Optimization Methods for Civil Engineering
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Lecture - 04
Linear Problems (LP)

Hello student. Welcome back to the course on Optimization Methods for Civil Engineering. So, in the last class, we discussed about Linear Problems, so I explained what is linear problem. So, in this case, the number of variables is more than number of equation; that means, the n is greater than m. So, we have more variables than number of equation.

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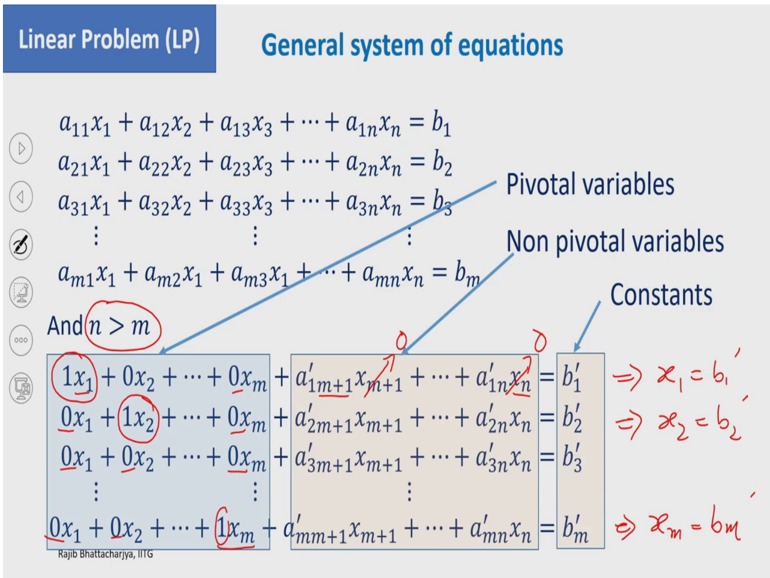

So, in this case by applying the row operation, so I can make the coefficient of variable x 1 is 1 and then I can make 0 in the other equations. And similarly, so for x 2 the coefficient in the second equation is 1 and for other equation it is 0 and similarly, up to x m. So, we have actually total m equation. And for the mth equation the coefficient of x m is 1 and for other equation it is 0.

So, one possible solution is, so if I consider that x 1 to x m these are basic variables and from m plus 1, ok from m plus 1 to n, so they are non-basic variables, ok. So, if I consider that; that means, that x m equal to 0 up to x n. So, these are all non-basic variables. And if I say that these are 0, then one possible solution is that x 1 equal to b 1 dash.

So, I may get from here that x 1 equal to b 1 dash. Similarly, from here I can get x 2 equal to b 2 dash. And for the last equation, so I can find out that x m equal to b m dash, ok. So, this is one possible solution. So, we call it these are pivotal variable and these are non-pivotal variables, ok. And these are the right hand side and you can say these are constants.

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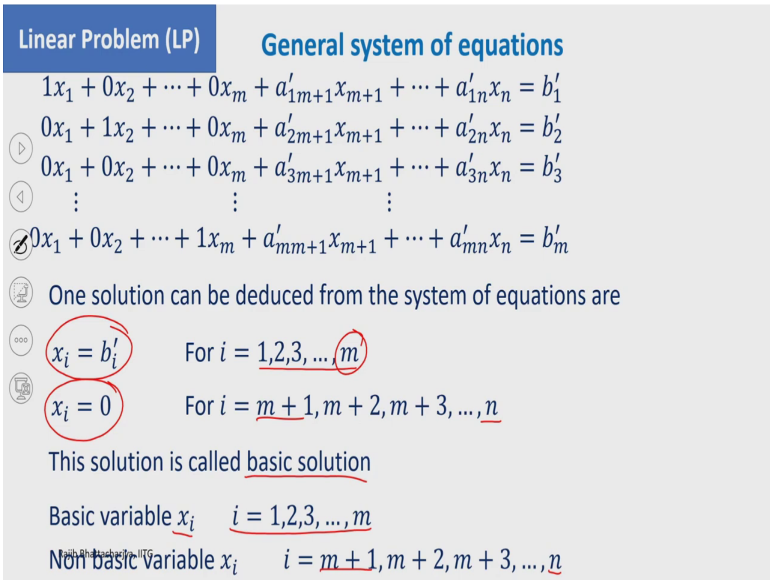

So, now as I said, so one possible solution is that x i equal to b i dash for i equal to 1 to m; that means, m is the number of equation, in this case. And for other variables that is from m plus 1 to n, m plus 1 to n, so these are all non-basic variable and we are putting x i equal to 0 in that case. So, this is one of the solution.

And what I can do basically, then I can have some alternate solution. Suppose in this case I have taken 1 to m. So I may take any m number of variables as a basic variable and other variables will be non-basic variable, so I can get an alternate solution. And so, what is optimization? So, optimization is basically to find out the best solution out of these combinations, ok.

So, as I said this solution is called basic solution. So, here as I said the basic variables are x i that is i equal to 1 to m, and the non-basic variables are for i equal to m plus 1 to n.

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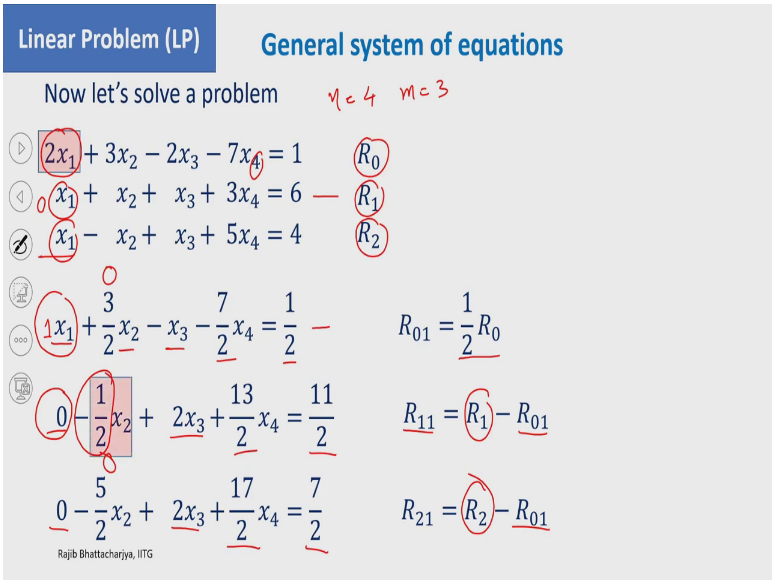

Now, let us solve a problem. So, in this case we have total 4 variable that is n equal to 4. And how many equations we have? So, we have 3 equations. So, m equal to 3. So, I would like to solve this particular problem. So, what I am doing here? So, I am telling this is row 0, this is row 1, and this is row 2, ok.

So, now how to solve this particular problem? So, what I will do, I will apply the row operation and I would like to make coefficient of this particular variable as 1 and then for other equations, so I would like to make it 0, ok. So, that is by using the row operation. So, now what I have to do here? So, for the R 0 row, so if I divide this particular row by 2 in that case the coefficient of this particular variable that coefficient of x 1 will be 1.

So, what I will do? I will take this particular variable x 1 in equation 1 and I have divided this equation by 2, ok. So, that means, I have multiplied this entire equation by half. So, in that case this will be x 1. So, now, coefficient is 1 here. So, coefficient is 1 here, and this is 3 by 2, then this is again x 3, ok divided by 2 and this is minus 7 by 2, x 4 and right hand side is half, ok.

Now, next step is that I would like to make the coefficient of x 1 in equation 2 as 0. So, what I will do here? So, I will do some operation; that means, I am now telling this particular row is row 11; that means, the iteration 1 and here I am doing R 1 minus R 11, ok. So, this is your R 1 minus R 01. So, if I do that, that means, the x 1 minus x 1, so you are getting 0.

Then this is 1 minus 3 by 2, x 2, so I am getting minus half. Then x 3 minus x 3; so minus minus plus, so I am getting twice of x 3. And for the x 4 I am getting 13 by 2 x 4, in right hand side I am getting 11 by 2. So, by doing this operation; so what I have done. So, I have eliminated this particular variable from equation 2. So, I have eliminated the x 1 variable from equation 2, ok.

So, similarly, I would like to eliminate this x 1 variable from equation 3. So, what I will do? So, in this case I have done this row operation that is R 2 minus R 01, ok, so R 2 minus R 01. So, what I am getting? I am getting that this is 0, then minus 5 by 2 x 2 plus twice x 3 plus 17 by 2 x 4 equal to 7 by 2. So, in this operation, so what I have done? I have made the coefficient of x 1 in equation 1 as 1 and for other equations the coefficient of x 1 is 0, ok.

So, now, I can repeat this process. So, now, what I will do? I will take the x 2 variable. So, I would like to make the coefficient of x 2 variable in equation 2 as 1. Suppose I would like to make this as 1, and then for this I would like to make 0 for the other two equations, ok. So, that means, I will take this x 2 variable now, ok.

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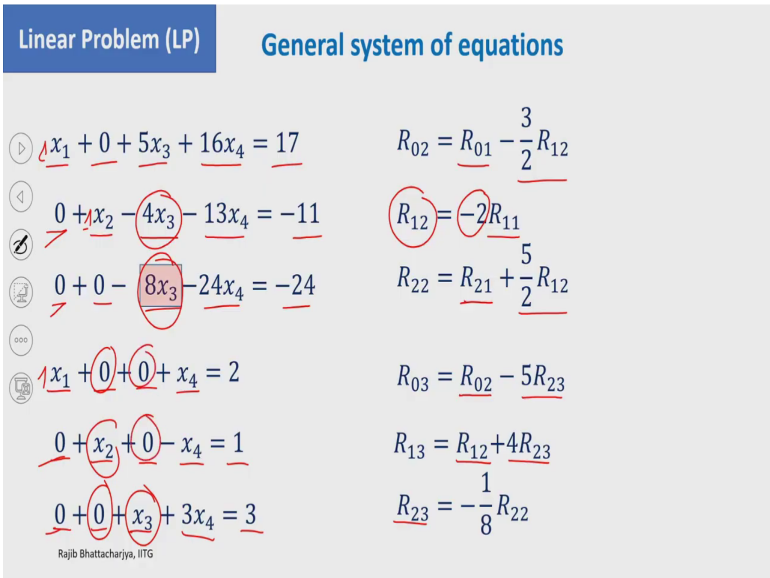

So, what I have done here? I have multiplied that R 11 by minus 2, ok. So, if you do that, so you are getting this is 0 already, this is now x 2, so the coefficient is 1 here, then minus 4 x 3, then minus 13 x 4 equal to minus 11, ok. So, now, if I do this operation that is R 01 minus 3 by 2 R 12, ok, so minus 3 by 2 R 12, so then I am getting here 0.

So, now, I am getting x 1 plus 0 plus 5 x 3 plus 16 x 4 equal to 17. And if I do R 22 which is equal to R 21 plus 5 by 2 R 12, then I am getting this is also 0 now, 0, and this is already 0. So, now, I am getting minus 8 x 3 minus 24 x 4 equal to minus 24, ok. So, now with this, what I have done? So, for x 1 coefficient of x 1 in equation number 1 is 1 and for others it is 0.

Similarly, for x 2 coefficient in equation 2 is 1 and for other equations that is 0. Now, let us consider x 3, ok. So, let us consider x 3 here in the third equation. So, I would like to make the coefficient of x 3 in the third equation as 1. So, what I will do? I will divide this equation that is I will divide this equation by minus 8, ok. So, if I do that R 23 equal to minus 1 by 8 R 22.

So, now here this is 0, this is 0, then coefficient of x 3 here is 1 plus thrice x 4 equal to 3. So, now, I would like to make the coefficient of x 3, in other two equation as 0. Now, for the second equation if I do R 12 plus 4 R 23, so in that case I will be getting 0 plus x 2 plus 0 minus x 4 which is equal to 1.

So, similarly, for the first equation if I do this operation that is R 02 minus 5 R 23, so in that case I will be getting x 1 plus 0 plus 0 plus x 4, ok. So, now, what I am getting here, you just see the coefficient of x 1 in the first equation that is 1 and for other equation these are 0. Similarly, for x 2 coefficient of x 2 in equation 2 is 1, for other equation that is 0. Similarly, x 3 in the third equation the coefficient of x 3 is 1, and for other it is 0, ok.

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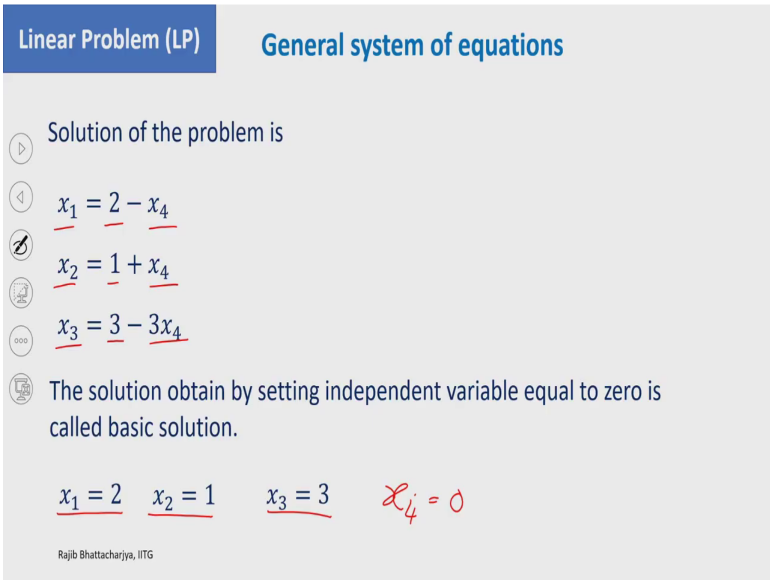

So, now, we are getting one solution. So, the solution is that x 1 equal to 2 minus x 4, x 2 equal to 1 plus x 4 and x 3 equal to 3 minus thrice x 4. So, now, if I consider that x 1, x 2, x 3, are basic variable and x 4 is a non-basic variable. So in that case the solution is that x 1 equal to 2, x 2 equal to 1, x 3 equal to 3 and that x 4 equal to x 4 equal to 0. So, that is your non-basic variable. So, now what we have done here? So, we have considered x 1, x 2, x 3, as basic variable and x 4 as non-basic variable.

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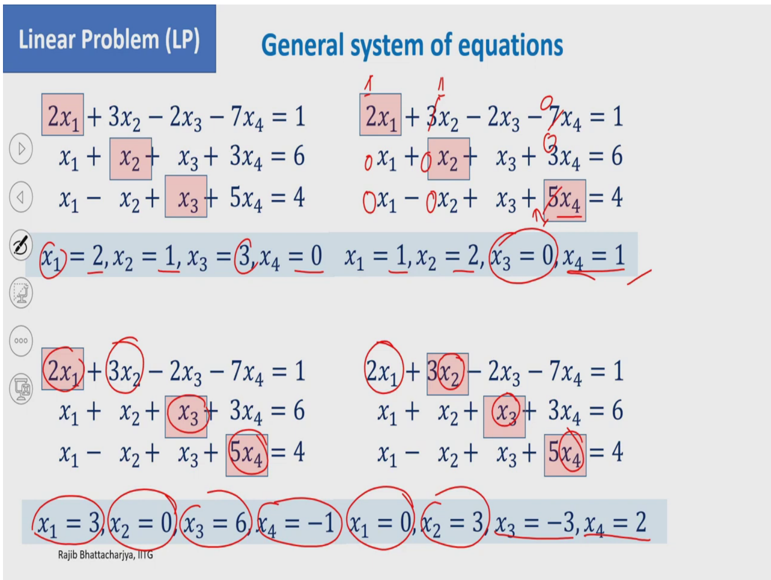

So, in this case I have considered that x 1, x 2, x 3, are basic variable. So, what solution I am getting? x 1 equal to 2, x 2 equal to 1, x 3 equal to 3 and x 4 as a non-basic variable thus the value is 0. Now, if I consider that x 1, x 2 and x 4 are basic variable and x 3 as a non-basic variable.

So, I can do the row operation, and finally, what I can do coefficient of x 1 I can make it one and for other it is 0. Similarly, here it will be 1 and for other it will be 0 and here this will be your 0, 0, and coefficient here is 1, ok. So, I can do that, and finally, what solution I am getting; that x 1 equal to 1, x 2 equal to 2, x 3 is a non-basic variable, so that is 0 and x 4 equal to 1.

So, this is another solution, ok. So, another solution you can say. So, this is the first solution where x 1, x 2, x 3, are basic variable and for the second case if I consider x 1, x 2 and x 4 are basic variable, then I am getting another solution. So, in the third case, what I can do? I can consider that x 1, x 3, and x 4 are as a basic variable.

So, these are basic variable and x 2 as a non-basic variable. So, I can do the row operation again. So, I can make the coefficient of x 1 in the first equation as 1 and for other equations that is 0. Similarly, for x 3 and x 4 also I can do that. And finally, these are the solution I am getting, x 1 equal to 3, x 2 as a non-basic variable, so that is 0, x 3 equal to 6 and x 4 equal to minus 1.

Now, in the fourth combination. So, if I consider that x 2, x 3 and x 4 are basic variable and x 1 as a non-basic variable, then solution is that x 1 will be 0 as it is non-basic variable and x 2 equal to 3, x 3 equal to minus 3, and x 4 equal to 2. So, what I can do? So, I can evaluate all the combinations and finally, I can tell that this is the solution, ok. So, this is the solutions.

So, in this case we have done total 4 combinations, ok. So, first we have consider x 1, x 2, x 3, and second we have considered x 1, x 2, x 4, and the third one x 1, x 3, and x 4, and the fourth one we have considered x 2, x 3, and x 4 as basic variable.

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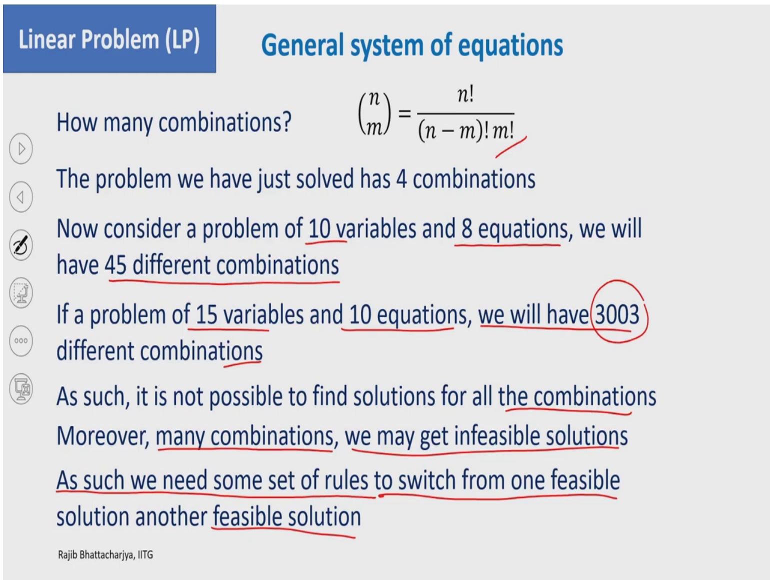

So, how many combinations we are getting? Factorial n divided by factorial n minus m into factorial m. So, this much combinations we are getting. The problem we have just solved has 4 combinations, ok. Now, consider a problem of 10 variable and 8 equations, ok. So, if you have 10 variables and 8 equation, we will have 45 different combinations.

So, if you have a problem of 15 variable and 10 equations we will have 3003 different combination. So, as such it is not possible to find solution for all the combinations. So, this is not possible. You just see, so once you have more number of equations and more number of variables, so you will have huge combination.

So, large combination you will get as such it is not possible to find the solution for all the combination and moreover many combinations, ok, so many times we may get infeasible solution, ok. So, if you are getting infeasible solution you need not evaluate them basically.

So, therefore, you need some basically technique, so that you are evaluating minimum number of combination.

So, as such we need some set of rules, to switch from one feasible solution to another feasible solution, ok. So, because we need not evaluate all these combination because some of the combination will be infeasible, ok. So, we need not evaluate that one. So, therefore, we need some set of rules, ok and to switch from one feasible solution to another better feasible solution, ok.

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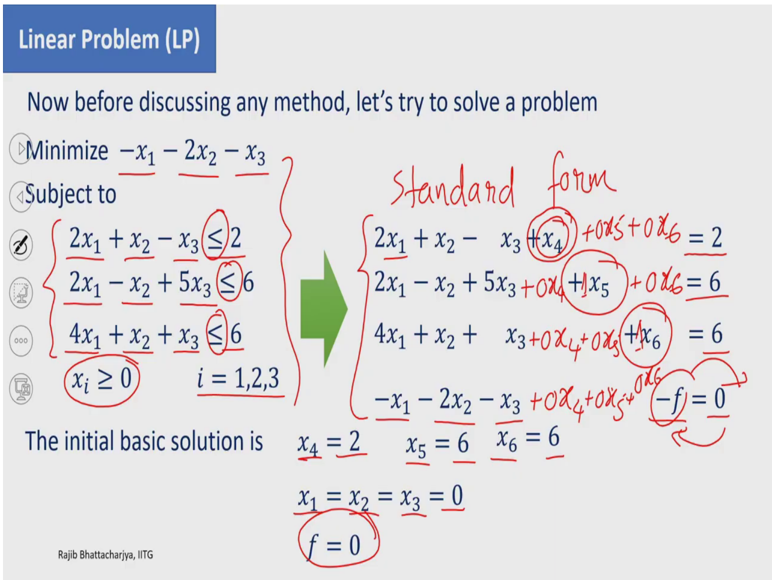

Now, before discussing any method, so let us try to solve a problem, ok. So, we will first solve a problem and then we will try to review the rules basically. So what rule you should follow and why you should follow. Before that I would like to solve this particular problem, ok.

The problem is minimize minus x 1 minus twice x 2 minus x 3, so that is your objective function. Objective function is minus x 1 minus twice x 2 minus x 3 and we have total 3 constraints. So, that is twice x 1 plus x 2 minus x 3 less than equal to 2, then second constraint is twice x 1 minus x 2 plus 5 x 3 less than equal to 6 and the third one 4 x 1 plus x 2 plus x 3 less than equal to 6, and that x i should be positive, all decision variable should be positive, it cannot be negative and i equal to 1, 2, 3.

So, in this case the number of variables is 3, and we have 3 constraint. So, what is the first step? Ok. The first step is that I have to convert this particular equation because it is a less than equally type equation. So, I have to convert them to equally type equation, equality type constraint. So, how we will do that? So, in the first equation I have added this slack variable, so by adding this slack variable I am making it equal, ok.

So, now, the equation is twice x 1 plus x 2 minus x 3 plus x 4, so this is the slack variable I am adding and now it is equal to 2. Similarly, the second one is also less than equally type constraint; so therefore, I am adding another slack variable that is x 5 to make it equal. And the third one is also less than equality type and therefore, I am adding x 6, ok. So, another slack variable to make it equality type constraint. And then I am writing the objective function. So, objective function is minus x 1 minus twice x 2 minus x 3 and I am writing minus f, ok. So, f also I am taking on this side which is equal to 0, ok or you can say that if you take f on the other side, so then f equal to minus x 1 minus twice x 2 minus x 3 equal to f, but I am putting f on this side. So, therefore, this is minus f equal to 0.

Now, I have these equations, ok. So, I have this optimization problem and I convert this optimization problem to standard form, ok, so standard form, ok. So, you are getting this optimization problem in standard form. Now, you just see here the beauty of adding the slack variable, the slack variable x 4 is added to equation 1 and therefore, the coefficient of x 4 in equation 1 is 1 and for others, ok.

So, this is 0 x 4, this is 0 x 4, this is 0 x 4, ok. So, you just see. So, now I am getting a variable x 4, so whose coefficient in equation 1 is 1 and for other equation that is 0. Similarly, for x 5 this is plus 0 x 5, ok, this is plus 0 x 5, and this is plus 0 x 5, ok. So, here also coefficient of x 5 is 1 and for other equation that is 0. Similarly, x 6, here it is 0 x 6 plus 0 x 6 and here it is plus 0 x 6, ok. So, here the coefficient of x 6 is 1 and for other equations that is 0.

So, therefore, if I consider that x 4, x 5 and x 6 are basic variable and other variables that is x 1, x 2, x 3, are non-basic variable. So, what is the initial basic solution? Initial basic solution is that x 1, x 2 and x 3 are 0 because they are non-basic variable, and basic variable x 4, x 5 and x 6, so x 4 equal to 2, x 5 equal to 6, and x 6 equal to 6.

So, this is one feasible solution, anyway. So, the optimal or objective function value is 0 here. Now, but this is one of the solution, it is not optimal solution, but this is one solution or we can say this is a initial basic solution or initial basic feasible solution.

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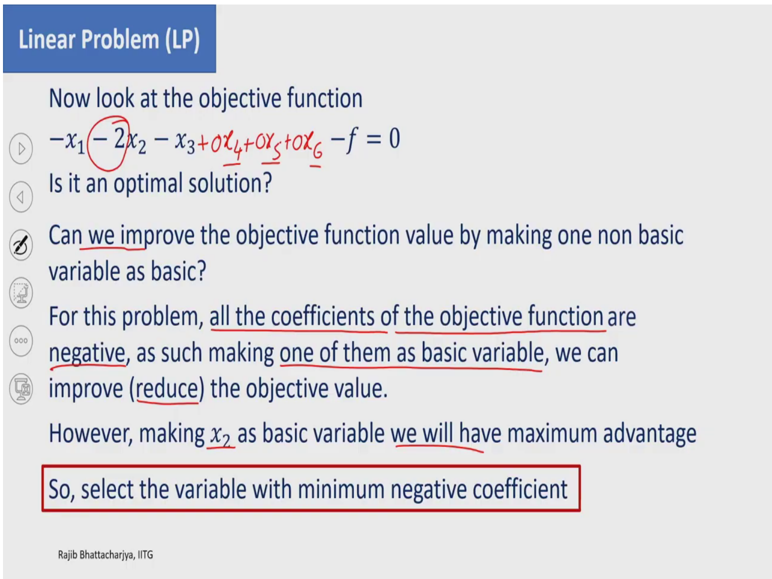

Now, look at the objective function. So, objective function is minus x 1 minus twice x 2 minus x 3 minus f equal to 0, ok. Now, we have here that is 0 x 4 plus 0 x 5 plus 0 x 6, ok. So, these are basic variable. Now, what I want to do basically, so I want to make one of them, one of them means one of x 1, x 2 and x 3 are basic variable and then one of them x 4, x 5 and x 6 will be non-basic variable.

So, if I say that I would like to make x 1 as a basic variable, I can also make x 2 as a basic variable or I can make x 3 as a basic variable, ok, so in place of x 4, x 5 and x 6. Now, I have to take a decision that which one will be the basic variable, ok. Now, let us look at this particular objective function.

Now, here you just see all are negative and this is a minimization problem; that means, if I make one of them as a non-basic variable my objective function will reduce, ok, so is not it. So, if I make one of them suppose x 1, x 2, because all 3 are negative, and coefficient of all 3 are negative. So therefore, if I make one of them as a basic variable my objective function will reduce, ok.

So, therefore, the question was can we improve the objective function value by making one non-basic variable as basic. So, this is the question. Yes, because coefficient of all of this x 1, x 2 and x 3 are basically negative. So, therefore, for this problem all the coefficients of the objective function are negative, ok, so is negative.

So, as such making one of them as basic variable we can improve or we can improve in this case because it is a minimization problem, we can reduce the objective function or value of the objective function, ok. Now, if we make x 2 as a basic variable, so we will have maximum advantage. Why we will get maximum advantage? Because x 2 has the largest negative coefficient out of x 1 and x 3, ok.

So, we will get more advantage if we consider x 2 as a basic variable in place of any other, suppose in place of x 4, x 5 and x 6. So, therefore, what we will do? So, we will select the variable with minimum negative coefficient, ok. So, what is the first rule? The first rule is we will select the variable.

So, we have to take a decision whether we will make x 1 as a basic variable, x 2 as a basic variable or x 3 as a basic variable. So, out of these 3 we have to select one, and we can select any one of them because. So if we select x 1 then also objective function will reduce, if we select x 2 then also objective function will reduce, and if we take x 3 then also objective function will reduce; that means, we will get a improved solution.

But question is that, what is the maximum advantage we will get? So, we will get the maximum advantage if we consider x 2 as a basic variable, is not it. Because the coefficient of x 2 is minimum out of these 3 variable, ok. So, therefore, the rule is select the variable with minimum negative coefficient, ok. So, what is the first rule? Select the variable with minimum negative coefficient, ok.

So, that is the first rule. So, out of these, so we will select the coefficient having minimum negative values, ok. So, this is the first rule.

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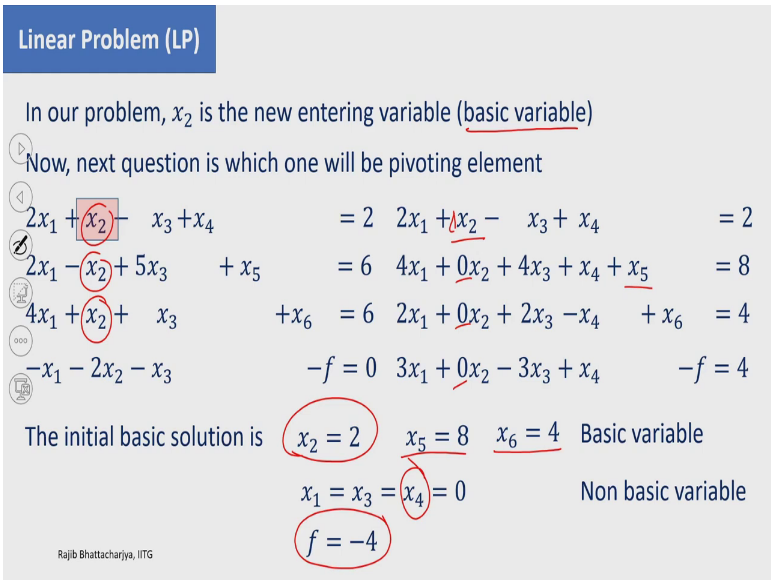

So, now in our problem x 2 is the new entering variable, so we will call the basic variable. So, this will be the basic variable. Now, the next question is which one will be pivoting element, ok. So, the pivoting element may be this one, this one or this one, ok. It may be this one, it may be this one, it may be this one. So, we have to take a decision at that.

Now, let us solve. So, if I consider this as a pivoting element, so I can apply the row operation as we have discussed already. So, if I do that, so the coefficient of x 2 in the first equation is 1 and for other equations it is 0, ok. So, now, this is another solution I am getting. So, what is the solution now?

Now, x 2 is a basic variable. So, therefore, from the first equation I am getting x 2 equal to 2, and then x 5 is another basic variable x 5 equal to 8, x 6 equal to 4, and x 1, x 3, and x 4 now; so, x 4 earlier it was a basic variable, now it is a non-basic variable, so x 1, x 3 and x 4 equal to 0. So, you are getting some improvement. So, what improvement you are getting? That f equal to minus 4 and earlier value of f was 0; that means, your objective function has improved. So, you have got a improved solution, ok.

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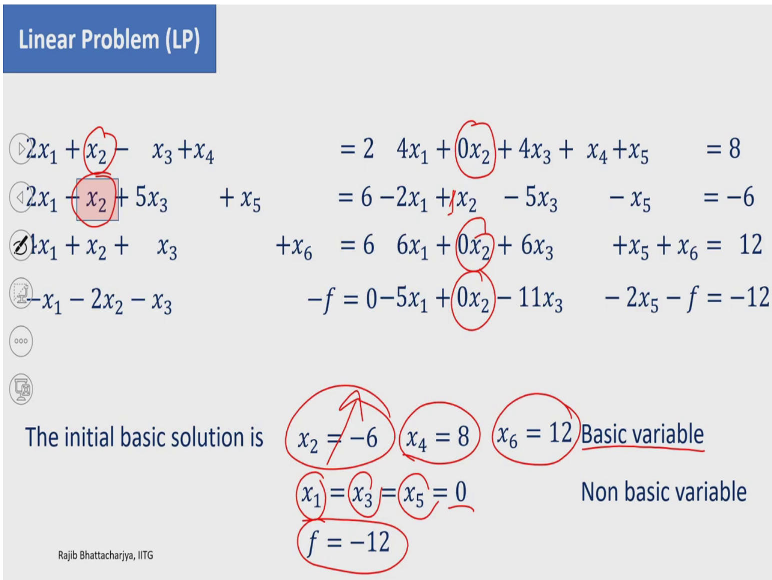

Now, let us consider the second one as a pivoting element. So, in this case, I am considering the x 2 in the second equation. So, x 2 is the incoming variable that will be the basic variable, but out of these 3 that x 2 in the first equation, x 2 in the second equation, x 2 in the third equation which one will be pivoting one. So, when we have taken x 2, so we got a solution.

Now, you consider the x 2 in the second equation. So, what we will do the coefficient of x 2 in this equation will be 1. So, let us do that. So, now we are getting this solution that here the coefficient of x 2 is 0, here it is 0, here it is 0, and here it is 1, ok. Now, if this is the condition that what is the solution I am getting? x 2 equal to minus 6, then x 4 equal to 8 and x 6 I am getting 12. So, these are basic variable. And non-basic variable is x 1, x 3, and x 5 which is equal to 0, and f equal to minus 12.

So, we are getting improvement. But what is the problem here? Here this is not acceptable, ok. So, all variables that is x 1, x 2 and x 3 should be positive, but in this case we are getting x 2 equal to minus 6. So, therefore, this is not a feasible solution, ok. So, if you consider this x 2 in the second equation as a pivoting element. So in that case we are getting a solution, but that is not a feasible solution because we are getting x 2 equal to minus 6.

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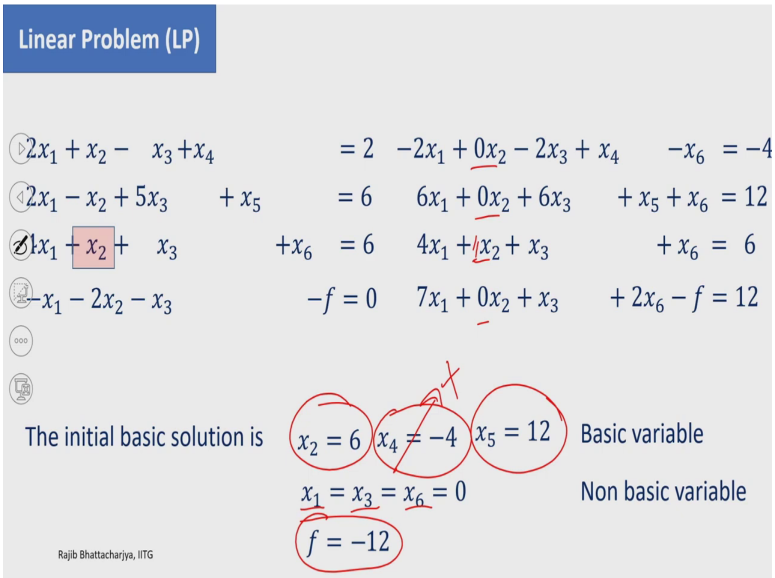

Now, let us consider the x 2 in the third equation as a pivoting element. So, in that case, what we will get? So, here we are getting 0, 0, and this is 1, ok, and this is 0 and now the solution is that x 2 equal to 6, x 4 equal to minus 4, x 5 equal to 12, and x 1, x 3, and x 6 they are non-basic variable, and f equal to minus 12. You just see.

So, now we are not getting a feasible solution again because this is not acceptable, ok. So, x 4 equal to minus 4. So, we are not getting feasible solution. So, you just see. So, we cannot take one of them as a pivoting element. So, we have to have some rules, so that we are not evaluating the infeasible solution, ok.

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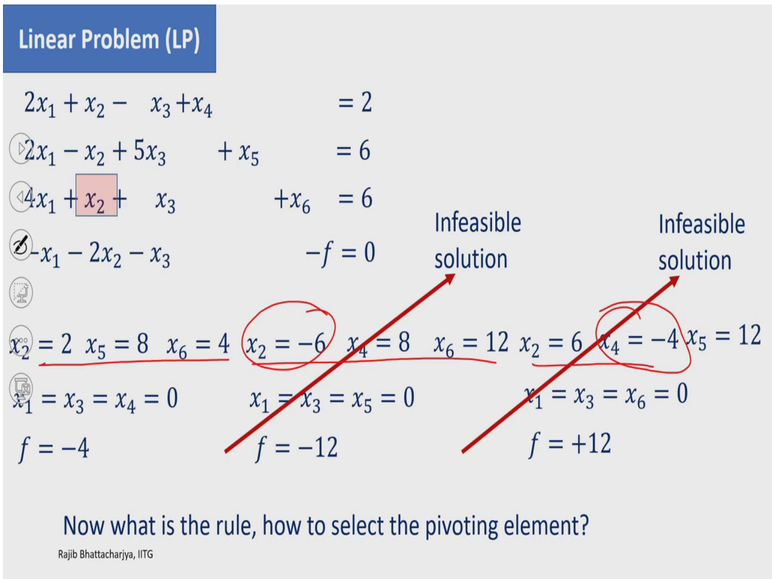

So, if I summarize this thing when we consider the x 2 in the first equation as a pivoting element, so we are getting this solution, we are getting this solution. And when we have taken x 2 in the second equation, so we are getting this solution. And when we have taken x 2 in the third equation as a pivoting element, so we are getting this solution, ok. So, as I said this is infeasible solution and this is also infeasible solution, because here the x 2 equal to minus 6 and here x 4 equal to minus 4.

So, that is not acceptable because all this decision variable should be positive. So, now, what is the rule then? How to select the pivoting element? So, we have to select the pivoting element in such a way, so that we are not getting infeasible solution, ok and we are getting only feasible improved solution, ok.

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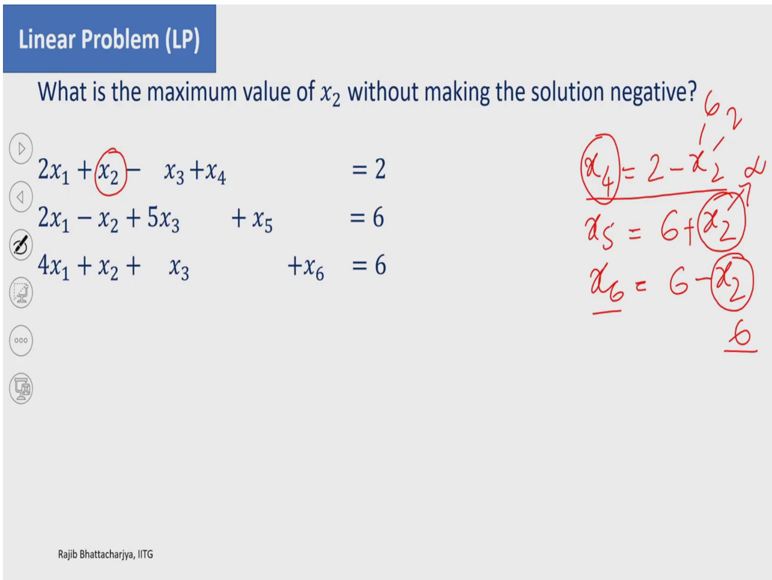

Now, what is the rule here? So, before discussing the rule, so let us see this one that I would like to make x 2 as a pivoting element. So, what will be the value of x 4? Ok. So, x 4 will be equal to 2 minus x 2, ok. So, here x 5 equal to 6 minus x 2 and x 6 equal to 6 minus x 2, sorry, this will be positive.

So, now if I consider the first equation, ok, so what is the maximum value of x 2? So, I can go up to 2. So, in that case x 4 will not be negative, ok. So, that is the maximum value. Now, in this case, if I consider this one, so in this case I can go up to infinity, is not it. I can go up to infinity and x 5 will never be negative. So, I can say in this case this is unbounded, ok.

And in the third, if I consider the third equation, the maximum value of x 2 is 6, so in that case x 6 will not be negative. But question is that if I consider this one then for the first equation that is x 4 if I put x 2 equal to 6, so in that case x 4 will be negative and then I will get the infeasible solution.

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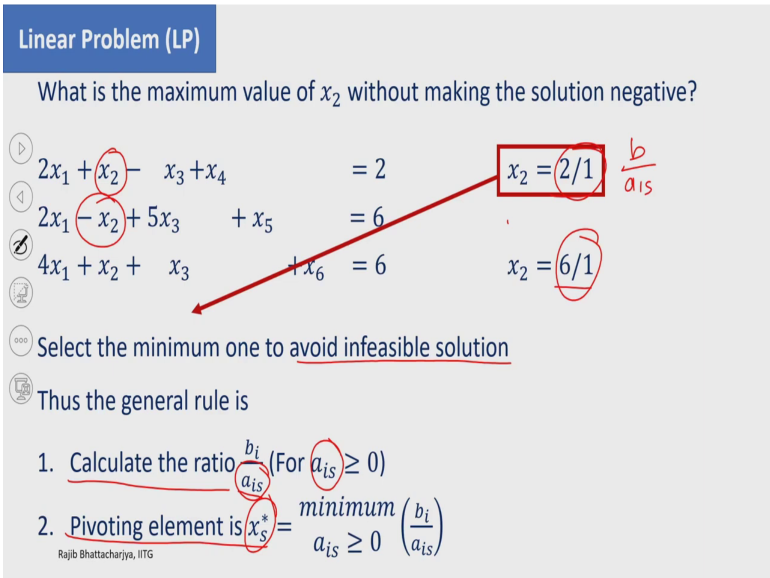

So, therefore, what is the rule? So, what we will do? We will take the ratio of b by a is, ok. So, this is the incoming variable x 2 and coefficient of x 2 in the first equation is 1, and so, b by the coefficient of x 2 is basically 2 by 1 in this case, and here this is 6 by 1, the coefficient of x 2 in this equation is negative. So, as I said, so we will get unbounded solution.

Now, the rule is select the minimum one because if we select this one then we will not get any negative value of the variables, ok. So, select the minimum one to avoid infeasible solution. So, what is the rule now? Thus, the rule is calculate the ratio b i by a is. So, a is the coefficient of incoming variable and a is should be positive. So, if it is negative, so we will not consider.

And then pivoting element, so we can say x s star, so that is the minimum of this one, ok. So, minimum of this ratio b i by a is. So, here in this case the minimum is this one, ok. So, this is 2 and this is 6, so we will consider the minimum one. So, otherwise what will happen? So, we will get the infeasible solution. I hope rule is clear.

Now, the second rule is we will calculate the ratio between the right hand side b i divided by a is for all these 3 equations and we will select the minimum one.

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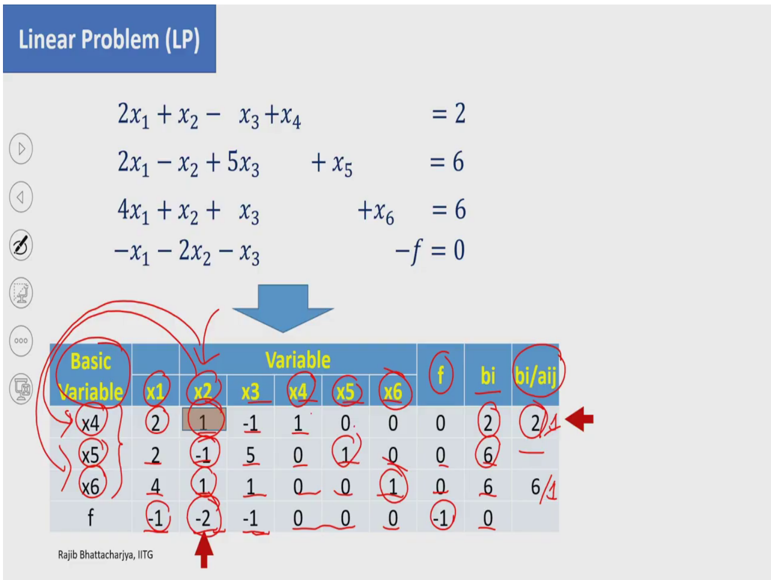

So, now, let us prepared a table, ok. So, here I am writing the basic variable. So, what are the basic variable? Basic variable is x 4, x 5 and x 6. I would like to do it from the beginning. And here I am writing the variable this is x 1, x 2, x 3, x 4, x 5, x 6 and this is the objective function, this is b i, and this is b i by a ij, ok.

So, what we are doing here? We are writing the coefficient of x 1 in the first equation that is 2, here it is 1, this is minus 1, and this is 1, this is 0, 0, 0. And similarly, equation 2 this is 2, then minus 1, plus 5, then this is 0, then 1, 0, 0, right hand side is 6. And the third equation that is 4, then 1, then 1, 0, 0, 1, 0, and 6. And the objective function, the coefficient of x 1 is minus 1, this is minus 2, minus 1, 0, 0, 0 and this is f which is minus 1 and this is 0, ok.

So, initially, so what are the basic variable? The basic variable are x 4, x 5 and x 6. So, why they are basic variable? Because the coefficient of x 4 in the first equation is 1, for the other equations that is 0. Similarly, x 5 the coefficient of x 5 in the second equation is 1, for other equations it is 0. And x 6, in the third equation, the coefficient is 1 and for other equation it is 0. So, therefore, x 4, x 5 and x 6, so these 3 are the basic variable and x 1, x 2, x 3, are non-basic variable, ok.

So, now, what is the rule? Rule is, first rule is you check whether there is any negative coefficient in the objective function, ok. So, you just see all of them are negative; that means, coefficient of x 1 in the objective function is minus 1 that is negative, for x 2 that is also negative, for x 3 that is also negative. So, all are negative first. Then second observation is which one is the maximum one.

So, maximum one is minus 2 and that is for x 2 variable. So, therefore, x 2 will be the incoming variable. So, x 2 will come as a basic variable now, then we have to take a decision whether x 4 is going out, x 5 is going out or x 6 is going out, ok, so which one will be replaced by x 2. So, x 4, x 5 and x 6, so out of that one will be replaced by x 2, but which one. So, what we will do? We will calculate the ratio, ok.

So, b i by a ij, so in the first equation that right hand side is 2 divided by 1, so I am getting 2. And second I am not considering because the coefficient is negative. So, therefore, I am not considering here. And the third one, the coefficient is positive that is 1, so 6 by 1, so 6 by 1, so I am getting 6 and this is 2 by 1 I am getting 2. So, what I will do? I will consider minimum one. If it is minimum then this will be your pivoting element. I hope this is clear.

So, if this is the pivoting element then x 2 will come here; that means, x 2 will replace x 4. So, x 4 will leave the basis. So, x 4 will no longer be a basic variable. Now, x 2 will be a basic variable; that means, x 2, x 5, and x 6 will be basic variable. So, now x 4 will replace x 2, so x 4, x 5 and x 6 will be basic variable and other variables will be non-basic variable.

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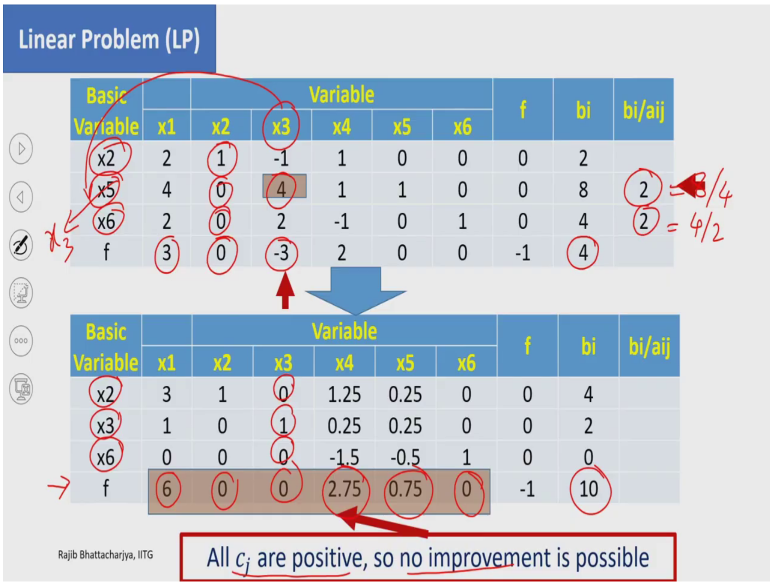

Now, let us see the table. So, if I apply the row operation. So, what is the purpose of row operation here now? So, I would like to make this is 1 and for other these are 0, ok. So, now, x 2 has entered as a basic variable. So, x 2, x 5 and x 6, so they are basic variable and as I said, so objective function value is minus 4, ok.

Now, are we getting the optimal solution? Now, what we have to say; so, is there any negative coefficient in the objective function, ok. So, here the coefficient of x 1 is 3, coefficient of x 2 is 0, x 3 is minus 3; that means, there is a negative coefficient. So, therefore, that means, we can get an improved solution. So, what we will do?

So, this is the negatives. So, therefore, the x 3 will come as a basic variable. Now, whether x 3 will replace x 2, x 5 or x 6, so what we have to do? We have to find out the ratio here, ok.

So, b by a ij ratio. So, now, x 3 is the incoming variable. So, here this is 8 by 4 that means, 2 and here it is 4 by 2, so 4 by 2 so that is 2.

So, you just see, both are equal. So, if both are equal, so as per rule you should take the minimum one, so when both are equal, so you take one of them, ok. So, you take one of them, so let us take this one, ok. So, in that case this will be now my pivoting element, ok. So, if it is pivoting element then x 5 will be replaced by x 3, ok.

So, x 3 will come as a basic variable. So, x 3 will come on the basis and x 5 will go as a non-basic variable. So, if I do that, so we are getting this table. So, now, x 2 is a basic variable, x 3 is a basic variable, x 6 is a basic variable, and here in x 3, so this is 0, this is 1, and this is 0, and this is 0. And what solution you are getting? Solution you are getting minus 10, ok.

So, now if you look at the last row that is objective function row, so you can see what is the coefficient of x 1, this is positive; coefficient of x 2, this is 0; coefficient of x 3, this is 0; x 4 that is 2.75, x 5 this is 0.75, x 6 is equal to 0; that means, there is no negative coefficient. So, therefore, you will not get any improved solution. So, whatever solution you are getting, so minus 10.

So, that is the optimal solution, you will not get any improved solution. So, what is the solution? The solution is x 2 equal to 4, x 3 equal to 2, and x 6 equal to 0, and x 4 and x 5 is also 0 because they are non-basic variable, ok. So, all c j are positive. So, no improvement is possible. So, no improvement is possible. So, what you will do basically? So, you will look at the coefficient of the variable in the objective function and if it is positive, ok, so no improvement is possible; that means, you got the optimal solution. I hope this is clear.

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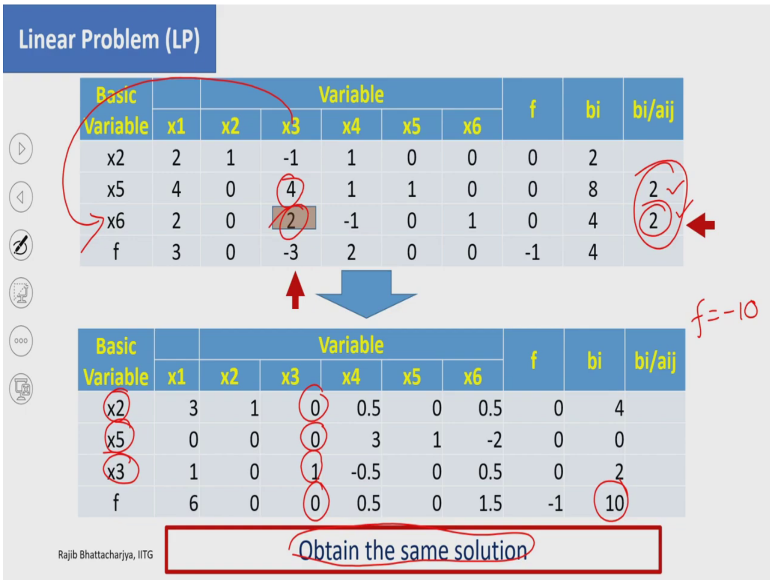

So, now what I can do? So, let us see if I consider this as an pivoting element. So, if I consider this as a pivoting element. So, earlier we have considered this, but now because as I said, so when this ratio is equal, so ratio is 2 and 2, so you can select one of them. So, if I select the second one, earlier we have selected the first one, if I select the second one then this will be your pivoting element. So, in that case what will happen? x 3 will come here, x 3 will replace x 6, ok. Now, what is the solution? Ok.

Now, if I do that, now x 3 is coming here x 3 is replacing x 6. So, x 3 is a basic variable, x 2 is already a basic variable, x 5 is as a basic variable and here I am getting 0, 0, 1, and 0. And now you look at, so what is the optimal solution? The optimal solution is 10, f equal to f equal to minus 10. So, in earlier case also you are getting minus 10, now also you are getting minus 10 basically.

So, what we can say, that now we are getting an alternate optimal solution. So, what we are getting now? So, we are getting the same solution, is not it. So, we are getting the same solution that is x 2 equal to 4 and x 3 equal to 2, ok. So, we are getting the same solution, so this is.

So, therefore, the rule says that you can take one of them; if there is a tie the both are equal, so you can take one of them and you will get the same solution. So, what is the rule now? So, rule now is that you check is there any negative coefficient in the objective function. So, if there is any negative coefficient then you will get an improved solution, and we will select the maximum negative value, ok. So, we will select the variable having maximum negative coefficient.

So, this is the first rule. So, that means, that variable will be an incoming variable. So, that variable will come as a basic variable. Now, question is that which one is the pivoting element. So, for deciding that one, so what we will do? We will calculate the ratio between b i divided by a ij, and we will take the minimum one. And if there is a tie we will take just one of them and you can take one of them as I have shown here that, if you take one of them you will get the same solution.

You can check actually, so you will get the same solution. So, therefore, you can take one of them. So, these two are the rules. So, first thing you will check is there any negative coefficient and we will select the maximum negative coefficient in the objective function, so that variable will be considered as an incoming variable. So, that variable will come as a basic variable and then pivoting element will be selected by considering this ratio, ok.

So, this is all about for today's class. So, I have explained the rule here. So, in the next class, we will discuss the simplex method. So, this rule will be followed, and we will try to solve some optimization problem.

Thank you.