} : q_2 = 0.622 \frac{e_a}{p_2}$

Now, substitute this expression for tau into 3.5.13. What is 3.5.13? If you go back, this equation \dot{m}_v , \dot{m}_v is what we are interested in, substituting into 3.5.13 what you would get is, I will give you the final expression as \dot{m}_v is equal to $K_w k^2 \rho a q v_1 \text{ minus } q v_2 \text{ times } u_2 \text{ minus } u_1$, this whole thing divided by K_m of natural log of Z_2 over Z_1 and all of that square. This is your final equation and we number it as 3.5.15.

So, this is the final expression for \dot{m}_v or the vapor flux, alright, which is moving upwards and which is taken away to the wind velocity, alright. It has considered two main concepts or two driving forces, one is the humidity gradient, other is the velocity gradient, which are the two other major factors affecting evaporation. Now, on the right hand side of this equation you see the lots of, you know, quantities, alright, and it is not very easy to apply in real life. So, what people have done is, people have tried to simplify this.

So, what we will look next is or analyze is how we can simplify this, so that we can use this equation for estimation, alright. But before we go to that I must point out, that this is your famous equation, which is named after Thornthwaite-Holzman; that is the name of this equation, Thornthwaite-Holzman equation. It was developed by this gentleman in 1939, alright. We need to measure q_v , we need to measure u at different heights, alright. Also, we need to know ρ and various k 's, different types of k 's.

So, there is lots of measurements to be taken of, or lots of quantities or you know, coefficients to be estimated. Now, what we do is then we look at certain simplifications of this Thornthwaite-Holzman equation, alright. What I will do is, I will list the simplifications one by one and then you know, finally, derive or write the compact form of this equation. The first simplification is, we get rid of these two coefficients k_w over k_m , alright. We say, that this is constant and fortunately, it is close to one, both of these constants are close to each other, their magnitudes. So, we can disregard their variations with respect to the different climatic conditions and we can say that this ratio is constant and almost equal to 1, so that we do not have to estimate both of them.

Number two, let us say u_1 is 0 at Z_1 is equal Z_0 , what do we mean by this? This is a fairly reasonable assumption wherein we are saying, that the wind velocity close to the ground, very close to the ground, is equal to 0 due to your boundary layer. You know that due to no slip condition. The velocity or the fluid velocity right next to a plate is 0 due to the no slip condition. So, we are trying to use that. We say that the u_1 is 0 at the roughness height. Number three, so that the u_1 will vanish from this equation.

We say that the air is saturated. This is another simplification or assumption, that air is saturated with moisture where at Z_1 is equal to Z_0 , close to your water body. You are trying to find out evaporation from a lake or from reservoir, so right next to your water body or Z_1 is equal to Z_0 . Let us say, in this case you are saying, that the air is saturated, so that you can measure the temperature close to the water surface body and then you can calculate what is called the saturation vapor pressure, alright and that will, you know, you can convert that into or we can find out what is q_v , alright. So, q_v and q_v is the thing that appears in this equation.

So, we are trying to simplify or estimate some of these things using these simplifications. Next simplification is, we use the expression for q_v , that is, specific humidity as $0.622 \cdot \frac{e}{p}$, we have derived this already, where e is your vapor pressure, which we can measure in the atmosphere. How we can measure the vapor pressure? Where we will measure the dew point temperature and we use the equation for saturation vapor pressure that will give you the vapor pressure. p , of course, is the atmospheric pressure, which we can, of course, measure, alright. So, measurement of e and p , then will give you the estimation of q_v , alright.

And then what is relative humidity? It will be nothing but e actual over e saturated. We have seen this earlier, alright, so that you can have $q \ v \ 1$ as 0.622 of your e sat close to the ground, cross-section 1 is very close to the ground. So, you have e s over p at $Z \ 0$ and then $q \ v \ 2$ will be what? It will be 0.622 of your e actual certain at height divided by p at $Z \ 2$. What is the change in pressure? As you go up at cross-section 2, we have this relationship. Once we know the pressure on the ground we can find out its variation as a function of height. So, we have all those things, alright.

And from here, actually, I can say then what is $e \ a$, is your relative humidity times $e \ s$. You can measure the temperature at a certain height. We can calculate the saturation vapor pressure corresponding to that temperature and once we have the relative humidity, we can find out what is the actual vapor pressure, alright.

Now, what we do is we put all these assumptions or these simplifications back into 3.5.15, which is your Thornthwaite-Holzmen equation and simplify this.

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Putting above values & eqs into (3-5-15), we get

$$m_e = \frac{0.622 k^2 \rho_a (e_s - e_a) u_2}{p [\ln(z_2/z_0)]^2} \quad (3-5-16)$$

Now $m_e = S_w \cdot E_a \ (A=1)$

$$E_a = B (e_s - e_a) \quad (3-5-17)$$

where $B = \frac{0.622 k^2 \rho_a \cdot u_2}{p S_w [\ln(z_2/z_0)]^2} \quad (3-5-18)$

Many empirical eqs are based on this Aerodynamic method.

- Meyer's Formula.
- Rowley's Formula.

So, if you say, that putting all of the above values or simplify the expressions and equations, that is, into 3.5.15, what do we get? You would have $m \ v \ dot$ is equal to 0.622 of your k square, k is the von Karman coefficient, 0.4 times $\rho \ a$, density of the air, times your $e \ s$ minus $e \ a$ times of your $u \ 2$, $u \ 1$ has gone and all of this divided by p is the atmospheric pressure, which can be measured, times natural log of $Z \ 2$ over $Z \ 0$, $Z \ 0$ is the roughness height corresponding to different surfaces and for water body it is given in

the table, which I just mentioned, whole square, and this is 3.5.16 now. So, this is your final expression or the Thornthwaite equation.

However, we still need to find the rate of evaporation. So, what we do is, we use the continuity equation, which we have seen already, alright. So, you have $m \cdot v \cdot \dot{}$ is equal to what? It is nothing but $\rho_w \cdot e_a$ and you are saying area is equal to unity from a per square kilometer or unit area. You are trying to find out what is $m \cdot v \cdot \dot{}$ or e_a is the rate of evaporation from a water body based on the aerodynamic method. So, A, A here refers to the aerodynamic.

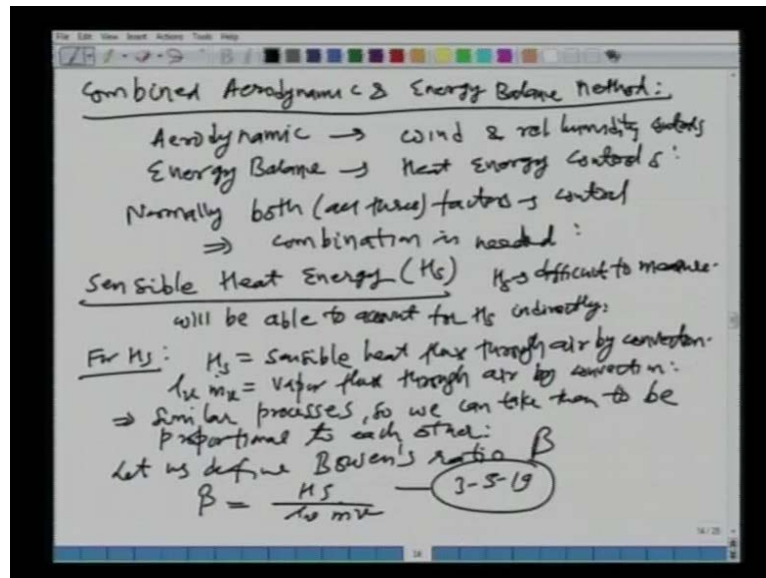
So, if you put this thing in here, so that you can have e_a , is equal to some constant B times e_s minus e_a . So, this is our compact final little expression, 3.5.17, where what is the value of B? B is called the constant, which combines all those things, $622 \cdot k$ square, von Karman constant, 0.4 square times, $\rho_a \cdot u^2$. So, it is taking care of the wind velocity, the density, lots of play factors over the atmospheric pressure, ρ_w and natural log of your Z^2 over Z_0 whole square and we say, that this is 3.5.18.

So, this combination of 3.5.17 and 3.5.18, so this is your final aerodynamic method or the Thornthwaite-Holzmen equation for estimating the rate of evaporation from a water body, earlier we called a reservoir in which we just need to calculate some over a measure, rather measure the humidity and temperature and pressure and vapor pressure, these things and we can calculate the rate of evaporation.

Now, see, that on the right hand side there are two things that appear, one is the vapor pressure deficit, that is, e_s minus e_a and other is the wind velocity, as far as the physical factors are considered. So, we have already seen the empirical equations. So, what I am going to say is, that many empirical equations are based on, many empirical equations are based on this, I am going to say aerodynamic method equation, aerodynamic method, Thornthwaite-Holzmen equation for example. What are those empirical equations? Well, yesterday we have looked at Meyer's formula, Meyer's formula and other one was the Rowher's formula, there are many more, but these are the two we had seen yesterday. You see, that this is the most general or the mother of all the evaporation equation, Thornthwaite-Holzmen equation and many empirical equations are based on this aerodynamic method.

Now, what we will do is, we will move on and look at what is called the combination of these two methods. So, we have looked at two analytical methods, one is the energy balance equation or energy balance method and the other one is the aerodynamic.

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So, now, we will look at what is known as the combined aerodynamic and energy balance method. As the name suggests, that we are combining two methods here, one method accounts for one factor, alright, physical factor, that affects the evaporation, which is the energy balance method, which takes care of the heat energy. There is another method, which accounts for two more factors, which is the humidity and the wind velocity, alright.

All three are important, but however, many times what happens is, sometimes the wind and the humidity gradient may not be that important or heat energy is the only important thing, alright, or the predominant thing. I should say, in those cases you can use the energy balance method, alright. In other cases, where the energy is not so important, but the winds and the humidity gradient are, you know, predominating factors in particular climatic conditions, then you can use the aerodynamic method. However, it is not easy to find out the conditions or situations where either of these two methods can be used very accurately because all the three physical factors are important, alright.

So, what we do is, we try to combine or we try to exploit the advantages of both the methods. So, let me write it down then. Aerodynamic method uses wind and relative

humidity controls or aerodynamic method is used when these factors control in the actual life, real life situation, and the energy balance method is used when the heat energy controls. However, normally, both or this all three factors control, means, some combination is needed.

Now, when we talk about this combination, alright, what we will do is, we will not only combine these two methods, but we will also look at one key parameter, which is the sensible heat, H_s , which we had not taken care of when we derived the energy equation. So, what we will do is this combined method, which we are going to develop, will not only be accurate as far as the being able to account lot of factors are right while deriving these, deriving this equation, but also it will take care of H_s , H_s , which is the sensible heat loss that to the atmosphere in certain form because that is very difficult to estimate. So, let us look at that.

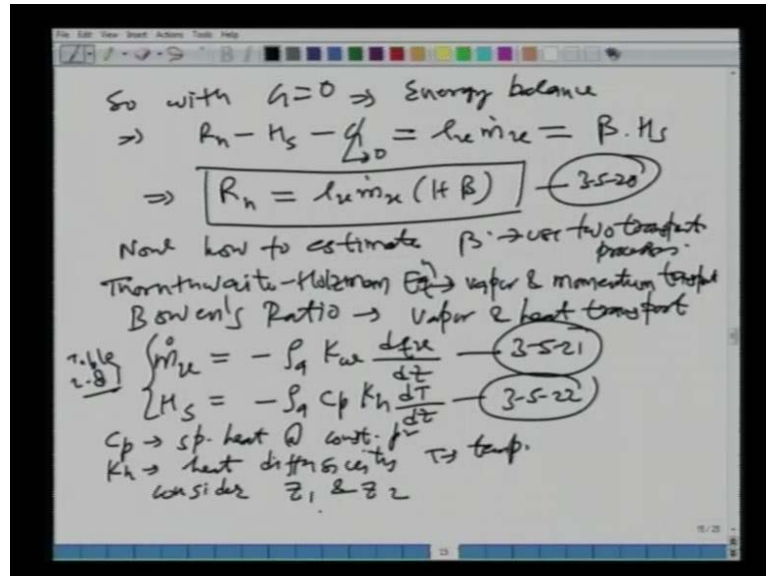
Sensible heat energy, which we had neglected while developing the energy balance method, H_s , as I said, is difficult to measure. So, what we will do is we will account for this H_s indirectly, alright. So, this combined method will be able to account for H_s indirectly, that is, we do not have to measure it, but we will account for that using some mathematical, you know, manipulations. So, let us look at that for H_s . What is H_s ? It is the sensible heat flux, alright, through the air physically by convection. It is the sensible heat flux, which goes back into the atmosphere out of the R_n that comes in, alright, through convection. So, that is physically what this physical variable is.

Similarly, to that is what we have $l_v m_v \dot{v}$, it is the vapor flux. We have seen that already in the aerodynamic method. It is the vapor flux through the air by convection. So, what we are trying to do is, we are trying to say, that H_s , that is, the sensible heat flux process and the $l_v m_v \dot{v}$ or which is the vapor flux process through the air, they are similar processes because they are caused by the process of convection. So, we say, that they are similar processes. So, we can take them to be proportional to each other. Why, because both of these processes being caused due to the convection in the air, alright. So, whatever is the convective condition, that will dictate H_s , that will also dictate the vapor flux, alright.

So, what we say is, that we if we take the ratio of these two, then that will be constant, alright, and then we try to find that constant. So, once we know one part, we know the

other one, alright. So, then we define or let us define a ratio, which is called Bowen's ratio, beta. How do we do that? It is the ratio of H_s and $l v m \dot{v}$, that is all, alright, which is we say, 3.5.19.

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So, with G is equal to 0 we still make the assumption, that the heat lost to the ground is 0 or we are not able to measure it, but that is actually quite, you know, reasonable assumption because G , as compared to H_s and R_n , is very small. So, we can make that assumption still, alright. So, that means your energy balance then looks like, looks like your R_n minus H_s minus G is equal to $l v m \dot{v}$. And we say that G is equal to 0 and what is $l v m \dot{v}$?

We have just said that, that is, beta times H_s using the definition of the Bowen's ratio. So, if you simplify that, then you will have R_n is equal to $l v m \dot{v}$ times 1 plus beta. So, you see, that in R_n we have incorporated beta, which incorporates H_s or the sensible heat laws, alright. So, we are indirectly accounting for this H_s without actually having to measure it and let me number this equation as 3.5.20, alright, alright.

So, now, the only problem is how to calculate or estimate this beta, right, alright or this, what we will do is, we will use two transport processes similar to the two transport processes we had used in deriving the aerodynamic method. So, in this case what we will do is use two transport processes. So, let me be more specific. In the Thornthwaite's method, Thornthwaite-Holzmen equation, in deriving that, which were the two transport

processes we used? One was the vapor transport and other was the momentum transport, right. We had said that vapor, first the liquid gets converted into vapor. So, there is a vapor transport taking place and then once the vapor gets accumulated, it is transported due to the wind action, alright. So, we, you looked at the momentum transport. And for the Bowen's ratio or estimating the Bowen's ratio we will use the two transport processes, which are involved here, which is vapor and the heat transport, alright.

So, we have already seen what is $m \cdot v$. It was, we had taken while deriving the aerodynamic method as this equation and I am going to number this as 3.5.21. And the H_s is minus $\rho \cdot c_p \cdot K_h \cdot dT/dZ$, alright. This is 3.5.22 and all of these are coming from your table 2.8.1 from page 44, which we had seen earlier.

So, these two equations correspond to the two physical transport processes for the transport of heat energy. One is the $m \cdot v$, which is for the vapor transport actually and H_s is for the sensible heat transfer. I will not go into the details of this, you can look at these equations and the associated material. However, let me define what are the quantities in the second equation? This c_p is the specific heat at what? Constant pressure; K_h is the heat diffusivity, alright; and temp, T is of course, the temperature, alright. So, this H_s actually depends upon what it is, a process of convection, alright. So, it depends upon the dT/dZ or how the temperature is varying as a function of height. So, H_s is directly proportional to dT/dZ .

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$$\frac{H_s}{m \cdot v} = \frac{c_p \cdot K_h \cdot (T_2 - T_1)}{K_w a \cdot (q_2 - q_1)} \quad \text{--- 3-5-23}$$

put $q_2 = 0.622 \frac{e_2}{p}$ & divide by $K_w a$

$$\beta = \frac{H_s}{m \cdot v \cdot K_w a} = \frac{c_p \cdot K_h \cdot p \cdot (T_2 - T_1)}{0.622 \cdot K_w a \cdot (e_2 - e_1)} \quad \text{--- 3-5-24}$$

or $\beta = V \left(\frac{T_2 - T_1}{e_2 - e_1} \right) \quad \text{--- 3-5-24}$

$V = \text{psychrometric constant} = \frac{c_p \cdot K_h \cdot p}{0.622 \cdot K_w a} \quad \text{--- 3-5-25}$

Now, it can be shown that

$$E = \frac{\Delta}{\Delta + R} \cdot E_r + \frac{h}{\Delta + R} \cdot E_s \quad \text{--- 3-5-26}$$

Now, what we do is we consider the two planes at Z_1 and Z_2 , alright and do a very similar analysis, that is, you will have H_s over $m \cdot v$. You take these two equations, divide each other, what you will have is this, $c_p K_h (T_2 - T_1)$ divided by K_w of your $q_v^2 - q_v^1$. And we go back, you divide this by this, $d q_v$ over $d z$ is, you have taken as $q_v^2 - q_v^1$ over $Z_2 - Z_1$. Similarly, dT/dZ you have taken as $T_2 - T_1$ over $Z_2 - Z_1$ and you divide these two equations, things will cancel out and this is what you will get. So, this is 3.5.23.

Now, let us put q_v as 0.622 of your e by p and divide by l_v , alright, this equation, alright. In 3.5.23, the operation we are doing is, we use q_v is equal to 0.622 e by p and then we divide this equation in the, on the left hand side if you see it is H_s over $m \cdot v$, alright. So, if you divide by this l_v , we will have on the left hand side, what, H_s over $l_v m \cdot v$. It is equal to what? The Bowen's ratio that is what we are trying to find out, alright. So, what we will have is Bowen's ratio β is equal to H_s over $l_v m \cdot v$ dot is equal to what, this whole thing, $c_p K_h p (T_2 - T_1)$ divided by 0.622 of your l_v . You can verify, that K_w and $e_2 - e_1$ or you say, β is equal to some γ , alright, times just the physical variable, which you will measure, $T_2 - T_1$ over $e_2 - e_1$. This you say your equation number 3.5.24 and this γ is called the psychrometric constant, which is $c_p K_h p$ over $0.622 l_v k_w$. This is 3.5.25, which is the definition of your γ .

So, what have we done so far in the combined aerodynamic energy balance method? The only thing we have done is, we have taken care of the sensible heat H_s , alright. We are, we have said, that in the energy balance method we ignored it and we said that we can take that equal to 0, but in many practical situations you cannot ignore the H_s , alright. So, what we have done is, we have gone around the two physical processes and then we have estimated H_s in the form of Bowen's ratio, which has come out to be equal to or given by this expression 3.5.24.

Now, in terms of all of this parameters what people have done is, we had, they had shown or we will write that now. It can be shown or derived, that your combined evaporation is given by this equation $\frac{\Delta}{\Delta + \gamma} E_r + \frac{\gamma}{\Delta + \gamma} E_a$. It is a simple combination actually. You see, that this is a weighted average, alright, of the two evaporation rates and let me number this as 3.5.26.

So, in this equation what we are saying is that this rate of evaporation e , alright, has a certain weightage. This is E_r ; E_r is what? E_r is the energy balance method. This E_a , E_a is what? The rate of evaporation calculated using aerodynamic method. So, we are combining these two and we are giving them different weightages, alright. One weight is $\frac{\Delta}{\Delta + \gamma}$, other is $\frac{\gamma}{\Delta + \gamma}$ and you see that the sum of these two weightages is equal to 1.

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$$\Delta = \frac{de_s}{dT} = \frac{4098 e_s}{(237.3 + T)^2} \quad ; \quad r = \text{already derived}$$

$$E_r = \frac{R_n}{\rho_w \cdot h_{e, \text{sat}}} \quad \text{Energy balance}$$

$$E_a = B (e_s - e_a) \quad \text{Aerodynamic method}$$

Evaporation over large Water Bodies:
Priestly and Taylor Method (1972):

- Energy balance considerations govern the evaporation:
- They found that second term is about 30% of the first one: $\alpha = 1.3$

$$E = \left(\alpha \frac{\Delta}{\Delta + \gamma} E_r \right) + \frac{\gamma}{\Delta + \gamma} E_a$$

Now what is this delta? Well, we have seen already, delta is nothing but the slope of the saturation vapor pressure curve as a function of this temperature. This we have seen already in the beginning of this chapter, alright, and gamma we have seen already derived above. And let me write simply that or simplify, that r , E_r is equal to R_n over ρ_w . This we had derived already and E_a , this your energy balance and E_a is capital B times e_s minus e_a . We have just derived, which is the aerodynamic method.

So, this is your final expression then 3.5.26, which combines the energy balance and the aerodynamic method, which is the most accurate method as it is able to account for both the methods. Not only that, but it is also taking care of the sensible heat, H_s .

So, what we are going to do the last thing in this chapter or the next thing in this chapter is, we would look at what is called the evaporation over large water bodies and this is what is actually called Priestly and Taylor method. And this was proposed by these two gentlemen in 1972. What they have done, these two gentlemen is, that they have

conducted many experiments, alright, across many different types of lakes of different sizes and shapes and in different climatic conditions. And what they have found is some interesting observation, alright. They have combined; they tried to combine the two methods. So, they, they would find out the evaporation from the lakes, alright, using the energy balance method and also the aerodynamic method and then they found, that the ratio of these two methods is a certain percentage of the other.

So, what we will do is, let me write it down, their observation is, that the energy balance considerations govern the evaporation. They said, the main factor for causing evaporation and is the energy, alright, after that you have the other factors, which will be certain fraction, although they will be important, but the major factor is the energy, alright. The other conclusion they have had was that they found, that the second term, the second term in your combined method is about or approximately 30 percent of the first one.

What is the second term? Well, the second term in your combined method is the aerodynamic term, alright, which is, which has the certain weightage times E_a , and E_a is the rate of evaporation estimated due to the aerodynamic method. So, what they are saying is, that the importance of the aerodynamic method is about 30 percent of the importance of the energy balance method, alright.

So, if you use that expression or their conclusion, then you can say, that the combined evaporation E is equal to α times $\frac{\delta}{\delta + \gamma}$. What is this factor $\frac{\delta}{\delta + \gamma}$? This is the weighting factor for the energy balance method in your combined method, you see. And we have put an α in front, what will be this value of this α if the weightage of the second term is 30 percent? Well, you have the 30 percent here and 100 percent here, if you sum the two it will be 1.3. So, you see that your value of α will be 1.3.

So, what they are actually saying is, that for the large water bodies or the large lakes, alright, you actually do not have to make lot of measurements, which are involved in the aerodynamic method, alright, your q_v , the wind velocities, pressure and so on. You can take advantage of the aerodynamic method by this observation, which they have made alright. So, if you look at this equation, alright, excuse me, and let me put, that this is equal to E_r . E_r is the due to radiation or the heat energies.

So, what they are saying is that the evaporation is equal to 1.3 times this factor times E_r , and E_r involves only one measurement, which is the net radiation, you see. So, we are able to account for all the factors without actually having to measure most of them. This is the major advantage of this Priestly and Taylor method, which gives you quite accurate results, as far as the rate of evaporation is concerned, and you do not have to measure a lot of these physical parameters.

So, with this I think we come to the conclusion of this chapter on the atmospheric hydrology or atmospheric water. In the next class we would like to start the next chapter, which is on the surface water. So, with that I would like to close today's lecture.

Thank you.