Advanced Hydrology
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Lecture – 13

Good morning friends and welcome to the video course on Advanced Hydrology. In the last class, we looked at the dynamics of what goes on inside the cloud. We, on the board we looked at a blown up picture of a cloud, the lifting of the air mass nucleation and what goes on and how the rainfall comes out of a cloud. Then we looked at the different forms of precipitation in which we said the precipitation can be in the form of liquid or solid and various different kinds.

Then we moved on to the precipitation characteristics in Indian conditions, and then we said that the season in India, the meteorological season in India is divided into four parts. And the most important one of them is what is called the South-West monsoon. And we looked little bit more detail into the South-West monsoon in India. Then we moved on and started looking at what is called the "Terminal velocity". As the rain drop forms in the atmosphere, it starts to descend towards the ground and then there are many forces acting on it.

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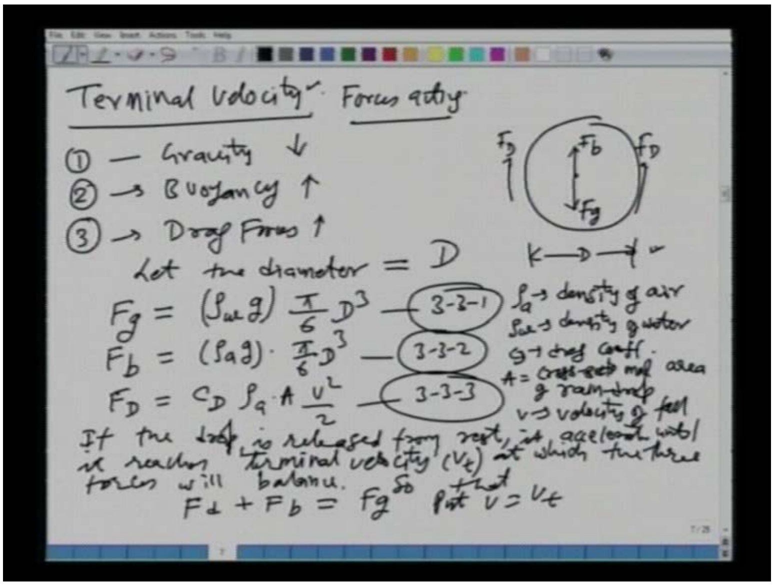

And then we started looking at how we can calculate, at what velocity the raindrop will fall. So, let us a look at this schematic, which we started looking at yesterday. So, we are trying to find out the terminal velocity.

Let us assume that there is a spherical raindrop and its diameter is D. And we said that there are three main forces that act on this spherical rain drop as it is falling through the air. First one is the gravity forces, which are acting in the vertical downward direction, then you have the buoyancy forces, which would be acting in the upward direction as shown in the figure. And then the third one is what is called the drag forces due to the resistance offered by the air.

Now, the diameter we have said is let us say D. And then, if you want to write down the expressions for these forces, so what will be F of g? Well, it is nothing but the density multiplied by the volume; alright, density of water multiplied by the volume of this sphere of diameter D. That is all it will be. Not exactly density, but the specific weight.

So if you see that, it would be rho w times g; that is your specific weight. And then, the volume of a sphere of diameter D, as you all know would be this. And I am going to number this equation as three three one. So, this is the force or the gravity force or the weight force which is acting or which is pulling this spherical raindrop below. What is the F of b? It will be nothing but the weight of the air or the fluid which is displaced by the object.

So, it will be the same expression; except that instead of the density of water, we will use the density of the moist air. So, it will be rho a g times pi by six D cubed. And this is three three and two. And what will be the drag force which will be actually acting on the surface area of this whole spherical raindrop? Alright, from the knowledge of our Fluid Mechanics, we will take this as C D times rho a times A times V squared by two. The drag force that acts on any object is directly proportional to the square of the velocity and there is a co-efficient of drag C D. And let me number this equation as three three three.

And then, let me define all the things here. Your rho a is the density of air, rho w is the density of water, C D as we know this is called the co-efficient of drag or drag co-efficient, D is the diameter we have defined and A is the cross sectional area of the rain drop. It is not the surface area actually.

So, that is how the drag force is defined. It is not rain fall, but the rain drop and V is the velocity of fall with which the rain drop would be falling. Now as the drop is released, let us from rest, initially it starts with the zero velocity, then it starts to accelerate. After some time what happens is, initially as it starts to accelerate, the drag forces are minimal and then they start to increase because the drag forces are directly proportional to the velocity. So, the drag force is increased. And after some time what will happen is all the forces will start to balance each other.

So, if you want to write it down in words, it will be if that raindrop is released from rest, it will accelerate until it reaches what is called the terminal velocity or fall velocity. And let us denote this as V t, which is what we are trying to find out now. At which the three forces will be in equilibrium, will balance out or will be in equilibrium. So that, what do we have? Your F d and F b are the two forces which are acting in the upward direction and their sum should be equal to F of g; alright, that is the balance of these forces. So, what we do is we have this expression for this and we put V is equal to V t and those three expressions. And then, what we are going to get will… we can simplify that and get the expression for the fall velocity. So, let me try and do that.

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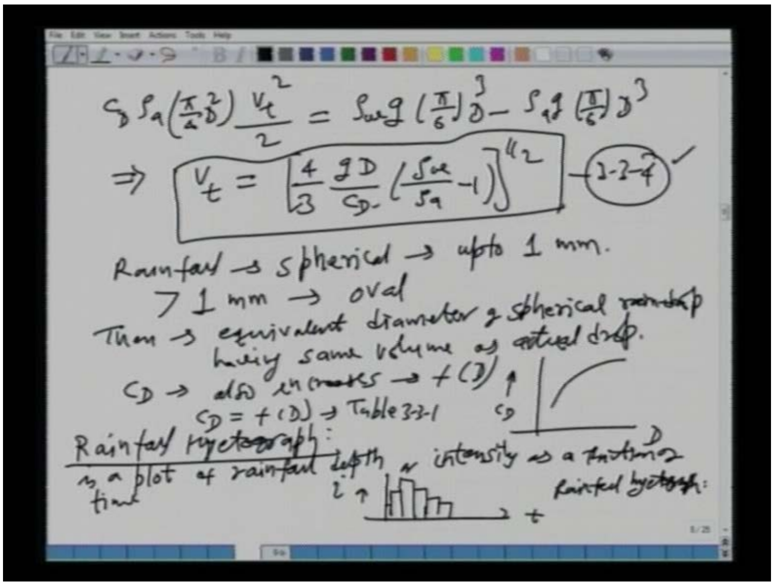

So, we have C D rho a pi by four D square is your cross sectional area, and let us say V t is your fall velocity, so V t square by 2. This is equal to rho w g times pi by six D cube minus rho a g times pi by 6 D cube. So, what I have done is I have taken the buoyancy forces on the other side. And then, I will not workout. We can see that easily that the expression for V t will come out like this… rho w or rho a minus one and this whole thing raise to the power half or square root of this. So, this is your three three and four. So, this is the final expression from which we can calculate the terminal velocity or fall velocity at which a rain drop of spherical diameter D would fall through the atmosphere.

So, you see that. If you look at the equation, the fall velocity is directly proportional to the diameter. So, as the diameter increases, its fall velocity will also increase. And then, if you see that the C D is in the denominator or the fall velocity is inversely proportional to the C D. So, if the drag force increases, the fall velocity will decrease and vice versa; which intuitively makes sense.

Now, this rain drop which we are actually assuming to be spherical; alright, it is true up to a rain drop of size approximately one millimeter. Alright, if the size of the rain drop becomes more than a millimeter, then it is no longer spherical. It is oval. If it is more than one millimeter, it becomes oval shaped; it is no longer a spherical. Well, how do you handle such situations? Well, all we do is we take an equivalent diameter of a spherical rain drop having the same volume. So, then all we do is we take equivalent diameter of spherical rain drop, I have the habit of writing rainfall, raindrop it is, having same volume as the actual drop which is oval in shape. Now, this is as far as the shape of the raindrop is concerned.

Now, let us say we are assuming that it is spherical; if the diameter of the rain drop increases, do you think that what will happen to the drag force? It will increase or stay constant? It will obviously increase because the cross sectional area is, you know one of the expressions on the right hand side. However what we are talking about is, it will not increase exactly in that ratio; the co-efficient of drag C D itself changes when the diameter increases.

So, what we have is C D or the co-efficient of drag also increases, alright, as a function of diameter. How does it do? Well, the relationship looks like this. If it is your diameter, it is a non-linear change like this.

So, the research has shown that the co-efficient of drag itself will increase as a function of the diameter of the rain drop. Now, this relationship of C D as a function of diameter, it is given in table three point three point one of the venty (( )). So, kindly you know, I request you to go through it and look at those numbers, how actually it is changing. Therefore, we have this final expression for finding of the fall velocity.

Now, what we are going to do is we will look at some of the basic review material, which you may have seen in your earlier classes. As when then rain is falling, alright, how do we measure the rainfall and how do we represent the data. Alright, as far as the measurement is concerned, we will not look at it at all because it is an advanced course and we have seen that already.

So, we will get started with how we represent this data which we have collected and how we store it and how we try to understand. So, there are two different forms of data representation. We had looked that earlier also. But, as far as the rainfall is concerned, we have the first thing is what is called a rainfall hyetograph. That is how we represent the rainfall data; rainfall hyetograph. I am sure all of you understand what is a rainfall hyetograph. Well, it is nothing but it is a plot of rainfall depth or intensity; it is a plot of rainfall depth or intensity, as a function of time.

And, how will it look like? Well, if you see that this is your time domain t and this is your, let us say i is represented as the centimeters per hour or inches per hour or something like that. Normally, it is represented like a bar chart. So, this is called a rainfall hyetograph. I am sure that you may have seen this earlier. So during a rainfall event, it gives us a very important information about how intensely the rainfall is occurring at different time, which can be very useful in some design purposes. So this is about the rainfall hyetograph.

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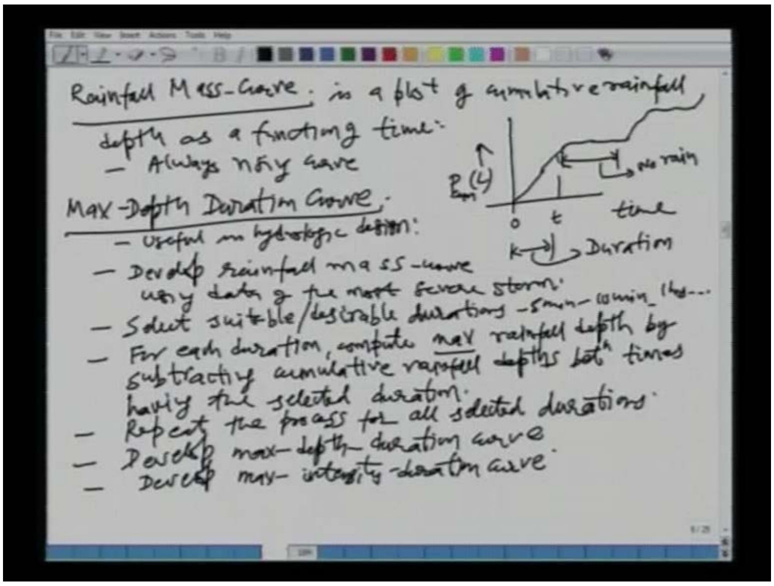

And then, you have a rainfall mass-curve. What is a rainfall mass-curve? Well, it is nothing but the cumulative value of the rainfall hyetograph. So, if we have a table in which we have time versus the rainfall intensities or the depth of rainfall in each interval, what we can do is we can just accumulate those values as a function of time. That is what will give us the rainfall mass-curve. So, it is the cumulative curve of the rainfall hyetograph.

So, if you want define it, it is a plot of cumulative rainfall depth as a function of time. And how will it look like? Let us say it is P cumulative and its unit would be some length units. It will be like this. In the rainfall mass-curve, one thing important we should notice that it is in always raising curve. That is, it will never drop down. So, it is always rising or it will remain constant, when it is constant like this, this period, what is happening during this period? Well, there is no rainfall during this period.

The rainfall depth is increasing as a function of time. Let us say from zero up to this time, whatever the time is… After that, the curve is constant. So, there is no rainfall. And what is this period? This is nothing but the duration of the rainfall. So, this is; you can say this is the duration of this particular storm event, which occurs between zero and t. Then, there is a period of no rain and then there is some more rain; then there is a period of no rain and then rainfall starts to increase.

So, it is a constant curve, continuous curve which gives you a lot of information about the duration of the rainfall, the depth of the rainfall in each interval and also the intensity. We can differentiate this curve, alright, and find out what will be the rainfall intensity at any given time. So, it is always very handy and very useful data representation method.

The next thing we will look at is very important; which is called the maximum-depth duration curve. This maximum-depth duration curve is useful in hydrologic design. As the name suggests, it gives you information on the maximum amount of rainfall depth that can occur in any given duration.

So, when we are doing some hydrologic deign, that is, you convert design or any hydrologic structure is to be design in a catchment, then we want to find out what is the maximum amount of rainfall that can occur which will, we can then easily convert into the run off depth. So, those things are designed for peak discharges which will depend upon the maximum intensity in the maximum rainfall. So, that is how it is very useful in hydrologic design of certain hydrologic structures.

Now, how do we find this maximum depth for certain duration? Alright, Well, what I like to do is I will give you a step by step procedure first to determine or to find the this maximum-depth duration curve from which we can deduce the maximum intensity duration curve also. And then, we will look at a solved example or a numerical example on this.

So, the steps that are involved in this are; let us say what we need is using all the data that are given to us, we develop a rainfall mass-curve using data of the most severe storm. So, when you are trying to find out the maximum depth that can occur in a duration, you want to take the data which is the most severe in a particular catchment. For example, you have a catchment in which you want to find out this maximum-depth duration curve. So, what we do is we just scan through all the data that are available to us. We may have twenty years of data available to us; we may have fifty years of data available to us. So, we scan through all the data and then we pick up one storm event which is the most severe as far as the intensities in depths of the rainfall is concerned. So, we select that and then develop a mass curve of that. And we have just seen what is a mass curve, and how we can calculate it?

So, once we have that mass curve, what do we do? The next step then is to select suitable or certain, I should say desirable, durations; alright, durations for which we want to determine the maximum depth. There can be different durations which are important from hydrologic design purposes.

For example, a two hour event is what is you know very important because you want to design a certain structure for a two hour event, which are predominate in a catchment. So, these durations can be, you know any duration; for example, five minutes, ten minutes or even you know hours, etcetera. So, we select those durations depending upon our need or the practical purposes.

Now, the next point which I am going to write is extremely important to understand; for each duration, compute the maximum rainfall depth. By how do you do that? By subtracting cumulative rainfall depths. And how do you select those? Between times having the selected duration. So, what you do? What this step saying is, let me read it once more and then come back to explain you. For each duration, compute the maximum rainfall depth. How? By subtracting the cumulative rainfall depths between times having the selected duration.

So, what we are doing is, you have the rainfall mass-curve with you. Let us say you have selected five minute duration, for which you want to find out the maximum depth. So, for five minute duration from the mass curve what you do is you subtract the two values which are separated by five minutes. There may be many of them. For example, you may have the values between at intervals of five minute. So, you have the cumulative rainfall at 5, 10, 15, 20 30, and so on minutes.

So, what you do is, you keep on subtracting the cumulative rainfall depth values at five minute. So, five minus zero, ten minus five and so on. So, you will have that column and in that column, you determine which is the maximum? So, that will be your maximum depth for a five minute duration. So, this is the procedure. I would like to take up an example to explain this further. And then, what you do is you repeat this exercise for other different durations, which you have selected. So, you say you repeat the process for all selected durations. And once you have the maximum depth for all the selected durations, you have the maximum-depth duration curve. And, you can convert those maximum depths for each duration into intensities. So, then you can develop easily maximum-depth duration curve. And then, also you can develop the maximum intensity that can be expected for each duration curve. So, this is the procedure for finding out the maximum depth duration or maximum intensity duration curve.

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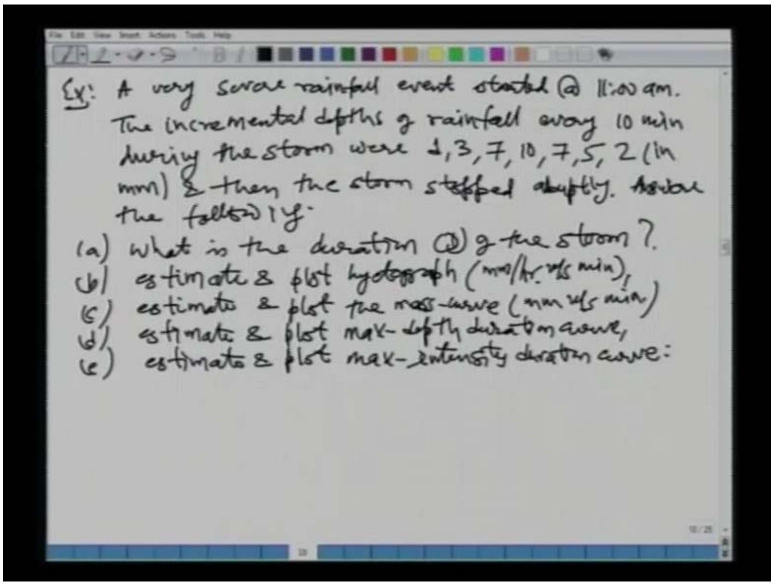

Now, what I will do is I take up an example. So for that, first I would like to write it for you, so that you understand what the problem is. Then, I will go to the board and try to solve it. So, we are looking for an example on this maximum-depth duration curve.

This is how it goes. A very severe rainfall event started at let us say 11 A. M. somewhere on a certain day. The incremental depths of the rainfall for every ten minutes, during the storm were observed as… This data is given to us. During the first ten minutes, it was one; during the next ten minutes, it was three and the next ten is seven and so on. So, you have ten, seven, five, and two. All these values are in mm or in millimeters. And then, the storm stopped and the rainfall event stops abruptly.

Answer the following: a) what is the duration of the storm? That is part a. Then b is estimate and plot the rainfall hyetograph or I will just say hyetograph in this case. And what are the units in which we have to do in millimeters per hour versus minutes? The third part is, estimate and plot the mass curve. And the units should be millimeters versus minutes. And d is estimate and plot the maximum-depth duration curve, which is the procedure we have just seen; the maximum-depth duration curve. And lastly e is estimate and plot the maximum intensity duration curve.

So, what I am going to do is I will demonstrate this on the board. And I am not going to plot all these different plots which have been asked. So, all we will do is we will just estimate or calculate these different curves, and then you can go ahead and plot them. We will just roughly catch the final two. Now, I am going to go on the board and look at this example how we can solve it.

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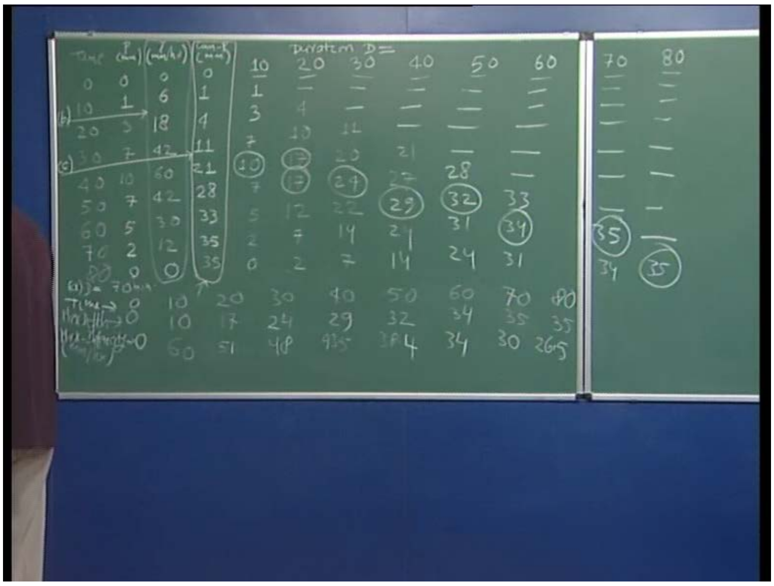

So, first we will put on all the data here, which are given to us as zero. All the data are given at ten minute interval. So, we will have 10, 20, 30, 40, 50, 60, 70 and 80. In each ten minute interval, the rainfall depth P; that is given to us in millimeters. During the first or zero, this is nothing; so it is zero. Then, I will just write all the numbers which are given in this problem; which is 1, 3, 7, 10, 7, 5, 2 and 0. So, the rainfall is up to 70 and after that in the next interval, there is no rainfall. And these are all the values given to us; 1, 3, 7, 10, 7, 5, 2.

Now, first thing we need to calculate is the intensity of rainfall and that is in millimeters per hour. So, what will be the intensity? At zero, of course you will have zero. So, you have one millimeter of rainfall occurring in ten minutes. So, what will be the rainfall occurring in one hour? Alright, you can easily calculate it. So, all we will need to do is, it will be six times of this; because this is ten minutes, you want to find out in sixty minutes. So, it will be six times of this column.

So, all you need to do is to calculate the intensity of rainfall during this storm event. You just multiply this column by six. So, I am just going to do that. So, you will have 6, 18. So, this is the intensity in millimeters per hour. You have 42, 60, again 42, 30 and 12 and then 0. It is nothing during the last interval.

The next column; which we will actually need for the maximum depth duration analysis. What will be the cumulative rainfall cumulative P in millimeters? At zero it will be zero of course. What will be the cumulative rainfall for the first ten minutes? So, now we will be looking at this column because this gives you the rainfall depth at each ten minutes and you want to accumulate these values. So, it will be this plus this; which is just one. So, the value of the cumulative rainfall depth at twenty minute will be equal to what this value plus what occurs between ten and twenty.

So, it is one plus three; that will be equal to four. This process should be very clear; how we can calculate the cumulative value of rainfall. Similarly, it will be four plus seven; that is eleven. And we keep on doing it. So, it will be 21, 28, you will see that they will be increasing all the time; it will not be decreasing; 33 because it is always a rising curve, 35 and then it remains 35. There is no rainfall between 70 and 80 minutes. So, the cumulative depth of rainfall is what? Thirty five plus zero, which is 35? So, it will remain constant, if it does not rain.

So, what are your answers then? As far as the first part is concerned, a is the duration D. what is the duration? Can somebody tell me? The duration of the total rainfall is what starts from zero; there is some rain between zero and ten. Then, at seventy minutes there is some rainfall. But, after that it does not rain. So, the duration as you see will be seventy minutes. That is your a part, what was b? b was your rainfall hyetograph. So, this is your answer b; time versus intensity of rainfall in millimeters per hour versus the duration. So, at these times these are the rainfall intensities. Part c, I think was what is the cumulative or the mass curve. So, it is this; your answer c. This is your… So, let me just write down somewhere that, this is your answer b and this one is your answer c.

Now, we move to d and e, in which we have to calculate the maximum-depth duration curve. So, alright… So, what we will do is, we will write this for all the durations we have; starting from 10, 20, 30, all the way up to 70 or 80. So, let me just write it and then we will see how we can calculate it. So we have 10, 20, 30, 40, 50, 60, 70 and 80.

So, we are looking at how we can determine the maximum-depth duration curve now. So, we will start with the duration of ten minutes and then we will do it for twenty, thirty and so on. So for ten minutes, initially when we have zero, there will be nothing. So for zero, you can put a dash for all of them. Now, we focus on that ten minute duration. What is the maximum depth that can occur in ten minutes and what is the procedure? Well, we take the mass curve which is this one. So, this is our mass curve. And now what we do? We subtract the cumulative depth which is separated by ten minutes. So, this is our focus of attention; is ten minutes. So, these successive values are at ten minutes. So, one minus zero is what you put here; then four minus one is what you will put here and so on; which actually will come out exactly this in this particular case.

So if you do that, you will have 1, 3 and 7. What is seven? It is eleven minus four. That is what we are doing. And we will keep on doing it for other minutes also. So this is 10, 7, 5, 2, should be easy to see, and zero. How is this zero coming? 35 minus 35. Thirty five minus thirty three is two; thirty three minus twenty eight is five and so on. So, these are the depths which occur in ten minutes. What is the maximum value? Just circle that. So, the maximum depth that occurs in ten minutes is ten millimeters. And when does it occur? Between thirty and forty minutes. So, this is how we find the maximum depth for any duration.

Now, we do it for twenty minutes. So for twenty minutes, the first ten minutes you do not have anything. So, you can put a dash everywhere; actually cumulative depths, which are separated by twenty minutes. So, you have four at twenty and zero add zero. So, you will have this are separated by twenty. So, then you have four minus zero, so that will be four will come here, the next one is eleven is at thirty and then one is at ten. The difference is twenty minutes. The difference in cumulative rainfall depth is what? Eleven minus one is ten.

Similarly the next one, it will be twenty one minus four, which will be seventeen. And you keep on doing this. Next one also will be 17, then you have 12, 7 and 2. What is the maximum? Well, it is seventeen. You do the same thing for thirty. So, one more row you can just say that there will be no value for twenty because we are working on thirty onwards. So, you have; for thirty, I am just going to write it directly. So it will be 11, 20, 24, 22, 14 and 7. The maximum is twenty four. The next one is forty. For that, it will be 21, 27, 29, 24 and 14. The maximum is twenty nine.

And, we can keep on doing this for the next one, which is fifty minutes. It will be 28. You can verify that. And we are using exactly the same procedure; 31 and 24. So, it will be 32 for fifty minutes duration and for sixty, you would have 33, 34 and 31. This is maximum. These two there is nothing. And for seventy you would have thirty five and thirty four. Maximum, of course is… Then for seventy minutes duration is 35, this nothing here. And then, the last one will be 35.

So, we have seen that how we can calculate the depths at different durations. And in each column we can find out what is the maximum depth? For ten minutes duration, the maximum depth is ten millimeters; for twenty minutes duration, the maximum depth is seventeen millimeters and then so on. So, what we can then do is, we can just write down this maximum-depth duration curve in a tabular form and then we can plot it.

So, you have time is equal to zero, ten, twenty, thirty, forty… So for ten, you have this maximum depth. This is in millimeters. For zero, obviously it is zero; for ten minutes, we have ten millimeter; for twenty we have seventeen millimeters. It is coming from here. For thirty minute duration, the maximum depth was twenty four and so on. We can keep on writing. So, I will just quickly write it; 29, 32, 34 and it was 35 and 35. So, this is your maximum-depth duration curve.

Now, the last part in this question is to find out the maximum intensity duration curve. And how do we do that? Well, we just divide these depths. These are the maximum depths. And if you want to find out maximum intensity, what you do? You divide the depth by the duration or the time frame. So, for ten it will be ten minutes. So, you divide this by ten; for seventeen it is twenty minutes, so divide this number by twenty and then so on. So, that will be your intensity in millimeters per minute. And if you want, then you can convert it into per hour.

So, the intensity we have to find is in millimeters per hour. So, you have ten millimeters; ten millimeters depth of rainfall occurring in ten minutes, so how much it will occur in one hour? Alright, that should be very easy to find out. So, if I do that, you will have maximum intensity duration curve as you can see. How much will it be? It is one millimeter per minute, so it is sixty millimeters in an hour. And then, you can keep on calculating this; so 51, 48, 43.5 and 38.4. You see that it will be constantly decreasing; 38.4, this is 34, 30 and for the last one it will be 26.5 millimeters per hour.

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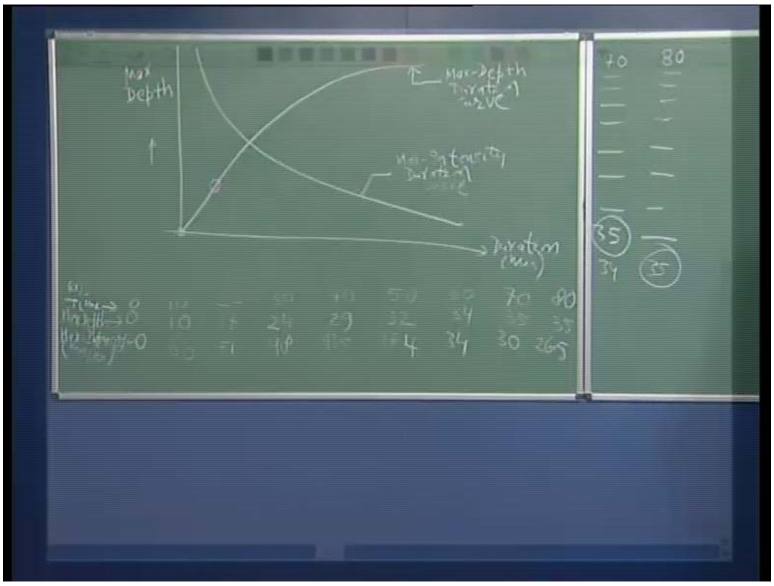

So, if you want to just look at how it will look like. So, what we can then do is, we can plot this maximum-depth duration curves and maximum intensity duration curves. So, this is your duration in minutes and on the y-axis we plot the maximum depth. Ok. So, you see that for zero the depth is zero. So, this is hour at one point. At ten minutes, we have ten. So, I am not going to scale. But, you can see that it will be somewhere here. And then, we can take all these points and it will be something like this.

If you plot all those points at different times, what is the maximum depth? You will have this maximum-depth duration curve. So, this is your maximum-depth duration curve. The other one is the maximum intensity duration curve and the intensity is in millimeters per hour. So, you will have; at zero, actually it will be infinity; it will not be zero. So, I will not plot it. So, at ten you will have the value is sixty, which will be maximum, and then it will in fact go down like this. So, this is your maximum intensity duration curve. So, we have seen that using the data from a rainfall mass-curve, how you can determine various storm characteristics such as the duration, the hyetograph, the mass curve and also the maximum depth duration and the maximum intensity duration curves.

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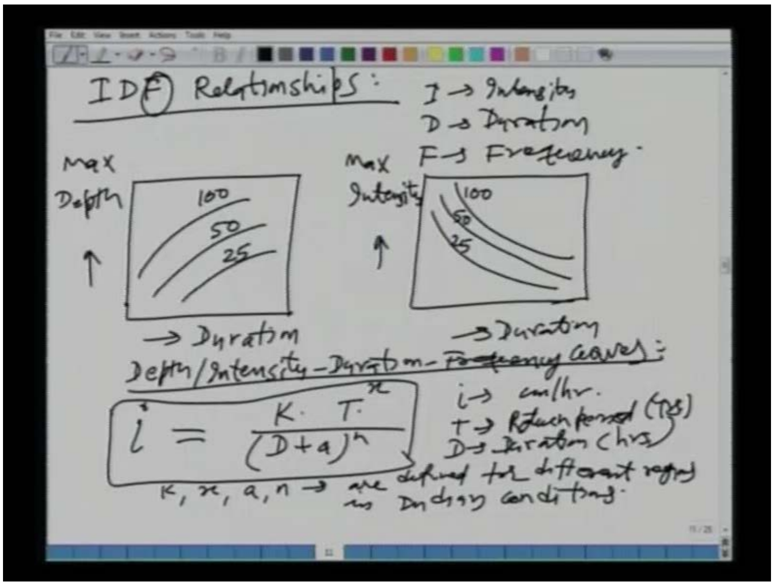

The next thing we will do is to look at another concept, which is called the I D F relationships. Until now, we have looked at the intensity duration relationship. Now, we are throwing another dimension which is called the frequency; I is intensity, D is duration and F is frequency. So, if I plot a curve which is duration and this is your depth.

So, first we look at the maximum depth duration frequency curve. And then, we will look at the maximum intensity duration and frequency curve. So, as we said we have thrown another dimension here, which is capital F. What is this F? F is frequency. So, we just looked at what is the maximum-depth duration curve. But, if we analyze any event which has a different frequency, and how do we measure frequency and hydrology? I am sure you might have seen something called return period or the recurrence interval. If you have an event which can occur once in ten years or once in fifty years or once in hundred years; so that is how we represent the frequency of an event.

So, let me plot three curves here. These are maximum-depth duration curves for three different return periods. And if I tell you that, one of them is for one hundred years, other is for fifty years and one of them is for, let us say ten years. Above twenty five years; which one will be which? Alright, there are three curves here. One is for a higher frequency; other is for a lower frequency and one is in the middle. And from your knowledge you understand that the event which is rare will have higher magnitude.

For example, a hundred year flood will occur less frequently. Is not it. Similarly, if you have a ten years flood, that will occur more frequently, but its magnitude will be smaller. So, you have the higher value here. So, you can say that this will be for one hundred years. Its frequency is less. It will occur once in hundred years. This one is fifty and this one is twenty five.

Now on this one, I plot the maximum intensity duration frequency curves, in which again you have three different frequencies with which these occur. The same logic will apply; the first one is for one hundred years flood or one hundred year recurrence interval rather, I should say. The middle one is for fifty and this is for twenty five and these are your depth intensity duration frequency curves.

So, you see that we can carry out the intensity duration analysis and then we throw another dimension of frequency. And the frequency, we measure in terms of the return period. And higher the return period, higher will be the magnitude of the maximum depth and the maximum intensity associated. Now, I M D has carry out last studies on Indian conditions in which they have tried to develop a relationship, which gives you this I D F curves or I D F relationships; that are called.

I will just review this. And then we would stop here today. So, this maximum intensity is given as this implicit equation K T x over D plus a to the power n; where i is in centimeters per hour, capital T is your frequency in terms of return period in years and D is the duration of your rainfall event in hours. So, this is the relationship the I M D has developed for Indian conditions, in which there are four parameters you can see; in which your K, x, a and n are defined for different regions in India.

So, if you have or if you want to find out the maximum intensity for a certain duration storm, for certain recurrence interval, you can just select the values of this parameters, that is K, x, a and n, and put those values in this equation. It will give you the maximum intensity. How these are been developed well? We look at this procedure. And all these procedures have been followed for lots of rainfall data for Indian conditions. So, what I like to do is to stop here today. And then, in the next class we will start looking at the special averaging of rainfall.