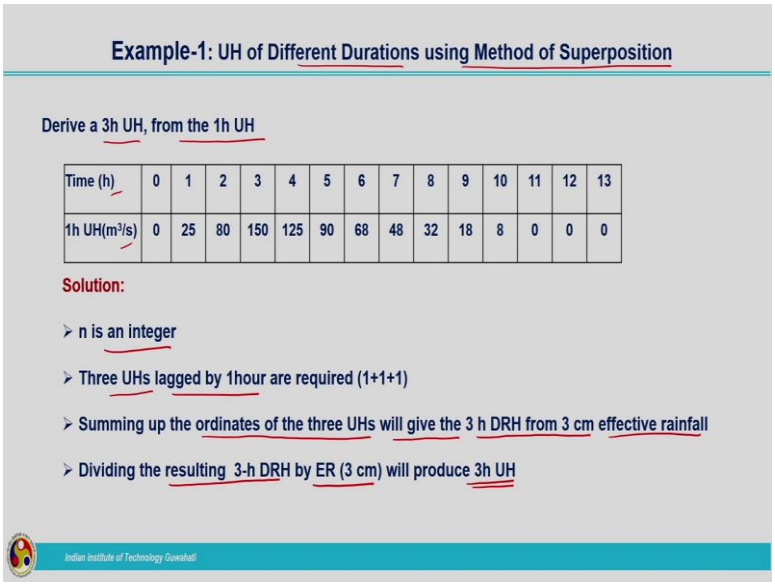
Engineering Hydrology
Dr. Sreeja Pekkat
Department of Civil Engineering
Indian Institute of Technology, Guwahati
Module 5
Lecture 59
Numerical Examples on Hydrograph of Different Duration

Hello all, welcome back in the previous lectures, we have seen the concepts related to S hydrograph and the methods by which unit hydrographs having different durations can be derived from the given unit hydrograph. So, today let us solve some of the numerical examples related to those concepts.

So, by deriving the unit hydrograph having different durations that that is *nD* hour duration, we have seen two concepts can be utilized method of superposition and method of S hydrograph. If the duration which we are going to derive is the integral multiple of the duration of the known hydrograph we can make use of the principle of superposition and in all the cases this may not be correct.

But sometimes we may have to derive a unit hydrograph having duration which is not an integral multiple of the given duration that is *n* can be a fraction in that case we can make use of S hydrograph theory. S hydrograph can be utilized in the case of integral multiple also, but in the case of fraction, we can make use of S hydrograph only, we cannot make use of principle of superposition. Today, let us solve some of the examples related to it.

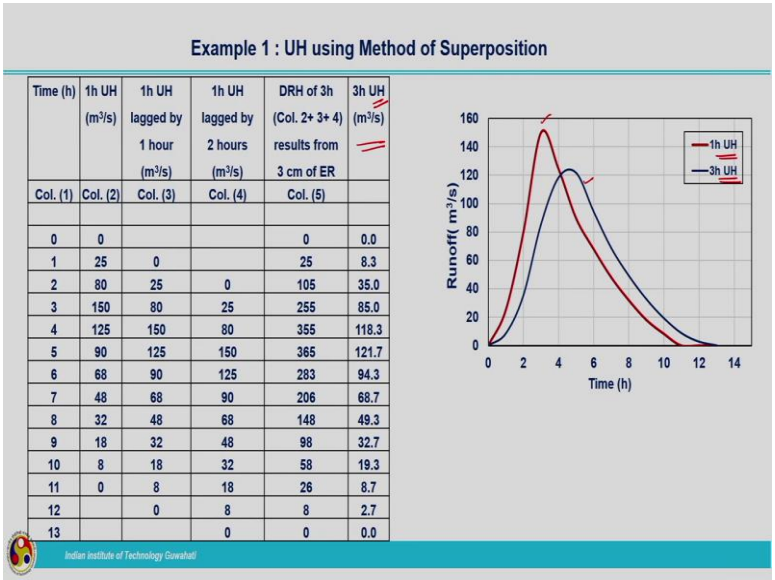
(Refer Slide Time: 02:19)



First example is unit hydrograph of different durations using method of superposition. Derive a 3-hour unit hydrograph from 1-hour unit hydrograph, time in hour and 1-hour unit hydrograph ordinates are given to us. That is, we are having the 1-hour unit hydrograph. The known unit hydrograph is having a duration of 1 hour, we need to derive the 3-hour unit hydrograph. Since the known duration is 1 hour, and we have to derive the 3-hour unit hydrograph. It is very clearly, we can understand that it is an integral multiple of the given duration. So, we can make use of the principle of superposition here to derive the 3-hour unit hydrograph from 1-hour unit hydrograph.

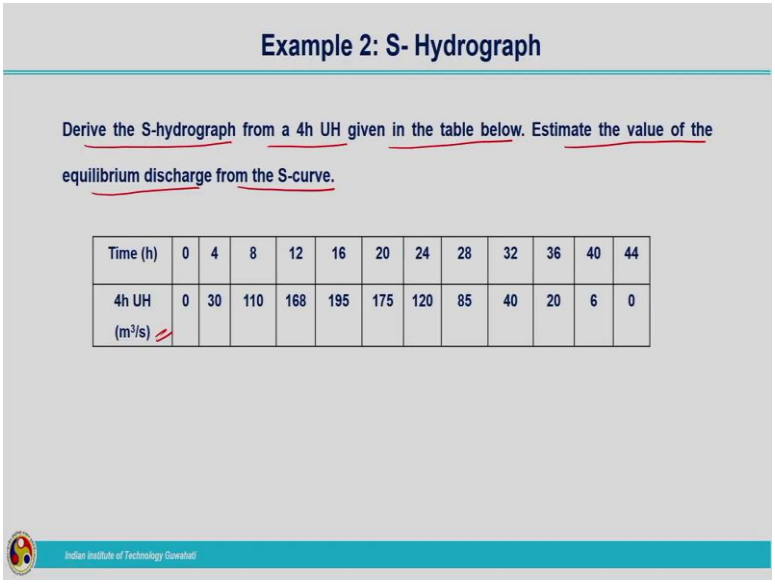
You know the steps related to it. Here in this case *n* is an integer and if *n* is an integer will be making use of the principle of superposition and 3-unit hydrographs will be lagged by 1 hour. 3-unit hydrographs are required, so these 3 units hydrographs superimposed together will be producing a 3-hour direct runoff hydrograph due to 3 centimeters of rainfall. Summing up the ordinates of the 3-unit hydrographs will give the 3-hour direct runoff hydrograph from 3 centimeters of effective rainfall. Now, by making use of the principle of proportionality, we can divide the resulting 3-hour direct runoff hydrograph by the effective rainfall of 3 centimeter that will be giving us 3-hour unit hydrograph. So, first what will be doing? We will be lagging the 1-hour unit hydrograph by 1 hour, then summing up the corresponding ordinates to form the direct runoff hydrograph of 3 hours which is produced due to 3 centimeters of rainfall. After that, we will be dividing each and every ordinate of the direct runoff hydrograph by 3 centimeters which will be producing the unit hydrograph of 3 hours.

(Refer Slide Time: 04:39)



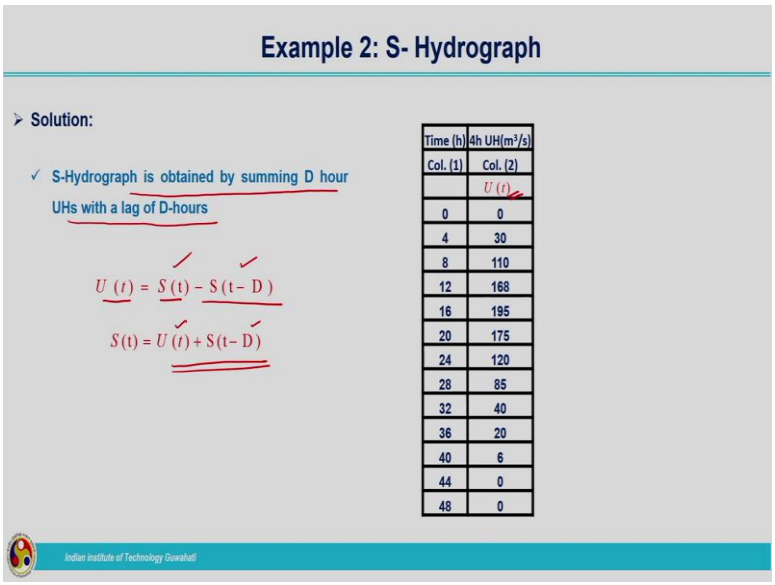
So, these are the data given to us. What we have to do? These are the ordinates representing 1-hour unit hydrograph it is meter cube per second. So now what we will do? We will lag these ordinates of 1-hour unit hydrograph by 1 hour. How it can be done. So, we will be lagging it by one row. So, it is starting from first chart, then we need to have the 3-hour unit hydrograph. So, we need to lag it again by 1 hour. So, this is the ordinates of 1-hour unit hydrograph, lagged by 2 hours. So, we are starting it 2 hours. After this what we will do, we will sum up the ordinates from column 2, column 3 and column 4, that will be resulting a direct runoff hydrograph of 3 hours, which is produced due to 3 centimeters of effective rainfall, this is the direct runoff hydrograph due to 3 centimeters of rainfall. Now, what we will be doing will be dividing the ordinates of the direct runoff hydrograph by 3 centimeters. So, it will be giving us 3-hour unit hydrograph So, these are the ordinates of 3-hour unit hydrograph which is produced from 1-hour unit hydrograph. Now, we can plot the unit hydrographs that is 1-hour unit hydrograph and 3-hour unit hydrograph can be plotted in a single figure, that is 1-hour unit hydrograph is represented by red graph. So, here we are having the plot showing 1-hour unit hydrograph and 3-hour unit hydrographs. So, you can observe the graphs, 1 hour unit hydrograph is having high peak and the peak is attained very fast and while looking at the 3 hour unit hydrograph it has produced a peak after certain time than that of the 1 hour unit hydrograph but the time base is the same that is we are having the one hour unit hydrograph and also 3 hour unit hydrograph which are having the same time base.

(Refer Slide Time: 06:55)



Now, let us move on to the second example on S hydrograph. While explaining S hydrograph concepts, we have discussed about the theory only. Now, let us solve one example related to a S hydrograph, then it will be clearer. Derive the S hydrograph from a 4-hour unit hydrograph given in the table below. Estimate the value of the equilibrium discharge from the S curve. Here what we have to derive, we have to derive a S hydrograph from a 4-hour unit hydrograph. While discussing about S hydrograph we have seen after certain time S hydrograph will be attaining an equilibrium discharge. So, what is the equilibrium discharge that also we need to determine in this example. So, these are the ordinates of 4-hour unit hydrograph.

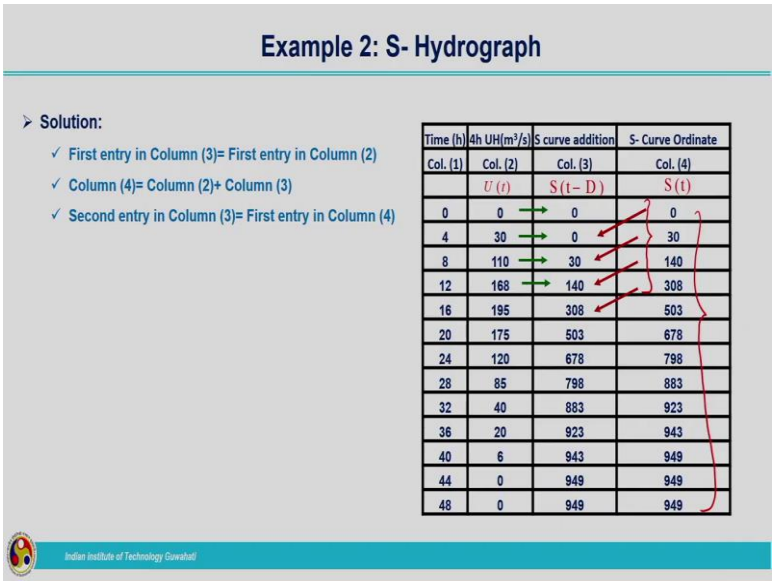
(Refer Slide Time: 07:55)



Now, let us move on to solve the example: the data given are 4-hour unit hydrograph ordinates. First what we will do we need to revisit into the concept related to S hydrograph, S hydrograph is obtained by summing D hour unit hydrograph with a lag of D hours. We are having the D hour unit hydrograph, we will be lagging the unit hydrograph by D hours. So, that way, so many numbers of unit hydrographs will be lagged and after that we will be summing up the ordinates to get the S hydrograph. So, here we are having the 4-hour unit hydrograph. So, definitely will be lagging that by 4 hours for producing S hydrograph and the ordinate of unit hydrograph is given by $U(t) = S(t) - S(t-D)$ i.e., the ordinates of unit hydrographs are summed up together to form the S hydrograph. So, for getting the unit hydrograph, we can get the ordinate of unit hydrograph by subtracting the ordinate of $(t-D)$, S hydrograph from $S(\mathrm{t})$. This $S(t-D)$ is nothing but the S curve addition. So, $S(\mathrm{t}) = U(t) + S(t\text{-}D)$, unit hydrograph ordinate at time t plus S hydrograph ordinate at

$(t-D)$, that will be giving us the ordinate of the S hydrograph at time *t*. $S(t-D)$ is the S curve addition. Now, we can proceed to solve the example

(Refer Slide Time: 9:46)



Time (h)	4h UH(m^3/s)	S curve addition	S- Curve Ordinate
Col. (1)	Col. (2)	Col. (3)	Col. (4)
	$U(t)$	$S(t-D)$	$S(t)$
0	0	0	0
4	30	0	30
8	110	30	140
12	168	140	308
16	195	308	503
20	175	503	678
24	120	678	798
28	85	798	883
32	40	883	923
36	20	923	943
40	6	943	949
44	0	949	949
48	0	949	949

The ordinates which are given in column 2 can be represented by the notation *U(t)* and next step what we are going to do we are going to find out the S curve addition, we need to find out the ordinates corresponding to S curve addition, then only we can find out S curve ordinates for time *t*. By deriving the S curve, you might have noticed the ordinate at the beginning time that is at time *t* is equal to 0, unit hydrograph ordinate and the ordinate of the S hydrograph both are 0. So, we can represent this S curve addition by the notation as $S(t-D)$. How can we get the ordinates corresponding to $S(t-D)$? You might have observed while discussing about the S hydrograph and unit hydrograph the ordinates corresponding to time 0, *t* is equal to 0 in both the cases that is in the case of unit hydrograph and S hydrograph is equal to 0.

So, first ordinate will be the same, first entry in column 3 is the first entry in column 2 that is nothing but it is 0. Now, we will move on to get the S curve ordinate represented by *S(t)*. How can we get *S(t)*? S(t) is given by

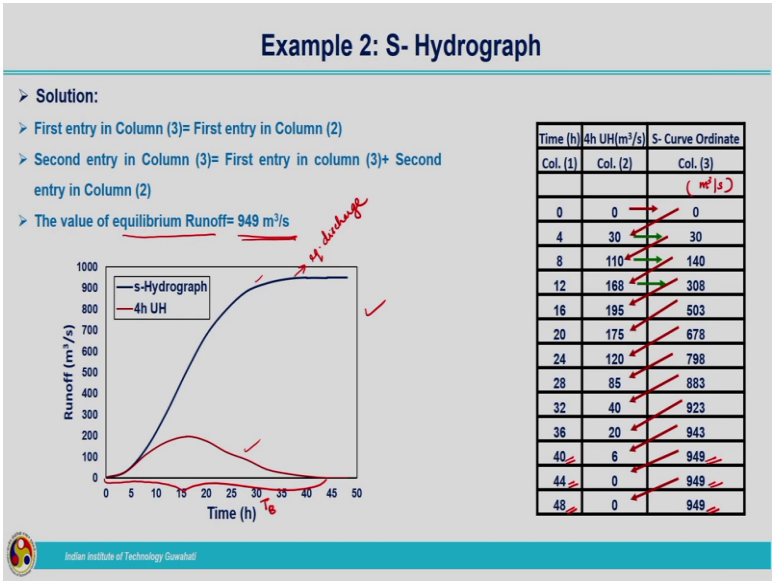
$$S(t) = U(t) + S(t-D)$$

So, we are having *U(t)* here, you are having $S(t-D)$ here that sum will be giving us S curve ordinate. So, the column 4 is obtained by summing up column 2 and column 3. What is our column 2? Column 2 is unit hydrograph ordinate and column 3 is the S curve addition. So, if

we are adding that we will get the ordinate corresponding to *S(t)*. So, here in the case of time *t* is equal to 0 we are having ordinates of unit hydrograph and the S hydrograph as 0 and now, coming to the second entry in column 3 that is the first entry in column 4, second entry in column 3 is the same as that of the first entry in column 4 here we are having 0 and same thing is written over here in column 3. Now, we will be summing up the ordinates of unit hydrograph and S curve addition, it will be giving us the S curve ordinate. This way we will be proceeding, this 30 will be returned in column 3 and 30 plus 110 will be giving us 140 that is S curve ordinate corresponding to time t is equal to 8 hours. Now, that 140 will be put in S curve addition column that will be added to the unit hydrograph coordinate 168 to get the S curve ordinate. So, it will be giving us 308. This procedure will be followed in order to produce the S curve ordinates for different time intervals. So, this way if we are proceeding, we can get the ordinates corresponding to the S curve for the entire time durations. So, you can see what we have done? We have written our unit hydrograph ordinates, then we have found out the S curve additions and that S curve addition is added to the unit hydrograph ordinates.

Here one thing you need to be careful is that we are adding the S hydrograph ordinate corresponding to a lag of *D* duration to the unit hydrograph ordinate of *U(t)*, $U(t)+S(t-D)$ we are doing that is why we are separately making a column for S curve addition then we are adding in order to avoid calculation mistakes. So, the same example can be solved by another way, you do not have to always write this column related to S curve addition, you can directly get the ordinates of S hydrograph.

(Refer Slide Time: 14:07)



Time (h)	4h UH(m³/s)	S- Curve Ordinate
Col. (1)	Col. (2)	Col. (3)
		(m³/s)
0	0	0
4	30	30
8	110	140
12	168	308
16	195	503
20	175	678
24	120	798
28	85	883
32	40	923
36	20	943
40	6	949
44	0	949
48	0	949

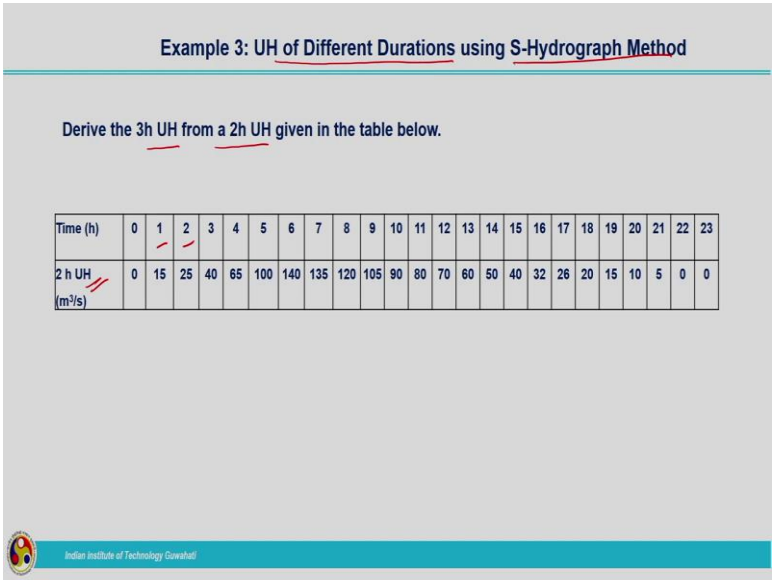
Let us look into that, the same example only, there is no change in the data points. Same 4-hour unit hydrograph is there that is written over here in column 2 and we are going to calculate the S hydrograph ordinates. So, first entry in column 3 is the first entry in column 2 itself because S hydrograph ordinate and the unit hydrograph ordinate at time *t* is equal to 0 is same as 0. So that will be written over and that is giving us S curve ordinate. After that we are summing up, that what we were doing in the previous case, we were writing it again as S curve addition, which has to be added to the unit hydrograph ordinate at time *t*, instead of making separate column we are directly adding and putting in the column 3.

So, second entry in the column 3 is the first entry in column 3 plus second entry in column 2. Here what we are going to do, we are going to add 0 and 30 it will be giving S curve ordinate corresponding to time *t* is equal to 4 hours. Same step will be repeated 30 plus 110 will be giving 140, 140 plus 168 will be giving us 308, this procedure we are continuing till the end of the unit hydrograph and we will get the ordinates corresponding to S curve. All the ordinates can be obtained. So, here instead of separately writing a S curve addition column, we have directly summed up the ordinates to get the ordinates of the S curve. Only thing is that you have to be careful about the interval by which the data is given. If your unit hydrograph is of 4 hour and the interval given is 2 hours, while adding you have to add corresponding to 4 hours that is first entry will be put in the cell corresponding to 4 hours it cannot be directly put just after the first cell. So, that care should be done, otherwise entire answer will be wrong that is why we are separately making S curve addition column, otherwise, this way you can easily find out the ordinates of the S hydrograph.

So, this is the way we are proceeding and we are finding out the ordinates. So, the first part of the question is over, i.e., derive the S hydrograph that we have done. Now, the second part of the question was what is the equilibrium discharge or equilibrium runoff value? Look at the curve, you can understand that at *t* is equal to 40 hours the ordinate of S hydrograph is 949-meter cube per second, the unit is in meter cube per second and corresponding to 44^{th} hour also it is 949 and 48^{th} hour again the same value. So, you can understand that as time is increasing there is no change taking place in the ordinate of the S hydrograph that is representing the equilibrium discharge. Here in this example, the value of equilibrium runoff is 949-meter cube per second. So, you look at the figure as the time base of the unit hydrograph is reached, that is around 44 hours, the S hydrograph has attained the equilibrium discharge. After certain time, how we are producing the S hydrograph? We are putting the unit hydrograph at a required lag or we are placing the unit hydrographs with the duration of

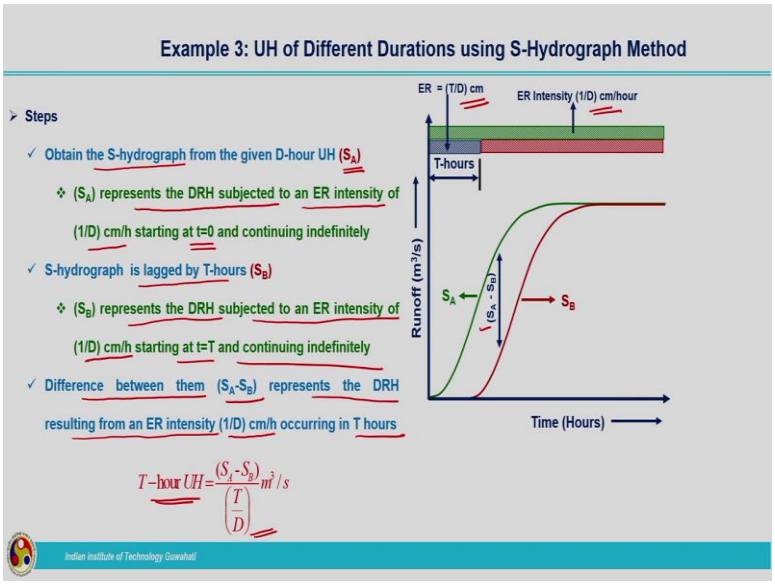
the unit hydrograph and summing up the ordinates. And we have seen that the S hydrograph will be attaining the equilibrium discharge after certain time that is corresponding to the time base of the unit hydrograph. That is clearly visible from this figure. S hydrograph is given by this blue curve and the given unit hydrograph that is the 4 hours unit hydrograph is represented by this red curve. So, now, it is very clear how to solve a question related to S hydrograph that is if a unit hydrograph is given to you, you can develop the S hydrograph by making use of these principles any of the ways you can follow you can make the column related to S curve addition separately or even if it is not there, you can directly add and find out the ordinates of the S hydrograph.

(Refer Slide Time: 19:06)



Now, we will move on to the third example, that is related to unit hydrograph of different durations using S hydrograph method. So, definitely unit hydrograph of different duration can either be obtained by S hydrograph method or superposition technique. So, here the question is derived the 3-hour unit hydrograph from a 2-hour unit hydrograph given in the table below. We have been given 2-hour unit hydrograph, ordinates of 3-hour unit hydrographs and the corresponding time is given to you. You look at that table carefully. This is the ordinates corresponding to 2-hour unit hydrograph but, the ordinates are given at an interval of 1 hour. So, in this case if you are not separately making the column related to S curve addition, you need to be very careful while making the additions.

(Refer Slide Time: 20:13)



Before moving on to solving the example, let us have a revisit into the steps involved with that. First from the unit hydrograph we will be finding out S hydrograph that is denoted by S_A. So, this is our effective rainfall intensity having $\frac{1}{D}$ centimeters per hour that is producing the S hydrograph S_A, and S_A is representing the direct runoff hydrograph subjected to an effective rainfall of intensity $\frac{1}{D}$ centimeters per hour which is starting at time t is equal to 0 and continuing indefinitely. Second step is to lag the S hydrograph by T hours, that is the catchment is going to experience the same amount of rainfall after T hour duration. So, same effective rainfall intensity of $\frac{1}{D}$ centimeters per hour is marked over here, which will be producing the S hydrograph with a delay of T hour that is marked by this red curve represented by S_B for that what we are going to do we are going to lag the developed S hydrograph by T hours and this S_B is representing the direct runoff hydrograph subjected to an effective rainfall intensity of same amount that is $\frac{1}{D}$ centimeters per hour starting at time t is equal to capital T and continuing indefinitely. These concepts you already know. But just before solving the example I am just refreshing the steps behind it. Now we will be finding out the difference between them, $S_A - S_B$ we will be finding out, that is representing the direct runoff hydrograph which is resulting from an effective rainfall having intensity

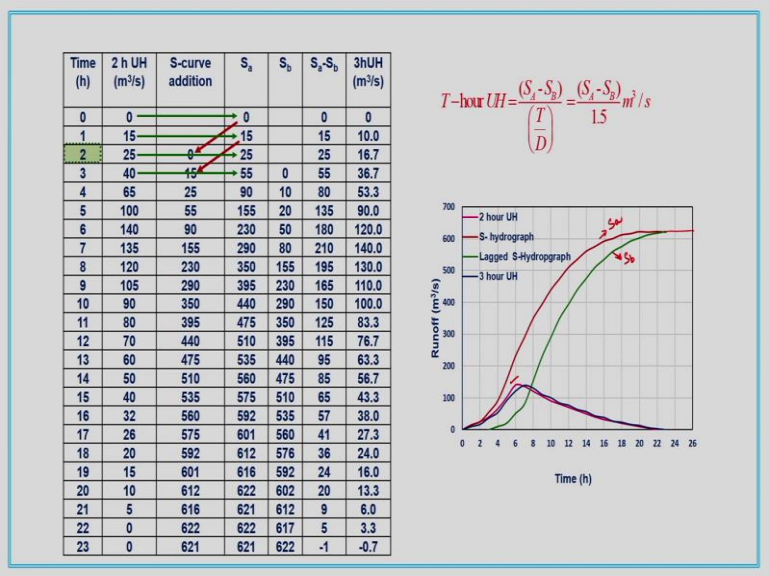
$\frac{1}{D}$ centimeters per hour. But you have to be careful, the duration corresponding to this is T hours that is we have found out the difference between $S_A - S_B$ so, that is representing an effective rainfall which is occurring for a duration of T hours, capital T hours, the rainfall depth corresponding to that will be $\frac{T}{D}$ centimeters. So, $S_A - S_B$ is the response of the catchment for a rainfall of $\frac{1}{D}$ centimeters per hour for T hours, this is the effective rainfall pulse input. The depth of that will be $\frac{T}{D}$ centimeters. Now, for getting the unit hydrograph corresponding to $\frac{T}{D}$ centimeters of effective rainfall, we will be dividing this $S_A - S_B$ by $\frac{T}{D}$.

So, T hour unit hydrograph is obtained by using this formula

$$T-\text{hour } UH = \frac{(S_A - S_B)}{\left(\frac{T}{D}\right)} m^3/s$$

Again, the ordinates will be in meter cube per second. These are the steps to be followed by deriving the unit hydrograph having different duration than that of the given duration.

(Refer Slide Time: 23:32)



Now, we can start solving the example, we have been given 2-hour unit hydrograph ordinates in meter cube per second is given but the ordinates are given at an interval of 1 hour. Here, I am not going to directly add this, I am putting the S curve addition column, after that will be

having the S hydrograph. So, first ordinate is same as 0 and after that we will be putting that S_A in the S curve addition column. But it should be put in such a way that the ordinate should be put in corresponding to 2 hours. So, for 1 hour, the same value will be repeated that is we will write 15 itself. Now, corresponding to 2 hours we will be having S curve addition that is nothing but 0. So, here the utmost care should be taken you need to put this S curve addition corresponding to the duration of unit hydrograph. You should not put it at 1 hour. So, that 0 we will be putting corresponding to 2 hours. Then we will be adding the corresponding unit hydrograph ordinates 25 plus 2 and 15 we will be putting corresponding to 3 hours that way the ordinates should be placed. Otherwise initial ordinate if you are putting at corresponding to 1-hour entire process will be leading to a wrong answer. So, this care should be taken care, even whether you are making the column related to S curve addition or not it does not matter how you are doing the addition matters. So, if you are directly adding the ordinate corresponding to S_A at time t is equal to 0 with 2 hours, then there will not be any mistake. If you are not sure about that, you need to make S curve addition column and put it corresponding to that time. So, 15 you will be putting over here, next step onwards there will not be any confusion and you can get the next ordinate corresponding to S hydrograph by adding 25 plus 0 and next one will be 40 plus 15 that is 55. In that way, we will be proceeding and we will get the ordinates corresponding to S hydrograph.

Next step is to lag the S hydrograph by the required duration that is what repeated over here as S_B we wanted to get the 3-hour unit hydrograph that is here in this case capital T is equal to 3 hours, given unit hydrograph is having a duration of 2 hours we are going to derive the 3-hour unit hydrograph. So, capital T is equal to 3 hours. So, we are lagging the derived S hydrograph by 3 hours that is why we have put the first ordinate at T is equal to 3 hours.

Now, next step is to find out the difference between S_A and S_B. The difference between S_A and S_B can be found out in this particular column. After that this difference is giving us the direct runoff hydrograph corresponding to an effective rainfall for a duration of T hours. So, the rainfall depth is not 1 centimeter, it is the direct runoff hydrograph corresponding to the rainfall depth of T by D centimeters. So, T hour unit hydrograph can be obtained by

$$T-\text{hour } UH = \frac{(S_A - S_B)}{\left(\frac{T}{D}\right)}$$

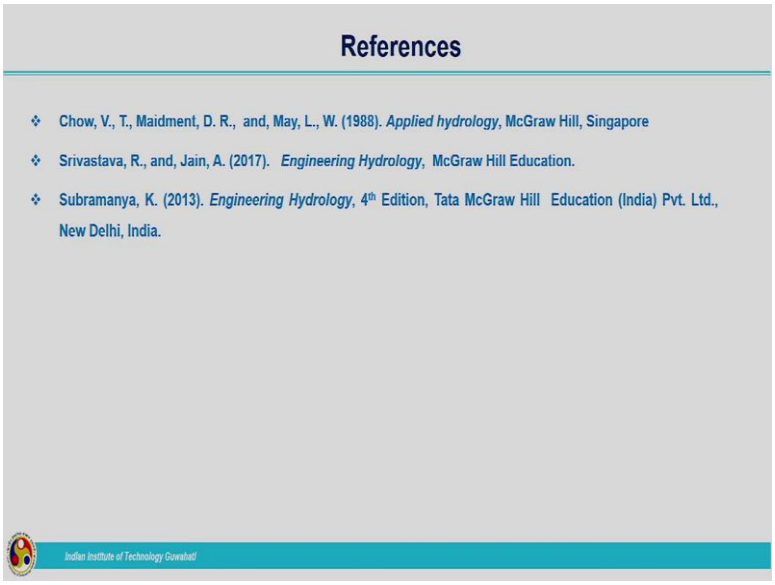
Here in this case $\frac{T}{D}$ is 1.5. T is 3 hours and D is 2 hours, D is the duration of the given hydrograph, T is the duration of the required hydrograph. So, $\frac{T}{D}$ is 3 by 2 that is 1.5. So,

$$T-\text{hour } UH = \frac{(S_A - S_B)}{1.5} m^3/s$$

Now, we are going to divide the ordinance of $S_A - S_B$ by 1.5. So, we will get the 3-hour unit hydrograph in meter cube per second after dividing $S_A - S_B$ with 1.5. So, this is our required unit hydrograph that is the 3-hour unit hydrograph. You look at the table carefully, here you can see certain ordinates are negative while coming to last ordinate for the time T is equal to 23 hours we are having minus 0.7. This way some fluctuations can be observed while solving the problems related to S hydrograph, while deriving the unit hydrographs of different durations by making use of S hydrograph technique. So, there you need to make certain adjustment or smoothing of the curve, so that it will reach 0 at the end time interval. So, instead of minus 0.7 we can assume it to be 0 and that is representing the 3-hour unit hydrograph.

If you are plotting the curve, you can see here our 2-hour unit hydrograph is represented by this pink curve and S hydrograph S_A given by this red curve and S_B is the lagged hydrograph which is represented by this green curve and you can see both are having an equilibrium discharge after certain time. Those ordinates we have not found out we have stopped there, because we were not interested in getting the S hydrograph ordinates, our intention was to get the ordinates corresponding to 3-hour unit hydrograph. And from this we have found out the $S_A - S_B$ that divided by $\frac{T}{D}$ was giving us the 3-hour unit hydrograph that is represented by this blue curve. So, whatever be the required duration you can find out by making use of the S hydrograph technique. If the required duration is nD hours from D-hour unit hydrograph, if this n is integer or fraction we can develop the nD hour unit hydrograph by making use of the principle of S hydrograph. If n is an integer, easily we can do by making use of the principle of superposition. So, these are the different types of problems related to S hydrograph and unit hydrographs having different durations.

(Refer Slide Time: 30:26)



So, here I am winding up this problem-solving session. You can get so many exercise problems from these textbooks, try to solve these for making the concepts very clear. Thank you.