Advanced Hydrology
Prof. Dr. Ashu Jain
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Lecture – 10

Good morning and welcome to the next lecture of this video course on Advance Hydrology. In the last lecture we looked at the few examples on the continuity equation, momentum equation and energy equation. What we would like to do today is we will start a new chapter on atmospheric water. So, if we see here.

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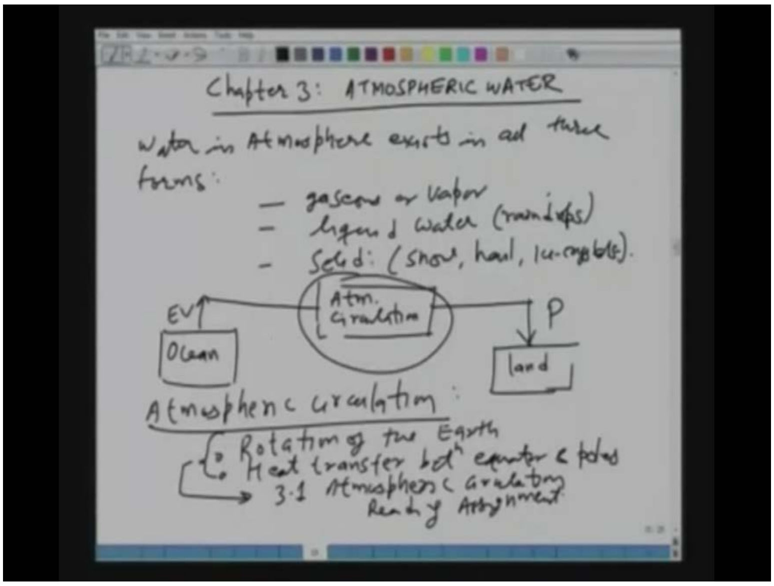

We will start chapter 3 in fact from your Wendi Chaos book and on atmospheric water. Atmospheric water as you know is very important, why because the developing countries like India they depend upon rainfall on its agricultural, its economy and everything the development of the country is related to atmospheric water.

In the atmosphere the water exists in all 3 different forms as you know, that is water in the atmosphere exists in all 3 forms, that is to say gaseous or vapor also liquid water which is in the form of rain drops and also solid. And how is the solid water occurring in the atmosphere, it is in the form of snow, hale, ice-crystals, etcetera. And as we have seen earlier that the residence time of the atmospheric water is very small only order of a few days. So, things are extremely dynamic in the atmosphere. So, we need to understand these processes very carefully.

How does the movement of this atmospheric moisture take place? So, we will look at the first thing in this chapter is what is called the “atmospheric circulation” and we will not go into the details of this except for saying that the water gets evaporated from oceans, all right, and that is due to the heat energy provided by the sun, these form some clouds and then due to some wind action and we will call it atmospheric circulation, all right, it gets transported over to the land masses and this water precipitates or falls on the land. Now, the key word here is the “atmospheric circulation”, the driving force of this atmospheric circulation or the moment of this water along with the winds there are 2 factors: One of them is the heat energy which is provided by the sun, all right and the other is the rotation of the earth itself. So, if we look at the atmospheric circulation, the 2 main factor that are responsible for that, one of them is the rotation of the earth and the second is the heat transfer, heat transfer between the equator and the poles. As you know the poles are very cold and the equator you know will have warm weather.

So, due to the difference in the heat, the transfer of heat energy takes place which sets up the pressure currents the winds take place and with that wind taking place from one place to the other we have the moment of moisture in the atmosphere taking place. all right. I will not go into the details of this as I said in the beginning of the course that I will need your help in completing this course all right in the form of some reading assignment. So, what I would like to do is this is your topic 3.1 in your Wendi Chaos book on atmospheric circulation. So, kindly go through yourself as a reading assignment. So, I expect you to just go through it and if you have any problems or questions you do not understand, you can always contact me.

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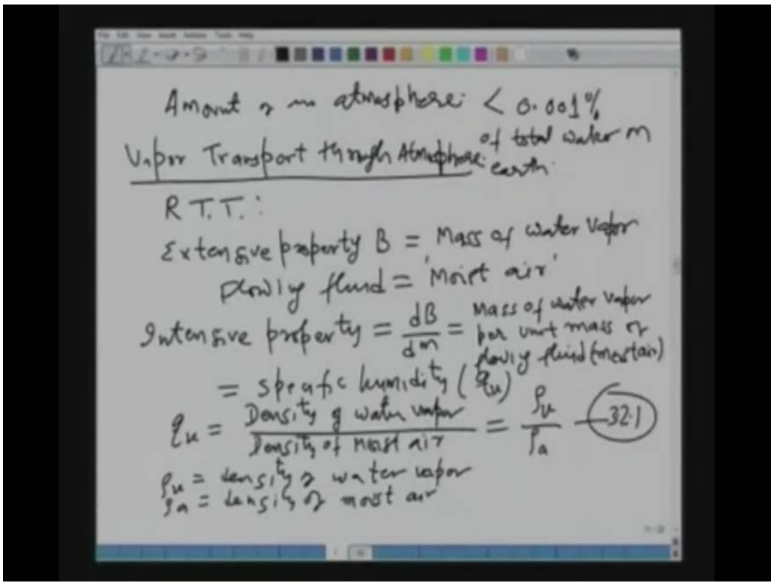

So, moving on we just look at the amount of water in the atmosphere is extremely tiny, it is very small quantity of water as a fraction of the total quantity of water that is available on the planet earth all right and the estimates tell us that it is less than 0.001 percent of the total water on earth. So, you see that it is a very tiny small fraction of water that is available on the earth. However, it is the life blood or it is responsible for providing us the water or on which our all most of the needs depend such as the agriculture, drinking purposes, industrial usage and so on.

So, it is important for us to understand and model the atmospheric water. Now in this chapter we are going to look at the movement of this water atmospheric water and how we can develop the equations, how we can model the evaporation the losses and so on. all right. Before we move to that we would first look would like to look at the basic Reynolds transport theorem and what are the elements involved in that. And I would like to go very slow because it is important to understand that. So, first thing we will do is what is called the "vapor transport" in this chapter through the atmosphere? How the vapor transport can be modeled? And for that what we will do is we will use a word regular Reynolds transport theorem.

For using the Reynolds transport theorem to model the movement of the fluid which is the moist air in this case. As you know we need to first define, what is the extensive property and what is the intensive property. So, in our case our focus is on the movement of the water in the air.

So, let us say the extensive property B is mass of the water vapor. So, we are focusing on the mass of the water vapor. And what is the flowing fluid in the atmosphere? It is a mixture of gases including moisture. So, it is moist air we will call it moist air which includes moisture as well as some other gases. So, it is not just the moisture itself it is a combination of many gases. Therefore, how do we define the intensive property in this case? I would like you to think about it very carefully, earlier we have been saying that the intensive property corresponding to the mass is always 1, dB by dm.

However, if we think about it what is this physically. Let me first write it and then we will come to it. dB by dm, is nothing but your mass of water vapor in this case or your fluid for unit mass of the flowing fluid right which in our case is the moist air mass of the water vapor per unit mass of the flowing fluid. What does that mean? Is it going to be equal to 1? In earlier cases for examples, channel flow or even the ground water flow, the flowing fluid had only water, it did not involve any other kind of fluid. So, the dB by dm quantity was equal to 1.

But here the extensive property is the mass of the moisture in a mixture of lots of gases so it will not be equal to 1. It is very important to understand that and this is what we call is the “specific humidity” and this is represented as q v. So, it is the moisture in overall volume of some moist air or mixture of gases. So, it some form of measure of the moisture content in the atmosphere and this is defined as the “specific humidity” and we denote it as q v. So, q v is some kind of ratio which tells you the relative importance or relative existence of the moisture in the air.

So, how can we define it or represent it or calculate it. Well, it should be very easy to see that q v is then equal to the density. If we take a unit volume of air, what is the density of the water vapor divided by the density of the moist air or the air which contains moisture? So, this q v is nothing but the ratio of the densities of the moisture and the moist air all right which is called the specific humidity and this can be represented as let us say row v is for vapor over row a is for air and this we will number our equation now as 3.2.1, and I will be following this numbering system as given in the book from now onwards.

Where you row v is the density of water vapor in the atmosphere and row a is equal to the density of the air or the moist air. all right. So, this is as far as the extensive property and intensive properties are concerned. Remember earlier when we had looked at the continuity equation we had said that in a single phase flow we can write this continuity equation very easily because there is only one phase. But in case of multi phase flow your fluid exists in 2 phases may in the vapor, may be in the liquid. We have to be very careful while writing the continuity equation. Before, we move on to that the first concept was about understanding the extensive and intensive properties.

So, with that we move on to the left hand side of your continuity equation.

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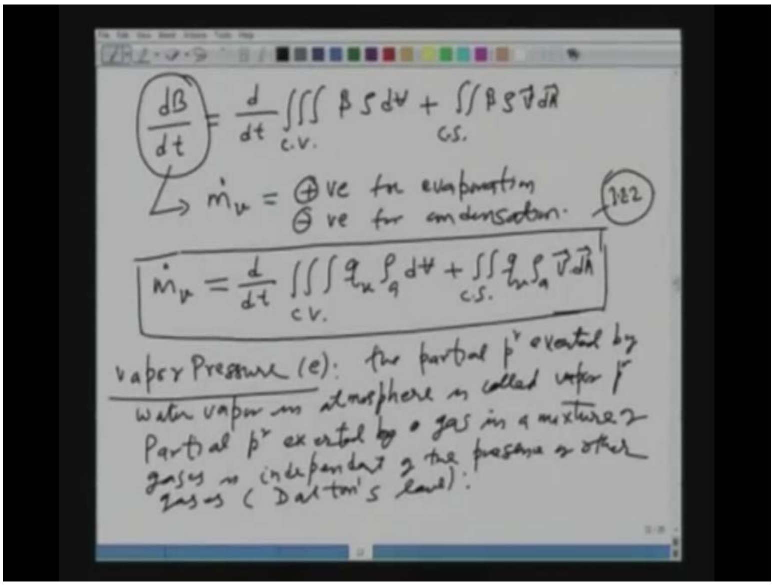

What is the continuity equation? As you all know let me first write it, you should remember this by heart. Your d B by d t is equal to the first term is the d by d t of triple integer over the whole control volume of beta row d v plus double integral over the whole cross section of beta row v dot d A. This is our continuity equation or the Reynolds transport theorem actually where the extensive and intensive properties we can write accordingly for the past. Now what will be d B by d t in this case in the atmospheric quadrant? This is the time rate of change of water vapor with respect to time, what is happening to the water vapor? Is it changing? Earlier, we had said let us say in a channel flow all right if a certain amount of water flow is taking place then there is no change in the mass of the flowing water per unit mass of the flowing water. That is why we have said that d B by d t is equal to 0.

Is this quantity going to be equal to be 0 in this case? No, what is happening to the water vapor in the atmosphere is that it is changing it is form from vapor it becomes liquid or from liquid it becomes vapor. We understand as per the law of conservation of mass that mass cannot be created or destroyed but it can change its form. So, d B by d t that is time rate of change of water vapor in the atmosphere is not 0 all right because condensation and evaporation are taking place. So, in this case we will denote this as certain rate which we need to work on somehow and we will define just say that it is equal to some rate m v dot. Where, m v dot is the rate of the change of time, rate of change of your mass and we will take this as positive for evaporation when evaporation is taking place in the atmosphere and we will take this as negative for condensation.

So, remember the important thing in this chapter now is that d B by d t on the left hand side of your continuity equation is no longer 0. It will be equal to some rate n v dot and we are going to have to find out what is this n v dot and it will be related to your evaporation as we will see later. all right So, with this you can write down all the things in terms of beta b n m v dot. If we did that, our continuity equation then will become left hand side is m v dot it will be d over d t of triple integral c v beta as we said is q v. What is row? Row is the density of the air moist, air which is the flowing fluid in this case all right and then d v as it is plus double integral over the control surface of your beta row v dot d A.

So, this will be your continuity equation for the atmospheric water or the transport of the water in the atmosphere. Let me number this equation as 3.2.2 and let us move on from here. What we will do next is look at certain definitions, certain basic properties of the water vapor which are essential for us to just review and understand I am sure all of you have seen this in your earlier classes, in thermodynamics or somewhere else. But none the less, it is important to just have a review of those. Let us see here you go, first thing we are going to look at is vapor pressure and we will denote this vapor pressure as e in this course. What do we mean by vapor pressure? I am sure you will understand what is a partial pressure in a mixture of gases? You remember Dalton’s law? I am sure you have seen Dalton’s law in your physics in your earlier classes. In a mixture of gases the partial pressure exerted by any gas is independent of the existence of the other gases. In the atmosphere you have moisture in the form of water vapor and there are other gases like hydrogen, oxygen and so on. So, the partial pressure exerted by the moisture in the atmosphere is called the “vapor pressure”.

So, let me define it the partial pressure exerted by water vapor in the atmosphere is called vapor pressure. Now we know that I think I do not need to write it but let me write it anyways, the partial pressure exerted by a gas exerted by a gas in a mixture of gases is independent of the presence of other gases right and this is what is called the Dalton’s law. This Dalton’s law or mathematically it is given by what is called ideal gas law all right and we use that even for the finding the vapor pressure in the atmosphere. So, what is the ideal gas law? Let me just review your knowledge or remind you.

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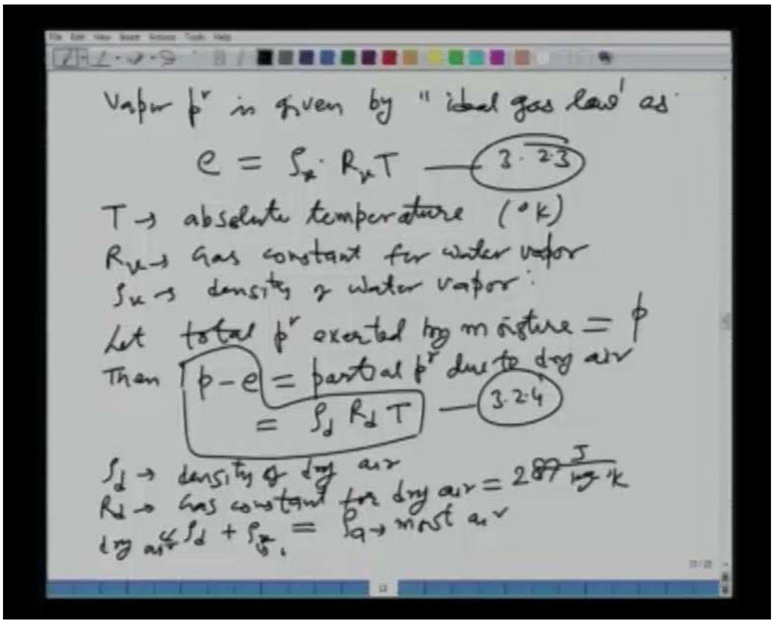

The vapor pressure is given can be given using or by ideal gas law as your e is the notation is equal to what row rt. So, it is row v R v and T this is your 3.2.3. Remember we use the proper subscripts all right we are trying to find out the vapor pressure. So, we are using the density of the vapor and the gas constant ideal gas constant for that particular gas all right whether it is oxygen, hydrogen or moisture whatever. So, we use the appropriate point piece for that. So, let me define these these things in the equation, e is the vapor pressure of course, and what is T? T is the absolute temperature in the atmosphere. In this equation, the temperature is what is called the absolute temperature and what do we mean by that, it is degrees Kelvin. R v is the gas constant for water vapor.

Ok. And row v of course, we have defined earlier is the density of water vapor. Now let us look at what will be the relationship between the atmospheric water, atmospheric pressure and this vapor pressure. We if we see that or if you say let, the total pressure exerted by the moist air which is nothing but the atmospheric pressure all right let us say this is equal to T. Then what will be p minus e can you tell me? The p minus e p is the total pressure exerted by all the gases all right or it is the total atmospheric pressure and e is the vapor pressure exerted by the vapor only.

You know that the total pressure exerted is the sum of the individual pressure. So, p minus e will be nothing but the pressure exerted by the dry air. So, if you want to write it, it is nothing but the partial pressure due to dry air and it will be given by use the corresponding quantities row d Rd and T and let me say that this is 3.2.4. And 3.2.4 is this quantity p minus e where, row d as you know is the density of the dry air and Rd is the gas constant for dry air and it is value is given by 287 joules per kg-k l Kelvin liters. Now what will be row d plus row v equal to? Can we think about that? What will be the sum of the density of the dry air and the density of the moisture? That has to be equal to the density of the moist air row a. Some of the densities of the individuals have to be the total density. So, this is your moist air density and this is for the dry air and this is for the vapor.

Now what I am going to do is I will throw a few equations at you which give you the relationship between many of these variables. So, let us not get worried about it too much. But none the less it is important to understand all these equations.

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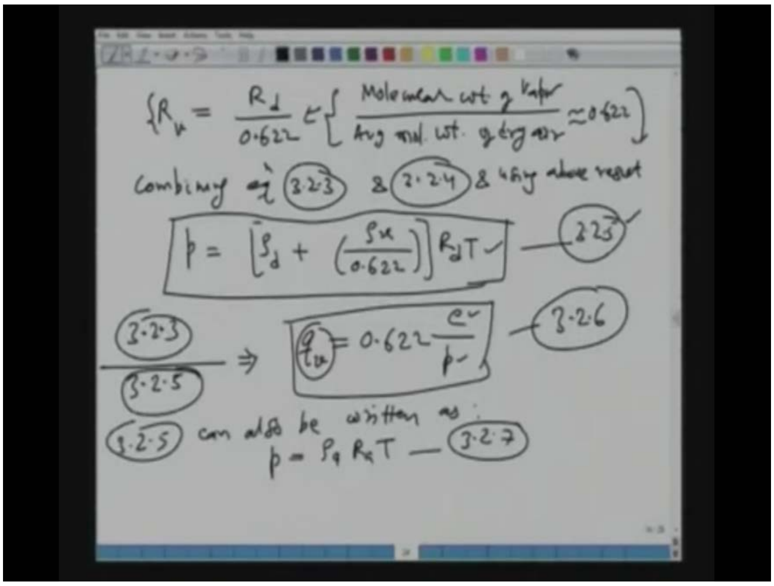

The gas constant for R v is given as R d over 0.622 and where it is coming from is from the fact that the molecular weight of vapor divided by the average molecular weight of dry air that ratio is approximately 0.622 from this fact we are able to write this equation. Now another equation is, if we combine 3.2.3 and 3.2.4. If I want to go back here what are these 2 equations, 3.2.3 is this and 3.2.4 is this. If you combine this 2 we should be able to write p is equal to row d plus row v over 0.622, the bigger brackets closed R d t and in fact I should write and using above result that is this one. So, when we combine the 2 equations 3.2.3 and 3.2.4 and use this relation this one we should be able to get this relation. So, should be very easy to see. Let us not worry about it too much, you all can derive this very easily and let me number this as equation 3.2.5.

Now, if you do 3.2.3 divided by 3.2.5 which we have just derived 3.2.5 we will actually not do all this mathematical or algebraic manipulations we are looking at the final results which is none the less very important for us, will be very important result 0.622 e over p. What I have done here is I am saying if we divide equation 3.2.3 by 3.2.5, it is this one 3.2.5 and 3.2.3. If I go back is this one we did that we get this relation and I will number this as 3.2.6. This 3.2.6 is a very important equation you should be able to realize. What is it giving us? It is giving us a measure of this specific humidity in the atmosphere UV. Remember, this is our intensive property we have defined in our continuity equation earlier. What is it? It is 0.622 in the ratio of e over p. What is e? e is the vapor pressure. Vapor pressure is something we can measure in the atmosphere. We have sophisticated instruments available which can measure the vapor pressure.

We also have directly or indirectly that is, we also have instruments which can give you the atmospheric pressure. So, these 2 quantities on the right hand side of 3.2.6 that is e and p are something which we can estimate or measure. That will give us an estimate of 2 that is why all these relations which we have looked at are extremely important because this will give us one of the important quantities which is needed in the modeling the movement of the moisture in the atmosphere. Now few more things I will throw at you here, equation 3.2.5 this one we just looked at can also be written as in a compact form.

That is, p is equal to row a Ra T and let me number this as 3.2.7. So, we are using the moist air, the density and the gas constant for the moist air which can be directly written as this and using the relations it can be written as this also moving further.

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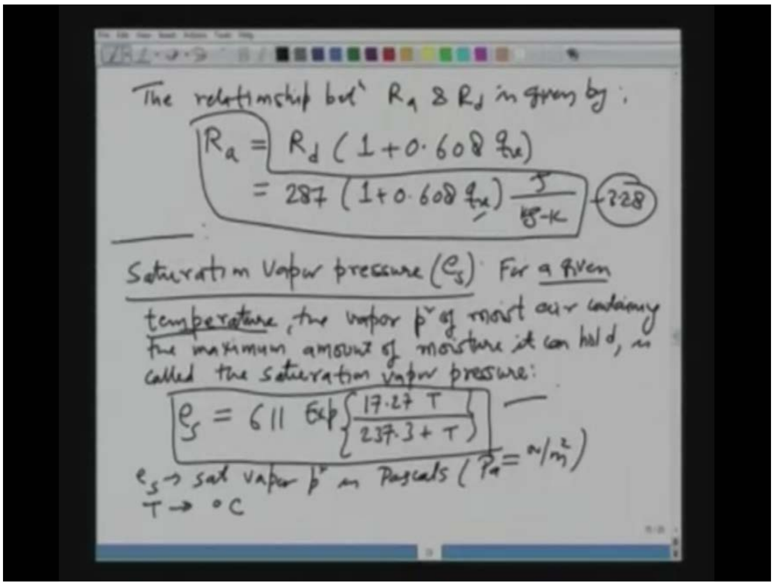

The relationship between Ra and Rd is given by this equation Ra is equal to Rd time 1 plus 0.608 q v and R d as we have said is 287 so it is 1 plus 0.608t of your q v and in its units and as we know are joules per kg- k l.

This is another important relation, if we want to be précised in our calculations in the movement of or modeling the movement of the atmospheric water. What is this equation giving us is that the ideal gas law sorry the gas constant for the moist air varies with respect to the moisture level in the atmosphere. On the right hand side you see the qv appears which we have just seen how to find out. So, if we can calculate the vapor pressure, if we can calculate or estimate the atmospheric pressure we can find out q v and knowing the q v we can find out R a, right which can be used in finding these things. So, this is 3.2.8. So, this was about the vapor pressure one quantity and various relationships related to that. The next thing we are going to look at is what is called a saturation vapor pressure and this we will denote as e s or e sat. What is e sat or saturation vapor pressure? As the name suggests it is the maximum value of the pressure which is exerted by the moisture in the atmosphere the maximum possible when the humidity level is maximum.

So, if we want to define it is defined like this, for a given temperature for a given temperature the vapor pressure the vapor pressure of moist air containing the maximum amount of moisture it can hold is called is the saturation vapor pressure. So, what is the saturation vapor pressure we are defining at a given temperature all right the important time here is for a given temperature. The vapor pressure of the moist air containing the maximum amount of moisture it can hold is called the saturation equation. What does it mean for a given temperature? If the climatic conditions change in the atmosphere and the temperature changes all right so as a function of the or as the change of the temperature the saturation vapor pressure will also change. So, saturation vapor pressure is a function of temperature in the atmosphere. So, its value will be different for different value of the temperature that is why out people has derived this equation for calculating e s.

If we know the temperature in the atmosphere you can find out what is the corresponding saturation vapor pressure. And it is I will give it to you directly without any derivation as our objective in this course is to use these equations as hydrologists not scientists, divided by 237.3 plus T. all right. This is an empirical equation as you see because there are many constants involved. Whenever you see some numbers 17.237 point something that equation is a empirical equation which takes care of the units themselves. So, we just put the value of temperature in certain units it will give you the pressure in the appropriate level so here in this equation where e s is the saturation vapor pressure in pascal’s.

Pascal is the unit of pressure as you know one pascal is given as one newton per square meters all right and the temperature is given in degrees centigrade. So, if you put the temperature in degree centigrade in this particular equation you will get the saturation vapor pressure in pascal’s and let me number this equation as 3.2.9. So, this was the saturation vapor pressure and related to that we will define another term which is the derivative all right of this saturation vapor pressure curve.

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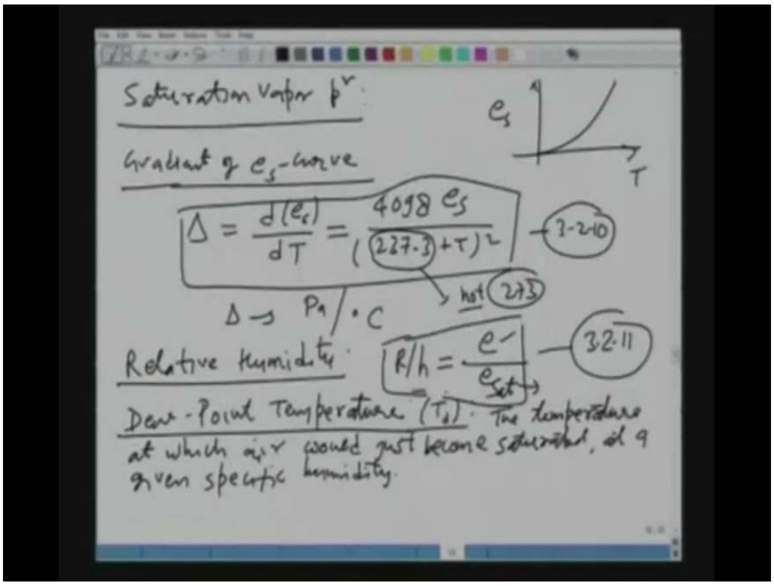

So, before I move on to that let me say that saturation vapor pressure the equations I have given you all right it can be given by the equation if you plot it, it will look like this. all right Where the x axis is the temperature and y axis is your e s. So, as the temperature increases the saturation vapor pressure in the atmosphere will also increase. Now, when we are doing the modeling of the moisture movement in the atmosphere, saturation vapor pressure will occur very frequently in our calculations. Associated with that what we also need to calculate or find out sometimes is the slope of this curve which we have just drawn. That is called the Gradient of e s curve or saturation vapor pressure curve, this is denoted as delta and this is nothing but the slope at any given temperature e s depends upon the temperature. So, first derivative of saturation vapor pressure curve as a function of temperature is given by this equation. You can do the first derivative of e s equation we have just written and we should be able to find this 237.3 t whole square this is 3.2.10.

What will be the units of delta can anybody tell me or think about it? The units of e s was, what it was pascal’s? It is a pressure. So, what will be the units of delta? It is vapor pressure per unit temperature. So, it has to be the units of delta will be per pascal’s per degree centigrade. Now, one thing I would like to stress here is at that both of these equations for saturation vapor pressure as well as the gradient there is something that appears in the denominator here. It is this factor is 237.3, it is not 237. Just like to draw your attention because many students I have seen in this course they make this mistake while doing the problems or applying this equation.

Because whenever you have temperature you add 237 to that and you get temperature in the degrees Kelvin. But that is not what it is in these equations it is 237, it is not 273, it is a very common mistake so keep that in mind. So, moving on the next thing we are going to look at is what is called a relative humidity.

How is the relative humidity defined? Well, it is defined as the ratio of vapor pressure to saturation vapor pressure. So, if we can find out what is the vapor pressure in the atmosphere and if we can find out what is the saturation vapor pressure, its ratio is nothing but the humidity the relative humidity in the atmosphere. And this is your 3.2.11. Now related to this we have seen how do we find this e sat or e saturation vapor pressure. We have seen the equation 3.2.9, how do we determine e or the vapor pressure corresponding to the existing present level of moisture in the atmosphere? We can do that with the help of another kind of temperature which I am sure you would have seen is called the dew point.

Dew point temperature and let us denote this as Tb. I would like to go very slowly because lot of you may be confused about what is exactly or how do we define this dew point temperature is very important to understand. So, let me first define it and then we will try to see this graphically what do we mean by that the temperature at which the air would just become saturated, saturated at a given specific humidity or a level of moistures in the atmosphere specific humidity. So, dew point temperature is what it is. First you look at the definition that is the temperature at which the air would just become saturated at a given specific humidity. So, it is defined for a specific humidity or what is specific humidity? It is a measure of the relative humidity at any given point of time in the atmosphere.

So, dew point temperature actually is nothing but a measure of humidity in the atmosphere, it is that is why very important to understand and there are instruments available that can measure the dew point temperature. So that is why I said we can directly calculate this relative humidity in the atmosphere all right and we will come to that all right. So, let me just explain it little further what is this dew point temperature using a diagram

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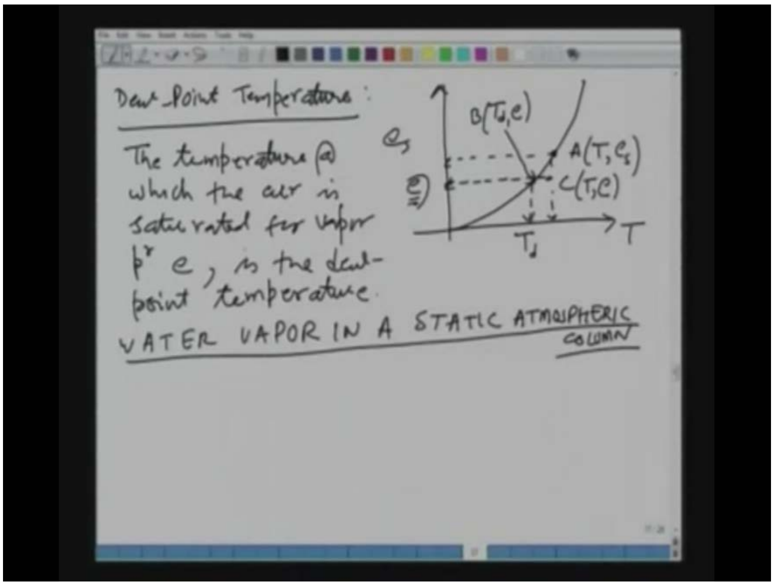

Ok. all right So, I did that slightly blown up diagram of what we just looked at this is a saturation vapor pressure curve. On the x axis you have the temperature and on the y axis you have the vapor pressure let us say e s and we have seen that this look like this. So, what I am going to do is I am trying to going to demonstrate is, what is a dew point temperature with the help of this saturation vapor pressure curve? So, let us first take any point on this and I will call this point as let us say A and what will be the coordinates of this point A let us say some temperature T and some value of e s right. What are these things? This is this temperature and this is this value of the saturation vapor pressure now, for a given value of e or given value of specific humidity.

Let us say this is e all right we are defining the dew point temperature for given specific humidity or the partial pressure exerted by the vapor in the atmosphere due to the level of moisture content in it. So, let us say that is e and this if you take that value of e and then we draw the horizontal line and let us say it cuts the vapor pressure curve at some point B and let us say this point is C. all right So, what will be the co-ordinates of C it will be the temperature in same and y co-ordinate is instead of e s it is e so this is at e and this is at same temperature T. Now what will be the co-ordinates of B that is what is important? So, this is a temperature at which the air would just become saturated for given e, given e is this point is on this curve so this pressure actually represents the saturation vapor pressure for this temperature which is called T.

So, the co-ordinates of this point B then would be let us say Td comma e so you see that the dew point temperature is a temperature at which the air would just become saturated for a given specific humidity or given value of e or in other words you can say that it is the temperature at which the air is saturated for vapor pressure e. This e corresponds to that specific humidity or related humidity is called or is the dew point temperature. What is a dew point temperature? In other words the temperature at which the air is saturated for vapor pressure e given that is called the dew point temperature.

So, these are some of the important properties of moist air we have seen, some of the climatic variables, how we can define them and how we can calculate, how we can measure them? The next thing which we are going to do in this course is we will look at the water vapor in a static atmospheric column all right. So, what will do is we will take a cylindrical column of atmosphere and see how the temperature and pressure will vary in it so that how we can calculate the water vapor and other quantities in it. So, I may not have time today to complete it none the rest we get started on this.

So, you have water vapor in a static, static means there is no movement of air taking place. Keep that in mind, we are looking at a static atmospheric column where there is no flow of air is taking place all right things are steady static atmospheric column. So, this is next topic and this is also very important to understand to do these calculations of the water vapor in a static atmospheric column.

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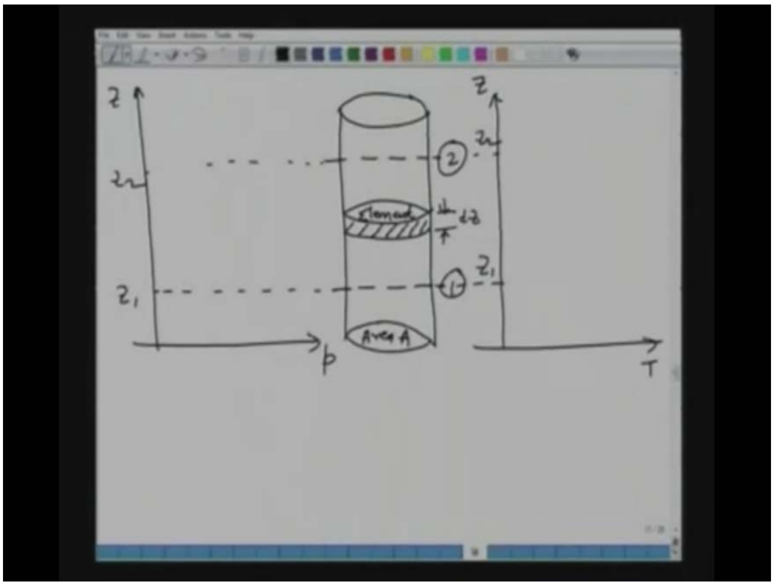

Let me first draw the figure which we will need. all right.So, let us consider the cylindrical element or cylinder of atmosphere. This area is A the cross sectional area of the cylinder is A. Now what we will do is we will look at 2 cross sections of this between 1 and 2. 1 is may be close to the ground or just above the ground, all right it does not show the way actually I have drawn but you understand 1 is close to the ground and 2 can be at any height all right which is important as far as the cloud dynamic is concerned all right. So, we can take this height of 2 at a few kilometers up to which your water vapor is important. What we will do then is, we will take a small strip we will take a small strip of height d z in the atmosphere and try to calculate a few things in it.

So, this is a cylindrical small element we will consider. Now corresponding to that I would like to be able to draw these two curves. One is the temperature and as a function of z. what is z? z is the height vertically going upwards. So, we want to find out what will be the variation of the atmospheric temperature as we go up and on the other hand we would like to find out what will be the pressure as a function of height z. Then we have these 2 quantities. So, our focus of attention would be let us say this is height z2 and this is height to z1. Similarly, here on the other side you have this as z2 and this is z1. So, our objective is to find out what is the variation of temperature and pressure in a static atmospheric column. Once we know that we can calculate the water vapor and many other things right of. So, let me go back then

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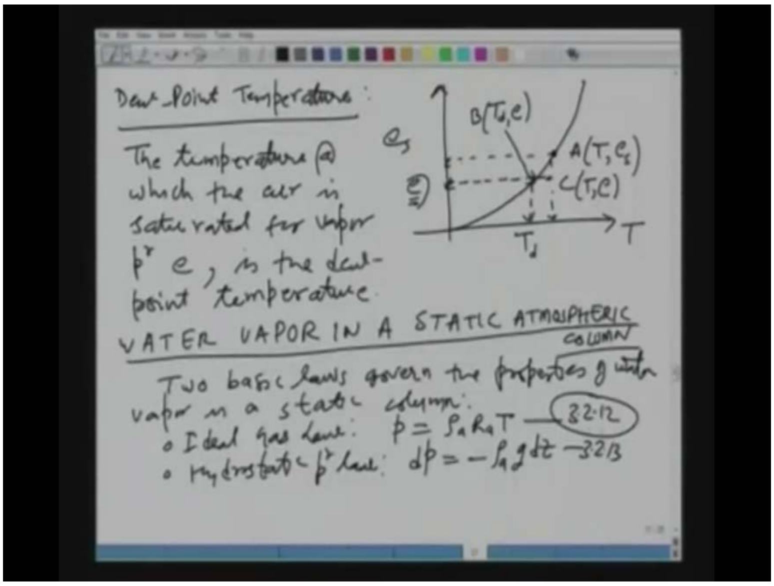

And to be able to do this we will need 2 basic laws. That governs the properties of water vapor that is in a static column. To be able to find out the quantity of water for water vapor in a static atmospheric column we will use two basic laws, all right we will combine these two and then generate a few equations and then I would like to look at an example of that. So, let me first write down these two basic equations. First one of them is ideal gas law which all of us have seen in. What is it? p is equal to row a Ra T. we have just seen that let me number this as 3.2.12 and the other one is what is called the hydrostatic pressure law, which is hydrostatic pressure law. As we go up the pressure is distributed as hydrostatic. What is the hydrostatic pressure law from your knowledge of hydraulics? Well this d p to p or incremental pressure, as we go up pressure will be decreasing which we take a negative. It is row a g d z or d z of these (( )) and this I am going to number as 3.2.13.

I am sure you have seen stationary water in your fluid mechanics earlier. What is the pressure? Pressure is OGZ, as we go down into a reservoir or in a pool of water the pressure increases as OGZ or of the height OGH. In this case we are going up in the atmosphere so the pressure will vary as OGZ we have take a strip. So, it is e p is equal to negative row a d z. all right. Negative we have take because as we go up the pressure will be decreasing. So, I think I would like to stop at this point of time. We will use these two basic laws ideal gas law and the hydrostatic pressure law to derive the expression all right for the water vapor in the static atmospheric column which will be very important for us to find out how much water available in the atmosphere for precipitation. So, we will take this up tomorrow.