

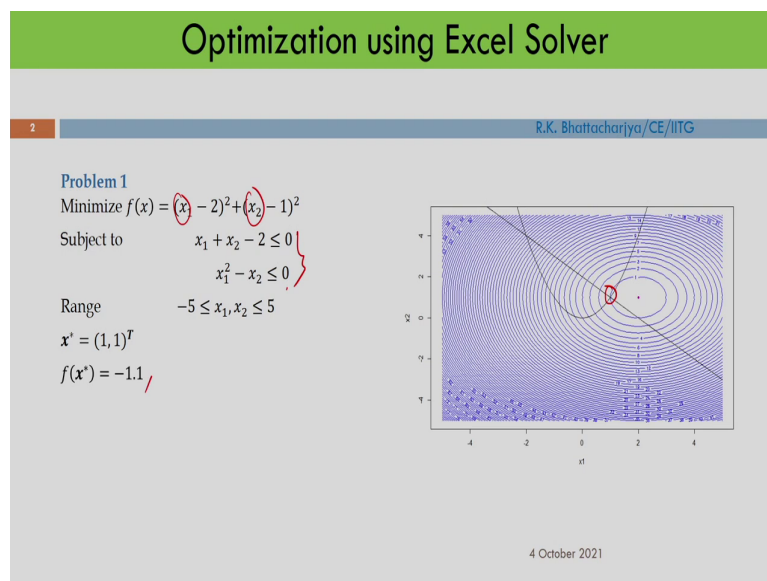
Optimization Methods for Civil Engineering
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Lecture - 36
Optimization Using Excel Solver

Hello student. Welcome back to the course on Optimization Methods for Civil Engineering. So, in today's class, so, we will learn how to solve a non-linear Optimization problem Using Excel Solver. So, we have already solved a linear problem using Excel Solver. And today we will see how we can solve a non-linear problem with constrain using Excel Solver. So, let us see.

So, we will consider few example problems and then I will show you how you can solve a partial differential equation using Excel Solver.

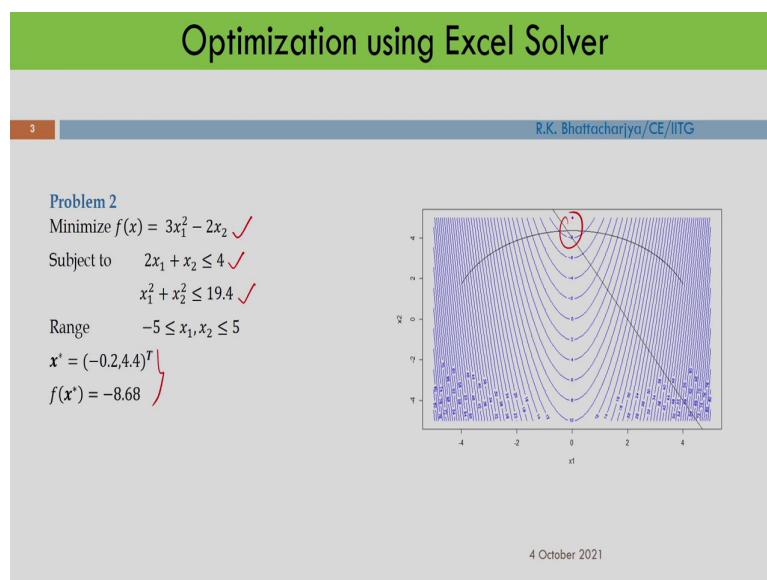
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So, this is the 1st problem I will consider. This is a two variable problem. So, we have x_1 and x_2 and there are two constrain in this particular problem. So, the first constrain is a linear constrain and second one is non-linear constrain. So, the solution is 1, 1 and I should get the optimal function value is minus 1.1.

So, this problem we have already solved using R software. So, you can see that this is the constrain solution of this particular problem and I should also get this particular solution when you are when you are solving this problem using excel.

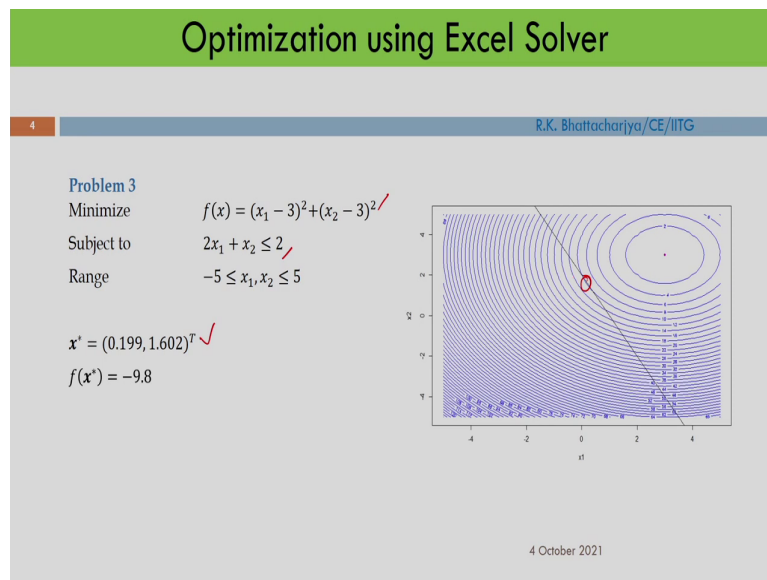
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The 2nd problem is also a two variable problem. So, we have a non-linear objective function. So, we have a non-linear objective function. So, this is your objective function and we have two constrains here. The first one is linear and second one is non-linear constrain. So, here I should get this particular solution and somewhere here this is the constrain solution of this

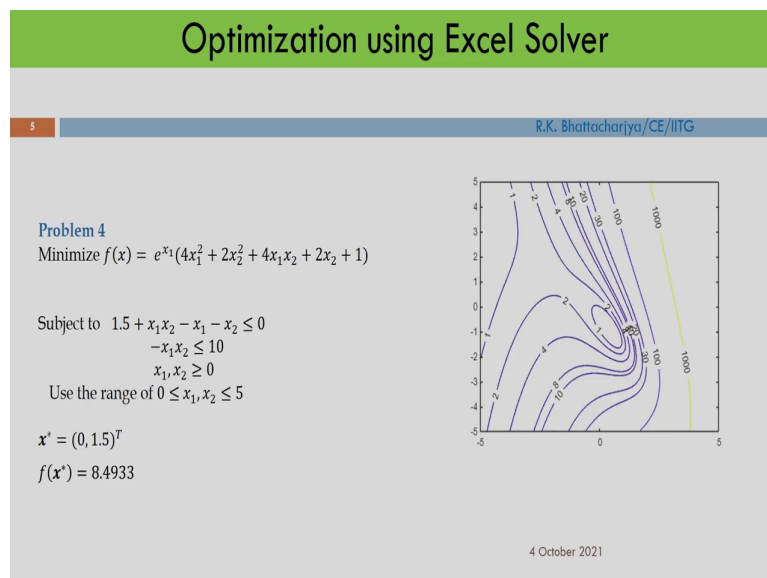
particular problem. So, we have also solved this problem using R. So, let us see whether we are getting the similar result in excel.

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The 3rd problem is also a non-linear problem with one linear constrain. So, objective function is non-linear and if you are solving this then you should get this particular solution and somewhere here is the optimal solution of this particular problem. So, I will try to solve this problem.

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And the 4th one is a little bit complicated problem to solve using classical optimization technique, but let us see whether we are getting the solution using Excel Solver. So, we have already solved this problem using genetic algorithm and we got the constrain optimal solution of this problem and the solution is 0 and 1.5 and objective function value is 8.4933 ok.

So, here this is the objective function and we have two constrains ok. So, this is first constrain this is second constrain and x_1, x_2 are positive and range is between 0 and 5. That means, x_1 is between 0 and 5 and x_2 is also between 0 and 5. Let us see how you can solve a partial differential equation using Excel Solver. So, I will show you, you can use the optimization technique for solving a partial differential equation.

(Refer Slide Time: 04:20)

Optimization using Excel Solver

Solution of partial differential equation using Excel Solver

R.K. Singh, Professor, CE/IITG

The 2-D steady flow equation for homogeneous isotropic confined aquifer may be written as

$$T \nabla^2 \phi + N(x, y) = 0 \quad (1)$$

Where, T is the transmissivity of the aquifer (m^2/day), ∇^2 is the Laplace operator, ϕ is the hydraulic head (m), N is the pumping or recharge value ($\text{m}^3/\text{day}/\text{m}^2$).

The finite difference approximation of the steady 2-D flow equation at cell (i, j) may be expressed as

$$A\phi_{i+1,j} + A\phi_{i-1,j} + B\phi_{i,j+1} + B\phi_{i,j-1} - (2A + 2B)\phi_{i,j} + N(i, j) = 0 \quad (2)$$

Where, $A = (T/(\Delta x)^2)$, $B = (T/(\Delta y)^2)$, Δx (m) is the grid size in x direction and Δy (m) is the grid size in y direction.

$T \frac{\partial^2 \phi}{\partial x^2} + T \frac{\partial^2 \phi}{\partial y^2} + N = 0$

flow equation

4 October 2021

Here I would like to solve a partial differential equation using Excel Solver then this is related to the flow equation. So, this is nothing but the flow equation for homogeneous isotropic confined aquifer. So, if you have gone through groundwater hydrology, so, I think you have seen this particular equation. So, this is the flow equation for a case of homogeneous and isotropic confined aquifer ok.

So, this is the equation here. So, this equation is $T \nabla^2 \phi + N = 0$. So, this is the source term and this is equal to 0. So, I would like to solve this equation using Excel Solver. Here T is the transmissivity of the aquifer the unit is meter square per day and ∇^2 is the Laplace operator, ϕ is the hydraulic head. So, that I would like to calculate and N is the pumping or recharge value. So, this is the source term and unit is meter cube per day meter square.

So, how I am solving this equation? So, I am solving this equation using finite difference approximation ok of this 2D flow equation and I am not showing how you can do the finite difference approximation, but if you are applying finite difference approximation. So, finally, you will get this equation ok you will get this equation and I would like to solve this equation and where A equal to T by del x square. So, this is the grid size and B equal to T by del y square. So, this is the grid size and del x and del y is the grid size in x direction and y direction.

(Refer Slide Time: 06:27)

Optimization using Excel Solver

Simulation Methodology

R.K. Bhattacharjya/CE/IITG

The simulation model solves the set of discretized equations written at each grid centre of the discretized aquifer. These discretized equations may be solved using the embedded optimization approach. The optimization model may be written as

Minimize $f = \sum_{i=1}^I \sum_{j=1}^J e_{i,j}^2$ (3) $i=1, \dots, I$
 $j=1, \dots, J$

Subject to

$e_{i,j} = A\phi_{i+1,j} + A\phi_{i-1,j} + B\phi_{i,j+1} + B\phi_{i,j-1} - (2A + 2B)\phi_{i,j} + N(i,j) = 0$ (4)

$\phi_{i,j} \geq 0$ $e_{i,j} \leq \epsilon = 0.001$
 ≥ 0.0001 (5)

Where, $i = 1, 2, 3, \dots, I$ and $j = 1, 2, 3, \dots, J$, I and J are the total number of columns and rows of the discretized aquifer.

The nonlinear optimization model can be solved using the Excel solver.

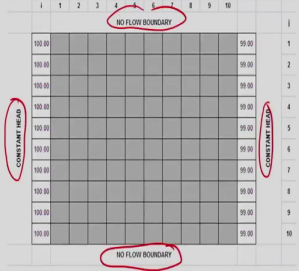
4 October 2021

So, how we will solve this equation? So, what I will do basically, so, let me go to the next slide and then I will come to this particular slide.

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Optimization using Excel Solver

B
R.K. Bhattachariya/CE/IITG



$T = 300 \text{ m}^2/\text{day}$

$\Delta x = 100 \text{ m}$
 $\Delta y = 100 \text{ m}$

$$A\varphi_{i+1,j} + A\varphi_{i-1,j} + B\varphi_{i,j+1} + B\varphi_{i,j-1} - (2A + 2B)\varphi_{i,j} + N(i,j) = 0$$

$A = (T/(\Delta x)^2), B = (T/(\Delta y)^2)$

If $A = B$

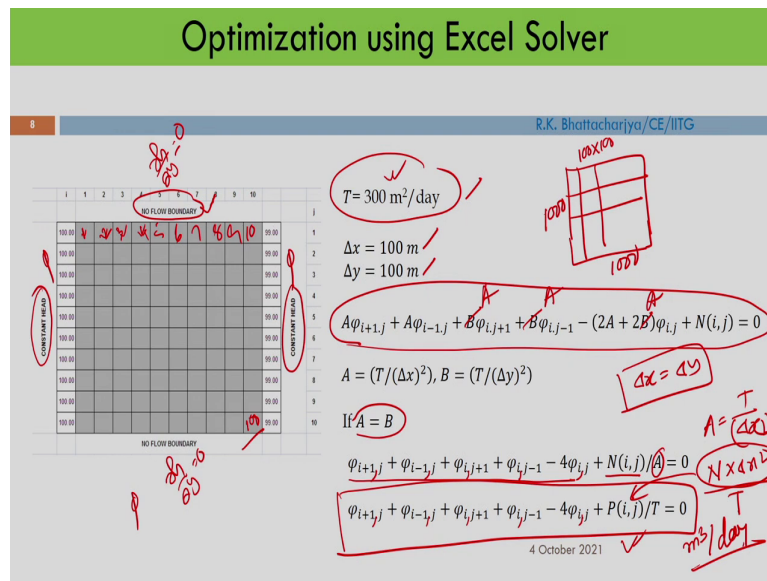
$$\varphi_{i+1,j} + \varphi_{i-1,j} + \varphi_{i,j+1} + \varphi_{i,j-1} - 4\varphi_{i,j} + N(i,j)/A = 0$$

$$\varphi_{i+1,j} + \varphi_{i-1,j} + \varphi_{i,j+1} + \varphi_{i,j-1} - 4\varphi_{i,j} + P(i,j)/T = 0$$

4 October 2021

So, in this case the boundary values are known. Let me explain how we will solve this problem. So, let us consider a rectangular aquifer.

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So, idea is that I would like to find out what is the value of phi at its grid point. So, entire aquifer, so, this is the aquifer and we have divided into some grids. And on the left side this is constant head boundary and we know what is the phi value here and right hand side this is also constant head boundary.

So, we also know the phi value here. And on this side, so, this is no flow boundary that means, the gradient is $\frac{\partial h}{\partial y} = 0$ and here also $\frac{\partial h}{\partial y} = 0$ ok. So, we know this thing now I would like to calculate the value at each grid centre ok. So that means whatever finite difference equation, so, this is the finite difference equation.

So, this has to satisfy at each grid point. So, if I write the finite difference equation suppose I am writing equation 1 here, equation 2 here and this is 3, 4, 5, 6, 7, 8, 9, 10 and this is my 100 equation. So, I will get total 100 equation and if I solve this 100 equation then I will get the

solution of this problem. So, if I can solve all these 100 equation then I will get the solution of this problem.

So, what I am doing here? I am formulating an optimization problem. So, and this actually so, this is the equation that has to be satisfied that has to be satisfied at each grid centre. Now for any arbitrary value, so, right hand side will not be equal to 0. So, I am considering this is e_{ij} . So, e_{ij} is the error. Now, I have formulate an optimization problem that means, I would like to minimize this error ok. So, here grid size is i equal to 1 to 10 and j equal to 1 to 10 ok and basically. So, I am getting e_{ij} .

And I am just squaring it just to make it positive. So, because this error should be positive I would like to minimize this error. So, I am squaring it or you can also use the absolute value. So, either I can write e_{ij}^2 or $|e_{ij}|$ ok absolute value. So, both are ok. So, I will use this one here and this is e_{ij}^2 and this should be equal to 0, but what I will do? I will make e_{ij}^2 less than equal to some absolute value ok.

So, may be this is 0.001 or 0.0001 something like that ok. So, it will not be 0, but it will be near to 0. So, I will solve this problem using Excel Solver. So, what I will do? I will use this particular I will use this particular equation. Now, let me go to the next slide. So, here this is the value given that T equal to 300 m square per day ok meter square per day.

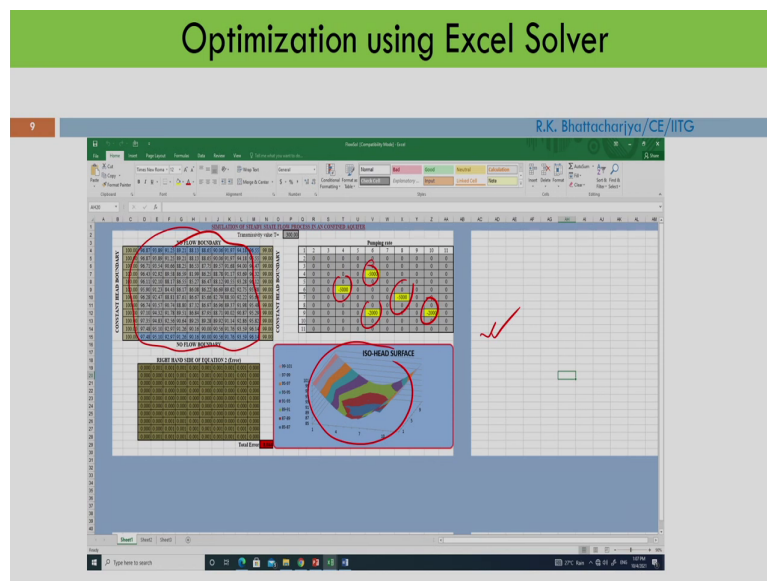
So, I have used Δx equal to 100, Δy equal to 100 and the aquifer size is 1000 by 1000. So, this is 1000 meter and this is also 1000 meter and I have divided in grid and one grid size is 100 by 100 ok and I have used T equal to 300 meter square per day ok. Now, in this case in my case, so, Δx equal to Δy just to simplify the problem. So, you can also use different Δx Δy , but here I am considering Δx equal to Δy .

So, if you are doing that so that means, A equal to B and if I put A equal to B here, so, finally, what I am getting that this will be your A . This will be your A , this will be your A ok. So, then if I divide this particular equation by A , so, I am getting this equation that is $\phi_{i,j} + 1$ then $\phi_{i,j} - 1$ $\phi_{i,j} + 1$ $\phi_{i,j} - 1$ minus 4 $\phi_{i,j}$ and this is $N_{i,j}$.

And finally, so, if I put what is the value of A, so, A is $T \text{ by } \Delta x^2$. So, what I am getting that what I am getting that N into Δx^2 by T. So, this is nothing but the pumping value that is in meter cube per day basically in this case ok. So, this is the pumping value that is the source term you are telling that is the source term and finally, I am getting this equation ok. So, I am getting this equation. So, I will use this equation here. So, basically using this equation I would like to solve the problem ok.

So, I will also explain. So, I will directly go to excel and I will show you how you can treat this particular problem or how you can solve this particular problem using Excel Solver.

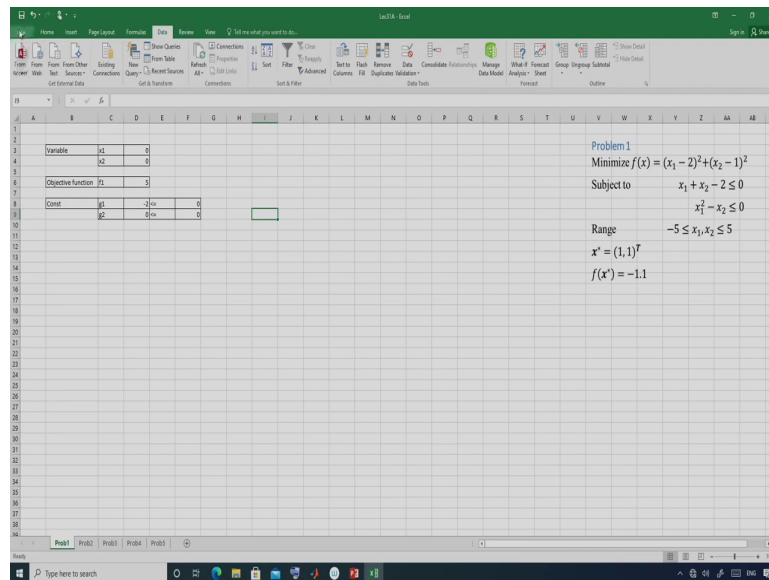
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So, this is the excel. I should get finally, this is the solution of this particular problem. So, I will explain you and this is the surface plot I have done and there is some pumping here, here, here and here. So, with this pumping, so, you should get this is the value of h ok. So, I am

getting somewhere here. So, I will solve this problem using Excel Solver. So, now let us go to Excel Solver and see how I can solve all this problem using Excel Solver.

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Now, let us consider the first problem. So, in the first problem we have in objective function. So, this is the objective function and objective function is x_1 minus 2 whole square plus x_2 minus 1 whole square and we have two constrain and these constrains are inequality constrain that means, less than equal to 2. So, what I will do? I will define the variable here ok. So, I will define the variable here.

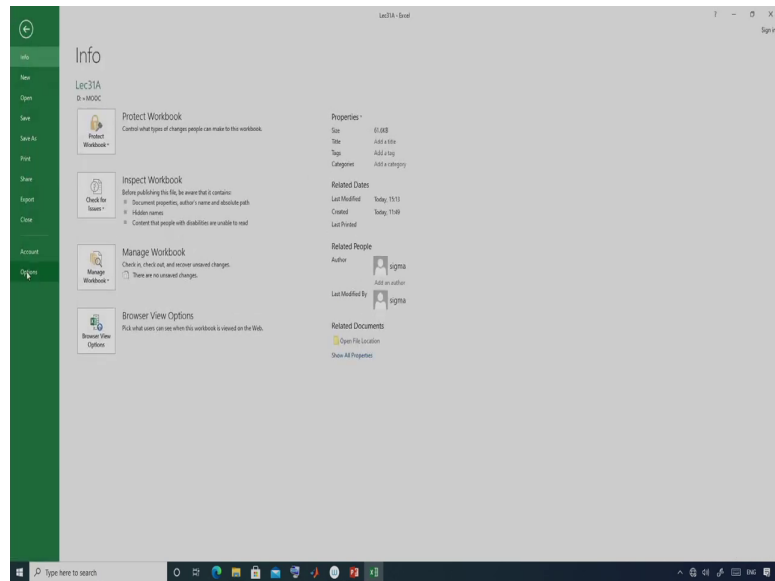
So, this is the variable and we have two variable that is x_1 and this is x_2 ok. So, this is x_2 and you consider some initial value let us start with 0. I would like to start with 0 here. So, this is these two are my variable ok. So, I am I am using these two variable ok and then let us write the objective function here. So, objective function is so, I am writing here.

So, let me write the objective function. So, this is objective ok function and objective function is I am writing f_1 here. So, write the objective function. What is objective function? Objective function is x_1 . So, this is x_1 minus 2 ok. So, this is square plus x_2 minus 1 and this is squared ok. So, this is my objective function. So, I am defining objective function here. So, I am getting for x_1 equal to 0, x_2 equal to 0 objective function value is 5 ok.

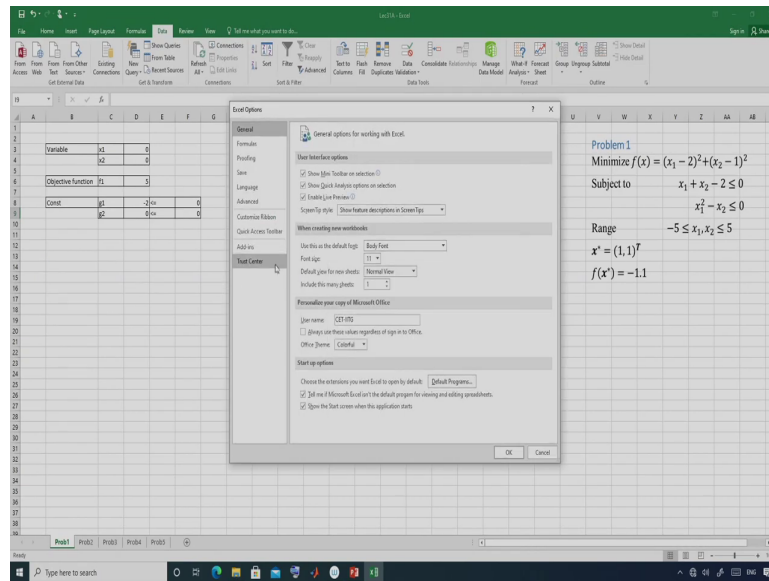
Now, you write constrain. So, we have two constrain here that is g_1 . So, you write g_1 . What is g_1 ? g_1 equal to x_1 plus x_2 minus 2 ok. So, g_1 value for x_1 equal to 0 x_2 equal to 0 this is minus 2 and you also write g_2 . So, g_2 is x_1 square x_1 square minus x_2 ok. So, it is 0. So, I am defining the objective function and constrain here ok.

So, I have defined my problem and this should be less than equal to 0. Is not it? Less than equal to 0 and this should be less than equal to 0. So, this is my constrain, ok. So, now, let us go to Excel Solver. So, just see whether Excel Solver is here. So, there is no Excel Solver here.

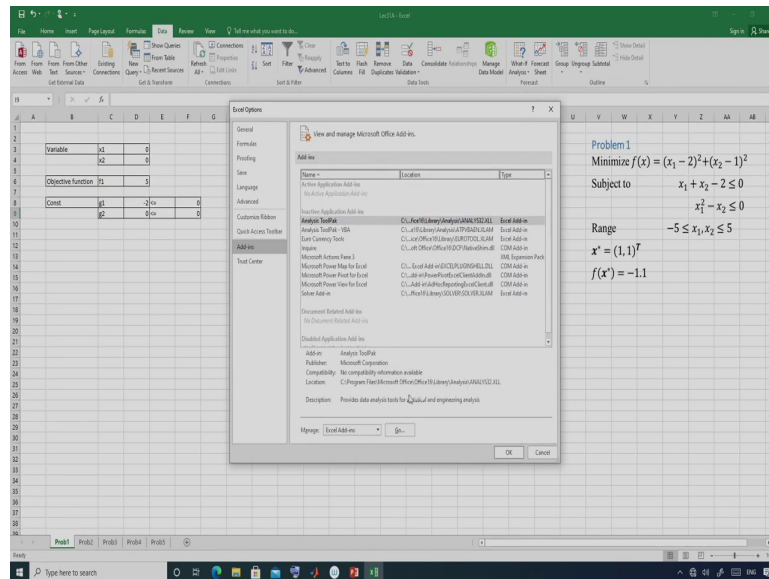
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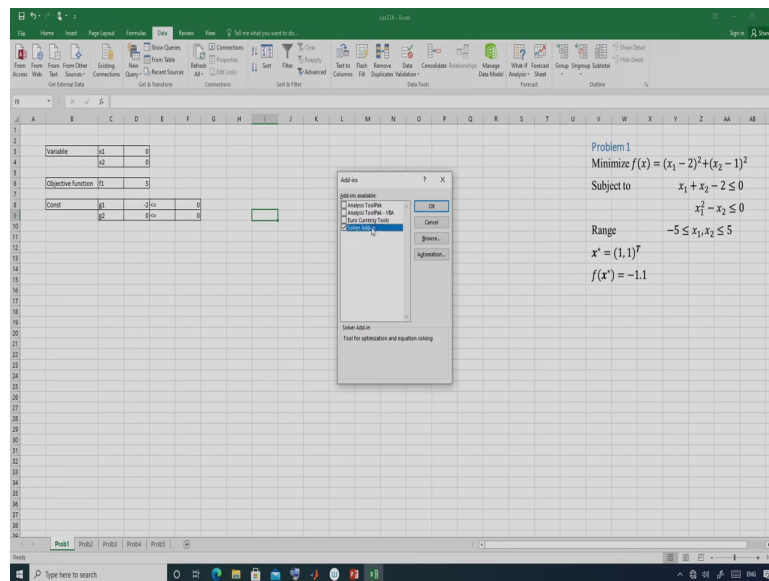
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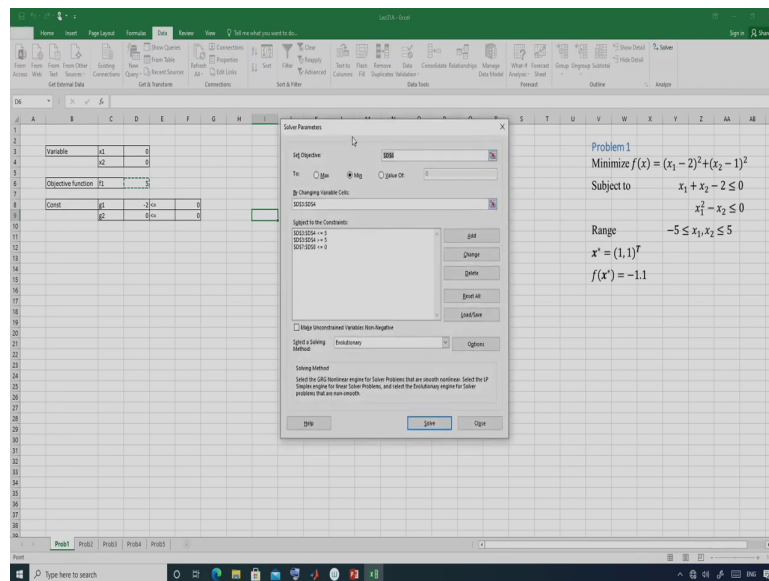


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So, then what I will do? I have to install that one. So, you go to Option. Go to Add-ins ok. So, you go to excel, Add-ins and excel within excel, Add-in. So, you will get solver Add-in ok. So, you just check this particular box and click OK. So, it has been installed and you can see that on the right hand side the Excel Solver is there now. So, this is under data ok. So, under data this is the Excel Solver.

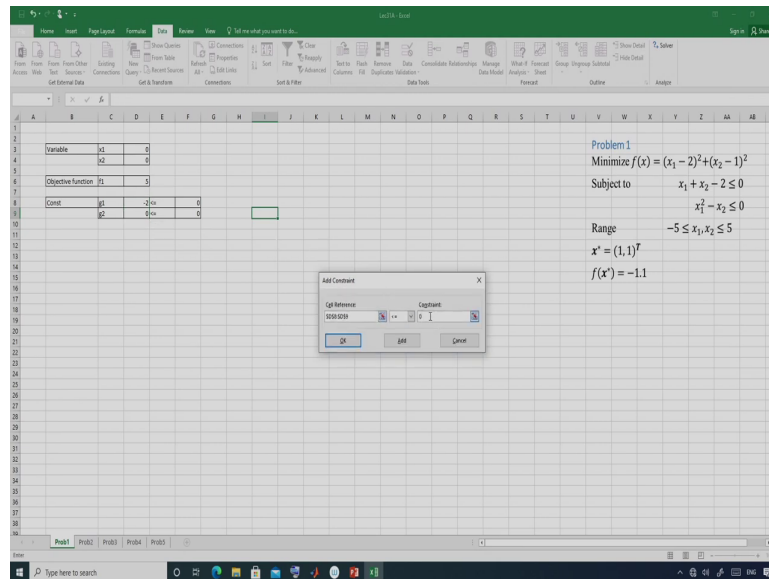
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So, let me go to the Excel Solver. So, already you know that one. So, what I will do? I will delete this portion ok. So, I will delete this, I will delete this portion. So, I will start from the beginning ok. So, here I have to be find the objective cell. So, objective cell is this and I would like to minimize the objective function.

So, I have selected minimization and if your problem is a maximization problem, so, you can select maximization or if it is a value of then you can also select this one. So, here I have to define the cell that means, my variable cell I have to define. So, here these two are my variable cell ok. So, D3 and D4; so, that is my variable cells.

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Then you define the constrain. So, I have two constrain here these two cell basically and this should be less than equal to 0 ok.

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The screenshot shows the Microsoft Excel interface with the Solver Parameters dialog box open. The dialog box is configured with the following settings:

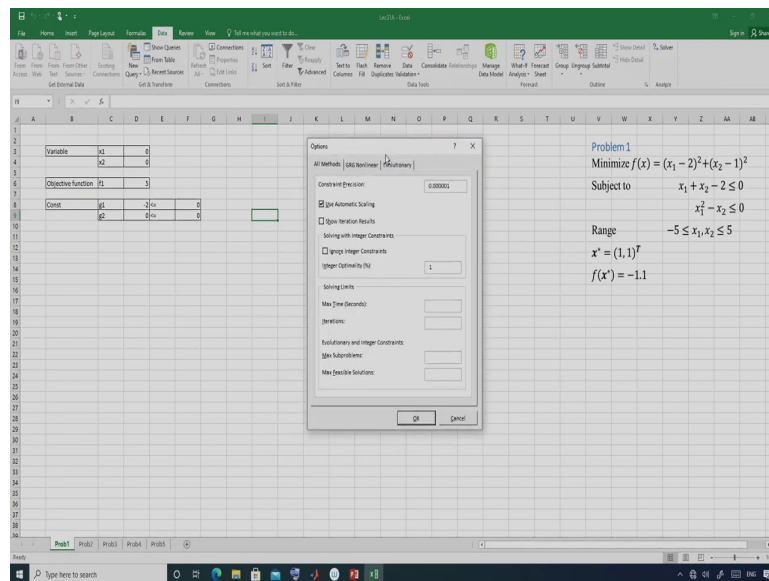
- Set Objective:** \$D\$5
- To:** ☐ Max ☒ Min ☐ Value Of: 0
- By Changing Variable Cells:** \$D\$2:\$D\$3
- Subject to the Constraints:** \$D\$2:\$D\$3 >= 0
- ☒ Make Unconstrained Variables Non-Negative
- Select a Solving Method:** GRG Nonlinear engine
- Solving Method:** Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for Linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons at the bottom of the dialog box include: **Solve**, **Load/Save**, **Options**, **Help**, **Load/Save**, **Options**, **Help**.

Handwritten text on the right side of the screen describes the problem:

Problem 1
Minimize $f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$
Subject to $x_1 + x_2 - 2 \leq 0$
 $x_1^2 - x_2 \leq 0$
Range $-5 \leq x_1, x_2 \leq 5$
 $x^* = (1, 1)^T$
 $f(x^*) = -1.1$

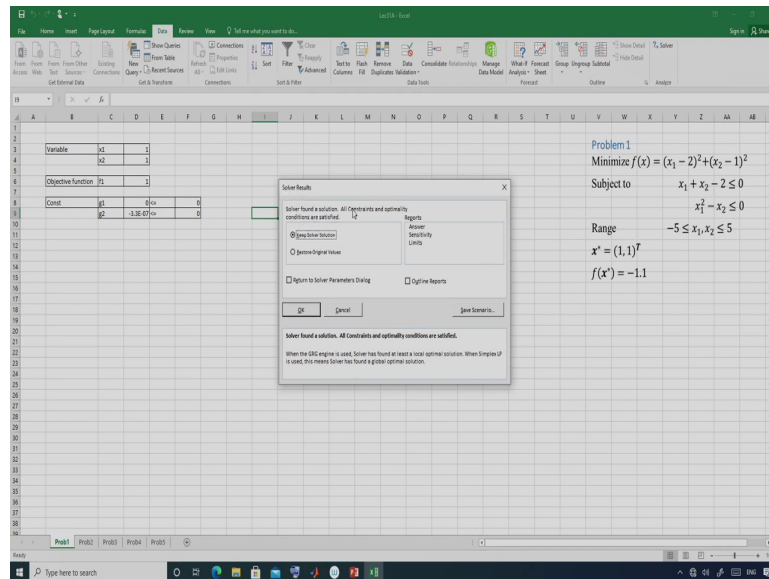
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So, now I have defined the optimization problem here and so, you can make unconstrained variable non negative. So, you can this and I will use mainly simplex sorry I will use the GRG non-linear method ok. So, that I will use you can change the option here, but I am not going to change right now here.

So, I will use the default value and just see whether I am getting the solution or not ok. So, let me run this particular or by I can execute this particular solver. So, using the solve button. So, let me solve this problem yeah.

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So, solver results solver found a solution. So, solver found a solution. All constrain and optimality conditions are satisfied ok. So, I can see the answer here. So, you go to answer and just see the answer.

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Microsoft Excel Solver Report

Worksheet: [Data] and [Sheet1]

Report Created: 04-10-2023 11:58:43

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Solver Engine

Engine: GRG Nonlinear

Solution Time: 0.032 Seconds

Iterations: 4 Subproblems: 0

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision: 0.000001, Use Automatic Scaling

Convergence: 0.0001, Population Size: 100, Random Seed: 0, Derivatives Forward, Assume Non-Negative

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance: 2%, Assume Non-Negative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$D\$6	F5	1	1.0000011

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$D\$3	x1	0	0.9999989	Constrain
\$D\$4	x2	0	1.0000011	Constrain

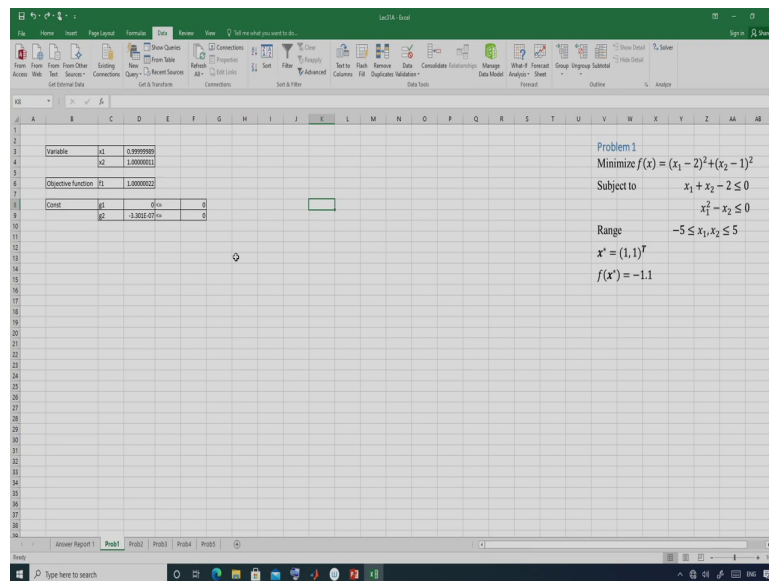
Constraints

Cell	Name	Cell Value	Formula	Status	Mark
\$D\$6	F5	0.9999989	<=	Binding	0
\$D\$3	x1	-3.902966E-07	<=	Binding	0

So, this is the answer report and you can see actually. So, here I have used GRG non-linear, solution time is 0.032 second and there are some other data about this particular algorithm. And you can see the objective function value is 1 then x_1 is equal to 1 and x_2 equal to 1. So, it is almost one that is here it is 0.999 and here it is 1.000011 something like that and both the constraints are satisfied ok.

So, you can see that one. So, this is the solution of this particular problem. So, I can also see here. So, I am getting the solution now 11 and objective function value is also 1 and all constraints are satisfied ok. So, if you go through this particular problem here, so, just see. So, this is the first problem and the solution is the solution is 11 ok.

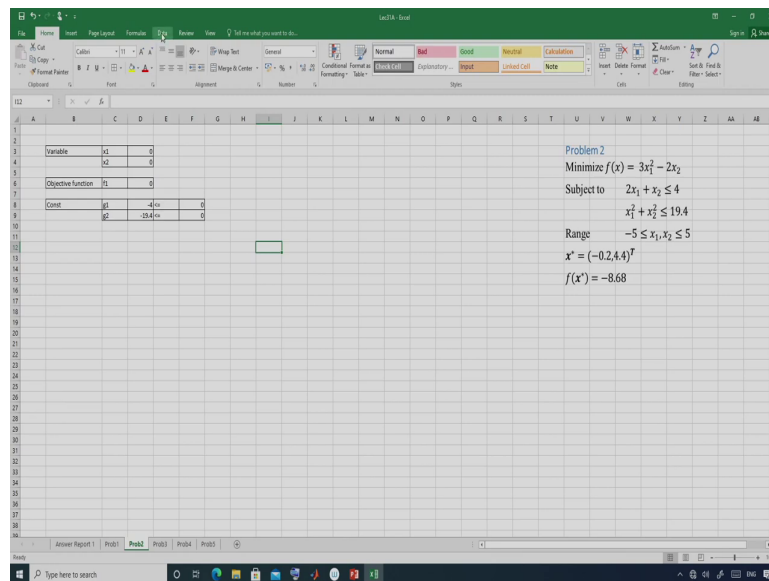
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So, solution is 1 and 1. So, here also I am getting 1 and 1 ok. So, this is the solution of this particular problem ok. So, I am getting this particular solution. So, I hope this is very simple. So, I can easily solve. So, without programming I can solve a non-linear problem here. So, you need not go for any programming. So, you need not download MATLAB or R programming here.

So, if you have a simple problem, So, that can be solved using Excel Solver ok. So, let me go to the next problem. In problem 2 we have two constraints and objective function is not non-linear ok. So, let me write it. So, I would like to copy this thing.

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So, this is we have two variables that is x1 and x2. So, I am putting x1 equal to 0 and x2 equal to 0 and here objective function is 3 star that is 3 star x1 square minus 2 star x2 ok 3 x1 square minus 2 x2 ok. Then you define the constrains. So, constrains are the first one is 2 star x1 plus x2 and minus 4. So, if I write minus 4 that means less than equal to 0 ok and second one is x1 square.

So, this is x1 square plus x2 square x2 square minus 19.4 ok. So, here it is less than equality type and here also this is less than equality type and this is less than equal to 0 and this is less than equal to 0 ok. I have defined the objective function and constrain here.

(Refer Slide Time: 23:22)

The screenshot shows an Excel spreadsheet with a linear programming problem setup. The Solver Parameters dialog box is open, showing the following settings:

- Set Objective:** \$D\$5
- To:** ☐ Max ☒ Min ☐ Value Of: 0
- By Changing Variable Cells:** \$C\$2:\$C\$3
- Subject to the Constraints:** \$C\$2:\$C\$3 >= 0
- ☐ Make Unconstrained Variables Non-Negative
- Select a Solving Method:** GRG Nonlinear
- Solving Method:** Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for Linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

The spreadsheet data is as follows:

Variable	x1	x2
Objective Function	19	0
Constraint	x1	-4/x2
	x2	12.4/x1

Problem 2

Minimize $f(x) = 3x_1^2 - 2x_2$

Subject to $2x_1 + x_2 \leq 4$

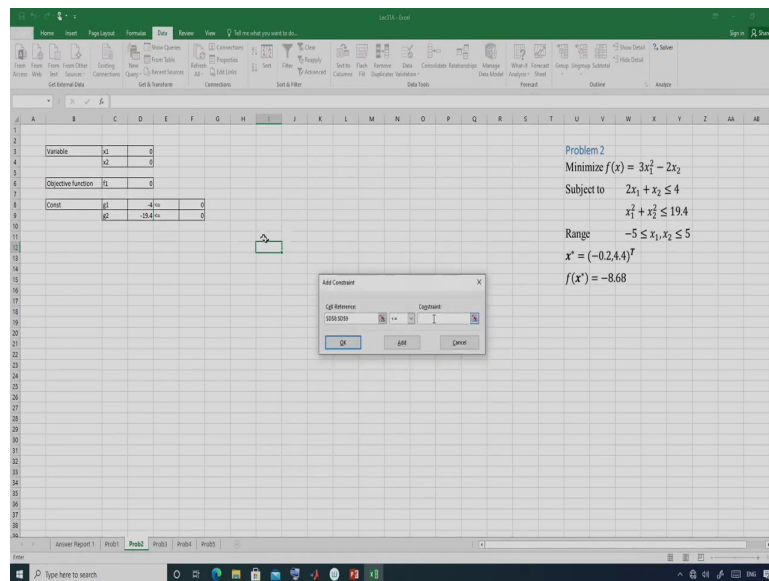
$x_1^2 + x_2^2 \leq 19.4$

Range $-5 \leq x_1, x_2 \leq 5$

$x^* = (-0.2, 4)^T$

$f(x^*) = -8.68$

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So, now I can solve this problem using solver. So, let us go to Data, use Solve button. So, here you define that this is your objective function cell now. So, this is the cell, this is minimization type and here this is the variable cells and so, this is the constrain. The constrain should be less than equal to 0. So, I have defined it. Now, let us solve this problem. So, if it is ok and if you have define everything correctly, so, you should get the solution.

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The image shows an Excel spreadsheet with a Solver Results dialog box open. The spreadsheet contains the following data:

Variable	x1	x2
Value	-0.2	4.4

Objective Function	F1
Value	8.68

Constraint	x1	x2	Value
1	0	1	4
2	1	0	5

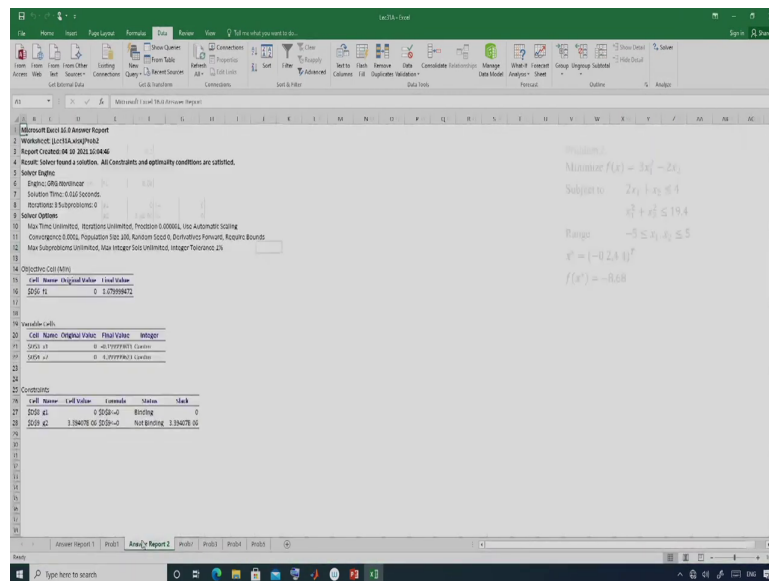
The Solver Results dialog box displays the following information:

- Solver Found a solution. All Constraints and optimality conditions are satisfied.**
- Report** (selected):
 - ☒ Show the Solution
 - ☐ Sensitivity Report
 - ☐ Limits
- ☐ Return to Solver Parameters Dialog
- ☐ Output Reports
- OK** **Cancel** **Load/Save...**

Below the dialog box, the problem statement is written:

Problem 2
Minimize $f(x) = 3x_1^2 - 2x_2$
Subject to $2x_1 + x_2 \leq 4$
 $x_1^2 + x_2^2 \leq 19.4$
Range $-5 \leq x_1, x_2 \leq 5$
 $x^* = (-0.2, 4.4)^T$
 $f(x^*) = -8.68$

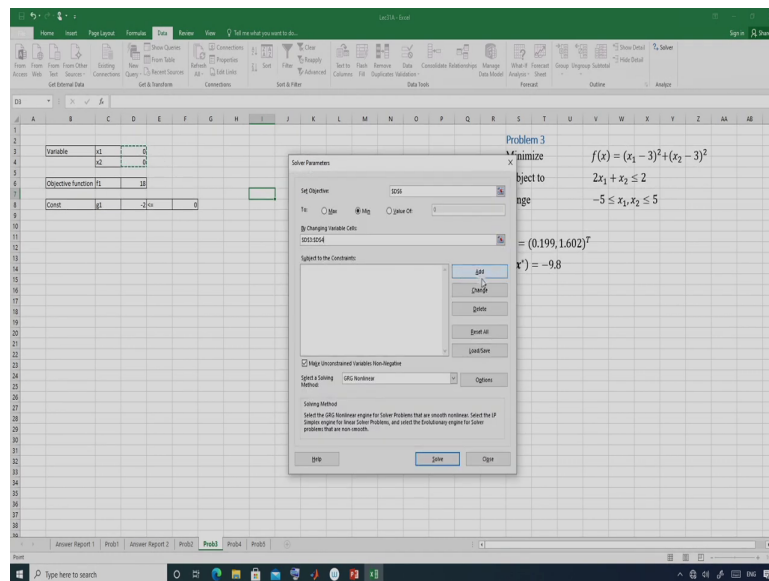
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So, let us let us solve it. You just see I can see the report also and so, this is report to the objective function. Objective function value is minus 8.67 and the solutions are that x1 equal to minus 0.1999 so that means, 0.2 and this is 4.399 and on all constrains are satisfied.

So, you can see that one that I am getting the solution that is x1 equal to minus 0.2 and x 3 x2 equal to 4.4. So, this is the solution you can see on the right hand side, the solution is minus 0.2 and 4.4. So, I am also getting the same solution and constrains are satisfied ok. So, it is quite easy to solve a non-linear problem using Excel Solver. Let us go to the next problem. The third problem we have one objective function and one constrain. So, let us solve it. So, before that I would like to copy this portion.

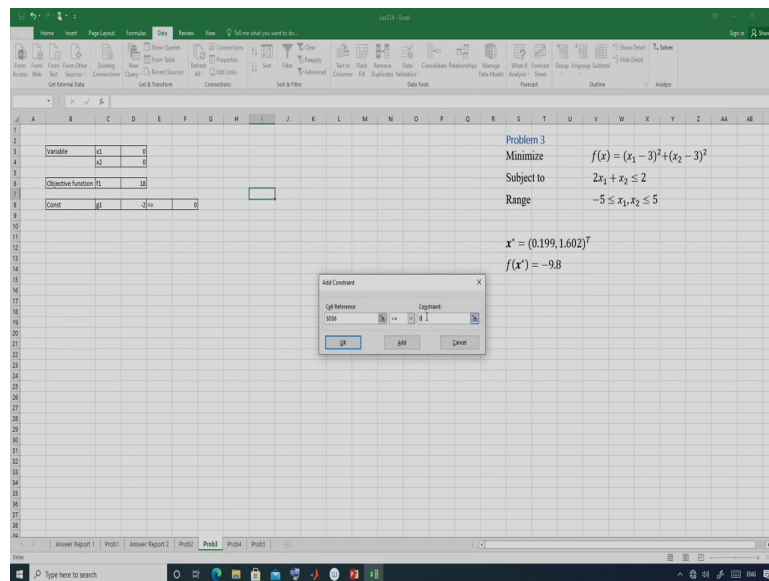
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So, I would like to copy this portion. Here I would like to start from 0 0 and objective function is x1. So, this is your x1 minus 3 whole square plus x2 minus 3 whole square ok. Sorry. So, this is x1 this x1 minus 3 whole square plus x2 minus 3 whole square ok and we have only one constrain that is 2 star x1 plus x2 minus 2 ok.

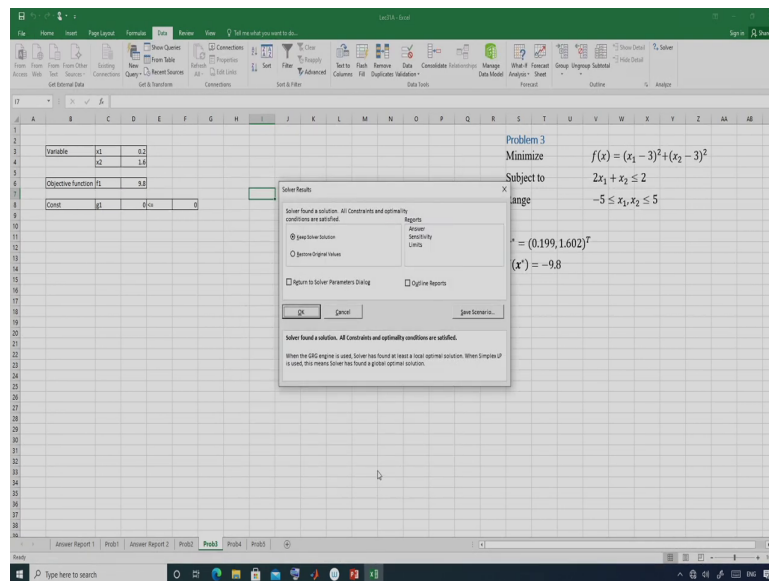
So, this is the constrain and ok. So, this is this is the constrain and this is less than equal to less than equal to 0 less than equal to 0. So, I have defined the problem here. So, let me go to solver. So, here I can define ok. So, I can define. So, I can define the objective function cells.

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So, this is objective function cell this is the objective function cell. Then changing cells are these then we have one constrain and that is these and this is less than equal to 0 ok. So, let me; so, now, I have defined this particular problem and if I solve it I should get the solution.

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So, let me solve it. Yeah. I am getting the solution. The solution is you can see the report solution is 0.2 and 1.6. So, I am getting and this is 9.8 ok. So, you go to answer and just see. So, this is 9.8. So, this is x will plus 9.8 ok. So, I am getting the solution.

(Refer Slide Time: 28:22)

Microsoft Excel Solver Report

Worksheet: [Lec31A.xlsx]Sheet1

Report Created: 04-10-2023 10:09:27

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Solver Engine

Engine: GRG Nonlinear

Solution Time: 0.025 Seconds

Iterations: 1 Subproblems: 0

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision: 0.000001, Use Automatic Scaling

Convergence: 0.0001, Population Size: 100, Random Seed: 0, Derivatives Forward, Assume Nonlinear

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 2%, Assume Nonnegative

Objective Cell (Min)

Cell Name	Original Value	Final Value
\$D\$6:\$F\$5	18	9.8

Variable Cells

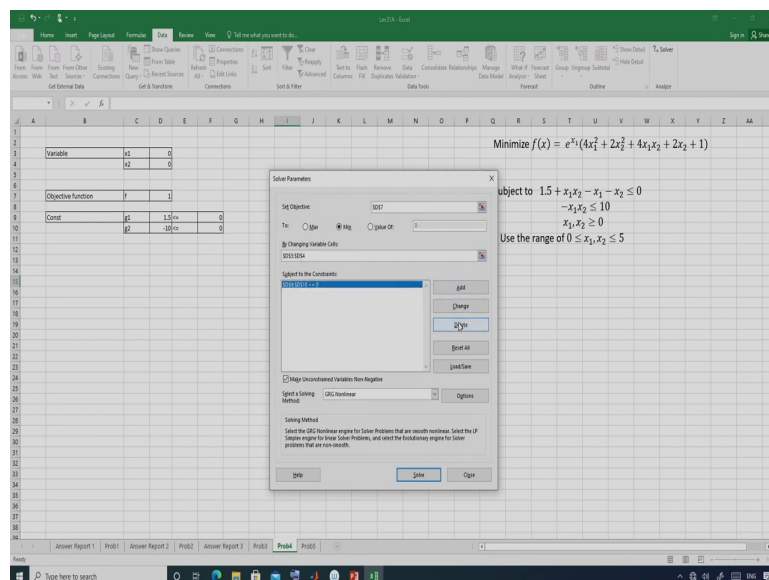
Cell Name	Original Value	Final Value	Integer
\$D\$3:\$X\$1	0	0.2000000000	Constr
\$D\$4:\$X\$2	0	1.5999999999	Constr

Constraints

Cell Name	Cell Value	Formula	Status	Slack
\$D\$6:\$F\$5	0.2000	=D\$6-d	Binding	0

So, you can see the objective function value is 9.8 and these are the variable value that x_1 equal to 0.2 and x_2 equal to 1.6 ok. So, I am getting the solution. This is your problem 3.

(Refer Slide Time: 28:51)

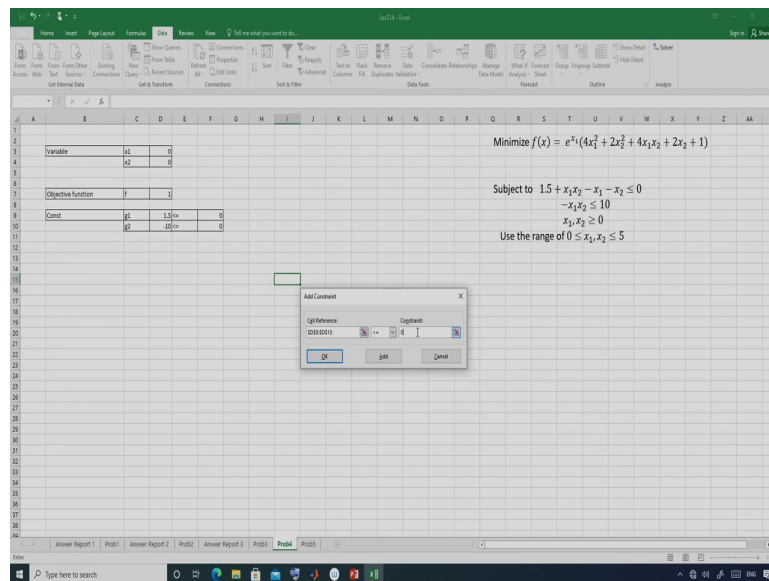


So, let me solve the 4th problem. The 4th problem is a complicated problem. So, I have already defined the problem here. So, here I have defined the problem. So, we have two variables. So, variables are x_1 and x_2 . So, this is the objective function value I have defined and this is the constrain ok. These are constrain and constrains are your less than equality type that means, this is less than equal to 0 and this is also less than equal to 0 ok less than equal to 0.

Now, let me solve this problem. So, this is a complicated problem to solve using Excel Solver. So, here every time you may not get the solution. The solution of this problem is so, I have defined here. So, just see that you should get 0 and 1.5 ok. So, 0 and 1.5 you should get. So, let us see whether you are getting. So, 0 and 1.5 is the solution, but let me start with 0 and 0. So, whether I am getting the solution or not.

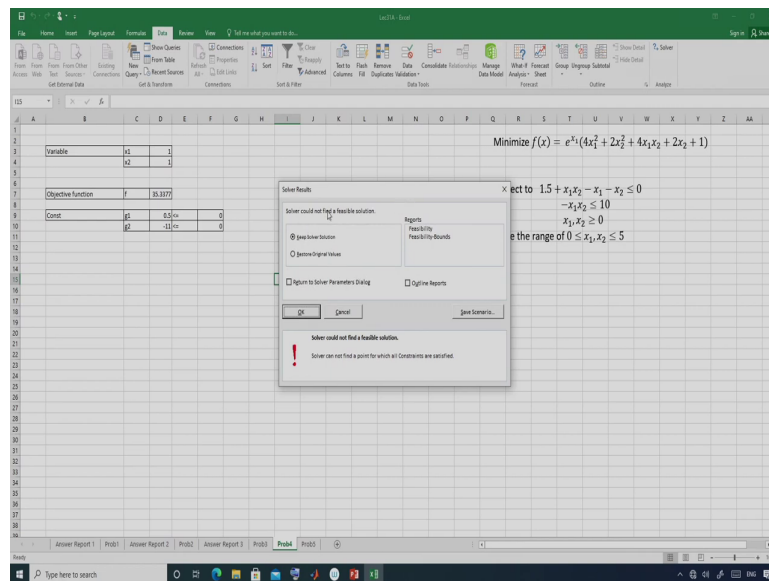
So, you go to data you go to solver and the problem I have already defined this problem. So, this is my objective function cell. So, this is objective function cell and these are the variable cells ok. The changing variable cell is these two.

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So, these two are the changing variable cell and then we have constrain here. So, let me add let me add the constrains. So, there are two constrains. So, these two are constrain and this should be less than equal to 0 ok. Now, let us solve this problem using solver. Just see whether I am getting the solution or not. So, I am putting 0 0.

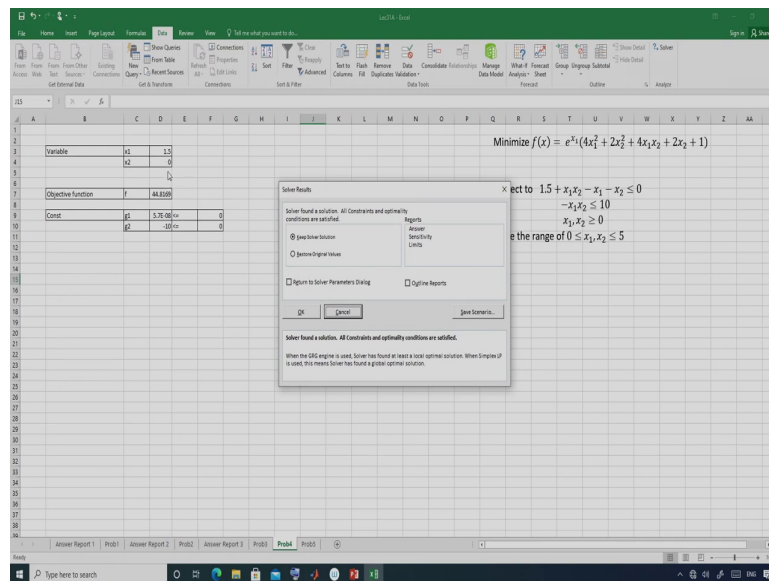
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So, no, I am not getting the solution. As I said this is a complicated solution and it may be difficult or it is difficult to solve this problem using Excel Solver ok. So, I am not getting the solution. Then what I will do if I am not getting the solution since the initial point? Suppose I am putting 1 1 now, so, in that case whether I am getting the solution, no, 1 1 also I am not getting the solution ok.

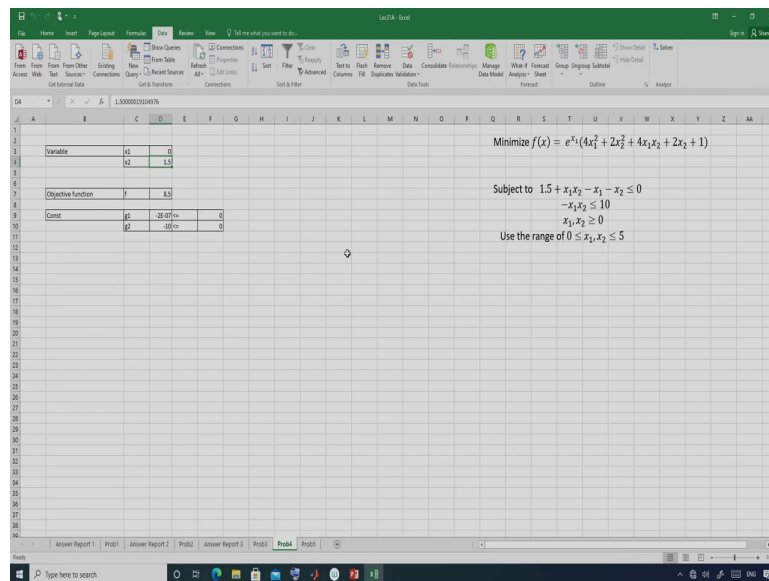
So, then you put 2 2. Are you getting the solution? No, you are also not getting the solution with 2. Then 2 1, you just see whether you are getting a solution with 2 1? Yeah.

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I am getting a solution. This is 1.5 and 0. So, this is also a solution, but this is not optimal solution maybe this is a another local optimal solution. So, you are getting this is 1.5 and 0, but excel solution is 0 and 1.5, but you are getting 1.5 and 0.

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So, then what I will do? I will put this is 3 and this is 1 and then you solve it and just see whether I am getting. Yes. This time I am getting the solution and the solution is 0 and 1.5. So, this particular problem is difficult to solve using the classical optimization technique. So, I have solved this problem using genetic algorithm and every time I am getting the global optimal solution of this problem. But when you are solving this problem using classical method.

So, as you have seen so, it is sensitive to the initial solution. So, in some time you are not getting any solution you are in the infeasible region. Sometime you are getting solution, but that may be a local optimal solution, but once you are changing your initial solution to 3 2 or then basically we are getting the solution ok, let me.

(Refer Slide Time: 33:45)

The image shows an Excel spreadsheet with the Solver Parameters dialog box open. The spreadsheet contains the following data:

Variable	x1	x2
Initial Value	1	0

Objective Function: $f(x) = e^{x_1}(4x_1^2 + 2x_2^2 + 4x_1x_2 + 2x_2 + 1)$

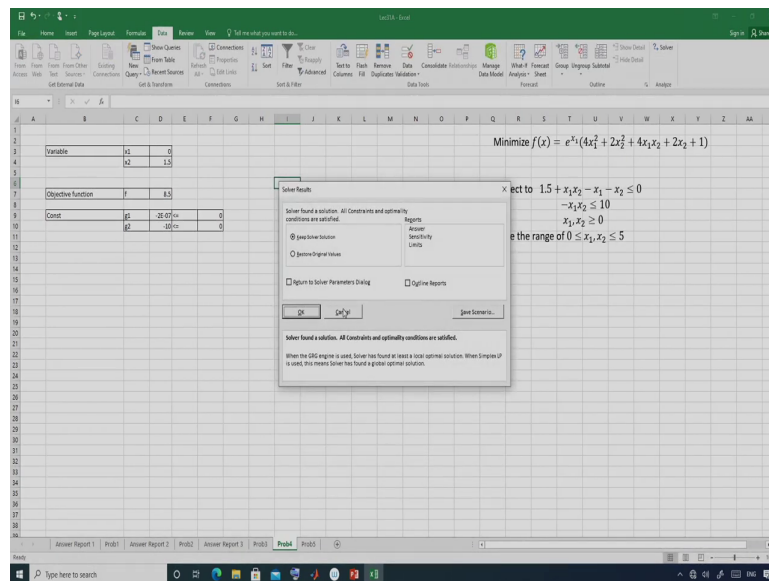
Constraints:

- $1.5 + x_1x_2 - x_1 - x_2 \leq 0$
- $-x_1x_2 \leq 10$
- $x_1, x_2 \geq 0$
- Use the range of $0 \leq x_1, x_2 \leq 5$

The Solver Parameters dialog box is configured as follows:

- Set Objective: $f(x)$
- To: ☐ Max ☒ Min ☐ Value Of
- By Changing Variable Cells: x_1, x_2
- Subject to the Constraints: $x_1, x_2 \geq 0$
- Select a Solving Method: GRG Nonlinear
- Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for Linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
- Solve button is highlighted.

(Refer Slide Time: 34:01)



So, once you are taking 1 and 3 as the initial solution then you are getting the solution. So, let me try with 1 and 3 again. So, this is 1 and 3 again and if you are solving, so, in that case you are getting the solution and that is 0 and 1.5 and objective function value is 8.5.

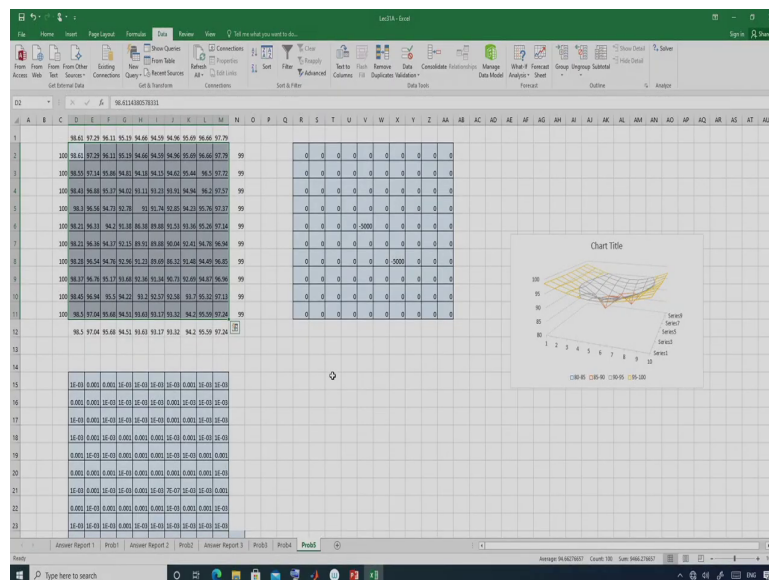
So, therefore, for if when you are applying the classical method to solve a non convex problem, so where you have more than one optimal solution. So, in that case what you will do? You change your initial solution ok. So, change your initial solution that is your x naught and try whether you are getting different solution or not ok.

So, once you are taking an initial solution which is in the global optimal region, so, you may get the global optimal solution of the problem. But if you are applying the non classical technique, so, like genetic algorithm or PSO or other methods, So, there is a high probability that you will get the global optimal solution of the problem. So, this particular problem I have

solved in the last class using genetic algorithm and I got the global optimal solution of the problem in a single run ok.

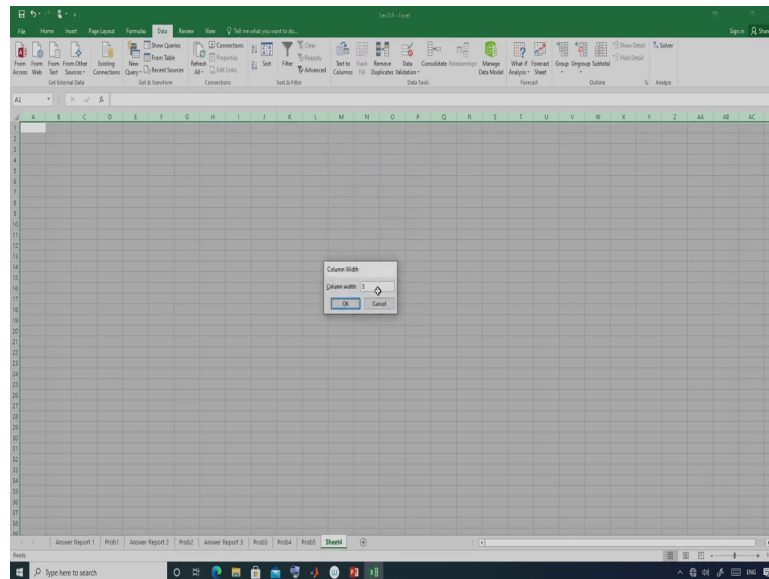
So, every run I am getting the global optimal solution of the problem. Now, let me go to the partial differential equation. So, I said that I would like to solve this partial differential equation using Excel Solver. So, I will show you how you can solve this problem using Excel Solver.

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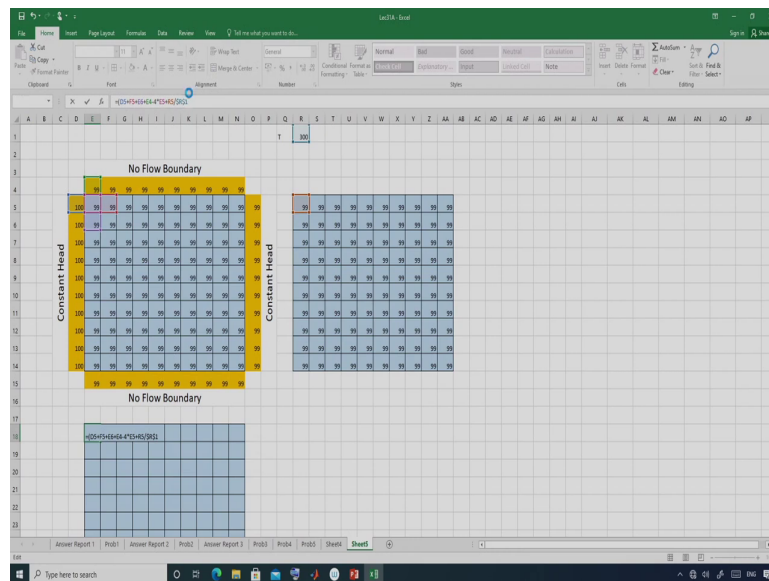
So, here already I have solved this problem, but let me try again ok. So, let me start it from the beginning.

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So, let me make it square ok and this also I make it square ok.

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So, here let me define my aquifer. So, aquifer size is 10 by 10 that means, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. So, this is the 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. So, this is my aquifer. So, I am putting a color here. So, a different color may be this particular color and I would also like to put that way. So, as you know that left side is your constant head. So, this value is given and this value is 100 ok. So, this value is 100.

So, I am putting this value here and right side value is also known and this is the also constant head ok. So, this is also given. Now, you consider some initial solution here ok. So, let me consider 99 as the initial solution. So, I am considering 99 as the initial solution. So, as you know when you are applying the classical method. So, you should start your iteration with an initial solution.

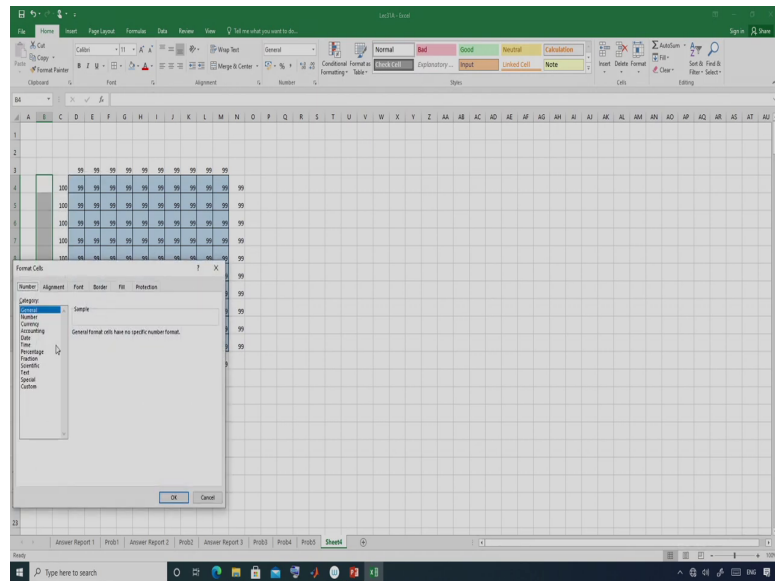
So, in this case I would like to find out what is the value at its grid point. So, initial solution is your 99. So, as you have seen so, we have total 100 grid point that means, in this particular problem we have 100 variables ok. So, we have 100 variables that is x_1 to x_{100} . So, I would like to show you that how you can solve a problem with 100 variables.

Suppose in the earlier problem we have solve. So, we have only two variables. So, but excel is also capable of solving a problem having your means having 50 or 100 variable. So, in this particular version of excel, so, this is the free version. So, there is only one restriction is that if your problem has more than 100 variable, so, you will not be able to this Excel Solver ok.

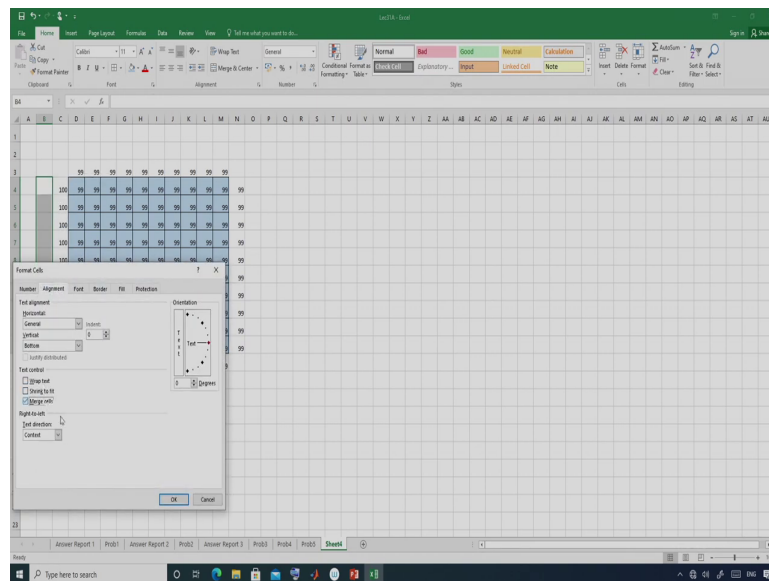
So, your number of variables should be less than 100, but in this particular problem we have exactly 100 variables. So, I can solve this problem using Excel Solver. So, I have considered the initial solution as 99 and I also need to define the boundary condition here. So, here boundary condition is that no flow boundary and that means, the value will be same gradient value the gradient value is 0. So, therefore, this cell value will be equal to this cell ok, so that means, this is equal ok.

So, similarly here also this will be equal ok. So, this will be equal. So, I can I can do that. So, I have defined here.

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So, let me define. So, this is the boundary condition. So, this is constant head ok. So, so I will rotate this x ok. So, this is the constant head here and here also this is the constant head ok. So, this is also constant head. This is also constant head and here this is no flow boundary no flow boundary ok. And this side also we have no flow boundary ok. So, this is the boundary values. I am putting a different color.

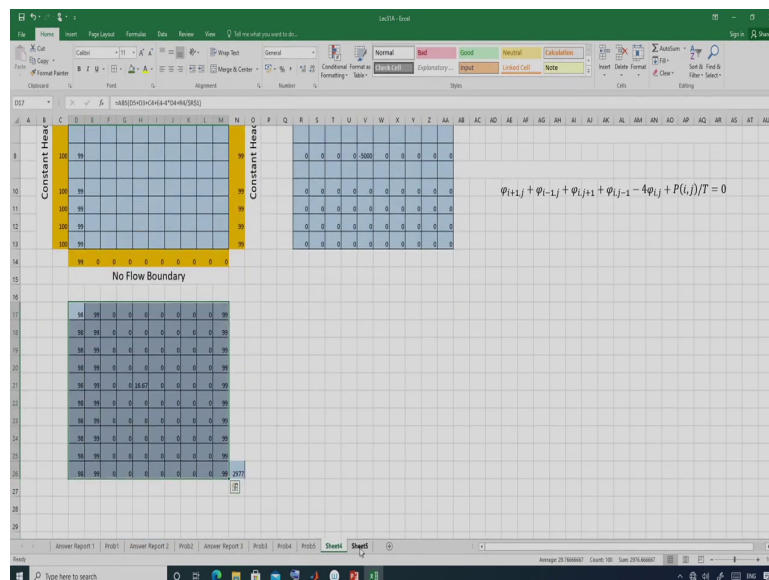
So, this is and this is also and this is also ok. So, now, I have defined my problem here. So, I have defined the constant head. So, these values are known. So, these are the boundary values. So, left hand and right hand side and here it is no flow boundary that means gradient is 0. So, I have copied, I have actually make this particular cell equal to this cell.

Now, let me define. So, this is the phi values I have. Let me define let me define the pumping value ok that source term here. So, suppose I have a source here and this is around 5000 ok.

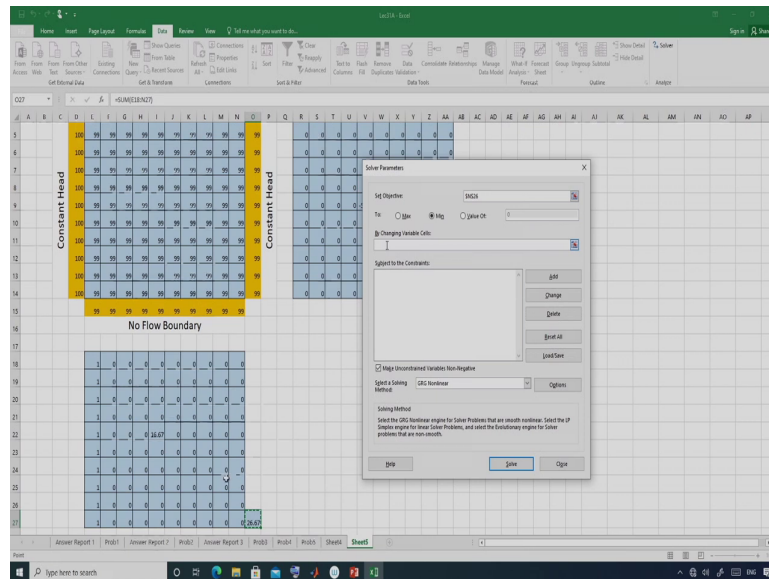
This is the source and for others, so, this will be equal to 0. So, you can put it or otherwise you need not put it also ok. So, this is fine so, this is fine. So, if you are not putting then also it will consider 0, so, these are all 0s. So, these are all 0. So, I have only one pumping here.

Now, what I will do? I will write the equation. So, which equation I will write? I will write this particular equation ok. So, this particular equation I will write ok. So, I will write this particular equation. So, let me write the equation here. The equation is this is this plus this plus this plus this minus 4 star i, j plus this divided by T.

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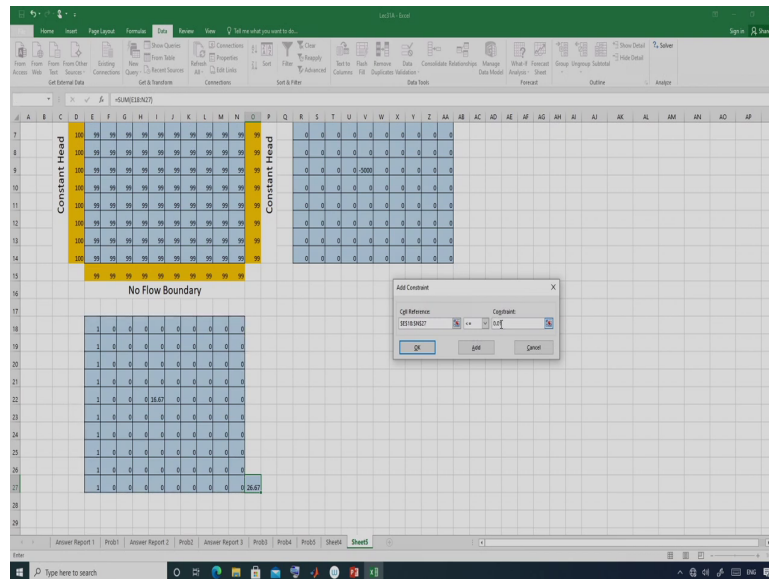
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So, here I am putting. So, these pumping values are 0. So, I am putting all 0 and somewhere here it is minus 5000 ok. So, now let me write this equation. So, I am just copying this equation. So, I am getting this. So, before that I have to put the absolute sign here ABS ok. So, if I put ABS absolute value, so, either you can put absolute or you can put square also ok and here I will make sum ok.

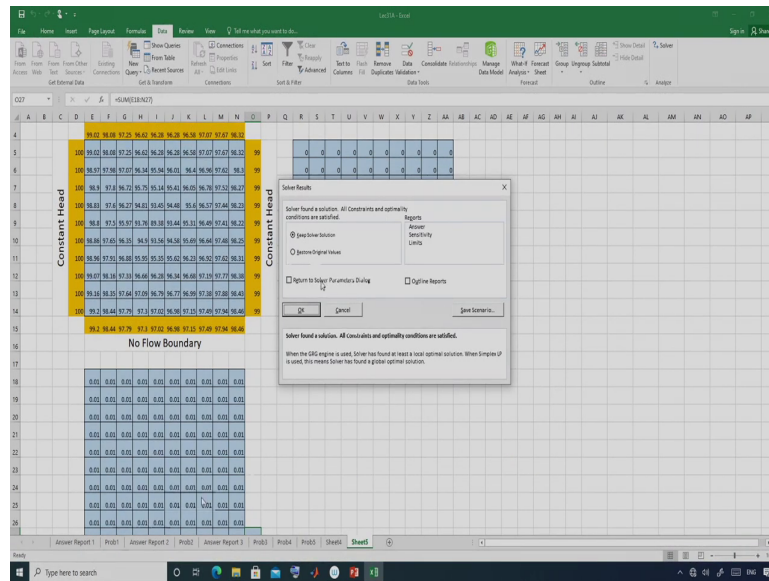
So, total error I am getting 26.67 and I would like to minimize this error in order to get the value of phi. So, let me use the Excel Solver. So, you go to Data then you use Solver. So, here the objective function is the error of function that is 027. So, 027 and these are changing cells. So, I have total 100 changing cells ok. So, all 100 I have considered and then I have constrain.

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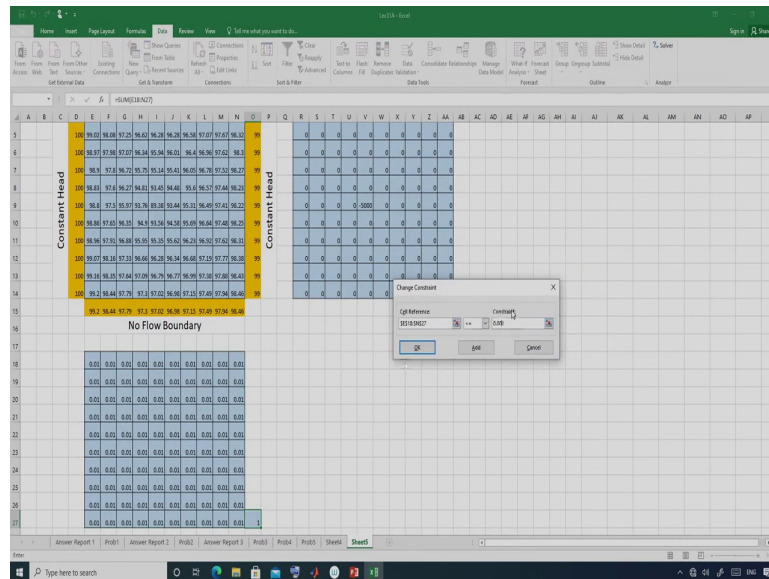


So, how many constrain I have? I have 100 constrain ok. So, these are all constrain and this should be less than initially you consider 0.01 ok. So, now, I have defined the problem. So, this is the problem I have defined and just see whether I am getting the solution or not. So, let me solve this problem. If it is fine then I should get the solution. So, what it will do? It will try to minimize the error.

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So, you can see it is running and solver found a solution all constraints and optimality conditions are satisfied. So, you can see the answer, but before that let me refine or let me increase the precision. So, what I am doing? I am just increasing the precision to 0 0 1. So, let me solve it again. So, now, you just see I can see the report yeah.

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[illegible]

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Microsoft Excel 36.0 Answer Report

Cell	Name	Cell Value	Formula	Status	Stack
S2512	No Flow Boundary	0.001	S2512=0-0.001	Binding	0
S2513		0.001	S2513=0-0.001	Binding	0
S2514		0.001	S2514=0-0.001	Binding	0
S2515		0.001	S2515=0-0.001	Binding	0
S2516		0.001	S2516=0-0.001	Binding	0
S2517		0.001	S2517=0-0.001	Binding	0
S2518		0.001	S2518=0-0.001	Binding	0
S2519	No Flow Boundary	0.001	S2519=0-0.001	Binding	0
S2520		0.001	S2520=0-0.001	Binding	0

So, I am getting a report here and you just see these are the value ok. So, these are the; so, original value was 99 and finally, I am getting these. So, these values; so, all 100 variables are here.

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[illegible]

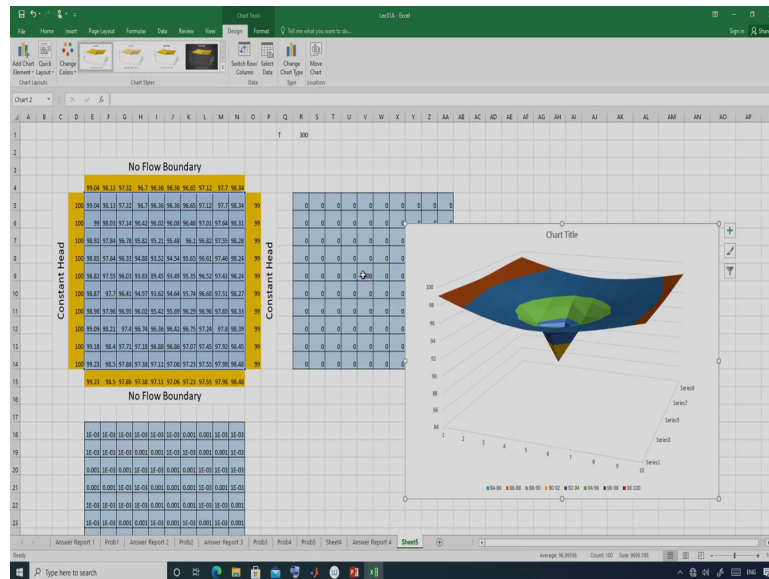
(Refer Slide Time: 47:12)

The screenshot displays the Microsoft Excel application window. The title bar reads 'Microsoft Excel - [Book1]'. The ribbon at the top includes 'File', 'Home', 'Insert', 'Formulas', 'Data', and 'Review'. The 'Formulas' ribbon is active, showing options like 'Calculation Options', 'Formula Auditing', and 'Formula Connections'. The main worksheet area shows a grid with columns labeled A through Z and rows numbered 1 through 26. The data in the grid includes dates and names, such as '0.000 0.000 0.000' and '0.000 0.000 0.000'. The status bar at the bottom indicates 'Answer Report 4' is selected.

And all 100 constrain you just see I have total 100 variables and 100 constrains. So, these constrains are there and all of them have satisfied. So, you just see constrain value is less than 0.001. So, I am getting the solution. So, now, this is the solution and as I have said this is a flow problem and there is a pumping cell. There is a pumping cell here and if I plot it so, I should get the cone of depression ok. So, cone of depression I should get.

So, I think you have already gone through well hydraulics sector of water resources ok. So, they are actually if we are pumping water from the confine aquifer, so, when you are pumping water from a confine aquifer a cone of depression will occur. So, I can see that cone of depression here. So, let me plot it. So, I can plot this surface.

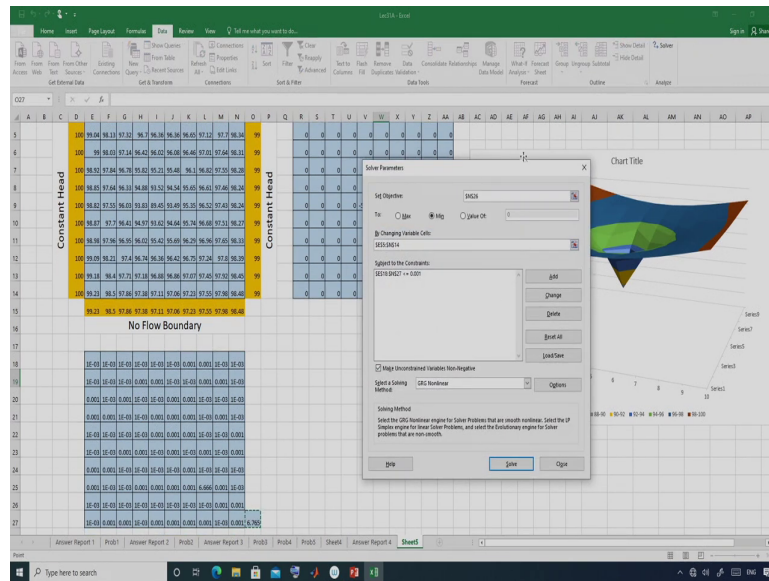
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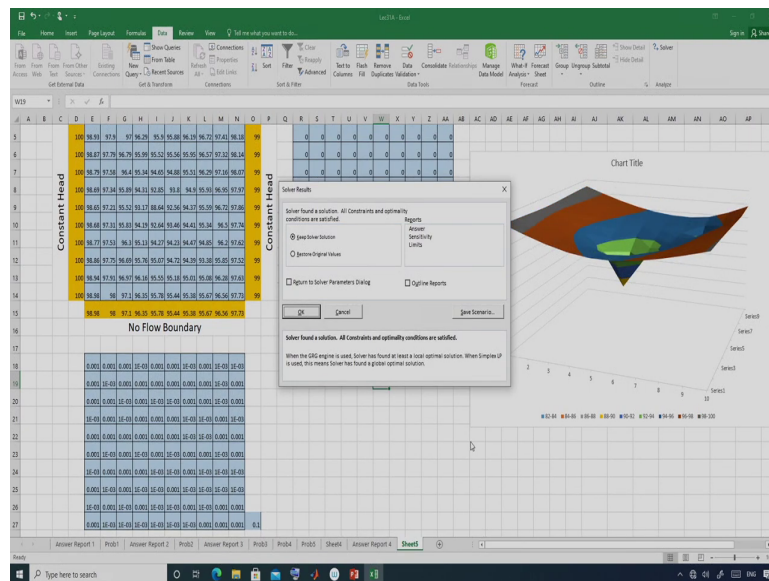
So, let me go to this one and here I can plot it ok. So, this is the plot you can see. So, this is the plot you can see either this is the plot ok. So, you can see that this is the plot. And here from here actually I have extracted 5000 meter cube per day and you can see basically that a cone of depression has occurred. So that means, I am getting or I got the solution ok. So that means, I got the solution of this particular problem.

So, you can also see suppose if I put another wheel somewhere here suppose this is minus 2000 ok. So, pumping is minus 2000 and now error is 6.76.

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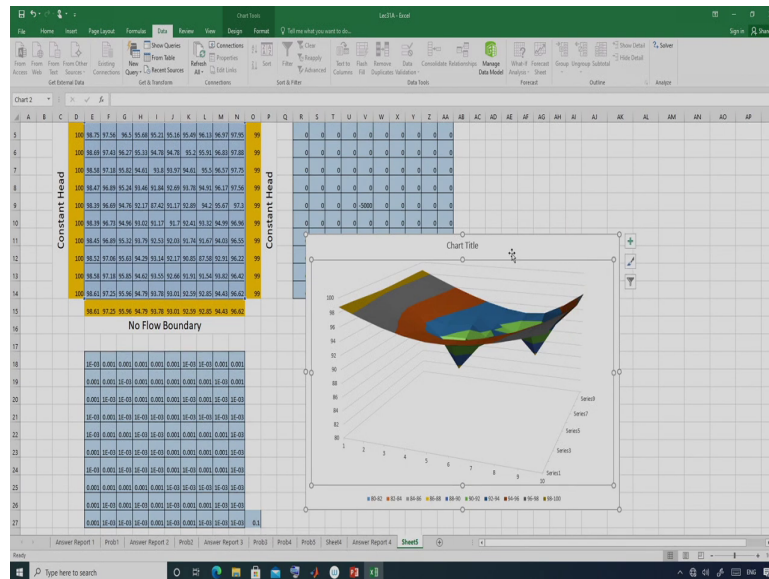


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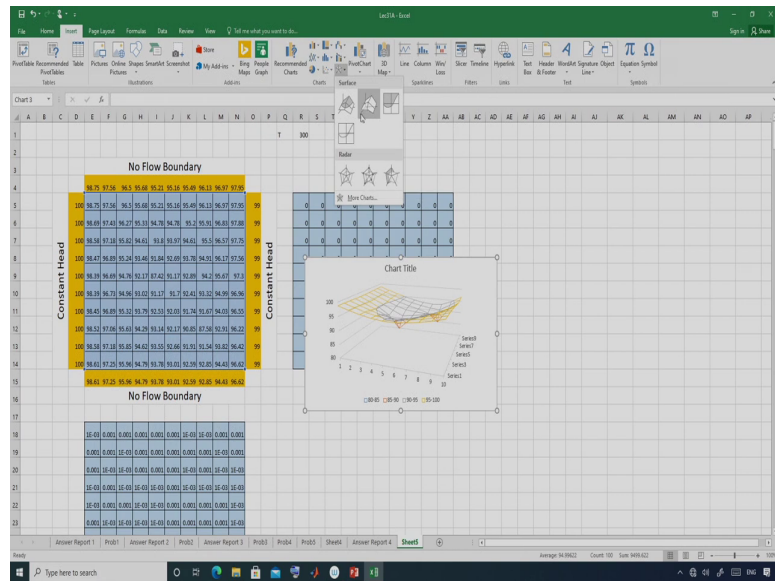
So, I can also minimize this error. Just go to Data and Solver then I will solve it again ok. So, let me solve it again. So, you can see that I am getting I should get another cone of depression here. So, this is the cone of depression. Maybe you can increase these things. So, minus 5000 and just see just solve it you should get yeah.

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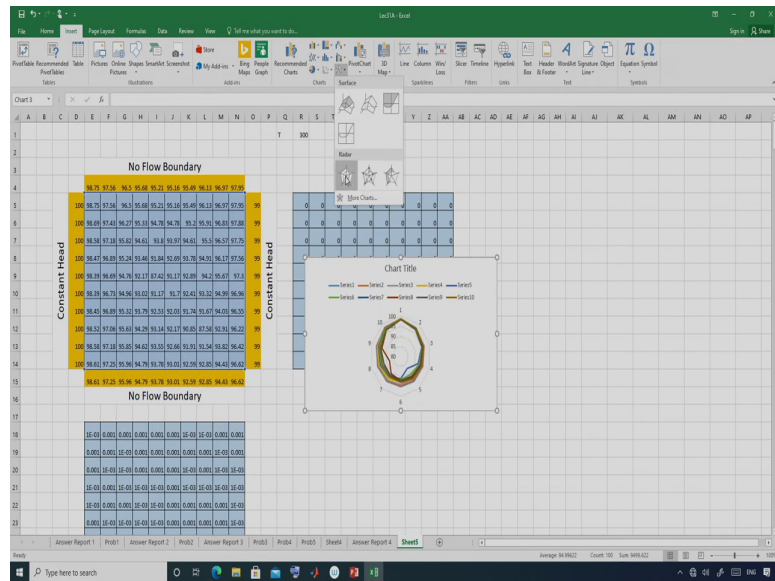


So, you just see you are getting two cone of depression. So that means, I am getting the solution and this problem I have solved using Excel Solver ok.

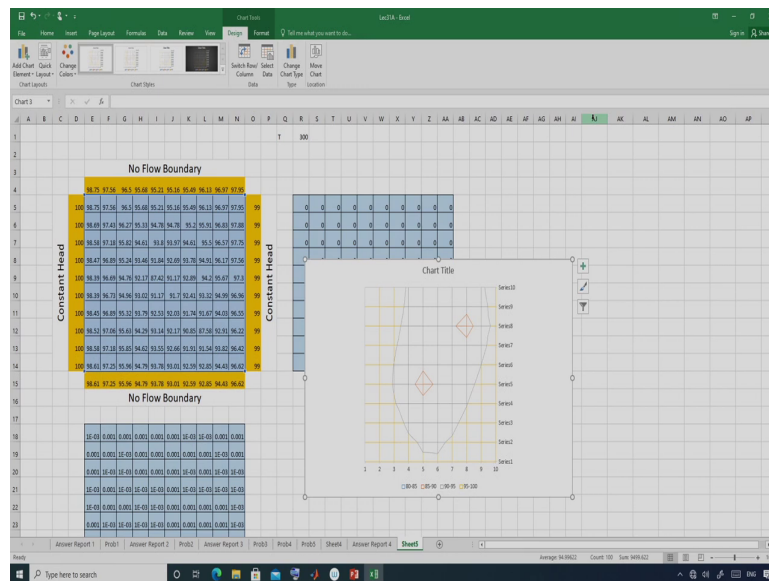
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So, you can also change the plot. Suppose I would like to plot a contour I would like to plot the contour that also you can do. So, Insert, suppose I can see that one ok, I can also see the contours ok. So, here you can see. So, there are two cone of depression. So, there are two cone of depression and because we have two pumping well in this case.

So, the problem I have considered. So, this is a problem having 100 variables. So, just to show basically that excel is also capable of solving a problem having more variables. So, in this case I have solved a problem with 100 variable. So, you can easily solve this problem, but as I said if the number of variable is more than 100 then you will not be able to use that on.

So, it can handle up to 100 and number of constrain also it can handle up to 100. So, you can see that in this particular problem we have 100 variables and also we have 100 constraints.

Then let us stop here. So, I hope you will be able to solve an optimization problem using Excel Solver.

Suppose if you are not comfortable with your MATLAB or if you are not comfortable with R programming then you can solve some problem. So, you will not be able to solve all the problem, but some problem you will be able to solve using Excel Solver ok. So, you do not need any programming skill. So, you just define your problem and you will be able to solve using the Excel Solver available in MS Excel.

Thank you.