Optimization Methods for Civil Engineering

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Lecture - 05
Linear Problem: Simplex Method

Welcome back to the course on Optimization Methods for Civil Engineering. So, we will continue our discussion on Linear Problems. So, in the last class I explained how we can solve a linear problem, what are the rules to be followed to go from one feasible solution to another feasible solution so, that we have discussed.

So, today I will introduce Simplex Method ok. So, this is the method for solving a linear problem. So, that we will discuss initially and then we will solve some example problems for different conditions.

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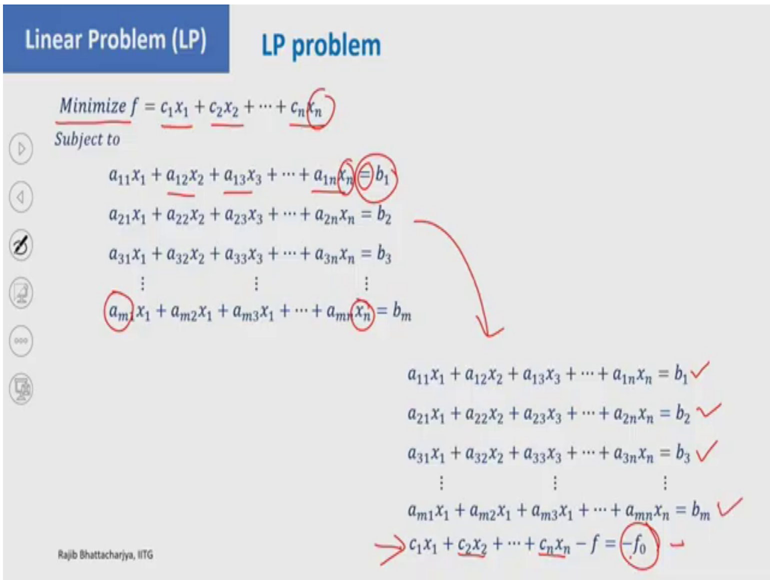

So, let us see this problem. So, it is a general form of a linear problem. So, here the objective function is a minimization type that is c 1 x 1 plus c 2 x 2 up to c n x n. So, we have total n number of variable up to x n. So, this is in equality sign we have. So, this is the first constraint that is a 11, x 1, a 12 x 2, a 13, x 3, a 1n x n. So, total n variable and right hand side is b 1 and already this is an equality constraint. So, we have total m constraint ok and n variable ok. So, this is general form of a linear problem.

Now, I can write in this form ok. So, I am writing the objective function here and that is c 1 x 1 plus c 2 x 2 plus c n x n minus f equal to f 0 ok. So, I am writing it as a system of linear equations. So, this is your first constraint this is your first constraint, second constraint, third one the m constraint then after that I am writing the objective function.

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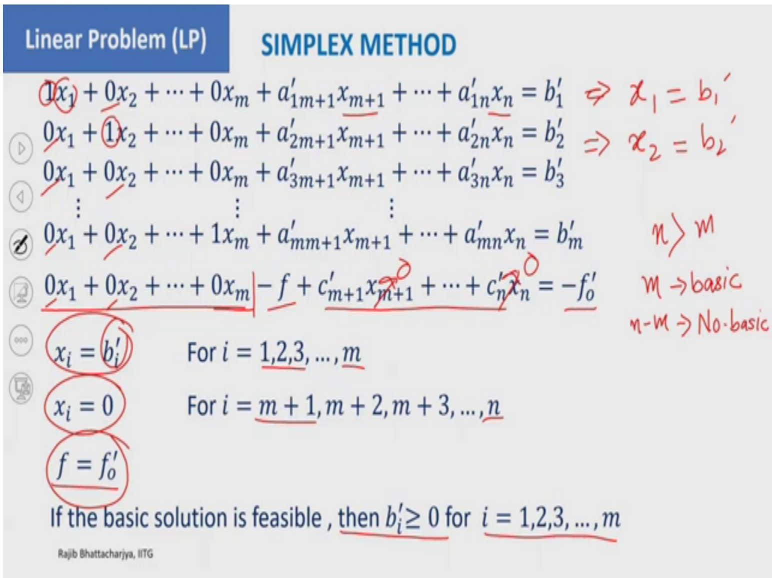

Now, if you apply the row operation I can apply the row operation here and I can make the coefficient of x 1 as 1 and for other that is 0 ok. So, coefficient of x 1 in other equations are 0 including the objective function. Similarly, coefficient of x 2 in the second equation that is 1 and for other equation including the objective function that is 0.

So, I am taking up to m because in this case the number of variable is greater than number of equation. So, what we are considering here? So, we are considering that up to x m is a basic variable that is up to number of equations. So, we have m basic variable. So, m basic variable. So, this is your basic variable and n minus m non-basic variable ok. So, we call non-basic basic variable ok.

So, here, so what we are considering that up to m. So, these are basic variable and from n m plus 1 to n non-basic variable. So, here we have arranged in such a way that coefficient of x 1 is 1 and for other equations that is 0. Now what is the initial solution here? So, I can get that x i equal to b i there. So, what I will be getting? So, from this equation if I consider that from x m to x n. So, these are 0 and this is not these are all non-basic variables if they are 0. So, in that case from the first equation I will get that x 1 equal to b 1 dash.

Similarly, from the second equation I will get that is x 2 equal to b 2 dash. So, what I am getting here that x i equal to b i dash for i equal to 1, 2, 3 up to m; m is the number of equation for other non-basic variable that is from m plus 1 to n. So, that x i equal to 0 ok. So, x i equal to 0 for non-basic variable and for basic variable x i equal to b i.

So, I am getting 1 solution here and f equal to. So, from the last equation that I can write f equal to that is f naught dash ok. So, I am getting the objective function value also from the last equation and because these are non-basic variable and these are equal to 0. So, I am getting f equal to f naught dash. So, if the basic solution is feasible then b i dash.

So, whatever you are getting this b i dash must be greater than 0 for i equal to 1 to m. So, i equal to 1 to m that is b 1 2 to b m this must be greater than 0. Then I will say that the basic whatever basic solution we got actually that is a feasible solution.

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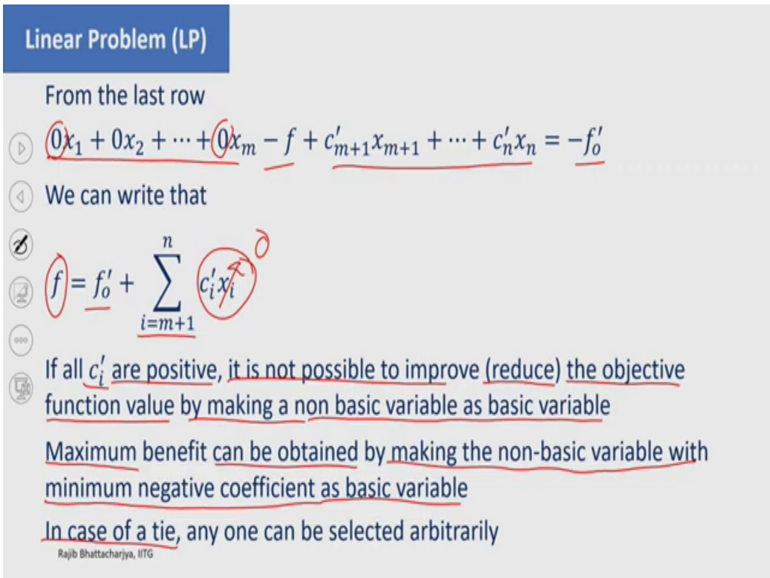

Now, from the last row ok. So, what is this is the last row that is the objective function row. So, coefficient of x 1 to x m that is 0 and we are getting minus f and these are non-basic variable ok and which is equal to minus f naught dash. So, we can write it that f equal to. So, from the last equation objective function equation I can write f equal to f naught dash plus summation of i equal to m plus 1 to n and this is c i dash x i.

So, that is this is the summation of all the component of non-basic variable ok or you can say non-basic variable components ok. So, now if these are equal to 0 then f equal to f naught dash. Now if all c i dash are positive all c i are positive as we have explained when we solve the problem in the last class that if all c i dash are positive, it is not possible to improve or we can say reduce in this case because this is a minimization problem. So, we it is not possible to improve the objective function value by making a non-basic variable as a basic variable.

So, as I said or as I have shown you in the last class. So, if c i dash are positive ok in the objective function row all c i dash are positive in that case, it is not possible to improve the objective function value by making a non-basic variable as a basic variable. So, maximum benefit can be obtained by making the non-basic variable with minimum negative coefficient as basic variable ok.

So, as I have explained in the last class. So, first condition is that if all are positive; that means, we are not getting any improved solution. We will not be able to reduce the objective function value in this case because we are solving a minimization problem. Now if there is a negative coefficient then what will what maximum benefit we can obtain is that if we consider minimum negative coefficient ok coefficient having minimum value.

So, maximum benefit can be obtained by making the non-basic variable with maximum negative coefficient as basic variable. So, in case of a tie, I also explained that one in the last class any one can be selected. So, if the coefficient of two variables are equal in that case I can take any one of them. So, in case of tie any one can be selected arbitrarily.

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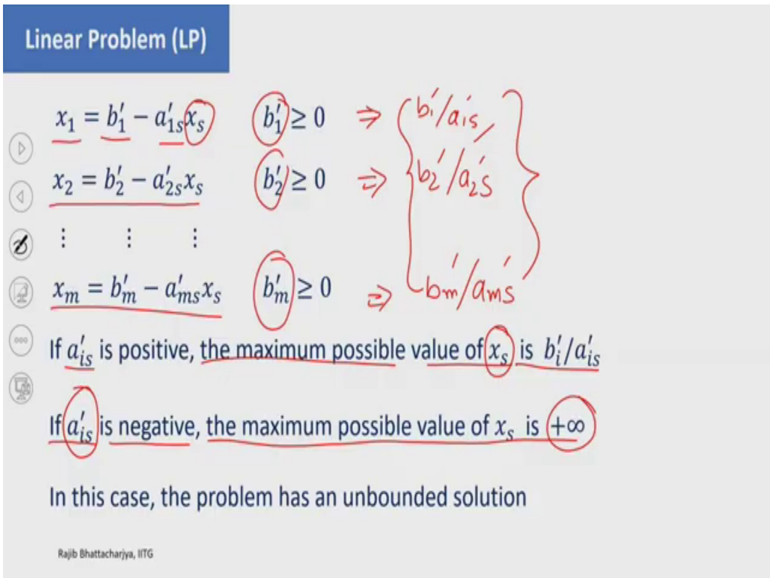

If I say that this is a variable which is basically I can say that this is a non-basic variable and I would like to make it a basic variable. So, from this equation what I will get that x 1 equal to b 1 dash minus a 1 s dash x s ok and here this is b 1 dash is positive because it has to be a feasible solution. So, that is positive b 2 is positive up to b m dash is positive ok.

So, this is from the first equation I can write x 1 equal to b 1 dash minus a 1 dash s. So, that is an x s similarly from x 2 I can write it and from x m I can write it like this. Now if a i s dash is positive the maximum possible value of x s is b i dash by a i s dash. So, what is the maximum possible value of x s?

Suppose from here suppose the maximum possible value of x s from the first equation that is b 1 dash divided by a 1 s ok or you can say a 1 s dash. Now from the second equation what is the maximum possible value of x s that is b 2 dash by a 2 s dash. Similarly, from the last equation I can write that what is the maximum possible value of x s is that b m dash divided by a m s dash ok.

So, now this is the possible values of x s. Now what we have to do? We have to take minimum of these two otherwise what will happen? The sum variable sum decision variable will be negative ok. So, therefore, we have to take minimum of this particular ratio as I have explained. So, if a i s dash is negative. So, again suppose if it is negative then maximum possible value of x s is infinity if it is negative ok. So, in that case so we can say that is an unbounded solution. So, in this case the problem has an unbounded solution ok.

So, therefore, what are the rules basically? So, rules is that I will calculate the ratio ok and we have to take the minimum of that one in order to avoid the infeasible solution. So, we have to take the minimum one and if any coefficient is negative. So, we have to ignore because if it is negative. So, in that case maximum possible value is x s is infinity; that means, x s can go up to infinity.

So, infinity means that is an unbounded solution therefore, we will not calculate or we will ignore the equation having a i s dash is negative. So, that will ignore and for other cases. So, we will calculate this ratios that is b i dash by a i dash. So, that we will calculate and we will we should take the minimum one in order to avoid the infeasible solution otherwise what will happen? You will get some infeasible solution. So, therefore, to avoid that one. So, we will take the minimum one. I hope the rules are clear to you.

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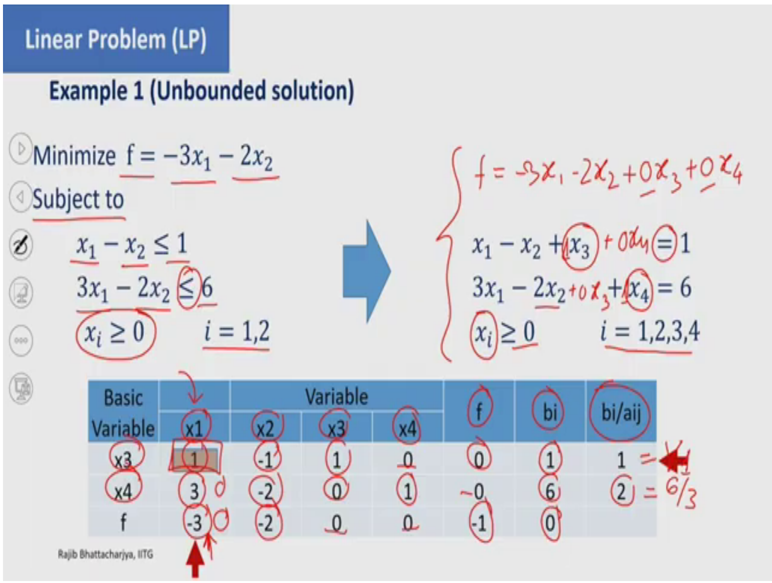

Now, let us go to this particular problem. So, first one I will show you a problem having unbounded solution ok. So, here the problem is minimize f equal to minus thrice x 1 minus twice x 2 subject to that x 1 minus x 2 less than equal to 1 thrice x 1 minus twice x 2 less than equal to 6 and all x i is positive and we have two variables. So, therefore, i equal to 1 to 2. Now what is the first step? The first step is I have to convert this problem to the standard form ok.

So, let us convert that one. So, if you convert it what is standard form? I have to make it equality sign. So, it is the first constraint is a less than equally type constraint. So, therefore, what I am doing? I am adding a slack variable here in order to make it equal. So, now, x 1 minus x 2 plus x 3 equal to 1, then the second constraint that is also inequality type. So, this is also inequality type. So, I am putting another slack variable that is x 4.

So, now it is thrice x 1 minus twice x 2 plus x 4 equal to 6 and x i should be greater than 0 and i equal to now 1, 2, 3, 4. So, with this I have converted this problem to a standard form. So, I can also write the objective function here. So, now, objective function will be this is minus 3 x 1 minus twice x 2 plus 0 x 3 plus 0 x 4 ok. So, this is my objective function. Now my problem is in standard form.

So, if I put this problem in this particular table ok I can call it a simplex table. So, here this is the variable x 1 x 2 x 3 and x 4. So, we have total 4 variable then the objective function and that is b i and I will calculate the b i by a i j. Now here when we are converting this problem to standard form; that means, I have added the slack variable x 3 and x 4 and if you look at this particular problem you can see that the coefficient of x 3 is 1 here in the first equation, this is 1 and for the second equation it is 0 ok.

Similarly, for x 4 this is 0 x 4 coefficient is 0 here and here it is 1 and in the objective function row also the coefficient of x 3 and x 4 are 0 ok. So, therefore, initially what I can consider that x 3 and x 4 that is the basic variable ok. So, basic variable and others x 1 and x 2 are non-basic variable. So, if I put this equation here. So, I am getting the coefficient of x 1 in the first equation is 1, then this is minus 1 the coefficient of x 2 then for x 3 this is 1 and then 0 and this is 0 f is not there and right hand side is equal to 1.

Similarly, for the second equation that is 3, then this is minus 2 then 0 and for x 4 the coefficient is 1, then 0 the right hand side is 6 ok. And I am also writing the objective function the objective function is coefficient of x 1 is minus 3, then for x 2 this is minus 2, then 0 0 this is minus 1 a right hand side is 0 ok. Now we have to look at the last row that is the objective function row is there any negative coefficient.

So, in this particular problem yes we have negative coefficient for x 1 and x 2. So, therefore, what I will do? I have to select the minimum values ok. So, here minimum is minus 3. So, we have minus 3 n minus 2 that is we have to select minus 3 ok. So, now, once you are selecting minus 3 ok so this particular column. So, in that case x 1 is the incoming variable is not it. So, x 1 now x 1 is a non-basic variable, now x 1 will come into basis basically. So, x 1 will be taking the place of either x 3 and x 4.

So, now we have to decide that whether it will replace x 3 or x 4. So, now, what we will do? We will calculate the b i by a i j ratio and here it is for the first equation this is 1 by 1. So, this is 1 by 1. So, I am getting 1 and the second equation this is 6 by 3. So, I am getting 2 ok. So, what is the rule? The rule is you have to select the minimum 1.

So, minimum one is 1 here. So, therefore, this will be your pivoting element ok. So, pivoting element means, now this will I will make it one, but in this case it is already 1 now, I have to make it this particular very coefficient should be 0 and this should be equal to 0.

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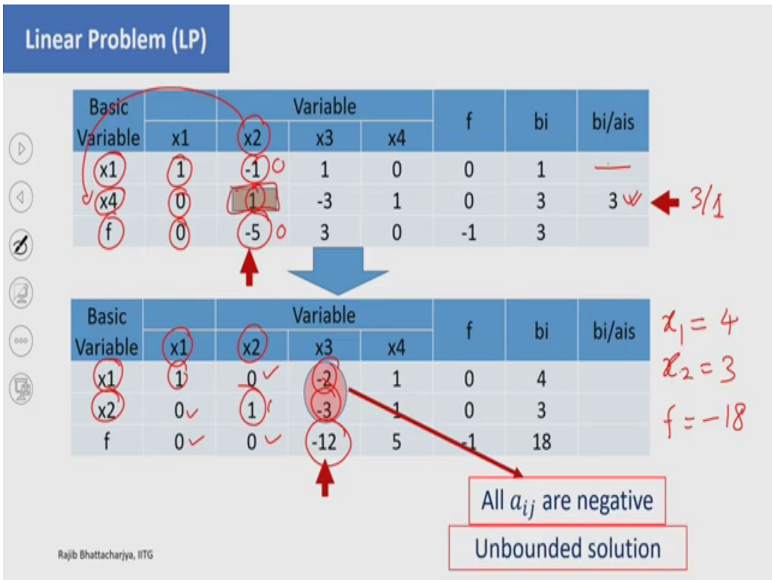

Basic Variable	x1	x2	x3	x4	f	bi	bi/ais
x1	1	-1	1	0	0	1	—
x4	0	1	-3	1	0	3	3
f	0	-5	3	0	-1	3	

Basic Variable	x1	x2	x3	x4	f	bi	bi/ais
x1	1	0	-2	1	0	4	
x2	0	1	-3	1	0	3	
f	0	0	-12	5	-1	18	

So, we can do the row operation and finally, I will get this particular table. So, I have not shown you the calculation part, but once you are implementing that one row operation. So, finally, you will get the coefficient here it is 1 and here it is 0 and here it is 0 and x 1 is coming as a basic variable and x 3 is now leaving the basis. So, it is now non-basic variable. So, now, what is the solution here?

The solution here is that x 1 equal to 1. So, after this iteration the solution is x 1 equal to 1, x 4 equal to 3 and x 2 and x 3 are 0, x 2 and x 3 are non-basic variable. So, therefore, the value of x 2 and x 3 are 0, x 1 equal to 1, x 4 equal to 3 and what is the value of f? f is equal to minus 3 ok. So, objective function value is now minus 3. Now what you have to do? Again we have to look at the objective function row, is there any negative coefficient? Yes.

So, we have one and that is for x 2 this is minus 5; that means, it is possible to improve the solution. Now x 2 will enter into the basis ok. So, x 2; x 2 will enter now whether x 2 will replace x 1 or x 4. So, what we will do basically, that we will calculate the ratio between bi by a i s. So, here the coefficient of x 2 in the first equation is negative. So, therefore, we will not calculate that one because if we consider that one so, that is actually will be an unbounded solution anyway.

And so, only coefficient of x 2 in the second equation is 1. Now if I calculate the ratio that is your 3 by 1. So, therefore, it is 3. So, we have only one value. So, therefore, we will consider this particular element as a pivoting element. So, if we are if you want to do that; that means, x 4 will leave the basis and in place of x 4. Now x 2 will enter into the basis ok. So, what you have to do? Now we have to make this particular one it is already one, but this is we have to make it 0 and this we have to make it 0.

So, finally, we are getting this particular table you just see that for x 1 this is 1 and 0 0 and for x 2 this is 0 and this is 0 and this is 1 ok. This is 1 and now here x 1 and x 2 are the basic variable ok. So, what is the solution now?

The solution is x 1 equal to 4 x 2 equal to 3. So, that is the solution that x 1 equal to x 1 equal to 4 and x 2 equal to 3 and what is the objective function value? The objective function value is now minus 18 ok. So, minus 18. So, this is the objective function value here and I am getting a solution that x 1 equal to 4, x 2 equal to 3.

Now, question is that is it the optimal solution? Is it possible to improve the objective function value? That means, can I reduce the objective function value in this case because this is a minimization problem. So, what I have to do? I have to look at the last row that is the objective function row ok.

So, is there any negative coefficient? Yes there is a negative coefficient ok. So, let us see what is this? That means, that minus 12 is there so; that means, it is still possible to reduce the objective function is not it because there is a negative coefficient here.

But question is these coefficients are also negative. So, therefore, that means, if x 3 is entering. So, in that case what will happen that x 1 can go up to infinity x 3 x 2 can go also up to infinity in this case. So, therefore, this is a problem of unbounded solution; that means, this is all a i j are negative in this case.

So, therefore, this is unbounded solution ok so; that means, you can go up to infinity because the coefficients are negative and coefficient in the objective function row is also negative here. So, in that case. So, you can go up to infinity. So, therefore, this is an unbounded problems that unbounded solution.

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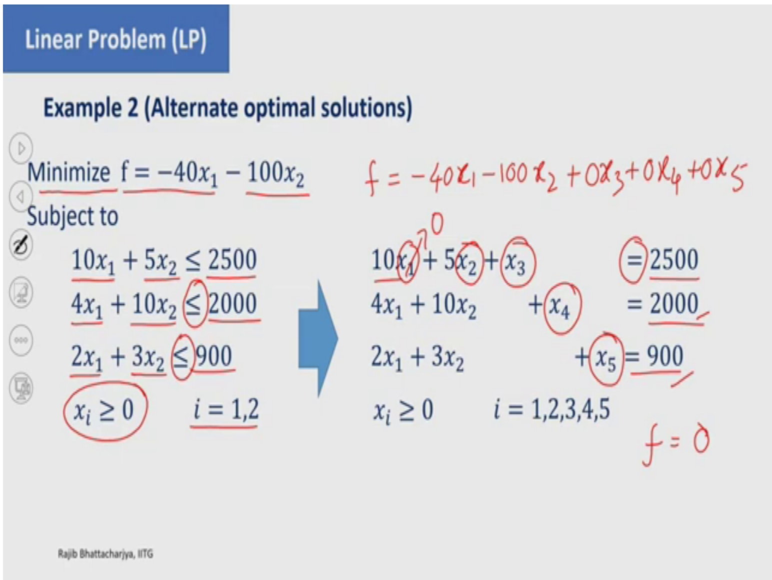

Now, let us see another example problem. So, in this case we will have alternate optimal solution ok. So, let us solve and then I will explain that this is a minimization function. Minimize f equal to minus 40 x 1 minus 100 x 2. So, there are three constraints the first one is 10 x 1 plus 5 x 2 less than equal to 2500, 4 x 1 plus 10 x 2 less than equal to 2000, twice x 1 plus thrice x 2 less than equal to 900 and all x i are positive i equal to 1 to 2.

So, here we have two variables and there are three equations. Now let us convert it to standard form. So, what I will do, I have to put 1 slack variable here in order to make it equal. So, now, the problem or the first constraint is 10 x 1 plus 5 x 2 plus x 3 equal to 2500, then in the second equation also as it is less than equality type. So, I have to put another slack variable that is x 4 and now it is equal to 2000.

And the third one again it is a less than equality type. So, therefore, I have to put another slack variable and just to make it equal to 900. So, now, this is in standard form. So, I can write the objective function here. So, this is minus 40 x 1 minus 100 x 2 plus 0 x 3 plus 0 x 4 plus 0 x 5 now you just see that if you look at.

So, what is the initial basic feasible solution that if I consider, x 1 and x 2 are non-basic variable. So, that will be equal to 0. So, in that case and x 3 x 4 and x 5 are basic variable. So, initial basic feasible solution is that x 3 equal to 2500, x 4 equal to 2000 and x 5 equal to 900 and what will be the objective function value that f equal to 0 ok. So, that is the initial solution I can have.

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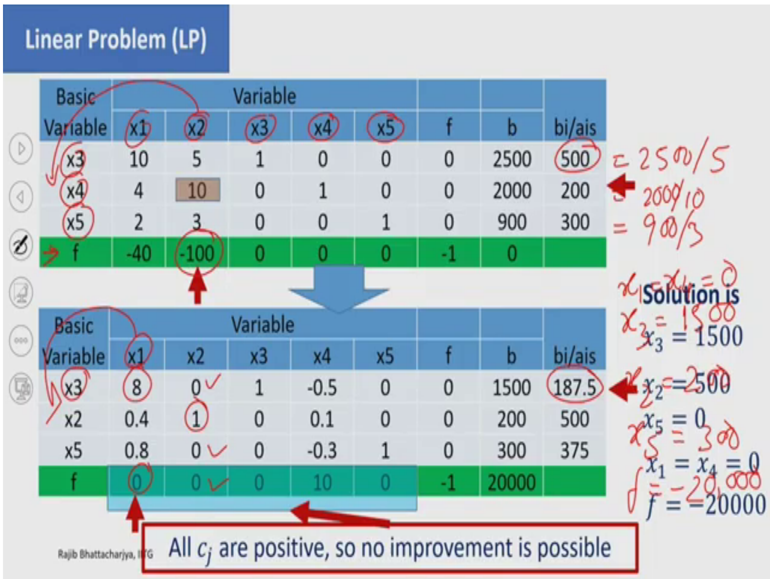

So, now, let us put in the table here. So, I am putting in this particular table. So, here as I said that x 3 x 4 and x 5. So, they are basic variable. So, they are basic variable and x 1 and x 2 are non-basic variable. So, now, after putting this. So, what is the first step? The first step is look at the objective function row ok. So, if you look at is there any negative coefficient? Yes for x 1 that is minus 40 and for x 2 this is minus 100.

So, what you have to do? We have to take the minimum 1; minimum 1 is minus 100. So, we will take that so; that means, the x 2 will enter into the basis and x 2 will replace one of these basically one of these variables that is x 3, x 4 and x 5. So, what one what it will replace? So, we have to calculate the ratio of b i by a i s.

So, the ratio is the for the first one that is 2500 divided by 5. So, I am getting 500 then the for the second one that is 2000 divided by 10. So, I am getting 200 and for the third one I should get 900 divided by 3. So, therefore, I am getting 300. So, what we have to do now? We have to select the minimum one here also the minimum one is 200.

So, therefore, this will be the pivoting element. So, if it is pivoting element. So, what we have to do? We have to make this one and for others the coefficient of x 2 will be equal to 0. So, if I do that. So, finally, I am getting this one that coefficient of x 2. In the second equation is 1 now and for other equation they are 0 including the objective function ok. So, now, what is the solution I am getting? The solution I am getting here that x 3 equal to x 3 equal to 1500 is not it. So, x 3 equal to 1500, then x 2 equal to 200 ok.

Then x 5 equal to I am getting 300 ok and what is the value of f? That f is equal to minus 20,000 ok minus 20,000 I am getting this. Now what we have to look? We have to look at the objective function row is there any negative coefficient? No. All c j are positive there is no negative coefficient. So, no improvement is possible. That means, I will not get any improved solution so; that means, I got the solution. So, here that x 3 equal to 1500, x 2 equal to 200, x 5 equal to 300, x 1 equal to 0 and x 4 is also equal to 0 ok.

So, x 1 equal to x 4 equal to 0 ok. So, this is the solution I am getting. Now question is that now if I consider x 1 ok. So, if I consider x 1. So, here you just see the coefficient is 0 so; that means, you are not getting any improved solution by converting x 1 as a basic variable is not it, but let us do that, but what we will get? Objective function value will not sense because the coefficient is 0, but let us do that.

So, if I consider that this particular column. So, here I would like to make x 1 as a basic variable. And I if I take the ratio. So, this is 187.5 now and that is the minimum one and therefore, this will be the pivoting element so; that means, x 3 will now be replaced by x 1 ok.

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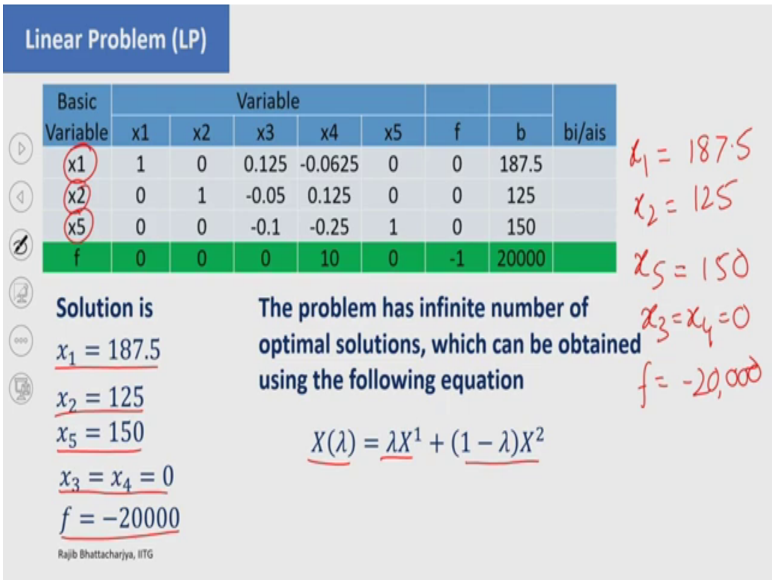

So, this is the pivoting element and if I do that. So, x 1 is now in as a basic variable then x 2 is already there a x 5 is there. Now I am getting a different solution, but if you look at what is the solution? The solution is x 1 equal to 187.5 ok. And x 2 equal to 125 and x 5 equal to 150 and x 3, x 4 equal to 0 and f equal to minus 20,000 ok. So, there is no improvement in the objective function value.

So, I am getting the same solution, but this is an alternate solution. So, what you are getting? x 1 equal to 187.5, x 2 equal to 125, x 5 equal to 150, then x 3 and x 4 equal to 0 and objective function value is minus 20,000. So, this problem has infinite number of optimal solution which can be obtained using the following equation that is x lambda equal to lambda x 1 plus 1 minus lambda x 2.

So, I will get infinite number of optimal or you can say alternate optimal solution. Objective function value will be same, but the value of the variable may be different or will be different.

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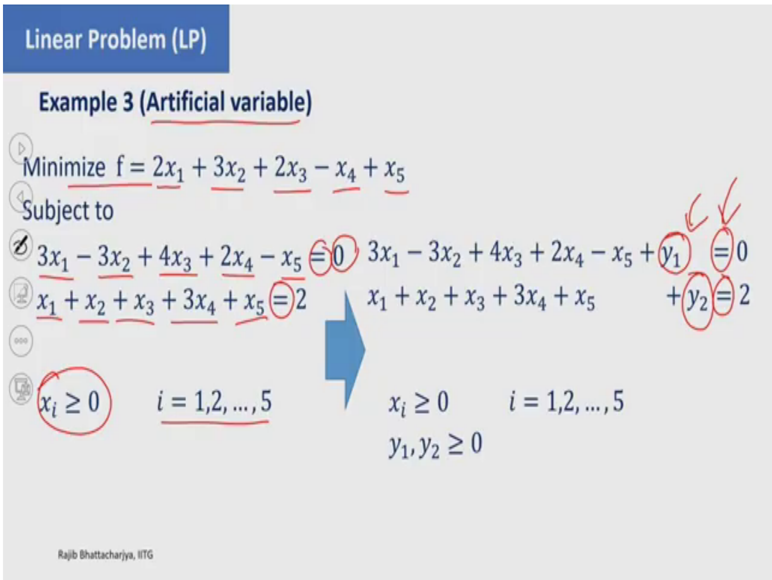

Now, let us see a problem with artificial variable. I have not discussed what is artificial variable, but in this problem I will discuss what is artificial variable. The objective function here is a minimization type objective function that is f equal to twice x 1 plus thrice x 2 plus twice x 3 minus x 4 plus x 5 and subject to there are two constraint the constraints are 3 x 1 minus 3 x 2 plus 4 x 3 plus twice x 4 minus x 5 equal to 0 ok and the second one is x 1 plus x 2 plus x 3 plus thrice x 4 plus x 5 equal to 2.

So, in this particular problem the constraints are equality type constraint ok and all x i should be positive greater than 0 i equal to 1 to 5. Now question is that I have to make it standard form. So, already this is an equality sign, but what I need basically. So, I need the initial basic feasible solution. So, what is initial basic feasible solution? That for a particular constraint there must be a variable whose coefficient is 1 for that particular constraint and for other constraint that is 0.

So, in order to do that what I have to do? We have to take we have to add two variables y 1 and y 2. Now once you are putting y 1 y 2. So, this equality sign is not actually valid and basically you have disturbed the equality sign. So, earlier we added a slack variable in order to make it equal, but in this case what you are doing? It is already equal, but you are adding a variable and this variable will disturb this equality sign.

So, you cannot actually write this is equal to 0, but in order to get a initial basic feasible solution. So, we are putting these two variable y 1 and y 2. So, we have put here two variables that is y 1 and y 2 as I said. So, that will disturb the equality sign ok. So, equality condition. So, therefore, these two variables has to be eliminated ok initially it has to be eliminated because I do not want this variable because these two variable y 1 and y 2 has disturbed the equality conditions ok. So, therefore, you have to eliminate these two variable and this y 1 and y 2 is known as artificial variable.

So, what we are doing here? We are just putting two artificial variables that is y 1 in the first equation y 2 in the second equation just to have a initial basic feasible solution. So, idea is that you eliminate as early as possible you eliminate this y 1 and y 2 from this equation.

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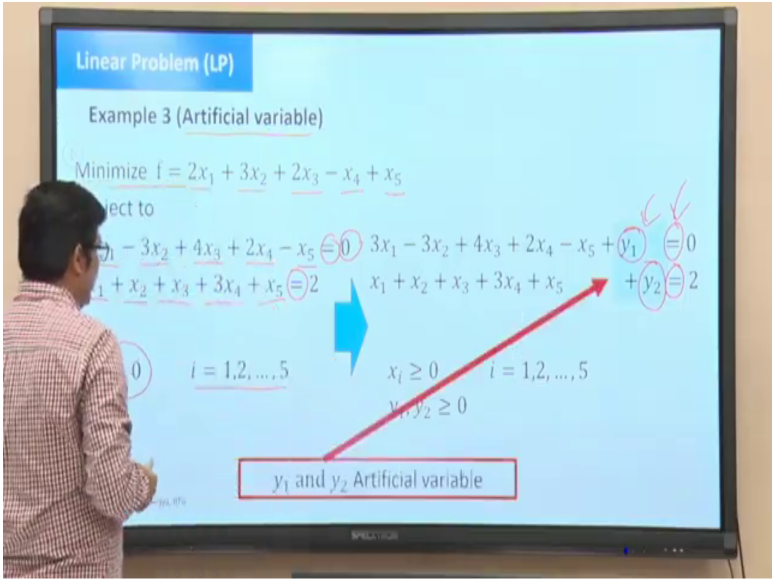

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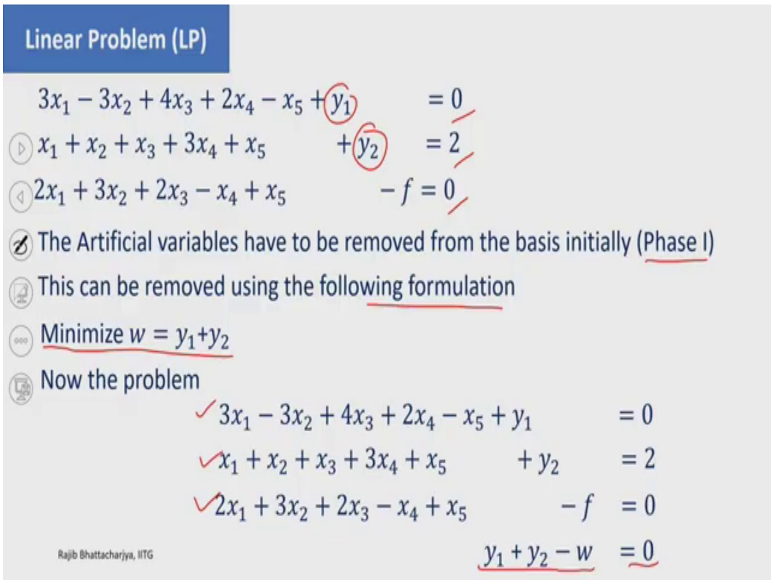

So, y 1 and y 2 are artificial variable. So, now, if I write this equations. So, this is my first constraint, this is second constraint and I have these two variables that is y 1 and y 2 and these two variable are basically I can consider the initial basic feasible solution ok. So, x 1 x 2 x 3 x 4 and x 5 are 0 they are non-basic variable and y 1 and y 2 are basic variable. So, the artificial variables have to be removed from the basis initially, this can be removed by using the following formulation.

So, we will have one formulation initially. So, we call it the phase 1 ok. So, phase 1 what is the objective? The objective is I would like to remove this y 1 and y 2. So, therefore, I am writing a different optimization mode problem or different optimization formulation where initially I would like to minimize w which is equal to y 1 plus y 2. So, this is the objective function.

So, if I put it here. So, this is my first equation, this is second equation, the third one is the objective function the original objective function and fourth one the phase 1 objective function that is y 1 plus y 2 minus w equal to 0. So, what I am doing here? I would like to minimize this particular function initially objective function initially that w equal to y 1 plus y 2. I would like to minimize it ok to remove y 1 and y 2 from the basis ok.

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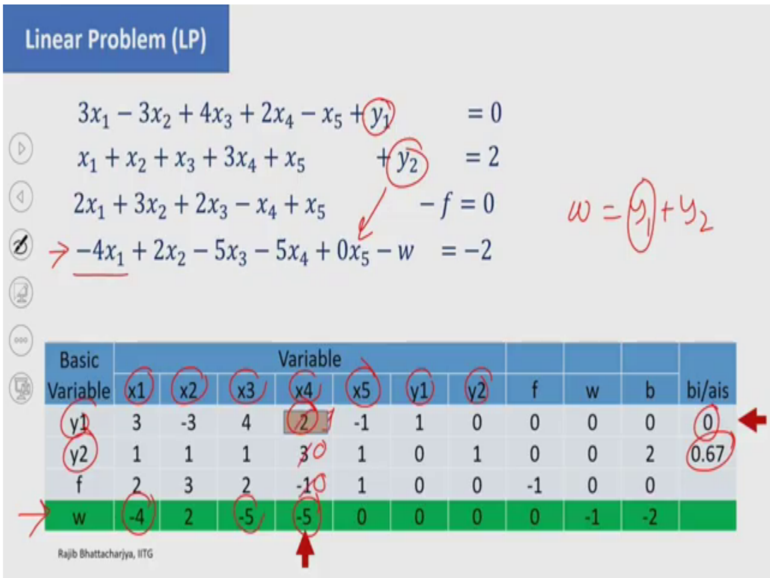

The last equation that is the objective function w equal to; w equal to y 1 plus y 2. So, what we are doing that we are writing this y 1 in terms of x 1 ok. So, what I can do? So, from the equation 1 that is the first constraint. So, I can write what is y 1. So, y 1 I can take on the other side of the equation.

So, y 1 equal to minus thrice x 1 plus thrice x 2 minus 4 x 3 minus 4 x 4 plus x 5. So, that is y 1 and similarly I can take what is y 2 and this y 1 and y 2 I am writing in terms of x 1 x 2 x 3 and x 4 and that I am putting in the last equation. So, if I do that. So, I will be getting the this w equal to y 1 plus y 2 which is equal to I am getting minus 4 x 1 plus twice x 2 minus 5 x 3 minus 5 x 4 plus 0 x 5 minus w equal to minus 2.

So, if I put on the simplex table. So, I am getting this particular table here y 1 and y 2 are basic variable. So, therefore, it is here and I and x 1 x 2 x 3 and x 4 and x 5 they are non-basic variable. So, now, what we have to do? We are minimizing this w function. So, we have to look is there any negative values. Yes there are negative values here it is minus 4 minus 5 minus 5, but what we have to consider?

We have to consider the minimum one. So, minimum one is here minus 5 and minus 5. So, therefore, you just take it arbitrarily any one either x 4 you can take or x 3 you can take ok. So, next is I will calculate the ratio. So, here ratio is 0 and it is 0.67. So, you have to take the minimum one and minimum one is the first equation. So, therefore, this will be your pivoting element. So, what I will do? So, I will make it 1 and for others. So, I have to make it 0 and 0 and this must be equal to 1.

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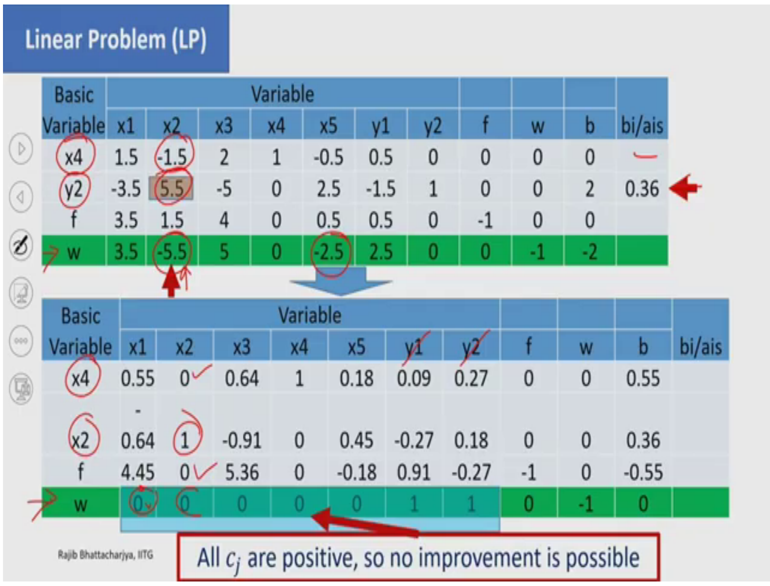

So, after implementing the row operation. So, finally, I am getting this particular table here now you just see that x 4 is now basic variable, but y 2 is still there ok. So, we have to look at in this row again is there any negative one yes. So, this is the negative value we have this is also the negative value, but this is the minimum one. So, therefore, x 2 will now enter into the basis and let us look at the ratio.

So, we have. So, first one is negative ok. So, first one is negative. So, therefore, we are not considering for the second one that is 2 divided by 5.5. So, we are getting 0.36 and that is the only one and therefore, this is the your pivoting element ok. So, this will be the pivoting element.

So, we will apply the row operation now we will make it 1 and for others that will be 0. So, what we are doing here? Now you just see this is your 1 and this is 0 and this is 0 and you just see now x 2 is here now and already x 4 here x 4 is already there. So, y 1 and y 2 are not there and if you look at the objective function row that is the phase 1 objective function that w equal to y 1 plus y 2 and there is no negative coefficient all coefficients are positive.

So; that means, that you cannot minimize this one further. So, you got the solution of fast space. So, what I can do, I can remove this particular row along with the variable y 1 and y 2 ok. So, all c j are positive. So, no improvement is possible.

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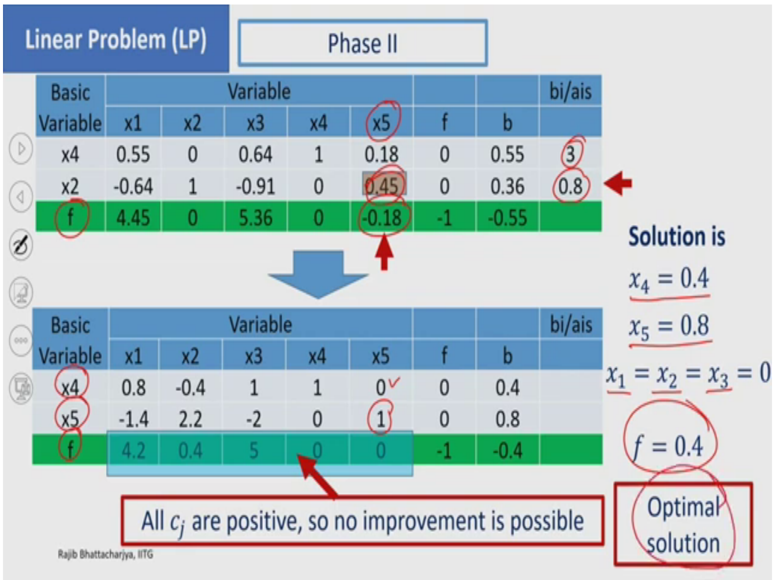

So, what will be the phase II now? The phase II I will remove the object that last objective function that phase I objective function w equal to y 1 and y 2 and now I will consider the original objective function ok. So, from the last table of the phase 1 ok. So, now, if you look at that the objective function row is there any negative value? There is 1 yes x 5 for x 5 the c j equal to minus 0.18 ok.

So, therefore, x 5 will be an incoming variable. So, x 5 will enter into the basis. So, now you calculate the ratios. So, here it is 0.08 and here it is 3. So, therefore, we will take the minimum one and now this will be the pivoting element. So, if you do that. So, finally, you are getting this 1 and this equal to 0 and the solution is now x 4 equal to 0.4, x 5 equal to 0.8 and f we are getting f equal to 0.4 ok.

So, you just see if you look at all c j are positive and so, no improvement is possible; that means, you got the optimal solution ok. So, this is the optimal solution we got in this case we have considered two artificial variables that is y 1 and y 2 and we call it 2 phase problem in the first phase the idea is to remove the artificial variable.

So, we have an objective function that is w equal to y 1 plus y 2 and I would like to minimize this objective function in order to remove the variable y 1 and y 2. So, these two are artificial variable and once we are removing that one once we are getting the optimal solution of that particular objective function then we will consider the original objective function ok.

And we got the solution and this is basically an example of your LP problem with artificial variable. So, this is the final solution that x 4 equal to 0.4 x 5 equal to 0.8 and x 1 x 2 and x 3 equal to 0 and objective function value is 0.4 and we got the optimal solution as there is no negative coefficient the objective function row.

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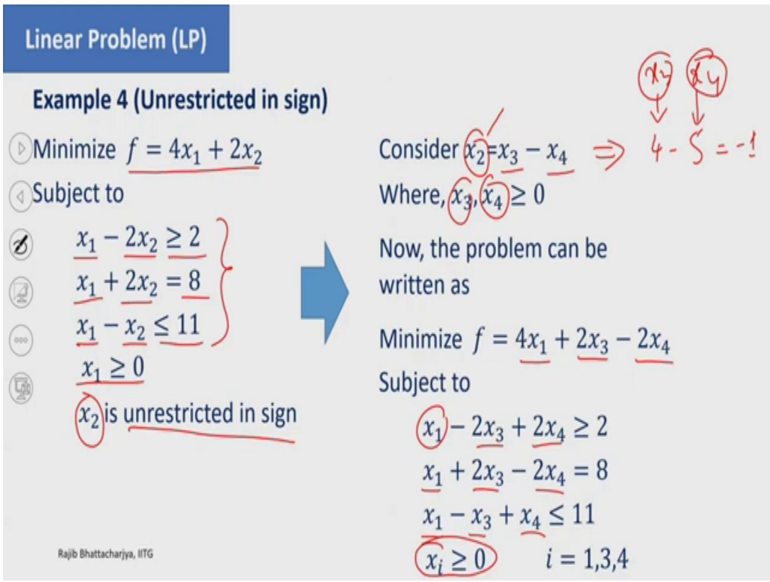

Let us take another example problem and the problem is a variable with unrestricted in sign. So, whatever problem we have discussed all variables are positive ok. So, sometime what may happen the variable value may be negative; that means, unrestricted in sign. So, it can take a negative value also ok let us see an example problem with a variable that is unrestricted in sign.

So, let us consider this particular problem. So, this is a minimization problem which is equal to plus 4 x 1 plus twice x 2 there are three constraints. So, x 1 minus twice x 2 greater or equal to 2, then x 1 plus twice x 2 equal to 8, x 1 minus x 2 less than equal to 11, x 1 is positive here, but x 2 is unrestricted in sign; that means, x 2 can be negative also.

So, what you have to do basically that I am considering x 2 is equal to x 3 minus x 4 what I am doing? I am writing x 2 in terms of x 3 and x 4. Now x 3 and x 4 are positive here. So, what I am doing? Because when you are considering the lp problem all the variables should be positive.

So, you cannot have a negative your variable ok. So, the variable value should be positive so, but in this case the x 2 can be negative also. So, therefore, I am writing x 2 in terms of x 3 and x 4 and x 3 and x 4 are positive suppose what I can do? Suppose if I want to have a negative value; that means, I can write 4 minus 5. So, this is your x 3 and this is your x 4. So, 4 minus 5 equal to minus 1, but x 3 and x 4 are positive ok now the problem can be written as.

So, if I replace x 2 equal to x 3 minus x 4. So, in that case I can write f equal to 4 x 1 plus twice x 3 minus twice x 4 similarly constraints can be written x 1 minus twice x 3 plus twice x 4 greater than equal to 2, then x 1 plus twice x 3 minus twice x 4 equal to 8 x 1 minus x 3 plus x 4 less than equal to 11 ok and here now x i are greater than equal to 0 i equal to 1 3 4. So, 2 is not there. So, now, we have variable 1 x 1 the variable x 3 and variable x 4.

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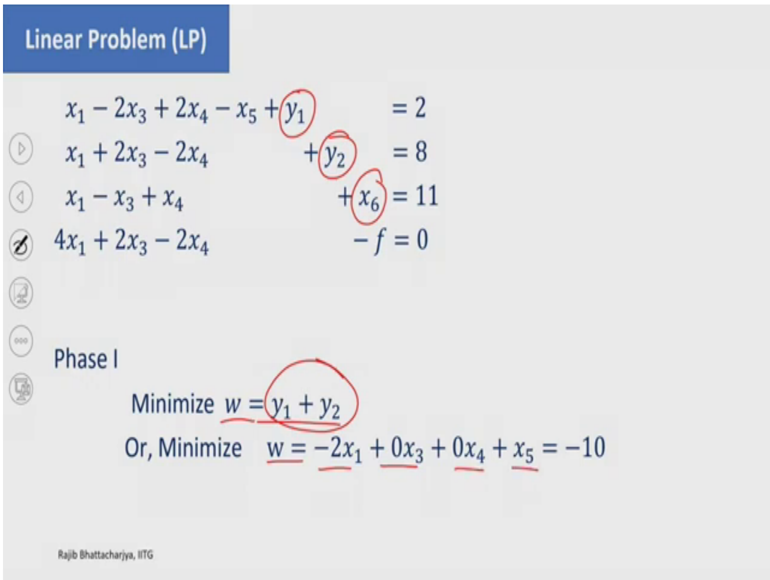

Now, let us convert this to a standard form. So, therefore, I am putting a artificial variable here in order to get initial basic feasible solution and here also I am putting an artificial variable and here I am putting a slack variable to make it equal. So, y 1 and y 2 are artificial variable just in order to get the initial basic feasible solution.

So, initial solution is now y 1 equal to 2, y 2 equal to 8 and x 6 equal to 11. So, therefore, this is a 2 phase problem. So, we will solve it using 2 phase method. So, what we are doing here that we have a new objective function the w equal to y 1 plus y 2. I would like to eliminate this y 1 and y 2 or I can write y 1 in terms of x 1, x 3 and x 4, x 5. So, if I write w equal to minus twice x 1 plus 0 x 3 plus 0 x 4 plus x 5 equal to minus 10.

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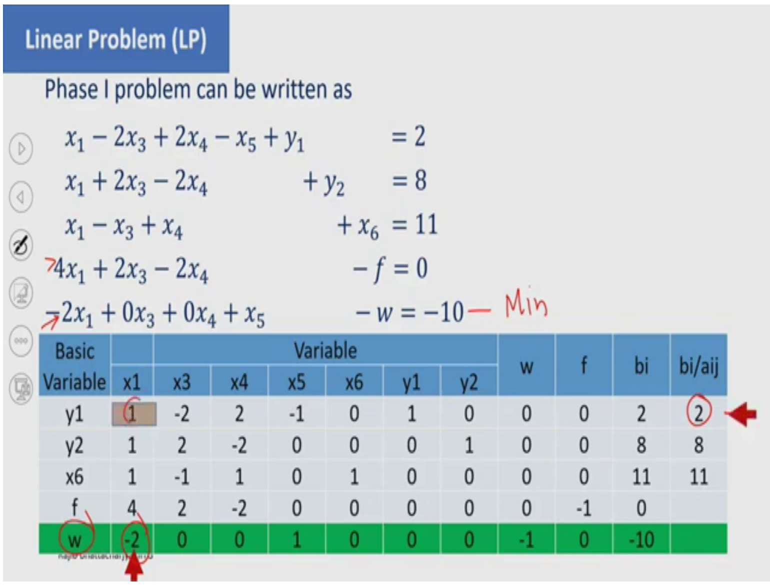

Let us put in the simplex table. So, here this is the original objective function and this is the phase I objective function. So, initially we will try to we will minimize this objective function ok. So, if I do that, I have to look at the last row, is there any negative coefficient? Yes, this is the negative coefficient. So, therefore, x 1 is the entering variable. So, x 1 will enter into the basis, I then I look at the ratio of b i by a i j. So, this is the minimum one. So, therefore, this will be the pivoting element ok.

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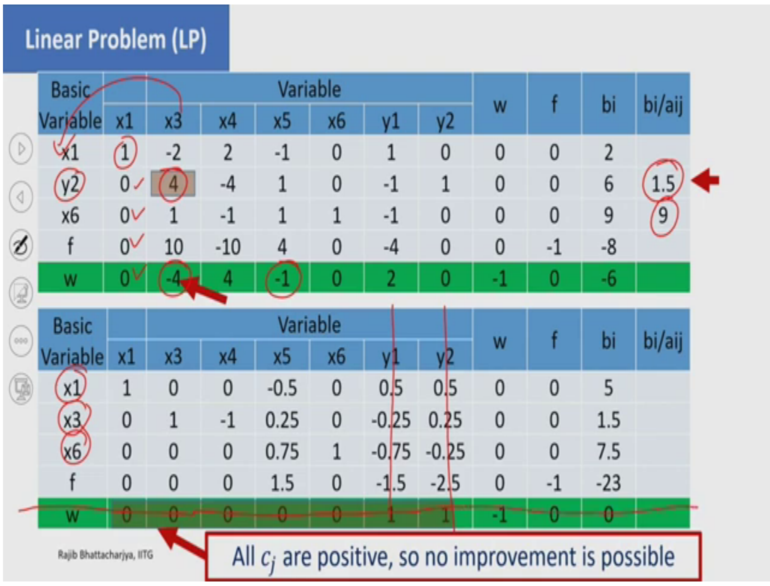

So, if it is pivoting element. So, then this will be equal to 1 and for others this will be equal to 0. Now x 1 is x 1 has enter into the basis now y 2 is still there. Now, next question is that can we improve the solution? Yes because there is a negative coefficient here there is a negative coefficient here.

So, this is the minimum one so; that means, x 3 will enter into the basis and we will calculate the ratio and this ratio is 1.5 and this is 9 and so, that is the minimum one. So, therefore, this is your pivoting element now. So, if it is pivoting element then y 2 will be replaced by x 3.

So, if I do that. So, what I am getting? So, now, x 1 is there x 3 is there and x 6 is there and y 1 and y 2 are not there. So, in the basis so and if I look at the last row that is the of phase 1 objective function here there is no coefficient with negative value; that means, I cannot improve this solution and y 1 and y 2 are not there. So, therefore, I will eliminate this particular column ok and this particular row also. So, I will remove this one.

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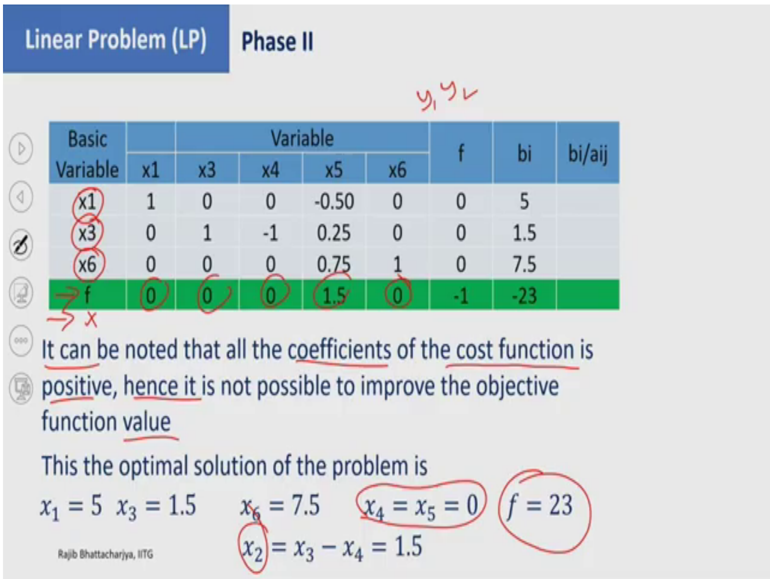

So, if I do that. So, I am getting this particular table here. Here now x 1 is a basic variable, x 3 is the basic variable, x 6 is a basic variable and whatever w row was there that we have eliminated and we have also eliminated the value of the row having y 1 and y 2. So, that is not there.

Now, question is that, can we improve this solution? So, there is no negative coefficient in the objective function row. So, all are positive ok. So, all are positive. So, therefore, you cannot improve this solution and so, I can say that whatever solution you are getting that is the optimal solution. So, what I have written here? It can be noted that all the coefficients of the cost function that is the objective function is positive hence, it is not possible to improve the objective function value.

So, you will not get any improvement because all of them are positive. So, therefore, the solution is now the x 1 equal to 5, x 3 equal to 1.5, x 6 equal to 7.5, x 4 and x 5 that is non-basic variable ok, so non-basic variable. So, therefore, it is 0 and what is x 2? x 2 is x 3 minus x 4 ok. So, x 3 is 1.5 x 4 is 0. So, therefore, x 2 value is 1.5 and objective function value is 23. So, that is the objective function value.

So, we got the solution and we are not getting any improved solution because all the coefficient at the objective function row are positive. So, in this class we have mainly discussed the simplex method. So, we have applied simplex method for solving different types of a linear problem. So, in the next class I will show you how you can solve a linear problem with using your excel solver ok.

Thank you.