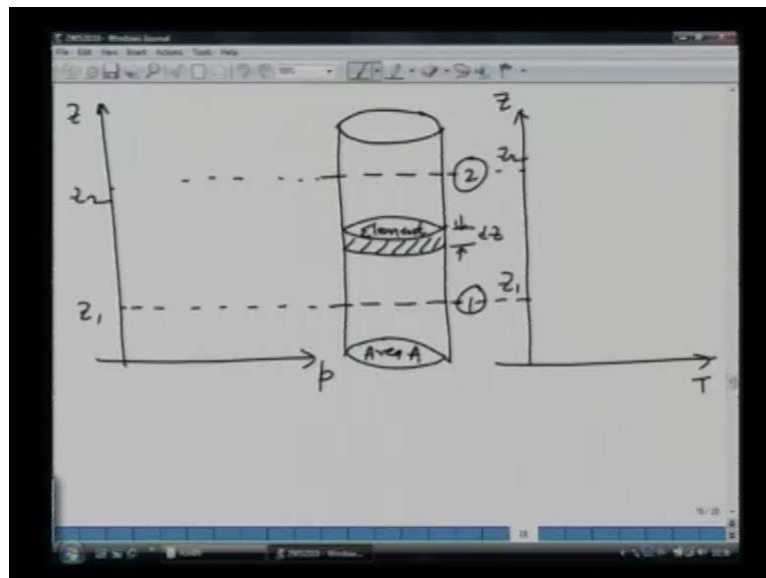


**Advanced Hydrology**  
**Prof. Ashu Jain**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 11**

Hello and good morning friends (( )) video course in advanced hydrology, we started the new chapter in the last class on atmospheric water. We looked at the concept or certain properties of the water vapor in the atmosphere, then we said that you will look at the atmospheric circulation or in the topic on your own as a reading assignment.

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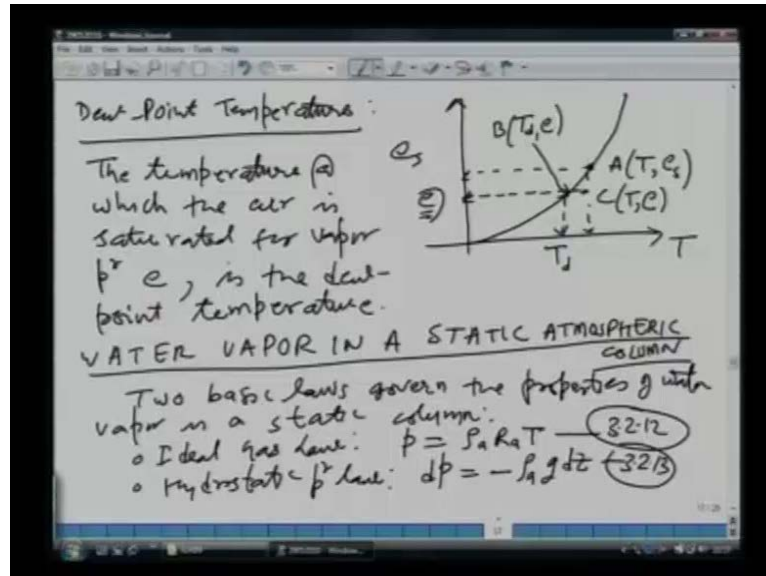


We also looked at the vapor transport, the Reynolds transport theorem or various properties or various variables that are involved in that, and how we can apply that to multiphase problem, we will come that little later. Then we looked at various inter relationships or various equations which give you the relationship between different parameters, such as the vapor pressure, the atmospheric pressure or the relative humidity and so on.

Towards the end we defined what is the saturation vapor pressure; the vapor pressure, the relative humidity and then we started looking at finding out the water vapor in a static atmospheric column. So if you look here, we drew this figure as we can see here, so with consider this cylindrical element of an atmospheric column. A cylinder basically and we

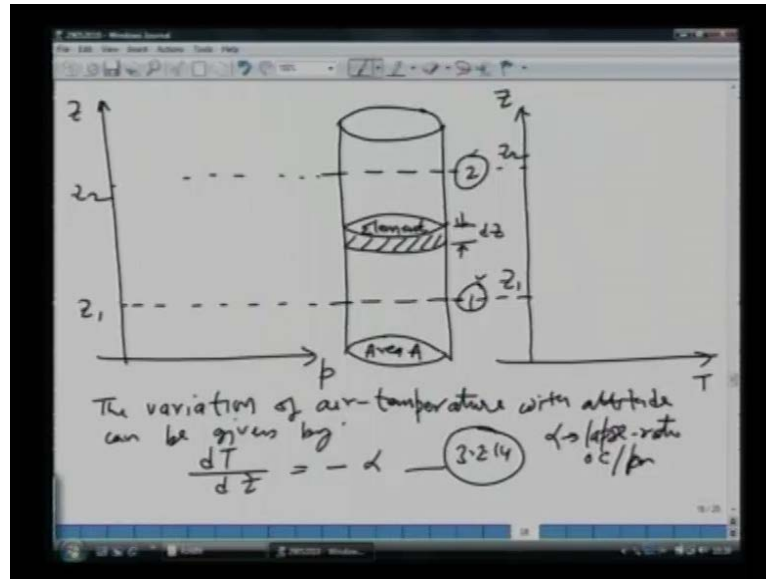
our focus of attention will be between 2 cross sections, that is 1 and 2. And we are trying to find out how the pressure will vary as a function of height and also how the temperature would vary as a function of height.

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To be able to find out the atmospheric water vapor in the static column, we said that we are going to use these two basic laws which will govern the properties of water vapor, one of them was the ideal gas law which we said is pressure is equal to rho r t, from the ideal gas law that is this was 3.2.12, and then we said the another thing we are going to use will be the hydrostatic pressure law, all right which is d p is equal to negative rho a d g z in this particular case.

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So let us get started today and then look at how we can do that, right. The first thing we will do is that as we go up into the atmosphere, the temperature changes, right; and the variation of air temperature in the atmosphere as we all know, as we go up the temperature decrease right, with attitude or height can be given by this equation that is  $dT/dz$  is equal to negative alpha and I am going to number this equation as 3.2.14. Where what is alpha, alpha is what is called the lapse rate, so alpha is the lapse rate. Normally it is express in degrees integrate per kilo meter of height and its value is between 6 and 7 and depends up on the climatic conditions and so on. So lapse rate is the rate at which the temperature lapses as we go up, right and this is 3.2.14. Now what we will do it will do certain algebraic manipulations and then try to find out the variation of pressure and temperature as a function of height.

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Handwritten derivation on a whiteboard:

$$(3.2.12) \Rightarrow \rho_a = \frac{p}{R_a T} \text{ put into } (3.2.13)$$

$$\frac{dp}{dz} = -\frac{p g}{R_a T} \text{ or } \frac{dp}{p} = \left(-\frac{g}{R_a T}\right) dz \sim dT$$

Now from (3.2.14)  $\Rightarrow (3.2) = -\frac{dT}{\alpha}$

$$\frac{dp}{p} = \left(\frac{g}{\alpha R_a}\right) \frac{dT}{T} \text{ Integrate on both sides:}$$

(both logs (3) & (2))

$$\ln\left(\frac{p_2}{p_1}\right) = \left(\frac{g}{\alpha R_a}\right) \ln\left(\frac{T_2}{T_1}\right)$$

$$\left[ p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\left(\frac{g}{\alpha R_a}\right)} \right] \quad (3.2.5)$$

So if you take equation 3.2.12 all right, it will give me rho as b over R a T; remember what was 3.2.12, it was nothing but the ideal gas law, and ideal gas law states what? b is equal to rho R T, so from p is equal rho R T, I have just written down the expression for the density rho a. And then we put this into equation 3.2.13 and what is 3.2.13, we go back, it this equation which is the hydro static pressure law; so what we are doing here basically we are combining these 2 equations, right. So we did that, this what we will get d p over d z is equal to negative of p g over R a T, or if you separate the variables so that we can integrate what we have is d p over p is equal to negative g over R a T of your d z. All right slightly rearrange this equation on the left hand side, now I want to replace this d z in the form of d t and there is relationship between z and t that is how the temperature changes and as a function of height.

So if you do that, if you look at equation 3.2.14 you have d z is equal to negative d t over alpha all right. What is 3.2.14? It is the laps rate, this is right so d t d z is this, so d z will be d t over alpha with the negative sign that is what we have done here. Now what will be put the value of d z from here into this and what you will then get is d p over p is equal to minus minus will cancel out, you will have g over alpha R a as a constant and then d t over t. Now we can see that we have the variables on either side which you can integrate, which are the climatic variable; so integrate on both sides. What are we going to get? And when you are integrating lets say between locations 1 and 2, remember what is the location 1 and 2, we have defined the height z 1 as 1 and z 2 as 2. So if you did that

you will have natural log and  $p_2$  over  $p_1$ , this will be natural log of  $p$  taken between  $p_1$  and  $p_2$  or  $1$  and  $2$  so that will be natural log of  $p_2$  over  $p_1$ ;  $g$  over  $\alpha R a$  is constant to that will remain outside and when you will integrate the  $d t$  over  $t$ , it will natural log of same thing it will be  $t_2$  over  $t_1$ .

Then you make slight readjustments which will give you  $p_2$  as  $p_1$  times  $T_2$  over  $T_1$  raised to the power  $g$  over  $\alpha R a$ . So this your relationship between  $p_2$  and other things, and I am going to number this as 3.2.15 all right. So this way we are able to write the expression in which we can calculate the pressure at any height from the ground, and what do you need to we need to know the pressure on the ground that is  $p_1$  and temperature on the ground  $T_1$  and also  $T_2$ ;  $T_2$  is the temperature at the same height right, which we would be able to know given the laps rate equation to us.

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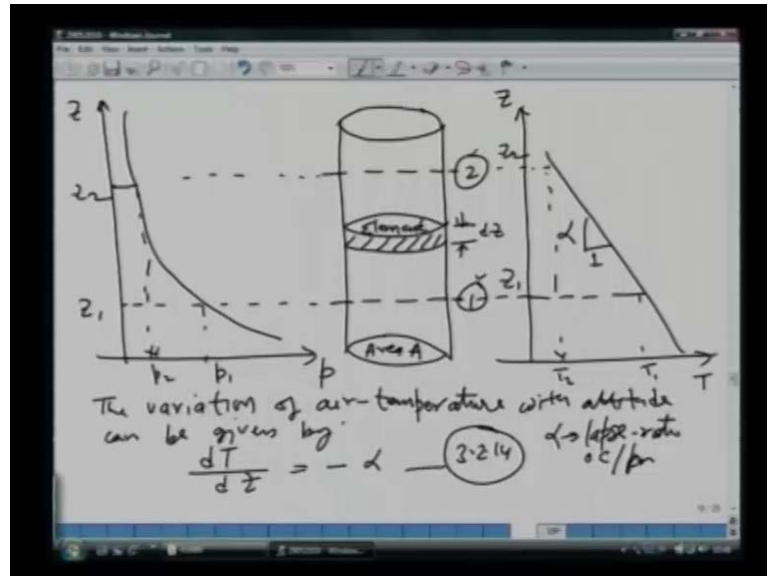
$$\left(\frac{dT}{dz}\right)_{air} = -\alpha \Rightarrow \frac{T_2 - T_1}{z_2 - z_1} = -\alpha$$

$$\Rightarrow T_2 = T_1 - \alpha(z_2 - z_1) \quad \text{--- 3.2.16}$$

So how you do that, so let me take the left side equation which is a  $d T$  over  $d Z$  is equal to minus  $\alpha$ , all right. What is  $d T$  over  $d Z$ ? If you write it between  $1$  and  $2$  it will be  $T_2$  minus  $T_1$  over  $Z_2$  minus  $Z_1$  from your knowledge of basic knowledge of your calculus we will have negative  $\alpha$ , right. So what is that give you in terms of  $T_2$ ,  $T_2$  is nothing but your  $T_1$  the temperature on the ground minus your laps rate times the difference in the height; which is  $Z_2$  minus  $Z_1$  and let's say then that this is our 3.2.16. So 3.2.16 will give you the variation of temperature as a function of height and we have already derived the equation or looked at the equation for 3.2.15 which is giving you the

variation of pressure as function of height. So with that what we are going to do is we will look at an example but, before I do that let me go back to my figure and see or try to plot how it will look like, here it is.

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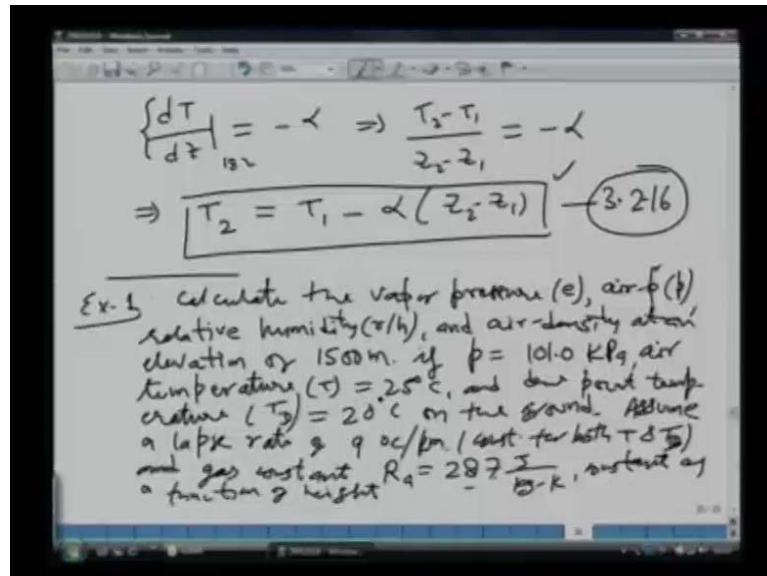


So we have considered this atmospheric column, right; height is  $Z_2$  minus  $Z_1$ . So let us first plot the temperature, right. Temperature as you have seen decreases as we go up right,  $T_2$  is equal to  $T_1$  minus  $\alpha$  and  $\Delta Z$  right; so you did that how will look like? Is the equation linear and non-linear? It's obviously a linear equation, right; and it will be sloping down ward right because it is decreases as the go up. So it would simply be something like this, so if the temperature here is this is  $Z_1$ , this  $T_1$  at here  $Z_2$ , this would be our  $T_2$ ; so as we go up the temperature decreases and the slope of this line as you see would be what, this would be our laps rate which is  $\alpha$ . The slope of this line is nothing but, the  $\alpha$  it's just equation of a striate line. What about the pressure? Pressure is as we have found out is a nonlinear relationship, pressure decreases non-linearly as we go up, right so its equation it plotted it will look something like this where this at height  $Z_1$  we have the pressure  $p_1$  which is close to atmospheric at height 1 and then at  $Z_2$  this is approximately  $p_2$ .

So that way we are able to plot the both here temperature and pressure, or variation as function of height. Now why we are doing all this is? This is important in calculating or

estimating the amount of ( ) water in an atmospheric column. What will do is will look at an example which will write to demonstrate this.

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So we take an example in this chapter which is our first example here, it says that calculate the vapor pressure which is e, air pressure which is simply p, relative humidity which is you are going to denote this as r/h and here density at an elevation of at an elevation of 1500 meters all right; so which is about one and half kilometers into the atmosphere. If you are p is equal to 101.0 kilo Pascals right, these are the data which are given on the ground. The air temperature capital T is equal to 25 degree centigrade and the dew point temperature, dew point temperature and we will denote this as T subscript D is equal to 20 degree centigrade on the ground. So the climatic data on the ground are given and what we have to find is what will be vapor pressure, the air pressure, our relative humidity and the air density at a height of 1 and half kilo meters.

You will obviously need some more data in this, so the other things that are given is assume a lapse rate of 9 degrees per kilometers right; which is constant or same which can be assumed for both T and T D. So this is the assumption, just T D right, so for air temperature and the dew point temperature for both we can take the same lapse rate which may not be the case ideally but, for this example purposes we take them to be same. And the gas constant R a which could be needed is equal to, is given to you as 87 joules per kilogram degree which is constant as a function of height. What does that mean that last

sentence it is constant as a function of height means that we have to ignore the variation of  $R_a$  as we are going up, so just use this value to addition ideally it will be changing as the humidity level will be changing. Remember these relationship between  $R_a$  and  $R_D$  which is related we to be but, we are to ignore that ok. So moving on we look at the solution of this.

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The image shows a handwritten solution on a whiteboard. It starts with given data:  $z_2 = 1500 \text{ m}$ ;  $p_1 = 101.0 \text{ kPa}$ ;  $T_1 = 25^\circ \text{C}$ ;  $\alpha = 9^\circ \text{C/km}$ ;  $R_a = 287 \frac{\text{J}}{\text{kg}\cdot\text{K}}$ ;  $T_2 = 20^\circ \text{C}$ . The goal is to find  $e_2$ ,  $p_2$ ,  $(\rho/h)_2$ , and  $\rho_{a2}$ . The solution uses the vapor pressure equation:  $e_2 = 611 \exp\left[\frac{17.27 T_2^2}{237.3 + T_2}\right]$ . It then calculates the dry-bulb temperature at height 2:  $T_D = T_D - \alpha(z_2 - z_1) = 20^\circ \text{C} - 9^\circ \text{C/km} (1.5 \text{ km})$ , resulting in  $T_2 = 6.5^\circ \text{C}$ . Finally, it calculates the vapor pressure:  $e_2 = 611 \exp\left[\frac{17.27 \times 6.5}{237.3 + 6.5}\right] = 968.3 \text{ Pa}$ .

And it is a good habit has said earlier that to summarize all the data that are given to us. So first write everything which is given in one place so that you can refer to this small corner in your note book if you any need data; your  $Z_2$  is 1500 meters that one off course is 0 you can take on the ground, pressure that is given is 101.0 kilo Pascals,  $T_1$  on the ground is 25 degree centigrade and  $T_D$  on the ground is given as 20 degree centigrade. Also given to us which is the alpha as 9 degree centigrade per kilometers,  $R_a$  is 287 joules per kg K and alpha is same for both  $T$  and  $T_D$ . So what you have to find is the vapor pressure at height to what is the  $p_2$ , the air pressure at height 2, what is the relative humidity at height 1500 meters and what is the air density at height 1500. So these are the 4 things we have to find. So will work on this in that order, so first we look at the vapor pressure.

How do we find the vapor pressure at a height of 1500 meters? If you remember the concept of saturation vapor pressure, there is an expression that is you know given to us  $e_2$  is equal to 6 and exponential so on we can put the value of air temperature in that and



it will give you the saturation vapor pressure. But a very important aspect is that because the vapor pressure gives you an indication of the relative humidity or the moisture content; right. So what we can do is we can substitute the dew point temperature in that expression to get the vapor pressure instead of the saturation vapor pressure. This comes from the definition of the dew point temperature which we have seen earlier. What we can then do is we can write, we can calculate  $e_2$  at the height 2 which is 1500 meters in this case; as  $611 e^{\frac{17.5(T_D - 293.15)}{243.5 - T_D}}$  this whole expression which you should have in your notes, this is  $T_D$  at height 2 on using another super script now. So don't get confuse that is the dew point temperature we are talking about.

Now what we have is dew point temperature on the ground but, we do not have the dew point temperature at height 2 or 1500 meters. But what is given to us is that the dew point temperature also follows the lapse rate, the same lapse rate. So using that we can find out what will be the  $T_D$  2. So let us do that, your temperature, dew point temperature at height 2 would be temperature at 1 right minus  $\alpha$  times of a  $Z_2$  minus  $Z_1$ . Now  $T_D$  1 and is given to as 20 degree centigrade minus we have 9 degree centigrade per kilometers lapse rate times  $Z_2$  minus  $Z_1$  is 1500 meters but, our lapse rate is degree centigrade per kilometers so we put the value of the  $Z_2$  minus  $Z_1$  in kilometers so that kilometers will cancel out.

Here to be careful while choosing the units of the particular equation. You can simplify this and it will give me  $T_D$  at height 2 as 6.5 degree centigrade. Now how can we find the vapor pressure? Well we already have the equation, so we put this value of let's say this  $T_2 D$  into this equation and we should be able to find the vapor pressure and height 2 right. So if we did that, you have  $e_2$  as  $611 e^{\frac{17.5(T_D - 293.15)}{243.5 - T_D}}$  you have 17.27 times 6.5. Remember in this equation we will put the temperature in degree centigrade, if you do that we  $e_2$  will come out in Pascals; it will be  $237.3 + 6.5$ , it is in degree centigrade. So once you simplify that calculate it will be 968.3 Pascals, so you are one of the quantities which have to find this vapor pressure at a height of 1.5 kilometers is 968.3 here Pascals. So that is your one of the answers

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Atm pressure ( $p_2$ ):

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{g}{\alpha R_a} \ln\left(\frac{T_2}{T_1}\right) \Rightarrow p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{g}{\alpha R_a}}$$

$$T_2 = T_1 - \alpha(Z_2 - Z_1) = 25^\circ\text{C} - 9(1.5) = 11.5^\circ\text{C}$$

$$\Rightarrow T_2 = 11.5^\circ\text{C}$$

$$p_2 = \frac{p_1}{\text{kPa}} \left(\frac{273 + 11.5}{273 + 25}\right)^{\frac{9.81}{(9 \times 10^3) \times 287}} = 84.6943 \text{ kPa}$$

$$\Rightarrow p_2 = 84.7 \text{ kPa} \quad \text{Ans.}$$

Relative Humidity:  $(\frac{e}{e_{sat}})_2 = \left(\frac{e}{e_{sat}}\right)_1 = \frac{e_2}{e_{sat,2}}$

$$e_{sat,2} = 611 \text{ Pa} \times \left(\frac{17.27 + 11.5}{237 + 11.5}\right) = 1357.4 \text{ Pa}$$

So let's move on and then we try to find out what will be the atmospheric pressure  $p_2$  at height of 1500 kilometers sorry 1500 meters. We have just derived an equation which relates  $p_2$  with  $p_1$  and temperature, right. Let me first write it down, you have natural log of  $p_2$  over  $p_1$  is equal to  $g$  over  $\alpha R_a$  a natural log of  $T_2$  over  $T_1$ . And that we had seen that, that is going to give you  $p_2$  is equal to  $p_1$  times  $T_2$  over  $T_1$  of raised to power  $g$  over  $\alpha R_a$ . If you see the right hand side; we know  $p_1$ , we now  $p_1 g$  is the constant acceleration due to gravity,  $\alpha$  is known,  $R_a$  is known, only thing unknown is the temperature at height of 1.5 kilometers which we can find out using the lapse rate again. So you find out the air temperature at a height of 1500 meters as this minus  $\alpha$  times  $Z_2$  minus  $Z_1$ , same thing which did earlier. So this is 25 degree centigrade minus 9 times 1.5, it will be 11.5 degree centigrade.

So that your  $T_2$  is equal to 11.5 degree centigrade so put this back into this equation. So that your  $p_2$  will 101.0 times, this is  $p_1$  in kilo Pascals remember that and then you have  $T_2$  over  $T_1$  all right, this expression. Remember in this equation  $T_2$  and  $T_1$  are in degrees so we have to be careful while choosing the units. So we have 273 plus  $T_2$  is 11.5 which we have just found out, divided by 273 plus what is it?  $T_1$  and  $T_1$  is the temperature, air temperature on the ground which is given to us as 25, right. So this is degree centigrade you have added to 73 so that the overall result is that degrees calculate. Now all of this is raised to the power of 9.81 is the acceleration due to gravity and then we have  $\alpha$ ,  $\alpha$  is 9 degree centigrade per kilometers all right. You have to be

extremely careful in choosing the units now, it is per kilometers but, this is an equation In the SI unit which will require that your distances are in meters.

So you have  $9 \times 10^{-3}$  is your  $\alpha$  which is degrees centigrade per meter and then we have 287 is your  $R$  in SI units. So please make note of this and I have seen many students committing a mistake, they just put the value of 9 here. So if you calculate this it will be approximately 84.6943 kilo Pascals. Why it is a kilo Pascals because this  $p_1$  is in kilo Pascals, you could have converted into Pascals by multiplying by  $10^3$  and the answer would be same in Pascals, so right. So this comes out in kilo Pascals therefore, pressure atmospheric pressure at height of 1.5 kilometers comes out to be above 84.7 kilo Pascals. So we see that the pressure decreases quite rapidly as we go up, that is why this tracking and mountaineering people as they are going up; because the pressure reduces the amount of oxygen reduces as we go up so they carry their oxygen cylinder etcetera and enough supply with them.

So this is the next answer, the next thing we have to find is what is the relative humidity at height of 1500 meters; so you have relative humidity and you know that this is given as simply  $e/e_{sat}$  or  $e$  saturation at height of 2 or 1500 kilometers. Now we have already found out  $e_2$  earlier and what we need is  $e_{sat}$  at the height of 1500 meters, what is  $e_{sat_2}$ . So what will be the saturation vapor pressure at second cross section or at a height of 1.5 kilometers? Well we know what would be the air temperature at that height, so once you know the air temperature we can always find we can put the value of that temperature in the expression here right. So if you did that, you have 611 exponential  $17.27$  times the air temperature that height which is 11.5 here we have just found out. This whole thing divided by  $237.3 + 11.5$  and if you solve it, it will come out to be 1357.4 and what will be the units? Well it will be Pascals.

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$$(r/h)_2 = \frac{e_2}{e_{2sat}} = \frac{968.3}{1357.4} = 0.7133$$

$$\Rightarrow (r/h)_2 = 0.7133 \text{ or } 71.33\%$$

$$\text{Air-Density: } \rho_{a2} = \frac{p_2}{R_a \cdot T_2} = \frac{84.7 \times 10^3 \text{ (Pa)}}{287(273+15)}$$

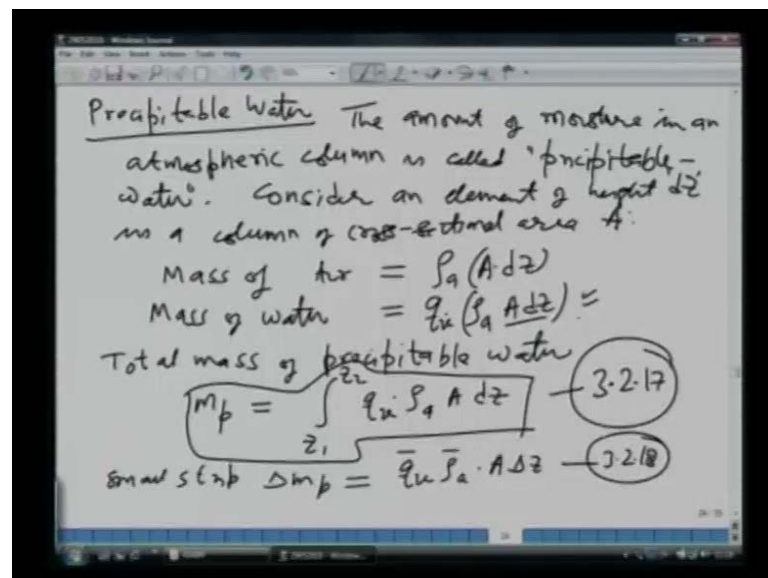
$$\Rightarrow \rho_{a2} = 1.03733 \text{ kg/m}^3 \quad \text{Ans.}$$

So moving on you have relative humidity at a height of cross section 2 or 1.5 kilometers is equal to your  $e_2$ , our  $e_2$  saturation which is this; we have found out what is  $e_2$ ?  $e_2$  if you go back is so 968.3 right. So it is 968.3 and the saturation vapor pressure at height of 1500 meters we just found out is 1357.4 and it will be 0.7133. So that implies your relative humidity at a height 1.5 kilometers is 0.7133 or in percentage you can express this as 71.3 percent. So see the all these equations which we have looked at yesterday they are extremely important in finding.

Our values of humidity and pressures and temperatures and all those climatic conditions at any height which becomes very important in climate modeling, ok right. So the next thing you have to find out is in this problem what is the density of the air at that height, which is also an important parameter;  $\rho_{a2}$  how can we find it? Well we can use the ideal gas law which is valid everywhere, at the ground or at any height right this was matter. So  $p$  is equal to  $\rho R T$  that will you give  $\rho$  is equal to  $p$  at the same location divided by  $R$  a times  $T$  at the same location is  $T_2$ . We have already found  $T_2$ , we found  $p_2$  and what is given to us in this problem is that the gas constant  $R_a$  is constant as a function of height; so we put that value 287 and we should be able to get what would be the air density. So it will be it  $84.7 \times 10^3$  what is why I am doing this  $10$  to the power 3?

$p_2$  is the pressure in Pascals all right, and what we find out was 84.7 kilo Pascals right. So that is why multiplied, so that decision SI units you have 287 times  $T_2$  in the situation is in Kelvin is 273 plus 11.5 was the temperature at height of 1.5 kilometers we found out. So that you are  $\rho_a$  at 2 will be 1.3733 kilo grams per meters ok. So that ends the a solution of this problem where when we saw that knowing the climatic conditions in terms of temperature, pressure, humidity that on the ground in terms of dew point temperature we can find out how all of these things will vary as function of height. And once we have done that you will move to a very important concept, remember that we are still in the phase or what we are considering right now is the static atmospheric column; things are not moving write now. Movement of the moisture (( )) modulator right. So the next thing we are going to look at is how much is the precipitable water in a static atmospheric column which is a very important concept.

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So let us move on and look at the precipitable water in this atmospheric column. The amount of moisture, the amount of moisture in an atmospheric column which we have just considered like a cylindrical column is called the precipitable water. The amount of moisture which is available in atmospheric column is called the precipitable water, the water that can precipitate as rain fall and any other different form. How can we calculate this calculation of the capacity of this water in a cloud or in a column is very important, because what will come out of that cloud of then we can module that everything will be

coming out of this precipitable water. So what we do is we consider an element of height  $d z$  like we had done earlier all right, in a column of cross-sectional area  $A$ .

So it is the same thing if you go back do not have to go back on that figure, so it's a cylinder; the cross sectional area is  $A$  and its total height  $Z_2$  minus  $Z_1$  what we are considering a small strip of height  $d z$ . So first we will write the amount of water in that strip design and then we can integrate between the limits  $Z_1$  and  $Z_2$  right. So what will be the mass of the air, what will be mass of air in the strip of height  $d z$  in that atmospheric column? The mass of air would be what, the volume of that particular element multiplied by the density of air right; density multiplied by volume is the mass like. So what is the volume? Area is  $A$ , height is  $d z$ , so that is your volume, ok volume that state. And if you multiplied by the density of the air that will be the mass of the air, once we have the mass of the air what will be the mass of water or moisture in that column?

It's very interesting and very simple concept, so it would be  $\rho A d z$  what do you multiply this by? If you multiply this by the specific humidity  $q_v$ , what is  $q_v$ ?  $q_v$  is nothing but, the measure of amount of moisture in atmospheric column right we find right. So this is the total mass in the volume  $A d z$ , so it will be equal to  $q$  multiply this by  $\rho$  so this gives you we mass of water in a small state. So if you want to find out what be the total mass of water in a height between  $Z_1$  and  $Z_2$  of an atmospheric column, what you do? Well integrate, total mass of precipitable water and then if you denote that by  $m_p$  it will be the integral of this quantity here between the limits of  $Z_1$  and  $Z_2$   $q_v \rho A d z$  ok, so this small little expression for finding out the amount of water or the precipitable water in a static atmospheric column. Now how do you actually calculate this precipitable water?

What we do is we discretize our whole space domain, and then we work with the average quantity all right. So if we actually do it this is how we in fact work it out. So if you take small strip  $d z$  we had seen earlier all right or  $\Delta z$  for that matter, because we are working with the discrete quantities now. So let me write this as  $\Delta m_p$  in a small strip desired will be equal to what, it has certain height lets say small  $z_1$  small  $z_2$ ; so what we do is we take the average quantity, the  $q_v$  is varying as a function of height so we take the bar represent the average quantity of the specific humidity. Similarly, we take  $\bar{\rho}$  which is the average density between those two heights and then  $A$  times  $\Delta z$

z is the volume right. This we say is then equation 3.2.18, so what we do is we find out the average specific humidity between any 2 heights, we find out the average density then multiply by the area that will be to the incremental amount of water in that increment all right. And then we keep on doing it for every 100 meters or for every 10 meter depending up on the accuracy desired for 1 kilometer, 2 kilometers and so on. So I will do is, just like to demonstrate how you can actually do that.

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Handwritten notes on a whiteboard:

- $\bar{q}_v = \text{avg. sp. humidity in a column of ht. } \Delta z$
- $\bar{\rho}_a = \text{avg. air density of moist air in col. of ht. } \Delta z$
- $\bar{q}_v = \frac{q_{v1} + q_{v2}}{2}; \quad \bar{\rho}_a = \frac{\rho_{a1} + \rho_{a2}}{2}$
- $\rho_a = \frac{p}{R_a T} \Rightarrow \rho_{a1} = \frac{p_1}{R_a T_1} \text{ \& } \rho_{a2} = \frac{p_2}{R_a T_2}$
- $q_v = 0.622 \frac{e}{p} \Rightarrow q_{v1} = 0.622 \frac{e_1}{p_1} \text{ \& } q_{v2} = 0.622 \frac{e_2}{p_2}$
- $T_2 \rightarrow \text{at } p_2 \text{ - note}$
- $p_2 \rightarrow \text{at } T_2 \text{ - note}$
- $e_1, e_2 \rightarrow T_D \text{ } ^\circ\text{C}$
- $q_{v1} \rightarrow T_1 \text{ } ^\circ\text{C}$
- $q_{v2} \rightarrow T_2 \text{ } ^\circ\text{C}$

Before I did that let me define your  $\bar{q}_v$  is the average specific humidity in a column of height  $\Delta z$ , any suitable  $\Delta z$  we can take and  $\bar{\rho}_a$  is the average air density; Average air density of moisture air in a column of height  $\Delta z$  same thing right. So how do we actually do it? This is the expression we have, what is  $\bar{q}_v$ ? Also we have 2 cross sections then decide is presented by 1 and 2 so average is what we are doing basically does in non-linear function but, trying to linearize. So it's like interpolation, so you are saying it is nothing but the average of the value  $q_v 1$  and  $q_v 2$  divided by 2.

So any between that  $Z$  you take the value of  $q_v 1$  and  $q_v 2$  and take the average. Similarly, you have  $\bar{\rho}_a$  will be  $\rho_{a1}$  and  $\rho_{a2}$  over 2, now how do we find this  $q_v 1$   $q_v 2$   $\rho_{a1}$  and  $\rho_{a2}$ ; you have already solved in example in which we have seen how we can do some of these things. For example, say  $\rho_{a1}$ , we know that  $\rho_a$  is given by  $p$  over  $R_a T$  ideal gas law right; this will give you  $\rho_{a1}$  as  $p_1$  over  $R_a T_1$  and  $\rho_{a2}$  as  $p_2$  over  $R_a T_2$  so ideal gas law is applicable at all the heights. So all we

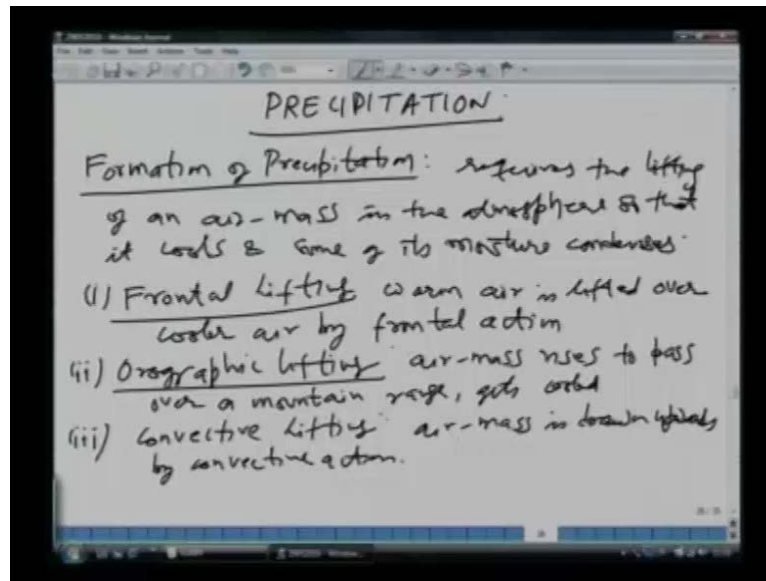
are doing is we are finding out the density of the moist air and different heights, how we can do that? We know the pressure and temperature on the ground, we know the ideal gas constant and then we can find out what will be  $p_2$  and  $T_2$  we just looked at an example in which we calculate the temperature and pressure as a function of height. This  $R_a$  in that example was constant but to be more accurate we can allow the variation of that right and then use slightly more accurate values right.

But for all practical purposes if you can keep that constant that should be fine. So you know how to calculate then  $\rho_{a1}$  and  $\rho_{a2}$ , we can keep doing that and we can find out what will be the average air density. Similarly, your  $q_v$  is given by  $0.622 \frac{e}{p}$  this expression we have looked at yesterday;  $e$  by  $p$ . So that will give you your  $q_{v1}$  is equal to  $0.622 \frac{e_1}{p_1}$ , which is basically at height  $Z_1$  or at cross section 1 and your  $q_{v2}$  would be  $0.622 \frac{e_2}{p_2}$  which would be at height cross section 2. Now there are few more things which we you need to calculate or that is your  $p_2$  and  $T_2$  for which we already have  $T_2$  by lapse rate, I am not going to write the equation again right and  $p_2$  by the equation derived earlier. And also this  $e_1$  and  $e_2$ , we use dew point temperature at those heights and if need to calculate this saturation but pressure we can use the temperature in degree centigrade.

So you see that we have a small little expression in which we should be able to find out what will be the amount of precipitable water in a static atmospheric column. It's a useful exercise, there is a solved example given in the book. I don't think I have time to take up any of the example on this so but, I suggest that you look at that example with which is given in the book, all right. So that you understand how you can actually calculate right, all of these things can be automatically in an excel program or computer program right. So with that what we would be like to do is will move on and we will move one to a slightly different kind of topic in which we will look at some more theoretical aspects, this was more practical. So we will look at the formation of precipitation.



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You will start out with precipitation and introduce the concepts or the mechanisms that are required for the formation of precipitation. In this topic we would look at different forms of precipitation, formation of precipitation and then its measurement and then data analysis, ok. So first thing we would look at is the formation. When we come to the formation of precipitation, I am sure all of you may have seen how the rainfall or precipitation is formed in the atmosphere. You had studied this in your 12<sup>th</sup> standard and earlier you may have seen this earlier under graduate hydrology course right, we will just review it quickly. What are the two basic mechanisms that are required for the formation of rainfall in the atmosphere or in cloud? There are two basic physical things that need to happen otherwise you may have noticed that with the same climatic conditions right same humidity same temperature, same everything right and two different days; 1 day it will rain other day it will not rain.

So there are certain mechanisms that are needed that are necessary for formation of rainfall, condensation of the moisture that is in the atmosphere and then it becomes heavy enough to fall. So what are the 2 basic mechanisms? Well first 1 is the lifting of an air mass, you have the moist air or air that contains moisture in the atmosphere; somehow that has to lift right as it lifts the temperature goes down as a temperature goes down the water vapor condenses. So lifting of a moist air mass is one basic mechanism right, what is the other one? Well we will that little later. So first one is the air lifting or lifting of an air mass. I am writing down, requires the lifting of an air mass

in the atmospheric or atmosphere rather I can say, so that it cools and some of its moisture condenses.

So for rainfall to form the one basic mechanism is the lifting of the moist air mass, all right. Now there are many ways in which an air mass can get lifted up, I am sure you have seen the different types of precipitation; you may have heard what is a frontal precipitation, what is an orographic precipitation right and what is a convective precipitation. They are associated or they correspond to different mechanism of lifting of the air mass all right. So there are three basic mechanism through which air mass will get lifted up, what are those? Well I just said them and will write it again. First one is what will call a frontal lifting of the air mass, what happens in a frontal lifting is there are 2 fronts one is cold other is warm there are travelling against each other right and when they come in contact the warmer will get lifted up due to the frontal action. So the warm air is lifted over the cooler air by what is called the frontal action, and the resulting precipitation is in then known as the frontal precipitation, all right. We will not go into the details of this because we have already seen them. We are just looking at three different mechanisms.

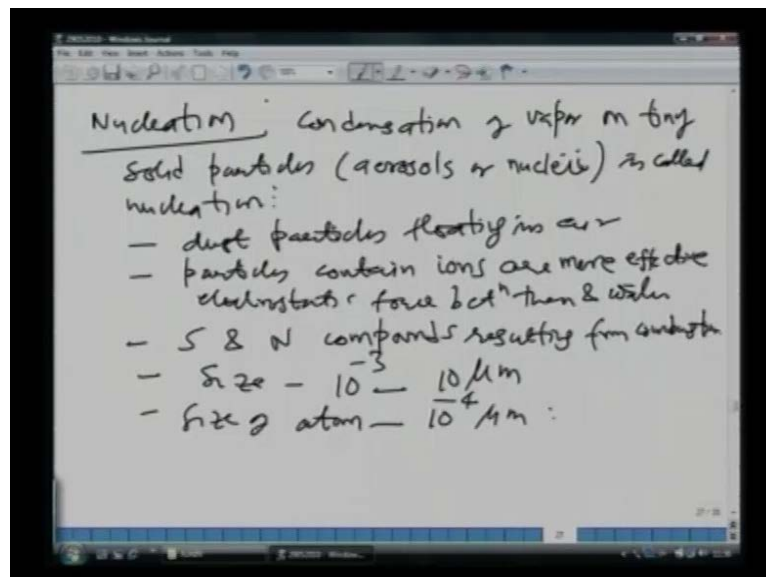
The second one what happens in the hilly areas or the mountainous regions? How does the rainfall form there? Well when the moist air is travelling there are certain obstructions in the form of hills and mountains, right, due to that the air rises that is called the Orographic lifting in which what happens, the air mass rises to pass over a mountain range and in that process it gets cooled. As we, the moisture goes up the temperature goes down it cools and because of that lower temperatures the condensation takes place. And in the hilly areas we call that what is called as orographic precipitation and the mechanism responsible with the orographic lifting due to mountains or barriers. The third one is what is called the convective lifting or convective precipitation; this is when the air mass is simply drawn upwards by convective action, convective action.

What is convective action? Well just the difference of temperature in the height due to the difference in temperature the temperature gradients set up the air gradients and then warm air gets lifted up. And the corresponding rainfall that occurs is called the convective precipitation right, so this is as for as the one basic required mechanism for formation of rainfall. What is the other one? Here we have seen that the lifting of the air mass is necessary fine but, many times we know that we have same climatic condition,

same temperature, moisture is there, humidity is there in the atmosphere, pressure is same or right all other things are same but one day it will rain other days it does not rain.

And important aspect, another important aspect for the condensation of moisture that is available in the atmosphere; we need to have a process which is called nucleation. What is nucleation? Nucleation is a process of condensation of water vapor in the atmosphere on very small tiny drops of some solid particles, it may be the dust particle, it may be some other things small; these are called nucleus which help or which acts as catalyst in condensation.

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So next thing we would look or say is nucleation. What is nucleation? It is basically the condensation of vapor, water vapor on tiny that is very small solid particles, which are also known as aerosols; some of my scientific friends call it aerosols or nuclei is called nucleation. I will, I would like to look at these nuclei, what are those things right? These are basically some dust particles which are floating in the air. These are particles which contain ions or which are ionized are more effective, because they have the affinity to attract the water towards them. They are certain ions due to various you know natural processes which are floating in the air, more effective due to the electrostatic force between them and water molecules or the water.

Then you have sulphur and nitrogen compounds resulting from combustion, due to various industrial activities we have lot of this pollutants floating in the air. I think, I will

write more and speak less. So you have size of these nuclei are very small of the order of  $10^{-3}$  to  $10^{-10}$  micro meters. And you know that what is the size of an atom; it is approximately  $10^{-4}$  micro meters. We can see that only a few hundred atoms can combine and make nuclei, which are very tiny small particles. I think, I would like to stop at this point of the time, wherein we have looked that the solved example; some examples of the calculation of various properties of water vapor.

Then we have come and looked at how the rainfall is formed, there are two basic mechanisms; one is the lifting of the air mass and another which the nucleation. And then we have looked that a few you know properties, some concepts about the nucleation. In the next class, you would actually look at how the rainfall is formed in a cloud, and then will go on to the different forms of precipitation.