

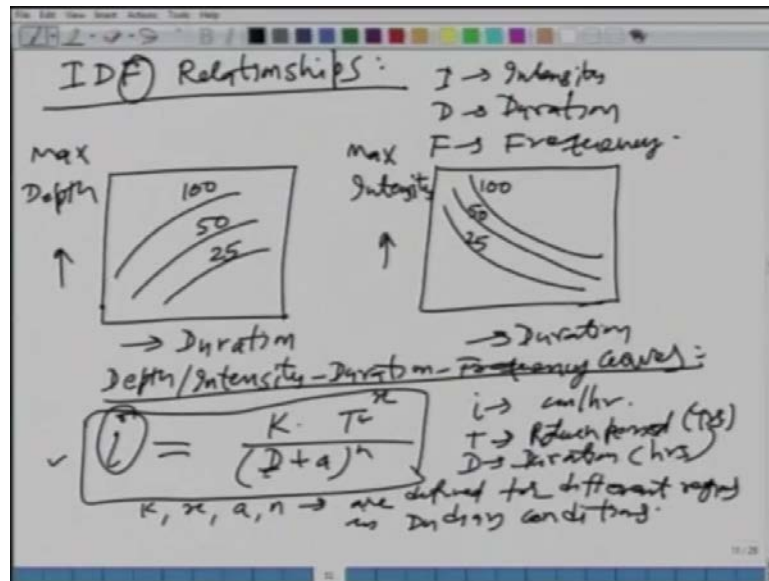
**Advanced Hydrology**  
**Prof. Dr. Ashu Jain**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 14**

Good morning and welcome to this video course on advanced hydrology. In the last class we looked at the expression derivation of expression for terminal velocity of a raindrop falling through the atmosphere in which we balance the 3 predominant forces which act on such a case. Then we looked at couple of data representation a method that is rainfall hyetograph and the rainfall mass curve. Then we looked at the procedure of developing what is called a maximum depth duration curve, and associated with that we also saw how we can develop the maximum intensity duration curve. Then we went had and looked at one numerical example of calculating the duration intensity the maximum depth duration curve, and the maximum intensity duration curve a given the data about a particular rainstorm.

After looking at that, we started looking at what is called the IDF relationships or the intensity duration frequency relationships in which we add another dimension which is called the frequency of occurrence. And this frequency of occurrence we measure in terms of the return period. We say that the higher the return period lesser is the frequency right a 100 year event would occur less frequently as compared to a 10 year event. Accordingly the magnitude of a lesser frequent event will be more severe or higher that is to say if you have a 100 year event it is flood magnitude would be much higher as compare to the magnitude of a 10 year event. So, we see that the frequency of occurrence also has a bearing on the magnitude of particular rainfall flood or any hydrologic variable. So, that is why it was important to add this frequency dimension into our analysis.

(Refer Slide Time: 02:16)



So, then what we did is we looked at this IDF relationships, if you go here we plotted few maximum depth duration curves. And the maximum intensity duration curves and then we said that India mythological department or IMD has come up with a relationship after studying a lot of data rainfall data of different durations and frequencies in different parts of India. And they came up with this relationship in which we said the  $i$  is equal to  $K T$  to the power  $\alpha$  divided by  $D$  plus  $A$  to the power  $n$ . In this  $i$  we said is the intensity of rainfall in centimeters per hour, capital  $T$  is the return period in years and capital  $D$  is the duration in hours and this 4 parameters  $K$   $\alpha$   $a$  and  $n$  are these are called the parameters of this particular model.

And these have been estimated or calculated by IMD and these are available in your Subramanian's book. So, I suggest that you have a look at these values of  $K$   $\alpha$   $a$  and  $n$  and once we have that for a particular region all we need to know is what is the duration in which we are interested, and what is the frequency of occurrence or the return period  $T$  in years? And then we can find out what will be the worst case scenario as far as the intensity is concerned. So, these intensities can be used in hydrologic decide.

What we will do today is we will start with another similar concept which is called a real averaging of the rainfall remember in the beginning of this course. We had looked at the classification of the hydrologic models in which we said that the hydrologic models can be either lumped or distributed. What was the difference in a lumped model in a distributed

model? Well in a distributed model we account for the variation in the hydrologic variables with respect to space, and in a lumped model we ignore those variations with respect to space. So, what we then need in a lumped model is the single value of rainfall, let us say if you are doing rainfall runoff modeling then you need the average rainfall in a very large catchment and you may have 2 or more a rainfall gauges are in a particular catchment. So, we need to calculate the average rainfall over the space. So, what will do is we look at 2, 3 different methods and there relative importance and applicability criteria about calculating the average rainfall.

(Refer Slide Time: 04:56)

AVERAGE AREAL RAINFALL

(1) Arithmetic Average Method:

$$\bar{P} = \frac{1}{N} \sum_{i=1}^N P_i$$

$P_i$  = rainfall recorded at  $i^{\text{th}}$  raingauge in the catchment  
 $N$  = no. of raingauges in the catchment  
 This method is rarely used in practice.

(2) Thiessen Polygon Method: assumes that

- (i) at any point in a catchment, the rainfall is same as that at the nearest raingauge
- (ii) depth recorded at a given raingauge is applied out to a distance half way to the next station in any direction

So, will get started that the new topic today which is called the average areal rainfall. And in this as we said there are many methods and we will look at them 1 by 1. And the first 1 of them is the most simple method it is called the arithmetic average method. Now all of these method what we are going to assume is let us say that this is your catchment you may have river running through it, these are different branches. And the flow is taking place from right to the left, then let us say you have these are the locations of the rain gauges. And there may be many different rain gauges depending up on the density of the rain gauges in that particular area. So, in the arithmetic average method all we do is we just take a simple average, that is your P bar is nothing but 1 over N summation of P i, i goes from 1 capital N is that make its simple average or arithmetic average of all the data in terms of P i where P i is needed.

Let us to say is the rainfall which is recorded at  $i$  th rain gauge in the catchment and clearly capital  $N$  is the total number of rain gauges in the catchment. So, there are  $N$  numbers of catchments,  $N$  number of rain gauges in a catchment. Then all we need to do is just take the data on a particular day or you know particular time period which is recorded those different rain gauges in the catchment. And just take the average of them. That will give the average rainfall or the special average rain fall in that whole catchment which can be used in rainfall run off modeling or any other purposes. Well the simpler the method less useful it is. And this method is hardly used although it is very simple, but is it is almost useless and it is not very popularly used at all. This method is rarely used in fact, in practice.

Why because it does not able to account for the amount of rainfall the area each rain gauge is represents and so on. It just simple average without accounting for many of the factors which are important well then what does we do? There are some more methods which people have developed. And one of the most famous methods which you may have seen earlier is called the Thiessen polygon method. So, the second method which we will see a quickly is called Thiessen polygon method as the name suggests we try to do some kind of construction in the form of polygons around the particular rain gauge in a catchment. And that in we say that that particular rain gauge use represented by the area which it represents, and how do will draw these thiessen polygons? Well there is a well defined procedure before I go to that this method follows certain assumptions or certain principles, can you tell me, what are those?

I am sure that you have studied this in your under graduate classes. So, what are the assumptions? So, what are the basic principle on which it is based on? So, let us look at that first. This method assumes that number 1 at any point at any point in a catchment the rainfall is same as that the nearest rain gauges, what does that mean? That means the rainfall in a catchment at any point in the catchment is same as the rainfall at the nearest rain gauge. So, whatever is the value of the rain rainfall at a particular rain gauge the area which is surrounded which surrounds this particular rain gauge we say that it is the same rainfall in that whole area. So, that is number 1; the second one is that the depth or the rainfall depth that is recorded at or given rain gauge or given rain gauge is applied out is applied out to a distance half way to the next station in any direction.

So, what is that the assumption mean? What is that useful mean that if you have 2 rain gauges and if you join those to rain gauges by a line. Then the depth recorded at one particular rain gauge is applied up to the middle of that line or the midpoint of to that line. After that point another rainfall which has been observed at the next rain gauge station is what is assumed in that particular area. That is why we do thus midpoint by sectors. And then that is how we develop this Thiessen polygon will seen a minute, well then let me a try to show you.

(Refer Slide Time: 12:15)

$$\bar{P} = \frac{1}{A} \sum_{i=1}^n A_i P_i$$

$$A = \sum_{i=1}^n A_i = \text{total catchment area}$$

$$w_i = \frac{A_i}{A}$$

$A_i$  = Area represented by the  $i^{\text{th}}$  polygon:  
 $P_i$  = Rainfall recorded at the  $i^{\text{th}}$  polygon

Average areal rainfall in catchment

**Disadvantages:**

- ① This method is not feasible as a new Thiessen network must be constructed each time there is a change in the gauge network.
- ② It does not account for orographic influences on rainfall.
- ③ Effect of rain-fall magnitude itself is not taken into consideration.

How we do this thiessen polygons I am show may have you know seen earlier I will not draw the river geometry here. So, what we do is we just join the locations of the rain gauges by striate lines such that they form triangles. The first step is I am taking only 3 rain gauges here for the simplicity for demonstration purposes, but there can be many number of rain gauges in a catchment here. So, whatever it is then you what you do is join them in such a manner that the form triangles. Then what will do is we draw what is called the perpendicular bisector on these lines. We draw the perpendicular bisectors; you take the midpoint and draw the perpendicular on them that is your perpendicular bisector. So, if this line it will be this line and will be like this line, because I am show exactly like that, but you understand that mean.

Then this whole area is represented by this rain gauge that say this is 1; this is rain gauge 2 and this is rain gauge 3. So, this is your area we call it A 1, we call this area A 2 and we

call this area  $A_3$  and so on. How many areas will be there as many numbers of rain gauges which represent in the catchment? So, what will be doing is we divided by the whole catchment area into  $N$  different number of Thiessen polygons which will be equal to the number of rain gauges. Once we have done that then what will do is assign rated to each rain fall observation in the ratio are the proportional of the area it represent with respect to the total area. So, then your  $\bar{P}$  will be equal to  $\frac{1}{A} \sum_{i=1}^N A_i P_i$  where  $i$  varies from 1 to capital  $N$ . See if I put the value, yes I did.

So, here what is  $A_i$ ?  $A_i$  is nothing but the area which is represented by the  $i$ th polygon or the  $i$ th rain gauge. Where another things are same actually the  $P_i$  is the rainfall recorded at the  $i$ th polygon or rain gauge or in the polygon where capital  $A$  is nothing but summation of all the  $A_i$ 's  $i$  varying from 1 to capital  $N$  total catchment area and  $\bar{P}$  of course, we did not define earlier. So, we can do that now is the average areal rainfall in catchment. So, you see that in the arithmetic average earlier we saw that the weightage assign to which observation was same it was  $1/N$ . So, we have  $P_1$  by  $N$  plus  $P_2$  by  $N$  plus  $P_3$  by  $N$  and so on. Here what is the weightage? If you see this formula above we are weightiest to the  $i$ th rain gauge is nothing but we can see  $A_i$  our capital  $A$ .

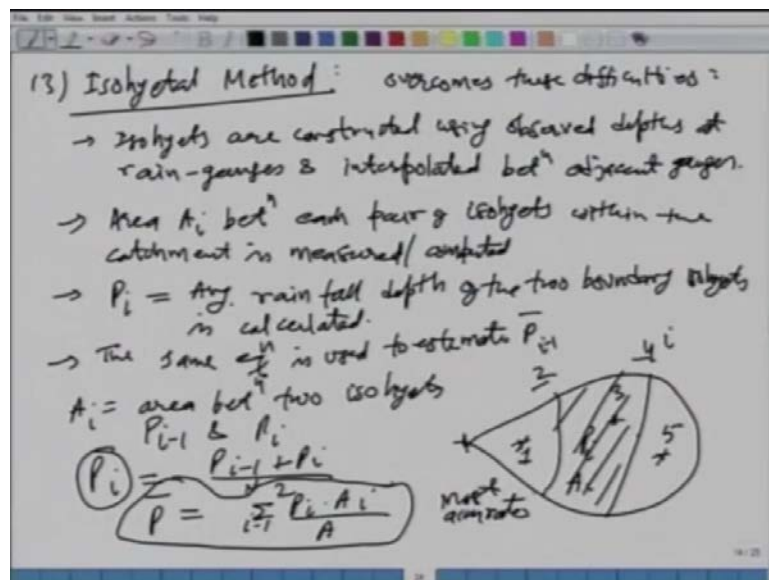
So, it is in the proportion of the area which is represented by that catchment. So, this is a method which does account for the area which is represented by the particular rain gauge. It is much better than using the simple average and this method is quite popularly use. However, this also suffers from certain weakness is our disadvantage this method. What are those, that is look at them you write these down, what are the disadvantages of this method? So, the first one is that this method is not flexible. Flexibility is very important has a new Thiessen network; new Thiessen network must be constructed each time there is a change in the gauge network. It should be very clear to understand, you see that polygons are constructed depending up on the locations of these rain gauges which are scatter throughout the catchment. If a person these an one of the rain gauge is the is not operating and it has to be replace by some other rain gauge by something or a new rain gauge is to be added, then this whole Thiessen polygon construction has to be done again.

So, it is not flexible which takes some you know time number 2 this method or it does not, it does not account for orographic influences, orographic influences on rainfall. What do you mean by that if you are very large catchment? One of the rain gauge is just may be close to the mountains are hails the rainfall patterns will be you know quite different there

are there due to the orographic you know patterns as compare to the plains. So, this method is not able to account for the orographic influences on that rainfall when we are doing the average in only with respect to the area it represents or represented by the theissen polygon. Third one is the effect of effect of rainfall magnitude itself, rain fall magnitude itself is not taken into consideration. What do you mean by that? Different rain gauges which as pattern in a catchment would observe different amount of rainfall? And there may be it pattern in a large catchment one once to graphical low local location may to higher amount of rainfall consistently.

And the other one you know small amount of rainfall if that is the case and if you are using theissen polygon then one of the rain gauge which as higher rainfall will not be represent at the, because it weight is always same or in the proportional to area which we not represent the proper importance in calculating the average. So, somehow of being able to account for the magnitude of the rainfall also in addition to the area rain gauge represents is also important. So, what we do is we have another method which is which takes care of some of these weaknesses or which over some of these limitations and then we call that what is called the isohyetal method.

(Refer Slide Time: 21:15)



So, the third method we will look at that is called isohyetal method which overcomes these difficulties or these limitations. Now, what is this isohyetal method? I am showing again that you may have seen this in your earlier classes. What is in isohyets well an isohyets is a

line joining in a catchment which represent equal amount of rainfall like isobars are the lines joining equal amount of pressure isotherms are the lines joining in a area which have equal temperature.

So, similarly, isohyets is a line or curve joining all the points which represents are which have the same amount of rainfall. So, this is a very important concept what we do is we construct the isohyets in a catchment using the rainfall magnitudes which are observed at a particular rain gauge station. So, let me give you this method and then and we will look at it. So, what we do is isohyets are constructed using the observed depths at rain gauges and then there interpolated between adjacent gauges if that is required. So, what we are doing basically is that we are using the rain fall observation magnitudes at the rain fall location or in gauge locations and then developing can this is similar to your contuse which may have continuing which may have done in your survey classes earlier. So, we have the bench marks as the rainfall depth scattered around the base in then what you do is you can develop these contuse of isohyets. There are software's available in these days to do this.

So, you can use it software in which you can you are catchment; you can put the location the rain gauge and put the bench marks at the as the rain fall magnitude. And then it will give you the contuse or the isohyets are some desired contuse interval. So, well not go into the details of that. Then what you do is the area  $A_i$  between each pair of isohyets within the catchment is measured are computed. Again you have the 2 isohyets in between the area is let us say  $A$ , you can either measure it using meters if you are working manually on a map or if you are working on a software it will automatically give you the area between 2 isohyets and the drainage boundaries or the catchment boundaries. Then let how we define is the  $P_i$  is then the average rainfall depth  $P_i$  is a average rain fall depth of the two boundaries of the two boundary isohyets is calculated it is not the overall average rain fall. This is  $P_i$ , what is  $P_i$ ?  $P_i$  is the average rain fall in an area which is bounded by 2 isohyets will look at it graphically inside the cycle. And then the same equation which you have d1 earlier portion polygon is used.

The same equation is used to estimate  $\bar{P}$ . So, if we look at this catchment are you may have these 3 different rain gauges in which the rainfall values let us say may be you know on a particular day or you know this may be normal range for, it may be let us 3 and let us say it is 5 or something I. I am trying to be a very simple here as you can see. So, between these 2 rain gauges you can select let us say 2 centimeters something like this it be 3 and 5



you may 1 to have can 2 corresponding to 4. So, this line represents of 4 centimeters, this line represents let us say 2 centimeters. So, this way you can have many numbers of rain gauges in this catchment. And then you can draw these isohyets which may have very complicated shape there may be circles there may be ovals and so on. So, just for demonstration purposes you have this. Here the rain fall may be you know very minimal or close to 0 whatever.

So, if you look at this one, let us say this is  $P_{i-1}$  or not  $P_{i-1}$  this is your  $i-1$  the isohyets and this is your  $i$ th isohyets and let us say this is  $P_i$  or this area is  $A_i$ . So, the area which is bounded by the 2 isohyets having 2 centimeters of rainfall and 4 centimeters of rainfall, that is your general  $i$ th area. So,  $A_i$  is the area between 2 isohyets which one which represent the rainfall  $P_{i-1}$  and  $P_i$ . And then you say the  $P_i$  actually is the average of these 2. So, you have  $P_{i-1} + P_i$ , so you are over writing this  $P_i$  divided by 2. So, this is the rainfall in this catchment which is the average of these 2 values, so which is 3 centimeters. So, you keep on doing that for all the areas and then you finally, use the same equation overall average is nothing but summation of your  $P_i A_i$  over total  $A$  where  $A_i$  varies from 1 to capital  $N$  where capital  $N$  is the number of areas which you have inter isohyetal areas that is what it is called.

So, you see that this method the isohyetal method is the most accurate method. Why it is most accurate method is that it is a it is accounting for the areas which is represented or the area which is represented by a particular rain gauge location number 1. And also it does take into account the amount of rainfall that occurs or that falls on that particular rain gauge. So, it will be able to take care of the ore graphic influence in any other pattern in the rainfall, you know special variation in the rainfall. So, this way you will have a better estimate of the average rainfall. So, this was the third method of areal averaging of the rainfall.

(Refer Slide Time: 30:01)

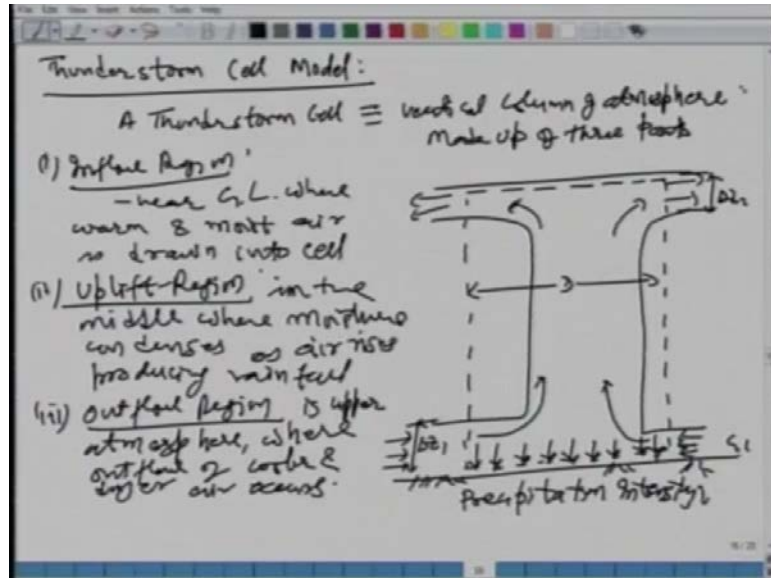
(4) Reciprocal-Distance Squared Method:  
Effect of rainfall on a gauge point on any other point in the catchment is inversely proportional to the square of the distance between the two:  
 $X_1(x_1, y_1); X_2(x_2, y_2)$   
 $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$   
 $W = \frac{1}{D^2}$   
 $\bar{P} = \frac{\sum_{i=1}^n P_i \cdot W_i}{\sum W}$

And the last one we are going to look at quickly is called the reciprocal distance squared method. This method is not very common in India actually this is used mainly in the United States in which what they do is the effect of rainfall on a gauge point or at a location on any other point, on any other point in the catchment is assumed to be inversely proportional to the square as the name of the method suggests to the square of the distance between the two between the two points. So, the rainfall on one gauge point and the rainfall on another gauge point, how are they are related? Well you take the distance between the two and the rainfall we say is the inversely proportional to the square of the distance between those two points. So, it should be very easy to compute also as you can see that if you have two locations.

Let us say 1 is x having coordinates  $x_1 y_1$  for example, you have another point I am call let me call to  $x_1$  and  $x_2$  another point has the coordinates  $x_2 y_2$  then you can easily calculate the distance between the two locations  $x_1$  and  $x_2$  which should be  $x_1$  minus  $x_2$  whole squared from the knowledge of your basic geometry. And then weightage as per this method is, what 1 over this distance square reciprocal distance square method? Once we have the weightage, then all we need to do is will be summation of your  $P_i w_i$   $i$  varying from 1 to capital N divided by summation of W or the total W's the summation of all the weight should be then equal to 1 So, these are the 4 different methods of calculating or estimating the special average rainfall in a large catchment Now, what will do is we will look at something extremely important in this chapter in which we will try to calculate the

intensity of rainfall coming out of a thunder storm cell. So, I will go back to the P G book in which we will look at what is called thunders storm cell model.

(Refer Slide Time: 33:15)



So, what is a thunder storm? First we will look at what do we actually mean by what is a thunder storm? It is basically convective cell rainfall or there is a convective cell or thunder storm cell that gets setup in atmosphere during certain conditions. And the rainfall that is coming out of that thunder storm cell is due to the convective processes. So, what happens is we will look at a thunder storm cell first. And then will go to the equations. What is it? It is nothing but a vertical column of atmosphere, and it is made of 3 parts. So, what I will do is a will look at the diagram first and then will come back to the equations. So, what it is? This is a cylinder vertical cylinder; the portion in the middle you see is a vertical cylinder. And then there is a vast expanse on the upper side which is into the clouds and 1 is close to the bottom of the ground.

So, what we have here is you see here this is the ground or the ground level the air gets into this thunder storm cell close to the ground inside it. Then it gets lifted as we have seen earlier that there is one important basic physical mechanism for rainfall to occur is that the air mass needs to get lifted. And in this particular case it is a convective or a thunder storm cell. So, air mass is getting lifted in this middle region. So, what we have is this of height  $\Delta z$  1 this is called the inflow region and I will come to that in a second the air the moist air enters into this thunder storm cell through this inflow region it rises in

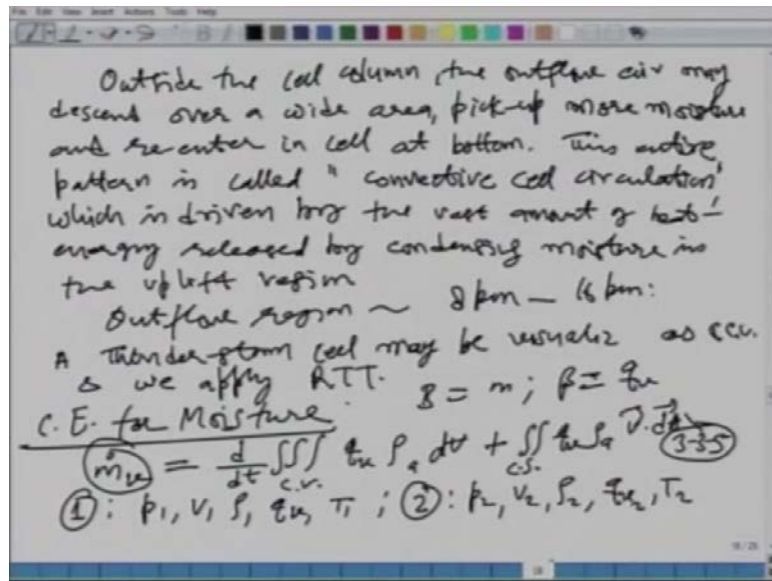
the middle, and then what comes out from here on either side. This is of height let say  $\Delta z$ .

So, the air that here comes out from what is called the outflow region this of height  $\Delta z$ . So, what we can do is we can take a cylinder of actually some diameter  $D$  when then ultimately what we are interested in is we want to find out the rate of rain fall or intensity of rain fall that will come out of it or that will be observed on the ground. So, you have the precipitation intensity high which is observed on the ground. So, let me quickly write down the 3 parts of the 3 regions which is one, one of them is the inflow region as we have seen it is near the ground where the warm and moist air is drawn into this thunder storm cell from the bottom.

Then the second portion is the middle portion which is called the up lift region which is called the up lift region. It is in the middle where the moisture condenses as the air rises. Remember we it says that for rainfall to occur it need to rise. So, that it can condense that condensation takes place in this middle of the up lift region producing rain fall or precipitation. Then you have the third region which is called the outflow region on the top is the upper part or the upper atmosphere, where the out flow of the cooler and dryer. Now, moisture has been taken out of it air the out flow occurs. So, this way we see that in a thunder storm cell that it consists of these 3 parts and what we will do next is we will look at how we can use our Reynolds transport theorem and apply these equations to this?

So, we see that the in the thunder storm cell there are 3 regions the warm and you know air the moist air enters at the bottom close to the ground through the inflow region it gets up lifted in the middle region. And then its comes out in the water condense comes out is some certain intensity. And the dry air and cooler air comes out from a top from the outflow region. The air that comes out from the top can this and down are at the outside it can pick up some more moisture and reenter into this thunder storm cell. And then this process again the actually get set up this is called the convective cells are circulation or the convective cell pattern which is responsible for rainfall intensity  $i$  coming out. So, what will do is will look at this mechanism.

(Refer Slide Time: 40:02)



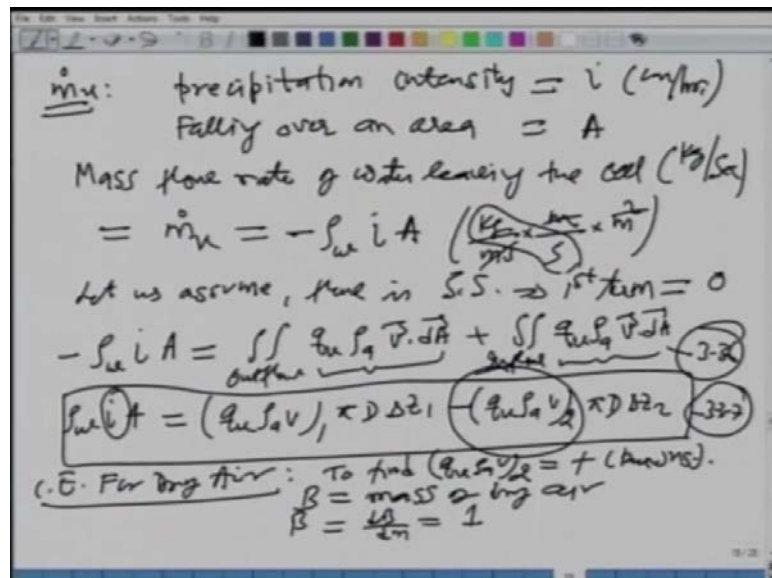
Basically what we are saying is that the outside the cell column what we just said the outflow air which has come out from the outflow region may descend over a wide area pick up more moisture pick up more moisture and reenter into the cell at the bottom or close to the ground. This entire pattern is called convective cell circulation which is driven by the vast amount of heat energy which is released by the condensing, by condense rather moisture. As the condensation takes place you know are the lot of heat that is released from this physical reaction moisture in the where is the seat in the middle portion or in the up lift region. So, this is what is called the convective cell circulation and this outflow region can be as high as about 8 kilometers or 16 kilometers into the atmosphere that is the maximum height of your convective cell circulation clouds. Now, a thunder storm cell what we will do is may be visualized has a CV or control volume. We apply our Reynolds transport theorem to see how the movement of the moisture is taking place and how we can calculate the rate of rain fall that is coming out of it?

So, what we will do is we will write the continuity equation for we moisture or the water wafer in the thunder storm cell. And we have already mention, what is the control volume? It is cylinder of diameter D. So, what we have actually is we have already seen that capital B is equal to your m beta was q e for this case, beta was not q e beta was m e dot we assumed and then we have already written that the beginning of this chapter that the continuity equation for the moisture in the mixture of air is m v dot is equal to V first term over the control volume of what it is going to be equal to I am sorry let me go back and

correct this that is actually your  $q_e$ , I was right and then  $\beta \rho D v$  that is the time rate of change of extensive property stored within the control volume. And the second term is the control surface of your  $q_e \rho A$  and  $v \cdot dA$ . Now, let me first define that you have an inflow region and an outflow region. So, let me say that the inflow region is cross section 1 and the outflow region is cross section 2.

And if we define all the property the climatic conditions at 1 or in flow region as say the pressure is  $P_1$  the velocity of the wind there is  $v_1$ , the density of air is  $\rho_1$ . Then you have  $q_v$  is the relative humidity there the temperature is  $T_1$ . Similarly, the other quantities at 2 is with subscript 2 we have  $P_2$   $v_2$   $\rho_2$   $v_2$  and  $T_2$  etcetera. So, everything is defined with subscript 1 and 2. Let me go back and number this equation as 3-3-5 that is the numbering we have been following. So, let us work on what will be this term  $m \cdot v \cdot m \cdot v \cdot$  is what physically what does it represent? It represents the time rate of change of mass, how is the mass changing is the mass is being created or destroyed within the control volume or it is changing in different form? So, we see that although the mass is not being created or destroyed, but it is changing its form from wafer to liquid and that liquid is coming down as certain intensity  $i$ .

(Refer Slide Time: 46:55)



So, if you want to write down expression for  $m \cdot v$  what would it be? Now, let us assume that the precipitation intensity is  $i$ . And let us say it is in centimeters per hour or some other suitable units its falling over a cross section area of that cylinder which we

have seen and let us say is equal to  $\dot{m}$ . So, with these notations what will be the mass flow-rate of water leaving the cell? And let us say the unit of that is mass let us say it is kilograms per second will be equal to what that is what is your  $\dot{m}$ ? That is what we want to find out is equal to if you see it will be  $\rho_w v_i A$  should be see that. Let me write the units in the brackets or  $\rho_w$  is density of water which is  $\text{K g per meter cube times high}$  let us say in meters per second area is let us say square meters. So, we have meter cube cancelling with this what is remaining is your  $\text{K g per second}$ . So,  $\rho_w v_i A$  represent nothing but your  $\dot{m}$ .

And we are going to put a negative sign in front just to be correct as far as the notations are concerned. Now, let us assume that the flow is steady state this convective cell circulation which we are assuming and the rain falling intensity  $i$  coming out. Let us say that all of this process is steady under that assumption the first term will be equal to 0 in your Reynolds transport theorem all this equation 3.3.5, so this quantity will be 0. So, if you want to write this down you will have negative  $\rho_w v_i A$  that is your  $\dot{m}$  left hand side is equal to the first term is 0 and the second term that is broken into 2 parts. So, you have the for outflow region you will have  $q_e \rho_a V \cdot dA$  term plus over the inflow region, what is the  $q_e \rho_a$  and  $V \cdot dA$ . Let me say that this is my equation 3.3.6. So, all we have done is we are say that because of the steady state nature of the flow first term is 0  $\dot{m}$  we have said is  $\rho_w v_i A$ .

And on the hand side the second term will be Reynolds transport theorem we have broken into 2 parts outflow region and inflow region separately. Now, we have already defined that there is various parameters. So, what will be able to do then is the write we will these terms in terms of those parameters 1 and 2. So, if take that you will able to see that  $\rho_w v_i A$ , I am taking negative on the other side will be equal to  $q_e \rho_a V$ . All those things at subscript 1 are the inflow region times the area, area through which the things are entry. So, if you look at from the top it is a circle of the diameter  $D$  and height is  $\Delta z$ . So, what is the area the perimeter is the  $\pi D$  and the height is  $\Delta z$ . So, it is let us say the area is going to be  $\pi D \Delta z$ , let me go back to some of you. So, this is diameter  $D$  at the bottom things are entering into a height of  $\Delta z$  you see that this is your  $\Delta z$  and it is a diameter of or it is a circle of diameter.

So, it is going to be the area is  $\pi D \Delta z$ . Similarly, then you have the inflow which is negative you have  $q_e \rho_a v$  at cross section 2 times your  $\pi D \Delta z$ . So, let me

number this equation then as 3-3-7. So, this is the equation that gives me the relationship between various parameters the specific humidity, the density of the air, the velocity or the diameter and the intensity and so on. What is my objective at this point of time? My objective is to determine this guy this  $\rho_w$  I know area I know and the right hand side I know everything close to the ground. Close to the ground I can measure various parameters. The pressure the temperature the humidity and so on. But what I do not know is this quantity  $q_e \rho_a V$  at height 2 at the outflow region. So, what I do is I actually need one more equation which can relate these quantities at cross section 2 or the outflow region. So, what I can do that is I can write the continuity equation for the dry air. Now, I have written the continuity equation for the moisture in the moist air.

So, what we do next is the we write the continuity equation for dry air that will relate this quantity with the other quantities, and why we are doing it to find  $q_e \rho_a V$  at 2 as a function of other known things. So, for the dry air where the flowing fluid is dry let us say for capital B is the mass of the dry air. So, what will be beta? Beta will be  $d_B$  over  $d_m$  which will be the mass of the dry air per unit mass of the flowing fluid which is dry air so it will be 1. So,  $d_B$  by  $d_m$  will be equal to 1 also assume last steady state process, so we do that.

(Refer Slide Time: 54:20)

$$0 = \int \int_{cs} \rho_d \vec{V} \cdot d\vec{R} \quad \text{--- (3-38)}$$

$$\rho_d = \rho_a (1 - q_w) \quad \rightarrow$$

$$0 = [\rho_a (1 - q_w) v \Delta z]_2 - [\rho_a (1 - q_w) v \Delta z]_1 \quad \text{--- (3-39)}$$

$$\Rightarrow (\rho_a v \Delta z)_2 = (\rho_a v \Delta z)_1 \left( \frac{1 - q_{w1}}{1 - q_{w2}} \right) \quad \text{--- (3-39)}$$

Substituting (3-38) into (3-39) &  $A = \frac{\pi}{4} D^2$

$$i = \frac{4 \rho_a v_1 \Delta z_1}{\pi D} \left( \frac{q_2 - q_{w2}}{1 - q_{w2}} \right)$$

Then what will have is the continue equation is left hand side is 0 will be equal to, because there is nothing coming out of that dry air as the dry air is moving there is no moisture



exchange that is taking place, So, this left hand side is 0. The first term from the right hand side is also 0. And then you will have the over the cross section or the control surface of your rho d here  $V \cdot d a$ . So, this is my 3 3 8; this rho d I will say is rho a times 1 minus q e, we put into this equation what I will have is 0 equal to rho a. And then I expand it 1 minus q e of your  $V \Delta z$  all of this quantities at 2 times pi D. So, I am I am opening out the  $V \cdot d a$  term like did earlier minus you have rho a 1 minus q e of your  $V \Delta z$  this quantity at 2 times of your i d. I would like to draw your attention the and the see if these things make sense to you or not all we are doing is rho d, we are replacing by this quantity and you are expanding your  $V \cdot d a$  term like we did earlier.

So, this will actually give you rho a  $V \Delta z$  at cross section 2 in terms of other known quantities, rho a  $V \Delta z$  at cross section 1 times 1 minus q e 1 divided by 1 minus q e 2. So, this is an important relation you get by applying the continuity equation to the dry air in this case and we number this as 3 3 9. Now, what I can do is we can put this rho d delta z at cross section 2 back into your equation are the continuity equation for the moisture, that way I will a write it down substituting 3 3 9 this equation which you have just seen into 3 3 7. What is 3 3 7? It is this 1; this 1 and then we take area as let us say pi by 4 D square it is a your circle or cylinder of diameter D. And then simplifying what we can get is this expression for i which will be 4 rho a 1  $V 1 \Delta z 1$  over rho w D times your q e 1 minus q e 2 you should be able to see that you can verify this 1 minus q V 2. This is your final expression.