Advanced Hydrology
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Lecture – 16

Good morning friends, welcome to the next lecture of this video course on advanced hydrology. In the last class we started the topic of evaporation in this chapter, we looked at the definition of evaporation, latent heat of vaporization and then we looked at what are the factors that affect the evaporation in terms of rate of evaporation as well as the total volume. Then we also looked at the other kinds of losses of water from a catchment in the form of transpiration and the combined term, you know where we combine the evaporation and the transpiration are the total water loss, we say is called the evapo-transpiration.

Then we look at the concept of what is actual evapo-transpiration and the potential evapo-transpiration. Then we said that there are all the methods, which we can use to estimate the evaporation. They can be classified into three types. First one was the experimental, the second was the empirical and the third one is analytical. We started looking at the experimental method in which we used, what is called an evaporation pan to estimate the evaporation losses from a lake or a reservoir.

We also looked at an example of how much water loss can be there for Indian conditions, we took the example of Heerakud dam in Orissa. We took some realistic numbers as far as the data is concerned and then we said or we found out that there can be significant amount of losses from evaporation from reservoir and it turned out that approximately 200 days out of a year of water supply for the whole state can be lost. And that is where I think we stopped, and what you would like to do today is look at very important method of estimation of evaporation through analytical methods. So, one of the analytical methods, which we are going to discuss in this course is what is called the energy balance method.

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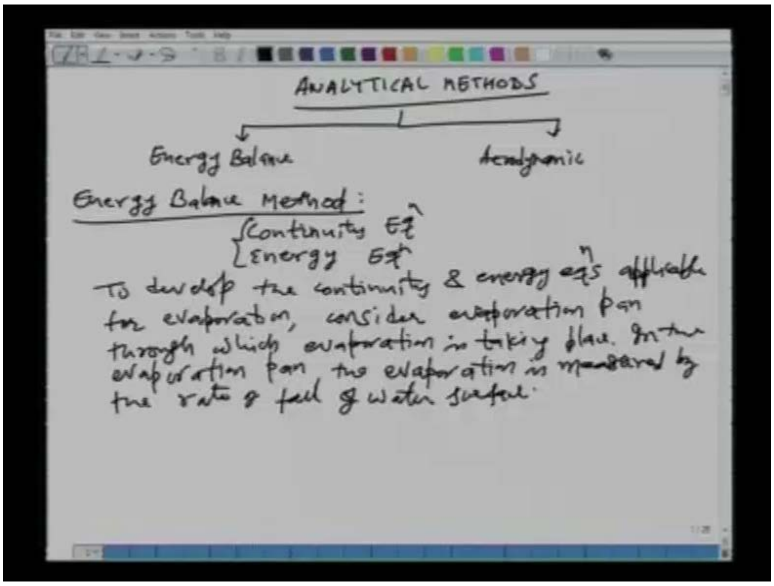

ANALYTICAL METHODS

Energy Balance

Aerodynamic

Energy Balance Method:

Continuity Eq^n

Energy Eq^n

To develop the continuity & energy eq^{n}s applicable for evaporation, consider evaporation pan through which evaporation is taking place. In the evaporation pan, the evaporation is measured by the rate of fall of water surface.

So, let me write down analytical methods. In that we will discuss two methods, as I have said earlier. So, let me first say, that one of them is the energy balance method and the other one, what we will look at, what is called as aerodynamic. And then there is a third one, of course, in which we will combine these two methods. So, let us start looking at the energy balance method.

In the energy balance method or what we will do is, we will apply the continuity and energy equation. As the name suggests, it uses the energy balance, so it is a combination of continuity equation, then also, the energy equation. So, the energy balance method consists of continuity equation and the energy equation. So, what we will do is to develop the continuity and energy equations applicable for evaporation.

Let us or we will consider an evaporation pan, alright, or we will consider evaporation pan though which evaporation is taking place and in the evaporation pan, in the pan the evaporation is measured, as we all know how, by the rate of fall of water, rate of fall of water surface. So, we will take the evaporation pan, we would look at all the components, which, which we need to consider in developing the continuity and the energy equations and we will apply the Reynolds transport theorem, which we have seen earlier.

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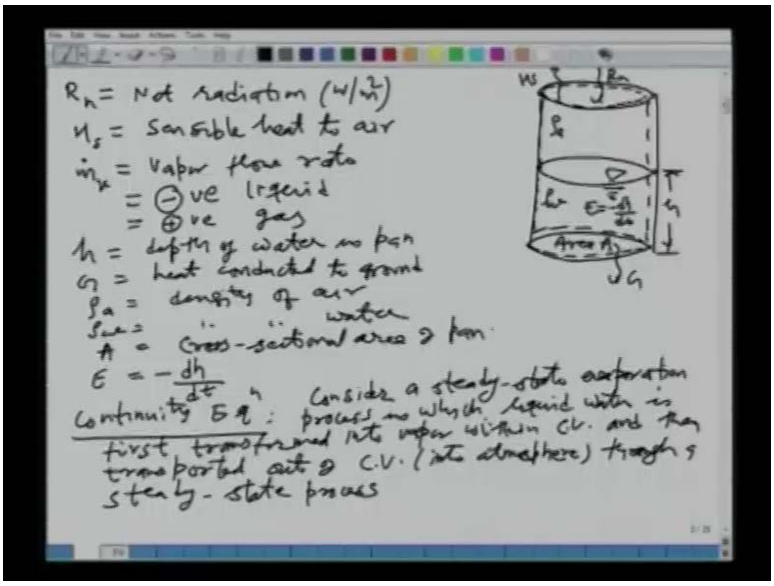

R_n = Net radiation (W/m^2)
H_s = Sensible heat to air
$\dot{m}_v$ = Vapor flow rate
= ⊖ve liquid
= ⊕ve gas
h = depth of water in pan
G = heat conducted to ground
ρ_a = density of air
ρ_w = " " water
A = Cross-sectional area of pan
$E = -\frac{dh}{dt}$

Continuity Eqn: Consider a steady-state evaporation process in which liquid water is first transformed into vapor within C.V. and then transported out of C.V. (into atmosphere) through a steady-state process

So, let us look at the schematic, alright, or the control volume and then define all the variables. So, this is a cylindrical evaporation pan and what we will do is, our control volume is right next to this surface or the walls of this pan is on the inside actually. And then let us say, there is a, this is the water surface elevation or the water in the pan, which is of height h. We will define all these things in a second. The area or the cross-sectional area is A, alright. The energy entering is R n; H s is what is called the sensible heat, that goes back to the atmosphere; and then there is some heat loss to the ground G; the density of the water is rho w and the density of the air or moist air is rho a, air. Then we say E is equal to negative dh over dt.

So, let us first look at all the basic physical parameters here. So, you have R n as the net radiation entering the pan and it is measured or its units are watts per square meter, so per unit area. H s is the sensible heat to the air, back to the air. And I will not worry about this too much because we have the sophisticated instruments, which should be able to measure this R n and H s. And if not, you know, we will see how to deal with these things later. Then this is important, m v dot is, what it is, the vapor flow rate; m v dot is the vapor flow rate within your control volume or in your pan.

Now, in the pan the water exists in two different phases, in liquid as well as in the vapor phase. So, the rate of increase or decrease of your water, alright, (( )), that is, for liquid it is negative, alright, m v dot is negative for liquid. What do we mean by that? The amount of liquid or the total volume of liquid in the pan is decreasing as a function of time because water is getting evaporated, right. And we will take this as positive for gas, this is for liquid and this is for gas or vapor.

So, m v dot is the vapor flow rate. It is, it has a positive value for the water vapor because it is increasing, alright, as function of time, and m v dot is negative for liquid. As per the law of conservation of mass, as we know, the mass cannot be created and destroyed with time, but it can change its phase. So, that is what is happening in this case, so that is, m v dot. h is the depth of the water in the pan; capital G is the heat conducted to the ground or heat loss to the ground. We need to estimate these things somehow.

Let us define rho a as the density of the air, we have done that earlier; rho w as the density of water; capital A, we have said, is the cross-sectional area of pan for the area, for the case of the lake it will be that over all surface, area of the lake; and then E is the rate of evaporation. What is the rate of evaporation? It is the rate of water loss from the pan. This is going to be measured or we can write in terms of dh by dt, alright, rate of fall of the water surface, alright. Now, if you look at this expression dh by dt, as the time increases what happens to h? The water surface elevation decreases. So, this quantity is going to be negative, but the evaporation if you wanted to be positive, what we do is, we say there is a negative (( )). So, the evaporation is nothing but minus of dh by dt, that is how we are measuring it.

Now, let us first write the continuity equation. So, this is the control volume we have defined and all the physical parameters in terms of energy and other things. Now, what we will do for doing this continuity equation is, they will consider a steady state process; consider a steady state evaporation process. This is important to understand because the Reynolds transport theorem, which we will write, we will have to take the different terms according to this assumption. So, we are considering a steady state evaporation process in which liquid water is, is first transformed into gaseous state or I can say, vapor within the CV, this process is taking place within the pan and then and then transported out of the CV, that is, into the atmosphere, into the atmosphere through a steady state, through a steady state process.

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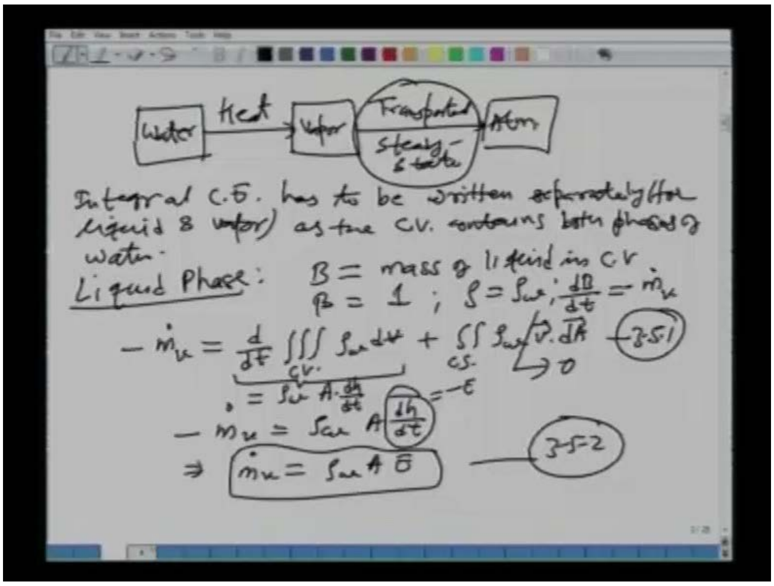

Integral C.E. has to be written separately (for liquid & vapor) as the C.V. contains both phases of water.

Liquid Phase: $B =$ mass of liquid in CV

$\beta = 1$; $\rho = \rho_w$; $\frac{dB}{dt} = -\dot{m}_v$

$$-\dot{m}_v = \underbrace{\frac{d}{dt}\iiint_{C.V.} \rho_w \, d\forall}_{= \rho_w A \cdot \frac{dh}{dt}} + \iint_{C.S.} \rho_w \underset{\to 0}{\vec{V}\cdot d\vec{A}} \qquad (3.5.1)$$

$$-\dot{m}_v = \rho_w A \boxed{\frac{dh}{dt}} = -E$$

$$\Rightarrow \boxed{\dot{m}_v = \rho_w A \bar{E}} \qquad (3.5.2)$$

So, let us look at it schematically what is happening. You have water in the pan, then heat is imparted to it. It is the heat energy from sun, due to that it is getting converted into vapor and this process is happening within the control volume or within the pan. So, you have water in the pan receiving heat from the sun and that heat is getting utilized into converting, transforming the liquid water into the gaseous state. So, that is one component.

And then what is happening to this water vapor? That is getting transported, alright. It is getting transported, how? By the wind action, somehow through a steady state process. Where is it going? It is going to the atmosphere. Remember, that this transport of the evaporated water is a steady state process. So, you have the pan water getting evaporated and then it is going away and all of this, the heat is coming in, water gets evaporated, vapor goes away. So, all this is a steady state process.

So, as you see, that in the or within the control volume or in the pan, the liquid is appearing in two phases. It is in the liquid phase as well as the vapor phase. And whenever we have a multi-phase flow, as we have said earlier, we need to write, we must write the continuity equation for both the phases. So, what we are going to say is, that the integral continuity equation has to be written, has to be written separately, that is for liquid and vapor phase of the water as the CV or the pan contains both phases of water, alright. So, let us look at this continuity equation one by one.

So, first we will take the liquid phase. I would like to do that quickly. We have seen some of these things earlier. So, we will say, that capital B is their extensive property, which is the mass of liquid in the CV. Beta will be 1 in this case; mass of liquid per unit, mass of liquid, rho, we are going to use is rho of the water. Then dB by dt, what will be this equal to? Alright, this will be m v dot, alright, the rate at which the vapor flow rate or rate at which the liquid is getting reduced in the CV to, let me put negative in front. So, it will be negative because the m v dot itself is negative for liquid we have assumed. So, we need to put a negative in front, so that dB by dt is positive, alright.

So, what will be the Reynolds transport theorem? dB by d t on the left hand side, which is minus of m v dot is equal to the sum of two components. What are the two components? First one is the time rate of change of extensive property stored within the control volume that is number one. Other one is the outflux, net outflux of the extensive property flowing across the control surface. So, if you write the first one, you have d over dt of over the control volume of your beta is 1, rho is rho w beta rho dB plus over the cross-section or control surface of your beta is 1 rho v dot dA. So, v dot dA and this we say, this equation number 3.5.1.

So, what would the first quantity be? It is the time rate of change of extensive property stored within the control volume. What is the extensive property? In this case the extensive property is the mass of the liquid water. Is the mass of the liquid water changing as a function of time in the pan? Yes it is, although your overall process, evaporation process is steady state, but you see, that the amount of liquid is reducing in the evaporation pan because it is getting converted into vapor.

So, as for as the liquid is concerned, that d by dt term will be, as you can see, equal to, rho w you can take outside, alright, d by dt of your volume total volume is A times, A is the cross-sectional area and the rate at which is reducing is dh over d t. So, if you take the rho w out you have v over t. So, you have A h over dt. Over all if you see these two things, the quantities of the units will be same. So, the first term will be equal to rho w A dh by d t.

What about the second term? If you think, physically, what does the second term represent? Well, it is the amount of or the outflux of extensive property flowing across the control surface. Is there is there any liquid water flowing across the control surface?

What are the control surfaces? Well, you have the cylindrical vessel, then there is a bottom, which is fixed, alright. No water, you know, can flow through the bottom; no water can flow through the pan, across it and no water can, liquid water can flow through the top. It is only the vapor, which is flowing upwards. So, as far as the liquid water is concerned, the second term is going to be equal to what, 0.

It is very important to understand and see that, but you should be able to see, that once you understand what is your extensive property to which your, you are applying this Reynolds transport theorem. So, the second term here will be 0 because there is no movement of liquid across any of the control surface. So, once we have understood that, then all you have is negative m v dot is equal to rho w A dh over dt. We have already said what is dh over dt. It is minus E or E is equal to minus dh over dt. So, that would mean, then your m v dot is equal to rho w AE. This is your equation number 3.5.2. So, this is your continuity equation for the liquid phase in the evaporation pan.

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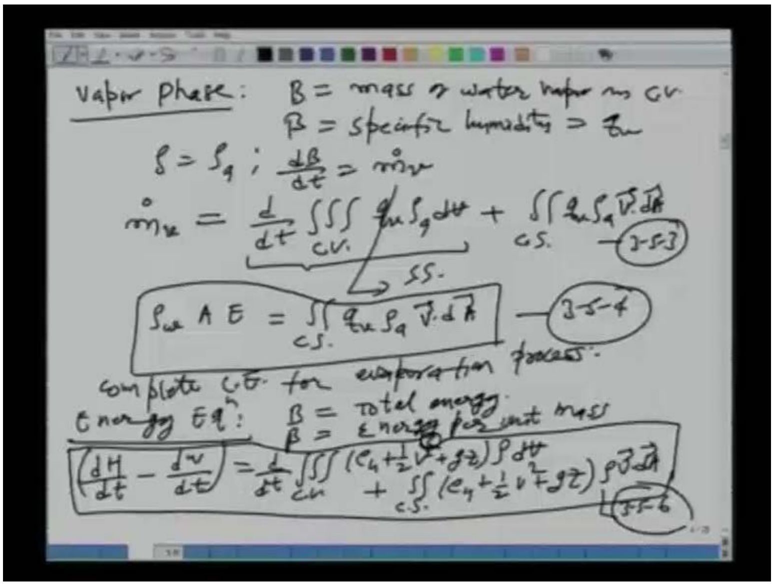

Vapor phase: B = mass of water vapor in CV

β = specific humidity = q_v

$\rho = \rho_a$; $\frac{dB}{dt} = \dot{m}_v$

$$\dot{m}_v = \underbrace{\frac{d}{dt}\iiint_{C.V.} \cancel{q_v \rho_a \, d\forall}}_{SS.} + \iint_{C.S.} q_v \rho_a \vec{V}\cdot d\vec{A} \quad (3\text{-}5\text{-}3)$$

$$\boxed{\rho_w A E = \iint_{C.S.} q_v \rho_a \vec{V}\cdot d\vec{A}} \quad (3\text{-}5\text{-}4)$$

complete C.E. for evaporation process.

Energy Eq^n: B = Total energy

β = Energy per unit mass

$$\boxed{\left(\frac{dH}{dt} - \frac{dv}{dt}\right) = \frac{d}{dt}\iiint_{CV} \left(e_u + \frac{1}{2}V^2 + gz\right)\rho \, d\forall + \iint_{C.S.} \left(e_u + \frac{1}{2}V^2 + gz\right)\rho \vec{V}\cdot d\vec{A}} \quad (3\text{-}5\text{-}6)$$

Now, let us look at the same thing for the vapor phase. For the vapor phase, the extensive property we will define as, the mass of your water vapor in the CV. What is beta for the mass of the water vapor in the air? We have seen that already, it is nothing but the specific humidity, which we denote as q v and then rho is equal to, what it is, moist air, density of the air and dB over dt is your m v dot, which is positive for the vapor, we have said already, alright.

So, when we put all these things in your Reynolds transport theorem, the left hand side is dB over dt, which is m v dot is equal to, again you have the time rate of change of extensive properties stored within the control volume. You have to remember this expression by heart. It should just come out automatically, alright. So, this is going to be equal to, beta is q v, rho is, rho, rho A and dv qv rho dv plus over the control surface of your beta rho A and v dot d A, and let me number this as 3.5.3.

So, all we have done is, we have written the Reynolds transport theorem continuity equation for the vapor phase in the evaporation pan, which is your control volume. Now, which term will be 0 let us see physically. See, the important thing is, we have to understand our physical process. What is actually in the nature we are trying to model and what are the extensive properties? What is your, you know, equations? Which term is representing what and then we should be able to see that. So, let us see first one. First term of this equation 3.5.3 represents what physically it is, time rate of change of extensive property, that is, mass of the vapor.

So, time rate of change of the mass of the vapor, alright, within the control volume. Is it changing? No, why, because we have said, that it is a steady state process. Remember, the assumption was, that the transport of water vapor, alright, is a steady state process. So, because we have assumed this to be steady state, so the first term on the right hand side is going to be equal to 0 because there is no accumulation of water vapor in your control volume. So, if you understand that, then all we would have is m v dot. We can write as rho w A E, alright, is equal to over the control surface of your q v of your rho a and v dot d A. So, this is a complete continuity equation, we will number it 3.5.4 and say, that this is your complete continuity equation for the evaporation process, which is very useful for finding the evaporation once we can estimate the values.

Now, as we said, that we are looking at the energy balance method alright and we have assumed the R n and h s and g and all those things, so we need to, we need to take care of the energies also. This is as far as the mass balance is concerned. So, we apply the energy equation, alright and then we can combine the energy and the continuity equation that will give you the final expression or formula for estimating or calculating the evaporation. So, let us quickly look at the energy equation for the control volume we have just defined.

So, let me write quickly, that B is the total energy. So, that is your total energy of the fluid, which is possessed by the fluid; capital B is the energy per unit mass. Now, we have defined all those things actually. So, what I will do is first I will write this energy equation which we have written at the beginning of this chapter. So, I will just go back and write it as, the left hand side, we will use the first law of thermodynamics. We say, the dB by dt is dH by dt minus dw over dt. We have defined this from the first law of thermodynamics is equal to d over dt, first term, same thing is your accumulation term or the amount of extensive property, the rate of change of that. So, it is a q beta is e u plus half of v square plus g z, that was the beta and then you have the rho and dv, this is the first term.

And second term is going to be over the control surface of your E u plus half of v square plus g z, that is your beta rho v dot dA, alright. And I will say, that this is your, this whole thing is 3 5 6, alright, 3 5 6, alright. So, these two actually let me make sure, that this is square here. So, let us look at this equation now term by term which quantity is going to be 0 and which quantity is not going to be 0 and how we can simplify dH by dt and dw by dt terms.

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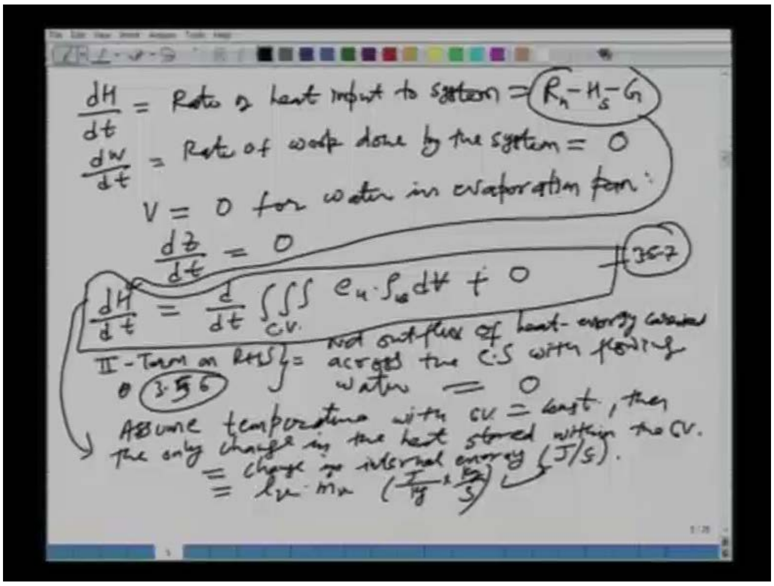

$\frac{dH}{dt}$ = Rate of heat input to system = $R_n - H_s - G$

$\frac{dW}{dt}$ = Rate of work done by the system = 0

V = 0 for water in evaporation pan ∴

$\frac{dz}{dt} = 0$

$$\frac{dH}{dt} = \frac{d}{dt}\iiint_{c.v.} e_u \cdot \rho_w \, d\forall + 0 \qquad (3.5.7)$$

II-Term on RHS of (3.5.6) = Net outflux of heat-energy carried across the C.S with flowing water = 0

Assume temperature with CV = const, then the only change in the heat stored within the CV.

= change in internal energy (J/s).

= $l_v \cdot m_v \left(\frac{J}{kg} \cdot \frac{kg}{s}\right)$

So, let us move on and use our knowledge and say, that on the left hand side what was dH by dt? It is the rate of heat input alright or the heat transfer to the system. What is a system? Our system is this evaporation pan. How much is the heat, which is or the net rate of heat input are going into your control volume? Well, in terms of the things we have it is nothing but R n goes in, minus H s goes out, minus G is the heat loss to the down. So, this is the net rate of heat input, you know, from the external sources we have it on the left hand side. What was the dw by dt term? It is the rate of work done, the rate of the work done by the system.

Now, I would just like to spend may be minute on this quantity dw by dt. When we have a pipe network system, for example, where the water is flowing in the pipes, or, or in the channels for that matter, what we have is, we have what is called as pump. Whenever you install a pump, what does a pump do? A pump imparts energy to the fluid or to the flowing water. Let us say you do not have the enough energy to do or enough head to take the water downstream, gravity is not available and we need to take the water to the higher level or higher the ground what do we do? We just pump the water. So, we put these artificial energy sources, alright, a pump for example, which will jump the energy and then you can take the water. So, it is that kind of work. So, pump is doing the work on the system.

Similarly, we may have turbine, alright, which will take the energy, alright. In the, in the dams what do we have? We have the turbines on which the water will fall and that energy gets converted into the first mechanical and then electronical energy. So, these are the kind of you know, work, or these are the kind of elements we are looking at here.

So, with that background in mind is there any source of heat, alright, either the heat being imparted to the system or taken away? Other than that we have considered R n, H s and G, that will constitute the rate of work done on the system. Obviously, there is no other element of source or sink in terms of the heat in the system. So, what will then say is, that this dw by dt term is nothing but equal to 0. There is no work done on the system in the natural process of evaporation, alright.

Now, what is v? v is the velocity of the water in the evaporation pan, alright, which is actually very slow, extremely slow. So, for practical purposes we say that it is 0 for water in the evaporation pan. Also, what is dz over dt? We are looking at one of the things in the right hand side, the first term actually is the latent heat or the latent energy of the overall system changing as a function of time. The evaporation pan is probably put at some altitude, alright. That altitude is not changing; latent heat is not changing as a function of time. So, dz by dt is also equal to 0. So, it should be very easy to see, that dz by dt is also equal to 0.

So, once we have put all these things in our equation, which we have just derived, that is this one 3.5.6, alright, then what do we get is dH by dt is equal to (( )) dt over dt of triple integral over the control volume of your e u times rho w and dv. If you go back you see, that the first term on the right hand side, the v is 0 and then dz by dt is 0. So, all you have is the e u term. So, that is out of the first term right. So, that is what I have written, d over dt of over the control volume of e u rho w dv.

What about the second term? This second term is this one, it is very important to understand this one, the second term in this equation. Physically what it is representing? it is representing, let me write it down or let me first save it. The second term on the Reynolds or in that Reynolds transport theorem is what net outflux of extensive property flowing across the control surface physically, that is what it is. And what is the extensive property? In this case the extensive property is the heat energy or the total energy. So, what does this second term represent then? It is the net outflux of heat energy flowing across the control surface with the flowing fluid.

Let me write it down and then we will see what it is. The second term on the RHS of, let me see the equation number, which is 3 5 6, let me go back, it will be 3 4 6. No, I am sorry, it is 3 5 6, is it? Yes, so it is 3 5 6, is physically, it is the net outflux of heat energy carried across or flowing across the control surface with, with what, with the flowing water. What is this equal to? What is the net outflux of the heat energy flowing across or carried across the control surface, alright?

We are saying, that the energy exchanges are R n, which is coming in H s, is the sensible heat out to the atmosphere. We have taken care of that to the ground, there is no heat energy taking place across the pan, nothing across the ground, which we have not taken care of and there is nothing to the atmosphere. So, with the flowing fluid including vapor and liquid, there is no flux. There is no heat energy, which is carried across the control surface. This is very important to understand that. Whatever are the heat energy exchanges, we have accounted for them. Net heat comes in, that is used up in converting the liquid water to the vapor, so that, that is all there is to it. So, we will say, that this is equal to 0. It is very important one to understand.

So, I am going to say, that this whole quantity is equal to 0. So, the second term on the right hand side of your energy equation applied to this case is equal to 0. Please see that on your own, if it is not clear try to think about it. So, once we have that, then I come to this equation, this one, the second term is 0, alright. So, that is all I will have and this I am going to number as 3.5.7.

Now, in this equation 3.5.7, which we have written, this one, that is, if we assume what does this represent? Basically, the right hand side represents the time rate of change of internal energy stored within the control volume. Now, out of the total energy, the latent heat and the kinetic energy, those things are taken care of. Only thing we have is e u, which is the internal energy, alright. So, the total rate of change of internal energy is what we have to write on the right hand side.

So, if we assume, that temperature within the CV is constant. If we assume, that the temperature within the control volume or in the pan is constant, then which actually is a reasonable assumption because during the process of evaporation we have seen, as the definition of evaporation, that the temperature is constant during evaporation, so if we assume the temperature within the pan to be constant, then the only change, then the only change in the heat energy stored within the pan or the CV is equal to what? It is the change in the internal energy, total heat is equal to your internal energy, alright, which is in joules per second. So, it is a time rate of change of internal energy and we are saying it is joules per second.

What will this B equal to, this internal energy is related to what kind of energy in the, kind of energy in the fluid in water? Obviously, during evaporation we are talking about the latent heat of vaporization, alright, and if we say, that the latent heat of vaporization is denoted by l v, then we can multiply this by m dot v, alright.

Now, see, that what is l v? The l v is the units are joules per kg. What is the unit of the latent heat of vaporization? It is the energy per unit mass of the substance. So, it is joules per kg. If you multiply this by m v dot and what are the units of m v dot? It is kilograms per second. It is the mass of the vapor flow rate, right, so it is kilograms per second. So, you see, that l v multiplied by m v dot will be nothing but joules per second. So, that is what we want here.

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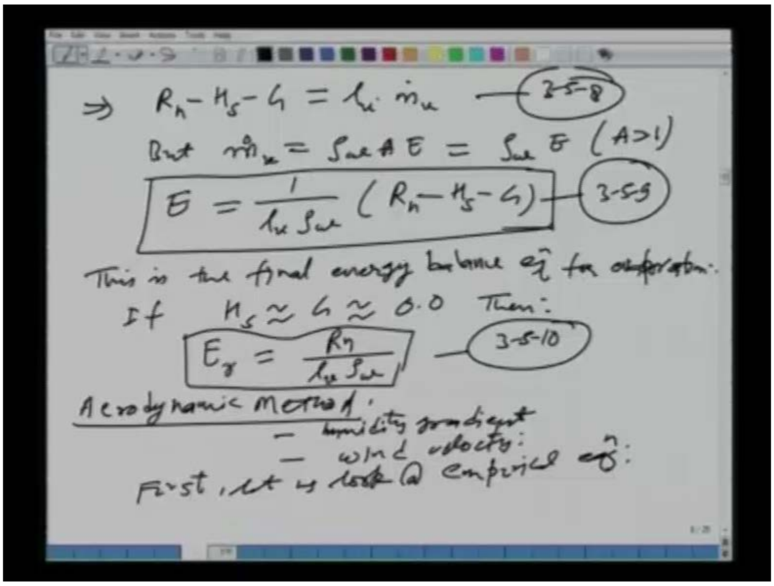

$$\Rightarrow R_n - H_s - G = l_v \dot{m}_v \quad \text{(3-5-8)}$$

$$\text{But } \dot{m}_v = \rho_w A E = \rho_w E \; (A>1)$$

$$\boxed{E = \frac{1}{l_v \rho_w}(R_n - H_s - G)} \quad \text{(3-5-9)}$$

This is the final energy balance eqn for evaporation.

If $H_s \approx G \approx 0.0$ Then:

$$\boxed{E_r = \frac{R_n}{l_v \rho_w}} \quad \text{(3-5-10)}$$

Aerodynamic Method:
- humidity gradient
- wind velocity:

First, let us look @ empirical eqn:

So, if you use this equation and let me see, that this R n minus H s minus G is your dH by dt. So, you are going to put this in this equation here and this whole thing d over dt of triple integral of e u rho w dv is l v m v dot. So, let me go to the next page and then say, that basically you have, R n minus H s minus G is equal to l v m v dot and let us say this is 3 5 8, alright.

But what is m v dot? Well, m v dot we have already said from the continuity equation and rho w A E, and this I am going to say for simplicity as rho w E. And if you assume A is equal to 1. If you are, you want to define in the evaporation per units square area, then you can take A is equal to 1 and then you have, will have the evaporation per unit area.

So, we put all these things into 3.5.8, what we are going to get is then E is equal to 1 over l v rho w times R n minus H s minus G. I am sure, that this equation sounds quite familiar to many of you. This is nothing but the energy equation you may have studied in your undergraduate classes of, this is your energy equation we have derived from the Reynolds transport theorem in which we have taken the movement of the fluid. We have written the continuity equation for the liquid phase and for the gaseous phase, then we have written the energy equation and we have combined those two and finally come up with this expression. So, let me say, that this is 3.5.9. This is the final energy balance equation for evaporation.

Now, you see, that this final expression for evaporation involves three things as far the energy is concerned: R n, H s and G. Normally, the incident, the radiation or the incident heat energy R n, we can measure using the instruments. In fact, most of the metrological stations will measure the heat coming in. So, R n is something, which can be easily measured.

However, there are practical difficulties in estimating or measuring the H s and G, alright. But the good thing is that there fraction or the amount of H s and G, as compared to the total heat coming in, is very small. So, as engineers or practical people what do we do? Anything that is small, alright, as compared to other quantity, let us say for example, R n is 100 and this is 0.1 and 0.2, we can neglect it. So, you say, that if H s and G we can neglect or we are close to 0, then your final expression or, we will call it E r, rate of evaporation due to your energy methods is going to be equal to R n over l v rho w. This is very small and sweet formula, I am sure you have seen in your earlier classes. So, this is the energy balance method for estimating rate of evaporation, alright, in which the physical factors we have considered is mainly the heat energy.

Now, the second method, as I said, we are going to consider is what is called the aerodynamic method, aerodynamic method, in which we will consider the expression, derivation of expression for evaporation, which considers not the heat energy, heat energy we already considered in the energy balance method, but two other main factors, alright, which is the humidity and the wind velocity, alright. But before I go to the aerodynamic method what I will, would like to do is, I would like to look at some empirical expressions, alright, which can be derived from this aerodynamic method.

The final equation of the aerodynamic method, which we will find is the mother of all the empirical evaporation equation, alright. So, let me just say, that it will consider humidity gradient or deficit rather and wind velocity, right, and we will develop this equation, but first let us look at a couple of empirical equations, which are based on this equation. I am doing this intentionally because you may have seen these equations and then once we derive the aerodynamic method you will be able to see, that, that is the most general one and how we can derive the other equations out of this empirical equation.

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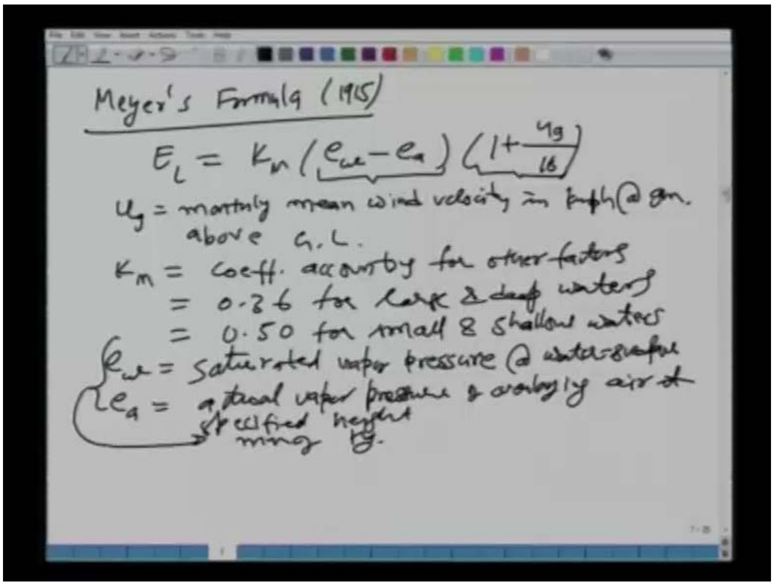

Meyer's Formula (1915)

$$E_L = K_m \underbrace{(e_w - e_a)}\underbrace{\left(1 + \frac{u_9}{16}\right)}$$

u_9 = monthly mean wind velocity in kmph @ 9m. above G.L.

K_m = coeff. accounting for other factors
= 0.36 for large & deep waters
= 0.50 for small & shallow waters

e_w = saturated vapor pressure @ water-surface

e_a = actual vapor pressure of overlying air at specified height

mm of Hg.

So, first one of the empirical equations, which we are going to see, is the Meyer's formula. It was developed by Meyer in 1915, and and if you remember it gives you the rate of evaporation by this expression, where K is some constant, alright. Let me first define the quantities, then I will come back to this expression.

So, what is U 9 is the monthly mean wind velocity in kilometers per hour. It is the wind velocity, where, at 9 meters above the ground level. And K m is your coefficient accounting for other factors, alright, that is, your size of the lake, etcetera, and this is given as 0.36 approximately for large and deep waters or lakes, and its value is about 0.5 for small and shallow waters.

What is e w? In this equation e w is the saturated, saturated vapor pressure, saturated vapor pressure at the water surface close to the water surface, which will depend upon the temperature and we have seen its expression already. And e A is the actual vapor pressure corresponding to the reality and humidity at that point of time, vapor pressure of overlying air at a specified height. And both of these things, these pressures are in millimeters of mercury. Pressures are, this we have seen, that can be measured in Pascals as well as in the, as a barometric unit and millimeters of mercury in this particular equation because it is empirical equation, it involves this coefficients. The pressure has to be the vapor pressure and the saturation vapor pressure have to be substituted in millimeters of mercury and once we have the value in Pascal, we can always convert into any other units.

So, this is the formula in which if you see, that the rate of evaporation E L or the monthly rate of evaporation E L is a function of two things. It is taken care of two physical factors, one is the deficit, what is it? e w minus e A. What is e w minus e A? It is the difference in the saturation vapor pressure and the actual vapor pressure, so it is the deficit, alright. Higher the e A, lower will be the rate of evaporation, alright; lower the value of e A, higher will be the evaporation and we have seen that earlier. And the second expression or the second factor, physical factor, which this method is accounting for, is the wind velocity, alright, because wind velocity is what is responsible for removing the evaporated water from the lake. So, it will depend upon the wind velocity also.

So, we have seen, that the Meyer's formula is accounting for the vapor pressure deficit and the wind velocity and our aerodynamic method also takes care of these two things, but it is it has been derived from that aerodynamic method we will look at that little later.

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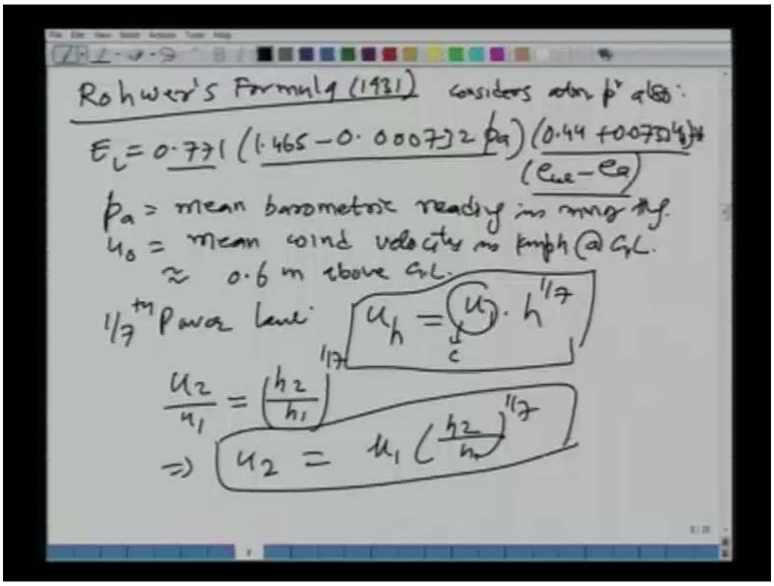

Rohwer's Formula (1931) considers atm. p' also:

$E_L = 0.771\,(1.465 - 0.000732\,p_a)\,(0.44 + 0.0733\,u_0)\,(e_w - e_a)$

p_a = mean barometric reading in mm of Hg.

u_0 = mean wind velocity in kmph @ G.L.

$\approx$ 0.6 m above G.L.

1/7th Power Law: $u_h = c \cdot h^{1/7}$

$\frac{u_2}{u_1} = \left(\frac{h_2}{h_1}\right)^{1/7}$

$\Rightarrow u_2 = u_1 \left(\frac{h_2}{h_1}\right)^{1/7}$

So, that first, number one or the first empirical equation and the second one, you may have seen is what is called Rohwer's formula for estimating evaporation and this was proposed by this particular gentleman in 1931. What it does is, it considers atmospheric pressure, alright, also. So, Rohwer's formula, what does it do? It not only considers the vapor pressure deficit and the wind velocity, those two are there, plus it also accounts for the atmospheric pressure for the given climatic conditions.

And then there is a long expression I am going to write it, so bear with me with that. E L is equal to 0.771 times 1.465 minus 3 0s and 732 times p a, bracket close. And then you have, 0.44 plus 0.0733 u naught and all of this multiplied by e w minus e A. We see that 0.77 multiplied by this whole thing, this whole thing multiplied by this and then this is multiplied by that. Here, your p a is the mean barometric reading at the location, again in millimeters of mercury. You see, that these empirical equations are, you know, designed in such a way, that they are very easy to use.

So, you can have a barometer, it will give you reading of pressure in millimeters of mercury, you can directly use it in this equation, so that is the purpose. And then u 0 in this equation is the mean wind velocity, wind velocity in kilometers per hour at the ground level, which is actually taken at 0.6 meter above GL at, right on the ground level due to the, in a boundary, that is, you know that wind velocity is actually 0. So, you take it about in 0.6 meters above the ground level, that is, where you know, most of the metrological stations put their anemometer. So, you see that the all other things I am not defining here. So, p a is the pressure and u is the wind velocity and e w minus e a is your saturation vapor pressure and the actual vapor pressure. So, this is another empirical equation in which this u 0 appears.

Now, one thing I would like to just point out here is that the both of these equations use the wind velocity, one is using at the ground, other is using at 9 meters. Sometime you may have metrological, you know, station, which measures the wind velocity at, you know, 10 meters or 5 meters or some other. So, we need a mechanism in which we can convert these velocities, you know, at any height.

So, I am going to give you something what is called as one-seventh power law. It gives the wind velocity at any height h, alright, as a function of wind velocity at height 1, which is a constant actually in this equation c times h to the power 1 by 7. This is actually called the one-seventh power law.

So, if you have the wind velocity at any height you can find out its value at any other height. For example, you have, let us say, u 2 over u 1, you can apply this equation at two heights, 2 and 1. So, we have u 2 over u 1 is equal to h 2 over h 1 to the power 1 by

7, that will actually give you your u at 2 is equal to u at 1 time h 2 over h 1 raise to the power 1 by 7. So, you can use this one-seventh power law to find out the wind velocity at any desired height.

So, I think I would like to stop at this point of time. And just to summarize, we have seen the energy balance method today in which we derive the expression for the rate of evaporation using continuity and energy, energy equations. Then we moved on to the aerodynamic method, which we have actually not looked at, that we will do it in the next class, but we have seen couple of empirical equations, which are based on this aerodynamic method. So, what I would like to do in the next class is look at this aerodynamic method.