Optimization Methods for Civil Engineering

Dr. Rajib Kumar Bhattacharjya
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Lecture - 09
Line Search Methods

Hello student. Welcome back to the course on Optimization Methods for Civil Engineering. So, in the last class we have discussed few line search techniques. So, I am calling it line search, because we are trying to find out an optimal solution of a single variable function.

So, in the last class we have discussed mainly region elimination technique. So, we discuss the interval halving method we discuss the golden section search method then also, we discuss the bisection method ok. So, before that we discuss the bracketing method.

The first step is to bracket the optima and after that we are applying the region elimination technique for finding the exact optimal solution of the problem. So, today I will discuss another algorithm for finding optimal solution of a single variable optimization function.

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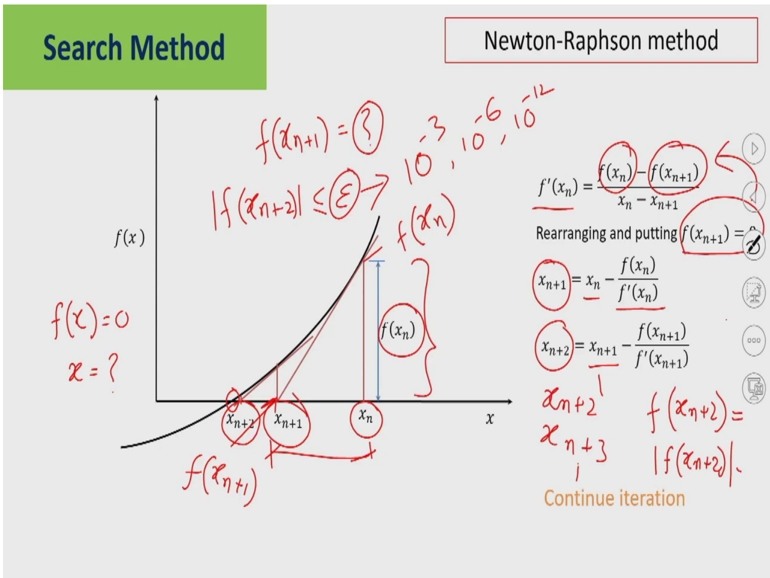

So, I hope all of you know Newton-Raphson method. So, this method is applied or this method is used to find the root or roots of the function ok. So, I can find out the root of this function that is f of x equal to 0. So, I have to find out the value of x and that is the root of this particular function.

So, I can apply Newton-Raphson method for finding the root ok. Suppose, if I take an initial point, we call it x n and this value is f of x n ok. So, this value is f of x n. And then, if I approximate or if I approximate the gradient at that point. So, I can calculate the gradient that is the first derivative gradient which is f of x n minus f of x n plus 1.

So, this is the line the tangent is touching the x axis at x n plus 1. So, f of x n minus f of x n plus 1 divided by this distance ok. So, divided by this distance. So, what is this is x n minus x n plus 1. So, from this triangle, I can find out the derivative. So, derivative is these values ok. So, this value what is this is f of here, it is f of x n and here, it is f of x n plus 1 ok.

So, this is this value is this value is f of x n plus 1 ok. So, now, f of x n plus 1 is equal to 0 and if I put f of x n plus 1 equal to 0 in this equation. So, and then if I rearrange the term. So, I am getting x n plus 1 which is equal to x n minus f of x n divided by the first derivative at x n.

So, what I am getting that using this equation, I can approximate that x n plus 1 ok, but x n plus 1 is not the root of this particular function. So, here I am getting the value of x n plus 1. So, I can say whether f of x n plus 1 is what is this value. So, if this value is not 0 or negative 0; that means, that is not the root of this equation.

Then what I will do? I will calculate the next value that is x n plus 2. So, I will take I will draw a tangent and this is the value of x n plus 2. So, x n plus 2 equal to x n plus 1 minus f of x n plus 1 divided by derivative at x n plus 1. So, and that way I can find out x n plus 3 n plus 4 something like that.

And every step I will check whether the, whether that function value is 0 or not or if it is near to 0 then, I can terminate this iteration. So, this is an iterative based process. So, next step is I will calculate x n plus 2 ok then, I will calculate x n plus 3 and then, I will continue and every step I will check whether x n plus 2 ok. So, it is 0 or not basically.

So, or what I can do that it should be equal to 0, but it will never be 0 here. So, what I can do? I can take that the function value ok that absolute value of x n plus 2 or whatever. So, this should be less than equal to this would be less than equal to some epsilon ok.

So, some epsilon value, so I will terminate my iteration once that f of ok x n plus 1 or n plus 2 or is basically, less than equal to some epsilon value. So, epsilon value maybe, you can put it the precision suppose 10 to the power minus 3 or 10 to the power minus 6 or 10 to the power minus 2 12.

So, as per your desired precision, so you can put the epsilon value and I can terminate this iteration. So, once the function value absolute value of this particular function is less than that precision ok. So; that means, that is the acceptable error epsilon is the acceptable error.

So, once the function value is less than acceptable error. So, in that case I can terminate my iteration. So, this is your Newton-Raphson method for finding the root of a function ok. Now, I can apply the same technique for finding the optimal solution of a function ok. So, how we can do that?

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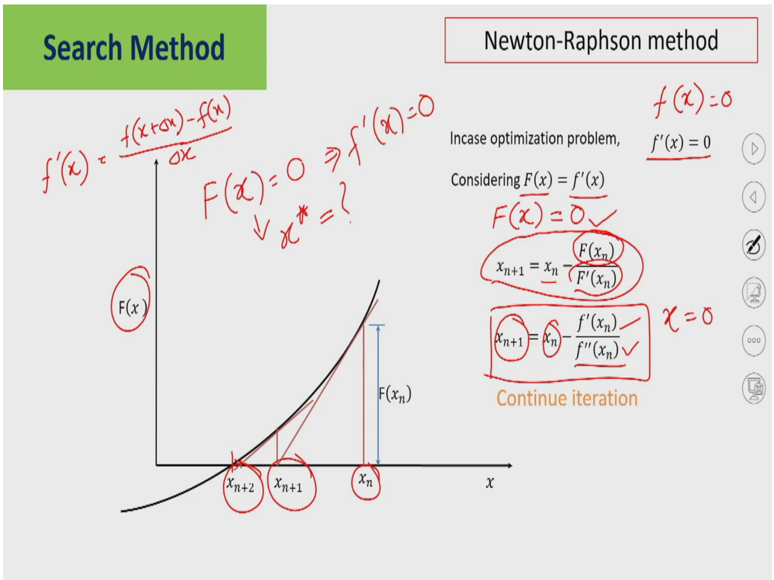

So, in case of optimization problem, the first derivative should be equal to 0. In case of root finding problem, the function value is 0 root finding problem that function value is equal to 0, but in this case the first derivative is equal to 0. So, if we consider capital F x which is equal to the first derivative of the original function small f x ok. So, if I consider capital F x which is equal to the first derivative of the original function.

So, therefore, in this case the capital F x. So, capital F x should be equal to 0; that means, what we are doing we are trying to find out root of this particular function root of this capital F x function. So, therefore, I can apply the same Newton-Raphson method for finding the root of this capital f x function and which is suppose, I can apply that f x n plus 1 which is equal to x n.

Now, that function value at x n and the derivative at x n derivative mean derivative of not the original function, but derivative of the derivative of capital F x. Now, if I put that f x n is nothing, but the derivative of the original function and if I take derivative of the capital F x function. So, that will be the second derivative of the original function. So, therefore, I can write this particular your equation.

So, this is an iterative base equation; that means I can find out x n plus 1 which is equal to x n. So, this is we will start with x n; that means, suppose we will start with x equal to 0 ok and x n minus first derivative at x n divided by second derivative at x n. So, I am getting this equation. So, this is your Newton-Raphson method and I can find out the root of the capital x f x function and that will be the optimal solution of the original function ok.

So, now this is your capital F x and this is the starting point and I will approximate what is x n plus 1 and then, I will approximate what is x n plus 2. Then, after some iteration, I will get the root of capital F x and that root of capital F x. That means, I will get the solution that capital F x equal to 0 and; that means, the first derivative is equal to 0 and whatever solution you are getting. So, that will be the stationary point ok.

So, that will be the stationary point ok. So, I can apply this Newton-Raphson method also, for finding the optimal solution of a single variable function. Now, question is that whether you will apply the region elimination technique or you will apply this Newton-Raphson method ok. So, if you compare with region elimination technique. In the region elimination technique, we are not calculating the derivatives ok.

So, derivative calculation is not there we do not we did not calculate there. So, what we are doing there. We are just calculating the objective function value ok. So, in the case of region elimination technique. So, we only calculate the function value, but here, in this method you have to calculate the derivative ok.

Now, question is that if you are working with a simple function then, derivative in the program itself you can give the derivative of that particular function, but when you are working in a very complex function, derivative calculation is not that easy then, it will be difficult for applying Newton-Raphson method. Another issue is that I can calculate the derivative ok using numerical method ok.

So, I can calculate the derivative using numerical method. I can apply the forward differentiation technique the backward differentiation technique or central differentiation technique ok. Now, if you are applying the forward differentiation. So, what is that suppose I can calculate the derivative of the function? So, derivative of this particular function which is equal to, I can calculate that f of x plus delta x minus f x ok minus f x divided by delta x.

So, this is the forward differentiation. So, I can apply and here, I can calculate the derivative using numerical numerical technique. So, in that case, using this equation I can calculate. So, here what you need basically. So, you need two function evaluation for calculating the derivative ok.

Similarly, you can calculate the second derivative also numerically, but you need more number of function evaluation ok. Therefore, if you compare with the region elimination technique. So, you have to compare that with that how much function evaluation you need basically, to reach the optimal solution ok.

So, therefore, depending upon the situation, we can choose one of the technique ok either region elimination technique interval halving technique golden section as you have seen the golden section method is more efficient than the interval halving method. So, you can go for golden section method you can also apply the bisection method ok.

Now, let us see some example problem. So, what I will do basically. So, I will solve a single example problem using different techniques. So, just to demonstrate how these algorithms are working.

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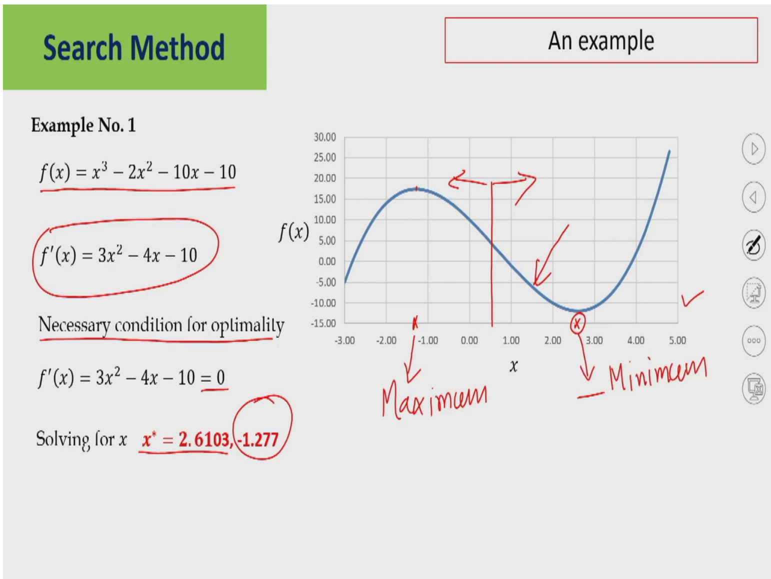

Let us take this particular function. The function is f of x equal to x cube minus twice x square minus 10 x minus 10. So, this is a very simple function. And I can plot it. So, if I plot it. So, this is the plot of this particular function and by looking at the plot, you can guess that there are two optimal solution between minus 3 and plus yeah plus 5 ok.

So, one solution is somewhere here. So, this is one solution and another solution is somewhere here. So, this is another solution and by looking at the curve itself. So, you can tell that this is the minimum ok. So, this is the minimum of this function and this is the maximum ok. So, maximum of this particular function. Now, if I say that the search space is between minus 3 and plus 5. So, in that case, this is not a unimodal function ok.

So, there are more than one optimal solution, but if I take suppose, if I take only this portion ok. So, only this portion; that means, from 1 or 0.5 to 5. So, if you consider that this particular function between your 1 and 5. So, that is a unimodal function ok and similarly, if I say that this particular function between 0 and minus 3; that is also a unimodal function.

So, as you have said that whatever method we have discussed ok. So, this is for a unimodal function; that means, there is only one optimal solution ok. So, I can apply the necessary condition ok. So, first derivative is this ok. So, this is the first derivative and I can apply necessary condition for optimality; that means, that first derivative should be equal to 0.

And if I put it equal to 0 and if I solve and then I am getting x star equal to 2.6103 and that is the minima somewhere here. And another solution is minus 1.277 and that is the maxima of this function and I can apply the sufficient condition ok. Sufficient condition means; I can take the second derivative ok. So, I have to check whether second derivative is 0 or non-zero.

If it is not 0 then, I have to look at the derivative value second derivative value if second derivative is positive. So, in that case this particular point is a minima and if second derivative is negative that particular point will be maxima ok. So, I can apply the necessary and sufficient condition and to find out the optimal solutions the first step you are getting the stationary points.

And after that you can apply the sufficient condition to check whether, that stationary points are a minimum point or a maximum point or an inflection point. So, I can do that, but in this particular example, we will try to apply the region elimination technique as well as

Newton-Raphson method for finding the optimal solution of this single variable function ok. So, let us solve this problem.

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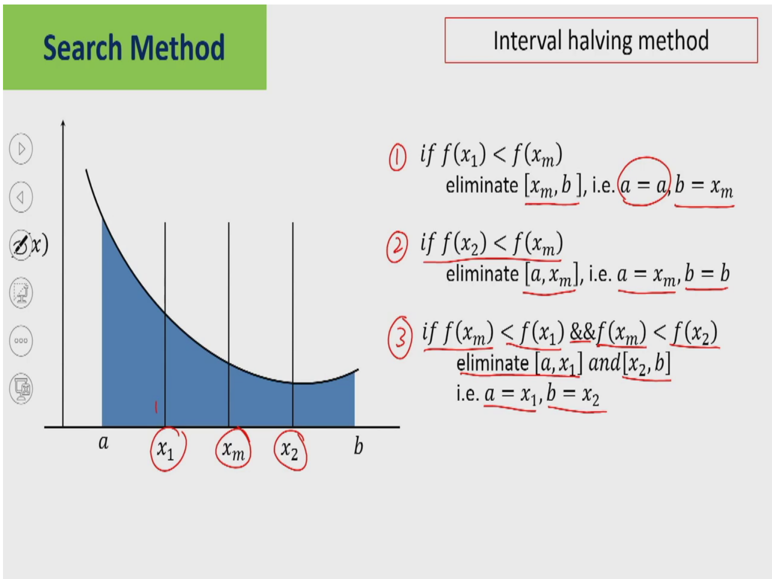

The first one is interval halving method ok. So, as we have explained the interval halving method. So, what we are doing we are taking three points and we are dividing this region in four equal parts; that means, 25 25 25 25 ok. So, therefore, x m is the mid and then x 1 is between a and x m at the midpoint and again x 2 is the midpoint between x m and b ok. So, one of this condition will be true.

So, what after taking these three points. So, one of these. So, this is condition 1 this is condition 2 and this is condition 3; that means, f of x 1 is less than f of x m.

So, f of x 1; that means, in this case anyway this figure is not showing that first condition, but. So, if the first condition is true; that means, f of x 1 is less than f of x m then, you will eliminate the region x m b. So, in that case what will happen that a equal to a. So, a will not change, but b will be equal to x m ok. So, in that case b will be equal to x m.

Now, in case of this one; that means, in case of the condition 2 if condition 2 is true; that means, if f of x 2 is less than f of x m then, what you will do you will eliminate a x m; that means, this from a to x m you will eliminate. So, in that case, a will be equal to x m and b will be equal to b will unchanged.

Now, if the third condition is true; that means, if f of x m is less than f of x 1 as well as f of x m is also less than x 2 f of x 2. So, in that case the region a x 1 and x 2 b. So, in that case a will be equal to x 1 and b will be equal to x 2. So, what will happen. So, if you take these three points. So, one of these three conditions will be true. So, accordingly so you will send your upper bound and lower bound ok.

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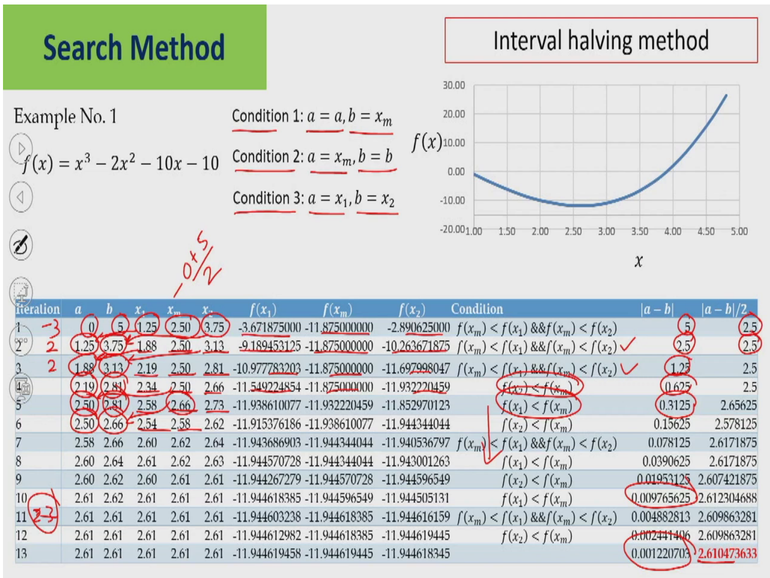

So, let us apply. So, in this case, as I said that I have taken this function between 0 and 5; that means, lower bound is 0 upper bound is 5 and within that region this particular function is a unimodal function and. So, if condition 1 is true. So, in that case a equal to a and b equal to x m.

If condition 2 is true, then a equal to x m b equal to b and if condition 3 is true then, a equal to x 1 and b equal to x 2 ok. Now, let us start this algorithm. So, first iteration, so this is iteration 1 ok. So, this is iteration 1 then, lower bound is 0 upper bound is 5, then I have calculated the midpoint that is 2.5. So, what is x m? x m is 0 plus 5.

So, I am calculating x m 0 plus 5 divided by 2. So, therefore, it is 2.5. Similarly, I have calculated x 1 and x 2 ok. Now, you calculate the function value at x 1 x m and x 2. Now, you check which condition is true. So, in that in this case, the condition 3 is true; that means, f of x m is less than f of x 1 and as well as f of x m is also less than f of x 2. So, in that case what will happen a equal to x 1 and b equal to x 2.

So, I have revised that 1 that a equal to x 1. So, you just see, this x 1 value has come 2 as in a and b equal to x 2. So, b is also coming like this ok. So, b equal to x 2 and I calculated what is a minus b just to check when we have to stop the iteration and then, the optimal value in this. So, it is coming 2.5 that is a minus b by 2 ok.

Now, I am getting new a and b that is now, in the second iteration the a equal to 1.25 and b equal to 3.75 then, I have calculated what is x 1 then x m x n x 2 ok and then, you calculate the function value you calculate the function value and in this case again, the condition 3 is true.

So, therefore, that a equal to x 1 ok. So, x 1 is coming as a and b equal to x 2 ok. So, now, again you calculate what is a minus b absolute value of a minus b and the optimal solution. So, in that case, it is 2.5 2.5. And now, in the third iteration again, you calculate what is x 1 x m x 2, then you calculate the function value and just see which condition is true. In this case also, the third condition is true and accordingly you sense what is the new a and b value.

Again, you calculate x 1 x m and x 2 and you also calculate the function value. And in this case, the second condition is true ok the condition 2 is true. So, second condition is true means a equal to x m ok. So, therefore, in the next iteration a equal to x m. So, this is coming as a x m and b will not change.

So, b is 2.81. Then again, you calculate your x 1 x m x 2 you calculate the function value. Now, you just see this condition 1 is true ok condition 1 is true 1 is true means a equal to a. So, therefore, a is not changing in this iteration and b equal to your x m. So, this x m is coming as a b here, then you again calculate x 1 x m x 2.

Then, you just continue this iteration and every iteration you just say what is absolute value of a minus v. Once it is reaching the precision you can stop your iteration. Suppose, this is coming here, this is suppose this value is less than 0.01 if I put 0.01 is the precision. So, you can stop there ok point. So, even you can stop here itself. So, you can if you are taking 0.01 is your precision. And then, the optimal solution is coming 2.6104.

So, as you have seen, the optimal solution in that particular region is 2.6103 and you are getting 2.6104 ok. So, if you are a doing it for another 2 3 iteration then, you will get 2.6103 ok. So, you can stop your iteration. So, this is this method is interval halving method. So, now, I would like to discuss one point here.

So, if you look at that the first iteration; how many function evaluation you are doing 1 2 3 function evaluations at the first iteration and after that you are doing two function evaluation; that means, if I calculate function evaluation, here it is 3 then 2 then 2 something like that.

So, you can see that if you are doing it for up to 11 ok. So, 11 into 2; that means, sorry 10 into 2; that means, 20 plus 3; that means, up to this step you have done 23 function evaluation ok 23 function evaluation.

So, as I have said that efficiency of the algorithm can be compared by using the number of function evaluation you have you are doing. So, in this case, if your iteration is up to 11 so; that means, 23 function evaluation you have done here ok. So, but anyway. So, we got the solution 2.6103 or 04.

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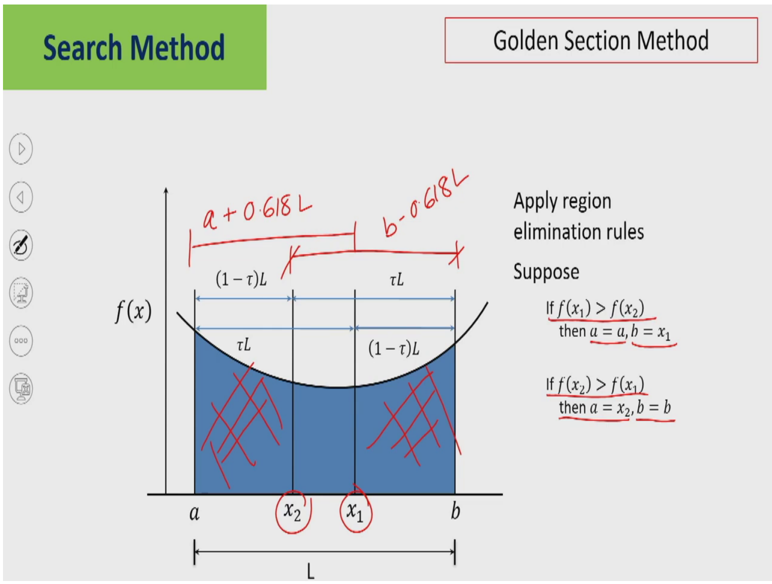

Now, let us see the next example next method. So, next method is the golden section source method. So, in this case, what we are doing we are taking two points; that is your x 1 and x 2 and x 1; what is x 1? x 1 is a plus tau. So, tau is 0.618 ok L ok. So, L is the absolute value of a minus v. And similarly, I am taking another point x 2 ok.

So, x 2 is b minus 0.618L ok. So, you are taking two points here, in case of interval halving so, you are you have taken three points, but here you are taking two points ok. And then, you will apply the region elimination concept.

So, in that case you will compare the function value between x 1 and x 2. So, the first condition is if f of x 1 is greater than f of x 2; that means, f of x 1 is greater than f of x 2. So, in that case what will happen; the reason x 1 to b will be eliminated. Therefore, a will not change. So, a will be equal to a then b will be equal to x 1, because this particular region will be eliminated ok.

So, if this condition is true. Now, if the next condition is true; that means, if it is not true then second condition is f of x 2 is greater than f of x 1. That means, the function value at x 2 is greater than the function value at x 1. So, in that case what will happen; this particular region will be eliminated ok. And therefore, a equal to x 2 and b equal to b.

So, b will be unchanged, but a will change to x 2 ok. So, in this case we are just applying these two condition. One of these conditions will be true ok.

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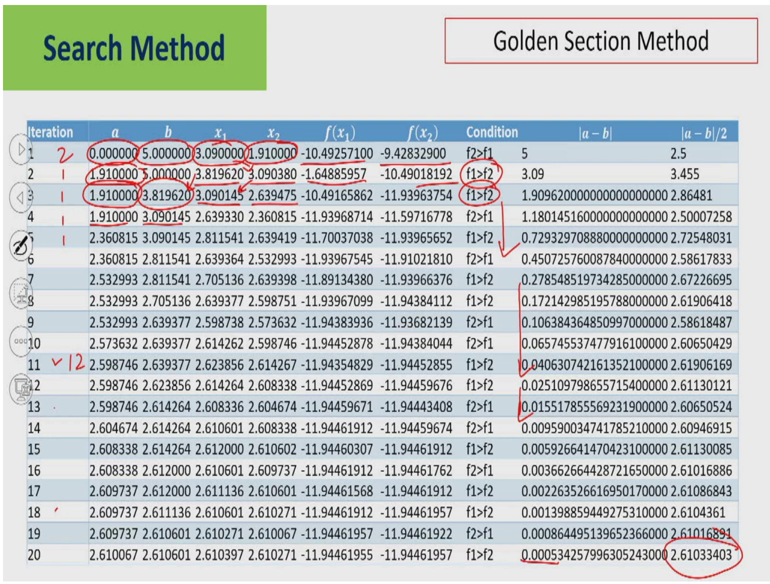

Iteration	a	b	x_1	x_2	$f(x_1)$	$f(x_2)$	Condition	$\|a-b\|$	$\|a-b\|/2$
1	0.000000	5.000000	3.090000	1.910000	-10.49257100	-9.42832900	f2>f1	5	2.5
2	1.910000	5.000000	3.819620	3.090380	-1.64885957	-10.49018192	f1>f2	3.09	3.455
3	1.910000	3.819620	3.090145	2.639475	-10.49165862	-11.93963754	f1>f2	1.90962000000000000000	2.86481
4	1.910000	3.090145	2.639330	2.360815	-11.93968714	-11.59716778	f2>f1	1.18014516000000000000	2.50007258
5	2.360815	3.090145	2.811541	2.639419	-11.70037038	-11.93965652	f1>f2	0.72932970888000000000	2.72548031
6	2.360815	2.811541	2.639364	2.532993	-11.93967545	-11.91021810	f2>f1	0.45072576008784000000	2.58617833
7	2.532993	2.811541	2.705136	2.639398	-11.89134380	-11.93966376	f1>f2	0.27854851973428500000	2.67226695
8	2.532993	2.705136	2.639377	2.598751	-11.93967099	-11.94384112	f1>f2	0.17214298519578800000	2.61906418
9	2.532993	2.639377	2.598738	2.573632	-11.94383936	-11.93682139	f2>f1	0.10638436485099700000	2.58618487
10	2.573632	2.639377	2.614262	2.598746	-11.94452878	-11.94384044	f2>f1	0.06574553747791610000	2.60650429
11	2.598746	2.639377	2.623856	2.614267	-11.94354829	-11.94452855	f1>f2	0.04063074216135210000	2.61906169
12	2.598746	2.623856	2.614264	2.608338	-11.94452869	-11.94459676	f1>f2	0.02510979865571540000	2.61130121
13	2.598746	2.614264	2.608336	2.604674	-11.94459671	-11.94443408	f2>f1	0.01551785556923190000	2.60650524
14	2.604674	2.614264	2.610601	2.608338	-11.94461912	-11.94459674	f2>f1	0.00959003474178521000	2.60946915
15	2.608338	2.614264	2.612000	2.610602	-11.94460307	-11.94461912	f1>f2	0.00592664147042310000	2.61130085
16	2.608338	2.612000	2.610601	2.609737	-11.94461912	-11.94461762	f2>f1	0.00366266442872165000	2.61016886
17	2.609737	2.612000	2.611136	2.610601	-11.94461568	-11.94461912	f1>f2	0.00226352661695017000	2.61086843
18	2.609737	2.611136	2.610601	2.610271	-11.94461912	-11.94461957	f1>f2	0.00139885944927531000	2.6104361
19	2.609737	2.610601	2.610271	2.610067	-11.94461957	-11.94461922	f2>f1	0.00086449513965236600	2.61016891
20	2.610067	2.610601	2.610397	2.610271	-11.94461955	-11.94461957	f1>f2	0.00053425799630524300	2.61033403

So, let us go to the table. Here also, we have started from 0 that a is 0 and b is equal to 5 ok between 0 and 5. Then I have calculated what is the value of x 1. So, x 1 is that a plus 0. 618L and similarly, I am also calculating the value of x 2 ok. Now, you calculate the function value at x 1 and function value at x 2.

Now, in this case, that f of x 2 is greater than f of x 1 ok. So; that means, this is the second condition is not it this is the second condition that is f of x 2 is greater than f of x 1, then what you will do? You will change a equal to x 2 and b equal to b ok. So, a will be equal to x 2 ok. So, this x 2 is coming here ok.

So, this x 2 is coming here, and b will not change, then you calculate what is new x 1 and new x 2. And you are calculating the function value. And in this case, the first condition is true what is the first condition? First condition is f of x 1 is greater than f of x 2.

So, in that case a equal to a and b equal to x 1. So, a will not change. Now, a equal to a, but b will be equal to x 1 ok. Then you are calculating again new x x 1 and x 2. And as I have said that one point will be the previous point either it will be x 1 and x 2. So, you can see that these two points are same and similarly, these two points are also same.

So; that means, in every iteration you are doing only one function evaluation, because one point will be a point which you have got in the earlier iteration ok. So, in this case that f of x 1 is greater than f of x 2. So, accordingly you will change your upper bound and lower bound and you just see if you just go this is iteration 3 4 then 5 6 something like that.

So, I have done it up to 20 iteration and you can see that the error is decreasing ok distance between a minus b ok distance between a and b is decreasing and once it is reaching the your that desired precision ok. So, you can terminate. So, I have done it up to 20 iteration and you can see the you have a very narrowed region and that distance between a and b is 0.0053 ok and you are getting the solution and that solution is 2.61033403 ok.

So, you got the solution using golden section search method. Now, if you compare the function value how much function value you have calculated ok calculation of function value you just see that in the first iteration. I am doing two and then every iteration I am just calculating one function I am doing just one function evaluation.

That means if you go up to eleven then how many function evaluation you have done you can see that function evaluation is; that means, 10 plus 2; that means, 12 function evaluation you have ok, but in case of interval halving method. So, you have done 23 function evaluation ok.

So, anyway. So, you did not get the solution as compared to the golden section search method in eleventh iteration, but I think you got the solution somewhere here ok. So, somewhere here you got the solution somewhere in thirteenth iteration ok. So, that is 0.009. So, that is your less than 0.01 ok. So, that way you can compare the algorithms. I hope this is clear to you, then let us go to the next algorithm.

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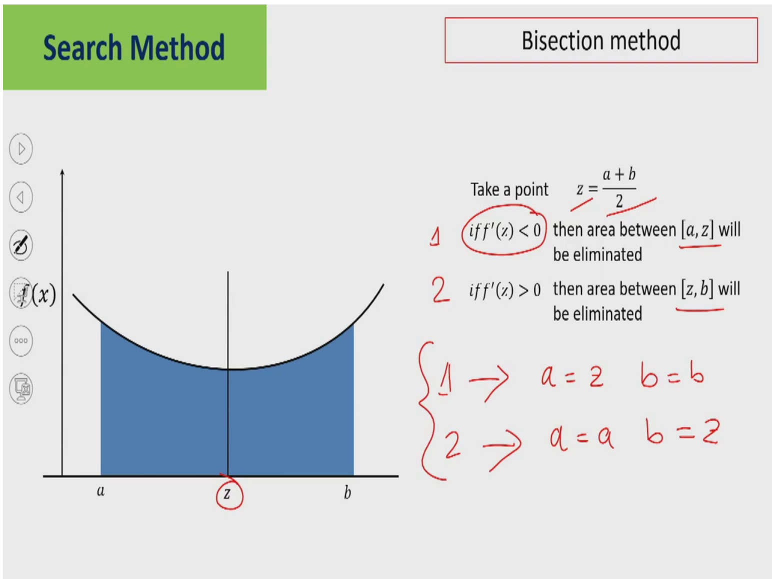

So, next algorithm is the Bisection method. So, in bisection method. So, what we are doing we are taking a point z ok. So, z and which is a plus b by 2; that means, at the midpoint ok. So, at the midpoint we are taking this point z. And then, we are just checking whether the derivative value at that particular point is less than 0 or greater than 0. If it is less than 0, then we are eliminating the region a z ok. So, a z and if it is greater than 0, then we are eliminating the region z b.

So, if condition 1 is true ok. So, this is condition 1 and this is condition 2. Now, if 1 is true. So, in that case what will happen that; that means, the derivative at z is less than 0; that means, we are eliminating a z. So, in that case a will be equal to z ok and b will unchange b will be equal to b. Now, if the second condition is true. So, in that case we are eliminating z b; that means, a will unchanged. In that case a equal to a and b equal to z ok.

So, we will just apply these two conditions and if we do it for some iterations and you will get the optimal solution of this particular functions ok.

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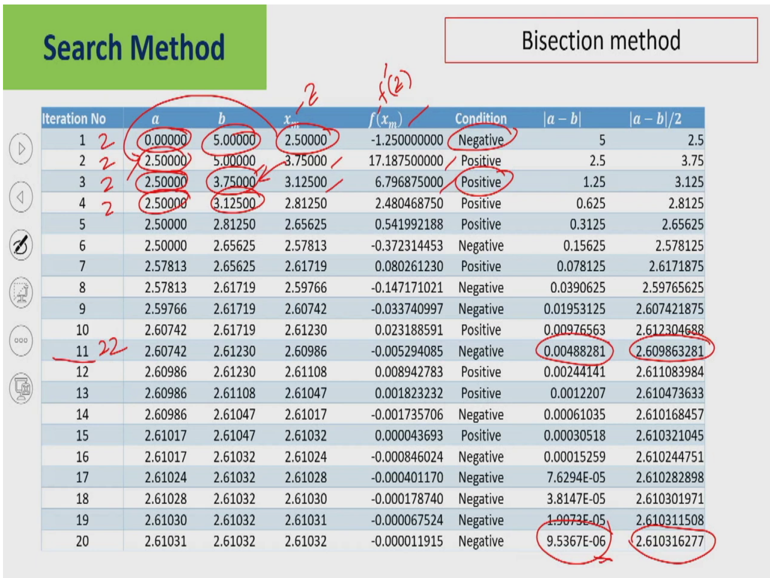

Iteration No	a	b	x_m	$f(x_m)$	Condition	$\lvert a-b \rvert$	$\lvert a-b \rvert/2$
1	0.00000	5.00000	2.50000	-1.250000000	Negative	5	2.5
2	2.50000	5.00000	3.75000	17.187500000	Positive	2.5	3.75
3	2.50000	3.75000	3.12500	6.796875000	Positive	1.25	3.125
4	2.50000	3.12500	2.81250	2.480468750	Positive	0.625	2.8125
5	2.50000	2.81250	2.65625	0.541992188	Positive	0.3125	2.65625
6	2.50000	2.65625	2.57813	-0.372314453	Negative	0.15625	2.578125
7	2.57813	2.65625	2.61719	0.080261230	Positive	0.078125	2.6171875
8	2.57813	2.61719	2.59766	-0.147171021	Negative	0.0390625	2.59765625
9	2.59766	2.61719	2.60742	-0.033740997	Negative	0.01953125	2.607421875
10	2.60742	2.61719	2.61230	0.023188591	Positive	0.00976563	2.612304688
11	2.60742	2.61230	2.60986	-0.005294085	Negative	0.00488281	2.609863281
12	2.60986	2.61230	2.61108	0.008942783	Positive	0.00244141	2.611083984
13	2.60986	2.61108	2.61047	0.001823232	Positive	0.0012207	2.610473633
14	2.60986	2.61047	2.61017	-0.001735706	Negative	0.00061035	2.610168457
15	2.61017	2.61047	2.61032	0.000043693	Positive	0.00030518	2.610321045
16	2.61017	2.61032	2.61024	-0.000846024	Negative	0.00015259	2.610244751
17	2.61024	2.61032	2.61028	-0.000401170	Negative	7.6294E-05	2.610282898
18	2.61028	2.61032	2.61030	-0.000178740	Negative	3.8147E-05	2.610301971
19	2.61030	2.61032	2.61031	-0.000067524	Negative	1.9073E-05	2.610311508
20	2.61031	2.61032	2.61032	-0.000011915	Negative	9.5367E-06	2.610316277

So, let us apply this bisection method. So, here again, we have started for with 0 and 5; that means, a equal to 0 b equal to 5 and this is x m. So, this is nothing, but z z is at the midpoint ok. So, z equal to 2.5. So, I am getting this z value. And now, you calculate the f dash z value ok. So, this is your f dash z ok. So, f z or x m.

Now, you just see whether it is negative or positive; whether it is less than 0 or greater than 0. If it is less than 0; that means, the condition 1; that means, a equal to z and b equal to b ok. So, in that case what will happen that; a will be, because it is a negative the first condition is true. So, therefore, a equal to z.

So, a equal to z means z here is 2.5. So, this value is coming here, ok a equal to z and b will unchanged. Then, you calculate new value of z calculate sorry this is the derivative value not your function value. So, the derivative is now 17.18 and that is positive now. So, if it is positive, what will happen?

If it is positive, a will not change b will be equal to z ok. So, a is not changing and b is equal to z. now, ok b is equal to z and then, you calculate new value of z new derivative value and in this case positive. So, positive means that a is not changing b is equal to z and then, you continue your iteration ok.

So, you can see that you are getting suppose, you are getting here, I have done up to 20 iteration and 20 iteration you are getting a minus b absolute value of a minus b is 9.5367 into 10 to power minus 6 and the solution is 2.6103 ok. So, anyway. So, we have some more decimals here, but solution is 2.6103 ok.

So, you got the solution. So, you can compare that in terms of efficiency with the reason elimination technique. Now here, suppose, if I calculate the derivative numerically. Numerical calculation means; at every iteration you have to do two function evaluation ok. So, to calculate the derivative.

Suppose, if I calculate this derivative numerically maybe, using forward differentiation or backward differentiation. So, in that case I have to do two function evaluation so; that means, every iteration. So, I am doing two function evaluation. So, if you are comparing with up to eleventh iteration. So, we are doing 22 function evaluation.

So, at eleventh iteration. So, we are getting the a minus b value absolute value of a minus b value is 0.00488 and the solution is 2.6098 ok. So, that way you I can compare and anyway. So, if you do some more iterations you are getting the optimal solution of this particular problem.

You just error is decreasing ok. So, in this case it is up to 10 to the power minus 6. So, I can also apply this bisection method for finding the optimal solution of a single variable function. So, as I have mentioned earlier. So, in that case you have to calculate the derivative of the function and many times we are not calculating the derivative, because derivative calculation is not that easy for some your problem ok.

So, therefore, if the derivative calculation is not that difficult. So, in that case this is one of the efficient method ok. So, and if you can put the derivative value directly in the iteration then this method is quite efficient ok. So, this is another method.

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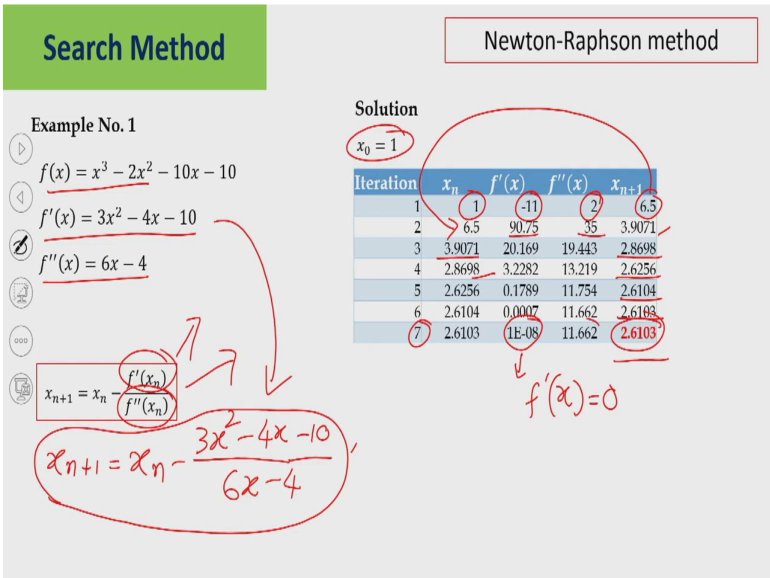

Then, let us check the Newton-Raphson method. So, I (Refer Time: 39:00) can also as I said that I can also apply the Newton-Raphson method for finding the solution ok. So, in this case what you have to do. To apply Newton-Raphson method. So, I need the first derivative as well as second derivative ok. Now, for this particular function. So, I can calculate the first derivative.

So, first derivative is 3x square minus 4x minus 10. And similarly, I can calculate the second derivative or that is 6x minus 4. So, in this case directly I can put the derivative here. I can write the equation something like that that x n plus 1 which is equal to x n ok minus the first derivative is 3x square minus 4x minus and the second derivative is second derivative is 6x minus 4 ok.

So, I can apply this particular equation iteratively for finding the optimal solution of this particular function ok. So, let us start with x naught equal to 1. So, in this case, we have to start our algorithm from a starting point. So, let us take x naught equal to 1 ok. Now, x this is the first point and then, the derivative here, is minus 11 and the second derivative is 2 and if I apply this particular equation this particular equation then x n plus 1 I am getting 6.5.

Now, this 6.5 I will put it here ok. So, I am I am calculating the second approximated value. So, in that case at 6.5 the first derivative is 90.75 the second derivative is 35 and then x n plus 1 ok so; that means, I am getting 3.9071. Now, I am doing the third approximation. I am getting 2.8698 then, the fourth one I am getting 2.6256.

Then, the fifth one I am getting 2.6104 and sixth one I am getting 2.6103 and you can see that the derivative value is decreasing ok. So, at optimal solution, the first derivative will be equal to it should be equal to first derivative should be equal to 0. So, in this case it will never be 0, but it will be near to 0 ok. So, we are getting. Suppose, if we stop our iteration at sixth seventh iteration.

So, seventh iteration I am getting is 2.6103 and the derivative value at that particular point is 1 to the power minus 8 ok. So; that means, it is near 0 and at optimal your point the derivative should be equal to 0. So, I am getting the solution. Now, if you look at if you compare with the region elimination technique you just see with a few iteration, I am getting a very quality solution ok.

So, with only seventh iteration I am getting optimal solution almost and the precision is 1 to the power minus 8, but question is that; compositional complexity can be calculated that how much function evaluation you are doing; that means, at every your first derivative. So, you are calculating you need two function evaluation and for the second derivative you need additional one; that means, every iteration you are doing three function evaluations ok.

So, that way compression complexity of this problem maybe more than the golden section search method will be more than golden section search method, but as I said that if you can directly. Suppose, for this function I am not calculating the derivative in every iteration. So, what I am doing I am giving the derivative value directly. So, in that case, this algorithm is quite efficient.

If you can put that one. Suppose, in your optimization problem. So, you have a very decent function and you can actually code the derivative value in your program. So, in that case the Newton-Raphson method is quite efficient ok. So, it will be more, but if you need to calculate the derivative using numerical methods. So, in that case, it may not be that efficient ok.

So, in that case, you can apply the golden section search method instead of Newton-Raphson method, but if you can calculate the derivative directly from the function the way I have done here. So, in that case, within a very few iteration you will get your optimal solution ok. So, in this case as you have seen that in the seventh iteration, I am getting the optimal solution of this problem with an accuracy level of 1 to the power minus 8.

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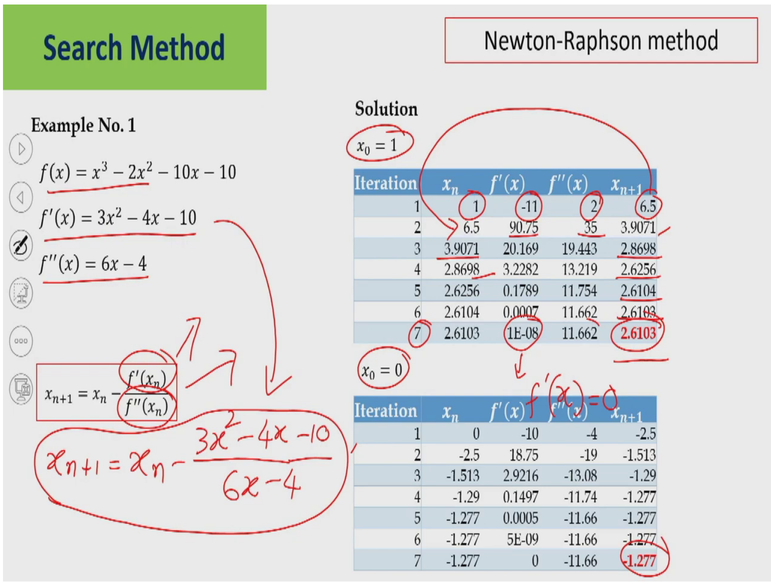

If I take another starting point. So, as you have seen that this particular function has two optimal solution ok this part has two optimal solution. If I start with x naught equal to 0. So, you just see. So, I am getting a different optimal solution ok. So, that is on the other side that is minus 1.277. So, that is the solution I am getting in seventh iteration. So, therefore, in Newton-Raphson method.

So, we are not specifying the lower bound and upper bound, but we are starting with a single initial solution. So, from the that that is we call it starting point and depending upon the starting point.

If you have a unimodal function, you need not worry. So, you will get that particular solution only, but if your problem has more than one optimal solution depending upon what initial solution you have chosen. So, you will get one of them basically ok.

So, in this particular class so, we have discussed Newton-Raphson's method for finding the optimal solution of a single variable function and then, we have solved this example problem using region elimination technique that is interval halving method, the golden section search method then, bisection method and also solve the problem using Newton-Raphson method ok.

Thank you.