

Optimization Methods for Civil Engineering
Dr. Rajib Kumar Bhattacharjya
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Lecture - 32
Optimization using MATLAB

Hello student, welcome back to the course on Optimization Methods for Civil Engineering. So, in the last class we have discussed about MATLAB. So, how to download MATLAB or how to create an account in math's work dot com and then we have learned how to write a vector or how to define a vector, how to write an expression, then how to plot a particular function a 2 D plot 3 D plot so that I have discussed in the last class.

So, today in this class we will discuss the solution of optimization problem using MATLAB. So, initially I will discuss or I will show you how you can solve a single variable optimization problem using MATLAB and then multi variable optimization problem in MATLAB so, mainly with constraint or without constraint. So, that I will be discuss in this class and then I will show you in MATLAB itself. So, I will show you live demonstration basically. So, I will show you how you can run a MATLAB code for solving an optimization problem.

(Refer Slide Time: 01:47)

Introduction to Optimization

Unconstrained single variable

Single variable equation

fminbnd to find minimum of single-variable function on fixed interval

$$\text{Min } f(x) = x^2 + \frac{54}{x}$$

$$0 \leq x \leq 11$$

```

clear variables
close all
% Define an objective function
objfun = @(x) (x^2+54/x);
% Define an initial point
x1=0;
x2=11;
% Output options
options = optimset('Display','iter','PlotFcns',@optimplotval);
% Implement optimization function
x = fminbnd(objfun,x1,x2,options);
disp(x)
disp(objfun(x))

```

Handwritten notes on the MATLAB interface:

- Red arrows pointing to the function definition and the interval [0, 11].
- Red circles around the function definition and the interval [0, 11].
- Red text: $x^2 = 2.25$ and $f(x) = 27$.
- Red text: $x = 2.25$ and $f(x) = 27$.

So, let us discuss, let us see how we can solve a single variable optimization problem. So, and it is an unconstrained single variable optimization problem. So, the problem is unconstrained. So, for that there is a function in MATLAB so, this function we will use this is fminbnd ok. So, this is we will use this function is used to find a minimum of single variable function with fixed interval ok with fix bound. So, I can use this function. So, if you have suppose this is an example of this function. So, I would like to find out the minimum of this particular function..

So, there is only one variable. So, I would like to find out the minimum of this function. And there is only one variable and that variable is x. So, f is a function of x and the function is x square plus 54 by x and the range of x is between 0 and 11 ok. So, I would like to find out the minima of this particular function. So, what I will do, I will use this particular your particular

function available in MATLAB that is `fminbnd` ok. So, `fminbnd` so, I will use and this is the syntax of this particular function ok. So, you can see that this is the syntax..

So, this is the keyword we have to use that is `fminbnd`. So, that I we have to use and then we have to what we have to put in this particular function. So, we have to put the function ok. So, that objective function and then the bound we have to pass that is lower bound and upper bound and here x_1 is the lower bound and x_2 is the upper bound ok. So, this is the syntax of this thing syntax of this particular your function and I can also give the options ok. So, there are some options available..

So, I can also use the option and then it will return x means the x^* basically it will return the optimal value it will return or you can also return the `fval` ok. So, this is the optimal objective function value. So, that also you can return or we can also see the exit function ok. So, exit flag actually. So, we can also see the exit flag and that why the iteration has been terminated. So, that also you can see or there are some other output also we can mention. So, this is very simple. So, I can use this function to find out the minima of an unconstrained single variable optimization problem.

So, let us see that how we will write this particular function. So, first three line they are clearing the variables and clearing the environment. And then you define the objective function. So, here objective function is to be define as a function so, I am writing at the rate and we have only one variable that is a x ok. So, this is a your scalar quantity x is scalar here and this is x^2 plus 54 by x . Then I have to define the initial point or interval I have to define; that means, x is range of x is 0 and 11. So, I am writing x_1 equal to 0 and x_2 equal to 11.

And then I can I can define the option. So, here options are. So, I would like to display the iteration then plot function. So, I am giving this particular option and then I am using this particular function. So, this function will or this line will calculate the value or minimum of this particular function $f(x)$ and basically it will give the value of x . So, once you are getting

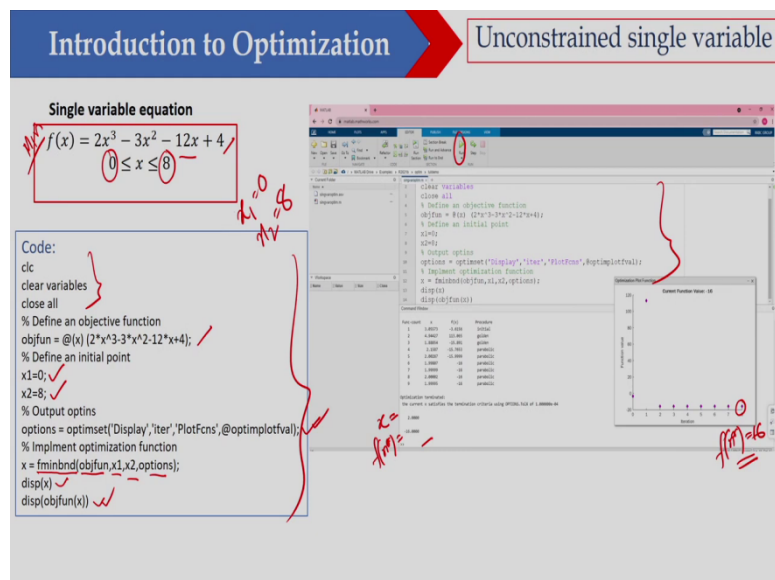
the value of x . So, you can display what is x and you can also display the objective function value ok.

So, I can also write `fval`. So, I can also use this thing. So, here I can write it x and `fval`. So, that is the value of the objective function. So, I can also this one. So, then I am getting x star value. So, this is the x star value optimal your point and similarly optimal objective function value also you can get. So, here I have shown you the MATLAB code here. So, I will also run this in MATLAB so, that I will show you here.

But here I have shown here. So, once you are running this particular your program then you will get the value of x . So, x star value is star 3 here. So, this is the x star value that is 3 and objective function value is 27. So, that I am getting and you can also see the iteration ok. So, this is the iteration you can see and finally, you are getting that x star equal to 3 and objective function value; that means $f x$ star equal to 27 ok.

So, here you can also see the plot. So, once you are giving this option and you can see that with iteration. So, this is with iteration how this objective function value is changing. So, initially this is the value and finally, objective function value is 27. So, you are getting this solution. So, you can also see how this objective function value is changing with the iteration in this particular plot. Now, let us see another example problem this is also a single variable optimization problem with bound so there is no constraint.

(Refer Slide Time: 01:42)



So, I will use fminbnd function ok. So, here the objective function is I would like to find out the minima of this particular function minimum of this particular function and the function is twice x cube minus thrice x square minus 12 x plus 4.

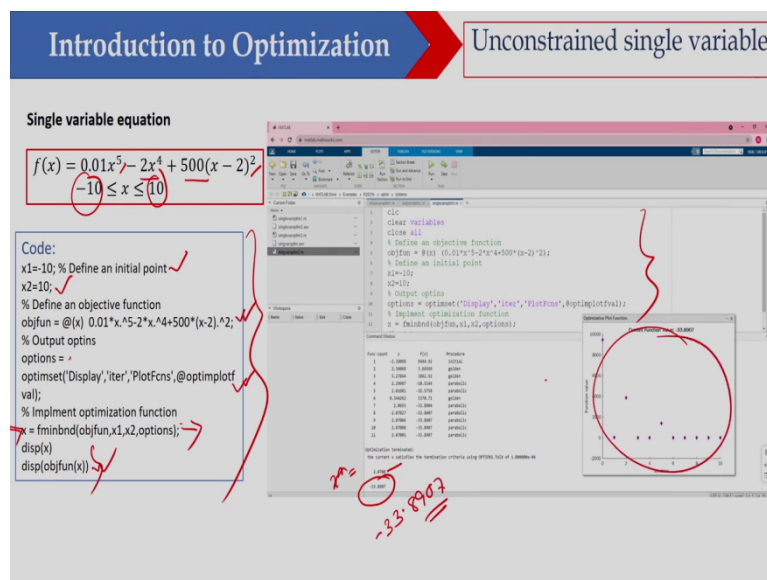
So, I would like to and the lower bound is 0 and upper bound is 8; that means, x 1 equal to 0 and x 2 equal to 8 ok. So, you write these three. So, to clear the environment and then I am writing the objective function. So, objective function is 2 x cube minus thrice x square minus 12 x plus 4 ok and then lower bound is 0 upper bound is 8 and then I am defining the option here.

And finally, I am using fminbnd and here this is the objective function, then lower bound upper bound and this is option and then I am displaying the value of x star and objective function value. So, this is the code and I have written this code here and if you are executing

this particular code and you should get the x star value that is x star value is 2 and f of x star is equal to minus 16 ok.

And you can see that how this objective function value is changing with iteration and finally, it is reaching the value f of x star equal to 16 ok. So, you can see that one. So, if you are executing this particular MATLAB code. Now, let us see another example problem. So, this is also a single variable optimization problem without constraint.

(Refer Slide Time: 09:31)



And the function is 0.01 x to the power 5 then twice x to the power 4 plus 500 x minus 2 whole square and lower bound is minus 10 and upper bound is 10 here.

So, I can write the code here. So, this is the lower bound x 1 is lower bound and x 2 is upper bound and this is the objective function I have defined here and then you define the option

So, you just see this is very simple code and if you are writing here. So, as an m file and if I execute then finally, I should get that x star is equal to 2.0700 and optimal function value is 33 minus 33.8907 . So, this is the optimal objective function value and you can also see with iteration how this function value is changing. So, that also you can see in this particular plot.

Introduction to Optimization

Unconstrained multivariable

Multi-variable equation

$f(x) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2$

$x_0 = (3, 3)$

```

> fminsearch(fun,x0)
> fminsearch(fun,x0,options)
[x,fval] = fminsearch(
[x,fval,exitflag, fminsearch_
[x,fval,exitflag,output] = fminsearch_
        
```

Code:

```

clc
close all
% Define the objective function
objfun = @(x)(x(1)^2*x(2)-11)^2+(x(1)+x(2)^2-7)^2;
% Set the initial value
x0=[3,3];
% Set an output option
options = optimset('PlotFcn','@optimplotfval');
% Implement optimization
[x,fminsearch_objfun,x0,options];
disp(x)
    
```

Handwritten notes in red ink:

- $x = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$
- $x_0 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$
- $x_0 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$
- $x_0 = (3, 3)$
- $x_0 = (3, 3)$

Now, let us see a multi variable optimization problem; that means, you have more than one variable and this is without constraint. So, I am not using constraint right now in the next example problem so, I will also use constraint, but here this is without constraint.

So, let us see I would like to solve this particular function. So, this function already you have seen the function is $x^2 + y^2 - 11x + 7y$ whole square and here I am giving an initial point this is the initial point I have defined; that means, the search process will start from this particular initial point ok. So, x_0 I have defined and x_0 is 3, 3; that means, the search process will start from 3, 3.

So, here what I will use, I will use a function call `fminsearch`. So, I will use a function and the name of the function is `fminsearch` ok. So, this is your classical your optimization method `fminsearch` and here what you have to define. So, you have to define the function value and then x_0 ok so; that means, I need to define the function ok which function I would like to minimize.

So, this is your minimize I would like to minimize this function, the function you have to define an x_0 ; that means, the starting point you have to define and you can also define the option here and similarly what you can get actually.

So, you can get the optimal value of this particular function, then optimal value of the objective function that is `fval` ok. So, that is the function value you will get and similarly you can also get exit function and other output ok so, that you can that you can get. So, here I have written the code here. So, this code is similar. So, first you define the objective function. So, objective function I have defined here.

The objective function is $x_1^2 + x_2^2 - 11x_1 + 7x_2$ whole square then $x_1^2 + x_2^2$ square minus 7 whole square. So, here I have defined x . So, this particular x as a vector and it has 2 value that is x_1 and x_2 . So, if I write that x_1 ; that means, this is x_1 and if I write x_2 ; that means, this is x_2 . So, already I have explained this thing in the previous class.

So, now you I think know how to define a vector and basically how to use that vector ok. So, here x is a vector and I can define this one. So, then I have defined the initial point the initial point is 3, 3 ok. So, this is also a vector and now you define the option. So, here the plot

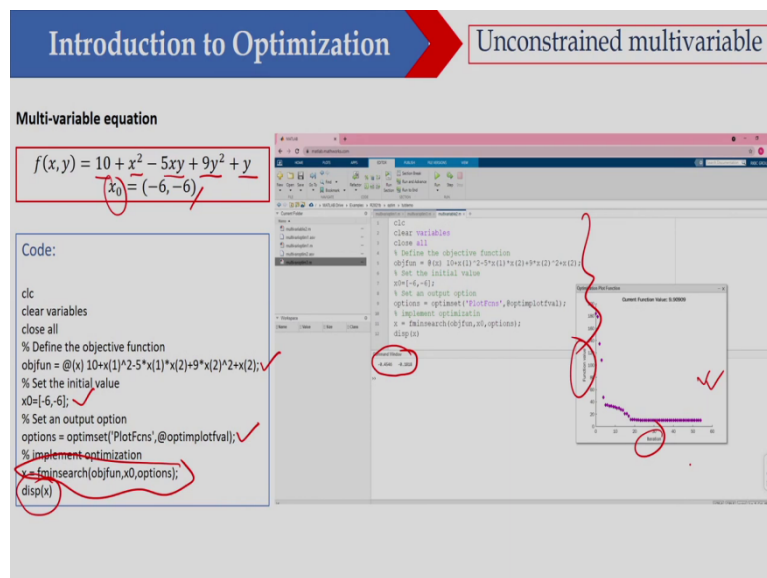
function option I have used. So, I would like to plot the your generation iteration versus function value.

So, that I have used and finally, I have written this particular line that is `x = fminsearch`. So, this is the keyword of this particular function and then you can use objective function then `x` naught an option and once you are executing this particular line then you will get the value of `x` and you can display that `x` using `display` function. So, here is the function here is the MATLAB code.

And if you are executing that one then you are getting 3, 2. So, 3, 2 is the solution. So, you know that this particular function has four optimal solution and these are all alternate optimal solutions and in this case you have taken `x` naught. So, `x` naught equal to you have taken 3, 3. So, from the here from the for this naught you are getting the solution 3 2 ok. So, this is the solution you are getting.

Now, let us see another function. So, this is also a multi variable function without constraint. So, I will use `fminsearch` your function to solve this particular optimization problem.

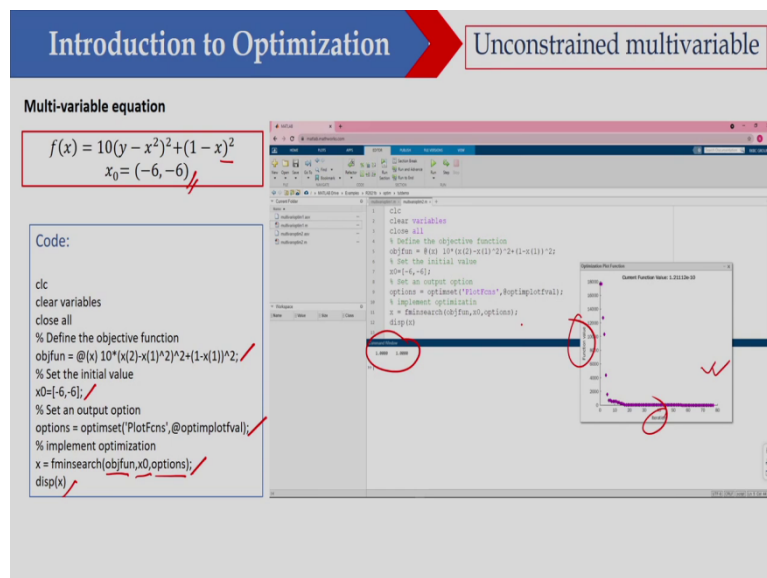
(Refer Slide Time: 15:19)



So, here the objective function is 10 plus x square minus 5 x y plus 9 y square plus y and the x naught is minus 6 minus 6 ok. So, this is your x naught. So, here I have defined the objective function value, here I have defined the objective function and then you define what is x naught and then you define option and finally, you use this particular line to solve this particular problem and then you just display the solution..

So, you just see this is only few lines code. So, we have only 12 lines here and finally, you just see this is the optimal solution of this problem and you can also see the iteration versus function value. How with the iteration function value is changing ok, that you can see.

(Refer Slide Time: 16:15)



Now, there is another function this is also a multi variable function; that means, 2 variable function and I will solve this function using `fminsearch`. So, here the function is 10 y minus x square whole square and then plus 1 minus x whole square and x naught value is minus 6 minus 6 ok..

So, you define the objective function here you define the initial solution, you define the option and finally, you execute this particular line `fminsearch` objective function x naught an option and then display the solution. And here I am getting the solution and that solution is 1 1 and you can also see the plot between iteration and function value.

(Refer Slide Time: 17:05)

Introduction to Optimization

Constrained multivariable

fmincon()

Description

Min $f(x)$ such that

$$\begin{cases} c(x) \leq 0 \\ c_{eq}(x) = 0 \\ A \cdot x \leq b \\ A_{eq} \cdot x = b_{eq} \\ lb \leq x \leq ub \end{cases}$$

Min $f(x)$
Subject to
 $c(x) \leq 0$

b and b_{eq} are vectors, ✓
 A and A_{eq} are matrices, ✓
 $c(x)$ and $c_{eq}(x)$ are functions that return vectors ✓
 $f(x)$ is a function that returns a scalar ✓
 $f(x)$, $c(x)$, and $c_{eq}(x)$ can be nonlinear functions ✓

```

x = fmincon(fun,x0,A,b,Aeq,lb,ub)
x = fmincon(fun,x0,A,b,Aeq,lb,ub,options)
x = fmincon(problem,x0,A,b,Aeq,lb,ub,options)
[x,fval] = fmincon(...)
[x,fval,exitflag,output] = fmincon(...)
[x,fval,exitflag,output,lambda,grad,hessian] = fmincon(...)

```

$c(x) \leq 0$ $c(x) \geq 0$
 $-c(x) \leq 0$

Now, let us see how you can solve a constrained optimization problem using MATLAB. So, here I will use fmincon function, this is a function which implements the classical optimization algorithms. So, I will use fmincon here. So, let us discuss the problem type. So, a optimization problem may be something like that. So, I would like to minimize $f(x)$ so; that means, I the objective function; that means, I would like to minimize $f(x)$ ok..

So, x is a vector here and then subject to subject to that is $c(x)$ is less than equal to 0. The first one is $c(x)$ is less than equal to 0; that means, this is a non-linear inequality constraint the first one is non-linear inequality constraint, then second one is $c_{eq}(x)$; that means, this is a non-linear equality constraint first one is inequality constraint and this is equality constraint. Then you have linear inequality constraint, suppose I have linear inequality constrain. So, I

can define it here and if I have linear equality constraint so, I can define here and finally, I can define lower bound and upper bound of the variable.

So, here what I can use, I can use non-linear inequality constraint, non-linear equality constraint, linear inequality constraint and linear equality and inequality constraint, linear inequality constraint and linear equality constraint. So, here we have to note one thing that when you are defining an inequality constraint; that means, the $c \cdot x$ is less than equal to 0; that means, the this sign should be less than equal.

That mean if your problem is that $c \cdot x$ is greater and equal to 0 then what you will do, you will multiply it by minus 1. So, minus $c \cdot x$ is less than equal to 0 ok. So, always this sign should be less than equality type and similarly in case of linear inequality constraint that is this also should be less than equality type constraint. So, if it is greater and equality type constraint. So, we have to convert it to less than equality type constraint. So, already we have discussed this part. So, I think you are aware about this thing..

So, I can convert a less than equality type constraint to greater than equality type constraint, similarly I can also convert a greater than equality type constraint to less than equality type constraint. So, that I can do so, in this case if you are using fmincon function then inequality constraint should be less than equality type, the inequality constraint should be less than equality type ok. So, if it is greater inequality type then you multiply it by minus 1 and convert it to less than equality type..

So, here I have defined what is b. So, b and b equality are vector and A equality type are matrix, then $c \cdot x$ and $c \cdot \text{equality } x$ are function that returns the vector and $f \cdot x$ is objective function and $f \cdot x$, $c \cdot x$ and $c \cdot \text{equality } x$ can be non-linear function. So, this is non-linear function and this is the syntax of fmincon function..

So, what you have to do suppose your problem has only objective function. So, then you are defining objective function then you are defining x naught then if you have linear constraint A, b. So, you have to define..

So, if you have linear inequality constraint then A, b you will define if you have linear inequality constraint then A e q and b e q you will to define and you if you want to define lower bound and upper bound you can define and if you have non-linear equality inequality constraint and equality constraint then you have to write a function a non-linear constraint. So, that you have to do and if you want to put the option that you can put the option..

So, if you are writing x; that means the optimal solution will be return and if you are writing x and fval so, in that case the optimal function value will also return. Similarly, I can see what is exit flag and output so, that also I can see and I can also get some other information like lambda, gradient, hessian that information I can also extract from here.

(Refer Slide Time: 22:00)

Introduction to Optimization

Constrained multivariable

Example 1

Minimize $f(x) = e^{x_1 x_2 x_3 x_4 x_5} - 0.5(x_1^3 + x_2^3 + 1)^2 //$

Subject to $\begin{cases} x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0 \\ x_2 x_3 - 5x_4 x_5 = 0 \\ x_1^3 + x_2^3 + 1 = 0 \end{cases}$ non linear
equality
const.

Use the starting point $x_0 = (-1.71, 1.59, 1.82, -0.763, -0.763)^T$

Now, let us see I would like to solve this particular problem using fmincon function. So, here we have the objective function. So, objective function is e to the power x 1 into x 2 into x 3

into x_4 into x_5 minus 0.5 within bracket x_1 cube plus x_2 cube plus 1 whole square ok. So, this is the objective function and we have three non-linear. So, this is non-linear equality constraint ok so, non-linear equality constraint.

So, we have three non-linear equality constraint and the starting point is this starting point is this. So, that has been defined. So, let us see how I can solve this particular problem using fmincon function.

(Refer Slide Time: 22:56)

Introduction to Optimization

Constrained multivariable

Example 1

Step-1: Create file nlcon.m for constraints

```

% create file nlcon.m for nonlinear constraints
function [c,ceq] = nlcon(x)
ceq(1) = x(1)^2+x(2)^2+x(3)^2+x(4)^2+x(5)^2-10;
ceq(2) = x(2)*x(3)-5*x(4)*x(5);
ceq(3) = x(1)^3+x(2)^3+1;
c=[]; % inequality non-linear constraint is EMPTY
end

```

equality type

nlcon.m

So, we have non-linear equality constraint ok. So, therefore, I have to write a function to define the constraint ok. So, here we have to we have three constraint that and this is equality type constraint. So, therefore, I am writing here this is c_{eq1} , c_{eq2} and c_{eq3} . So, what is this? These are equality type ok. So, equality type constraint. So, this is the first constraint, this is the second constraint, this is the third constraint. Now, I have defined these as a

function ok the function will return c value and c equality value and these are vector and the name of the function is n c con ok. So, you can change this name ok.

So, here I have written n l sorry n l nlcon and the input argument here is x basically. So, x means x is a vector and it is actually giving x 1 x 2 x 3 x 4 and x 5 ok. So, this three constraint I have written and finally, there is no inequality constraint. So, therefore, I am returning null. So, null is returned using this so; that means, there null this is inequality constraint and this is empty and then this function is n here ok. So, this is the function I have written..

So, now, I can do in two way; one is there I can write a different m file using n nn I can define it as a function or I can write in a same problem . So, in the same M file. So, I can also define this particular your function. So, I will show you how you can do that one. So, this is the function I have written and this function I have save as nlcon ok dot m. So, you can see this way I have save this particular file.

(Refer Slide Time: 25:08)

Introduction to Optimization
Constrained Multivariable

Example 1

Step-2: Create an objective function and use function fmincon()

```

clc
clear variable
close all
objective = @(x) exp(x(1)*x(2)*x(3)*x(4)*x(5))-
0.5*(x(1)^3+x(2)^3+1)^2;
% initial guess
x0 = [-1.71,1.59,1.82,-0.763,-0.736];
% variable bounds
lb = -5.0 * ones(1,5);
ub = 5.0 * ones(1,5);
% show initial objective
disp(['Initial Objective: ' num2str(objective(x0))])
% linear constraints
A = []; %linear inequality constraint is empty
b = []; %linear inequality constraint is empty
Aeq = []; %linear equality constraint is empty
beq = []; %linear equality constraint is empty
% nonlinear constraints
nonlcon = @nlcon; % call function from step-1
% optimize with fmincon
[x,fval] = fmincon(objective,x0,A,b,Aeq,beq,lb,ub,nonlcon);
% show final objective
disp(['Final Objective: ' num2str(objective(x))])
% print solution
disp('Solution')
disp(['x1 = ' num2str(x(1))])
disp(['x2 = ' num2str(x(2))])
disp(['x3 = ' num2str(x(3))])
disp(['x4 = ' num2str(x(4))])
disp(['x5 = ' num2str(x(5))])

```

```

clc
clear variables
close all
objective = @(x) exp(x(1)*x(2)*x(3)*x(4)*x(5))-0.5*(x(1)^3+x(2)^3+1)
% initial guess
x0 = [-1.71,1.59,1.82,-0.763,-0.736];
% variable bounds
lb = -5.0 * ones(1,5);
ub = 5.0 * ones(1,5);
% show initial objective
disp(['Initial Objective: ' num2str(objective(x0))])
% linear constraints
A = []; %linear inequality constraint is empty
b = []; %linear inequality constraint is empty

```

Now, I have written another M file. So, here I have defined the objective function and then initial guessed and then lower bound and upper bound. So, that I have defined and then I have just display the initial objective function value, but that is not required, but I have just display here and these are all empty ok..

So, A is not there, that b is not there, A equality is not there, b equality is not there, and then finally, I have used this particular line to call this function and here I have just used. So, this is actually I have call here and or I can directly write this here also this is f equal to fmincon the objective is defined here. So, this is the objective. So, I have defined and then x naught is the initial solution A, b, A eq, beq, then lower bound, upper bound I have defined and this is the constraint ok.

So, then once this line is executed so, you will get the optimal solution of this problem and then you can show that one display the solution x_1 x_2 x_3 x_4 and x_5 and you can also see the optimal objective function value ok. So, here I have written this code and once you are executing this particular code. So, you can see the solution is x_1 equal to minus 1.7171, x_2 equal to 1.5957, x_3 equal to 1.8272, x_4 equal to minus 0.76364 and x_5 equal to minus 0.76364 ok. So, this is the solution you should get.

(Refer Slide Time: 27:09)

Introduction to Optimization

Constrained multivariable

Example 2

Minimize $f(x) = \underline{(x_1^2 + x_2 - 11)^2} + \underline{(x_2^2 + x_1 - 7)^2}$

Subject to $\frac{(x_1 - 5)^2 + x_2^2}{4x_1 + x_2} \leq \frac{26}{20}$ (3, 3)

$x_1, x_2 \geq 0$

Use the starting point $x_0 = (3, 3)^T$

Now, let us see this particular problem. So, this is also a constrained optimization problem and this problem I will solve using fmincon function of MATLAB. So, here the objective function is x_1 plus x_2 minus 11 whole square plus x_2 square plus x_1 minus 7 whole square.

And we have total two constraint and the first constraint is $x_1^2 - 5x_1 + x_2^2$ less than equality type. So, this is less than equality type constraint and this is less than 26 and $4x_1 + x_2$ and this is less than equal to 20 and x_1, x_2 they are positive; that means, greater than 0 and initial solution here is x_0 and which is equal to 3, 3; that means, I would like to start the solution from 3, 3. So, this is my x_0 ok. So, let us see how I can solve this problem using fmincon function.

(Refer Slide Time: 28:15)

Introduction to Optimization

Constrained multivariable

Example 2

```

clc
clear variables
objective = @(x) (x(1)^2+x(2)-11)^2+(x(2)^2+x(1)-7)^2;
% initial guess
x0 = [-3,-3];
% variable bounds
lb = -5.0 * ones(1,2);
ub = 5.0 * ones(1,2);
% linear constraints
A = []; %linear inequality constraint is empty
b = []; %linear inequality constraint is empty
Aeq = []; %linear equality constraint is empty
beq = []; %linear equality constraint is empty
% nonlinear constraints
nonlincon = @nlcon;

% optimize with fmincon
x = fmincon(objective,x0,A,b,Aeq,beq,lb,ub,nonlincon);
% create file nlcon.m for nonlinear constraints
function [c,ceq] = nlcon(x)
ceq = []; % equality non-linear constraint is EMPTY
c(1) = (x(1)-5)^2+x(2)^2-26;
c(2) = 4*x(1)+x(2)-20;
end

```

So, here I have defined the objective function this is the objective function, then this is the initial guess x_0 , then lower bound I have defined, upper bound I have defined and then A, b, A eq, beq I have defined. So, these are all empty ok. So, that I have defined and then this is the constraint ok. So, now, I have defined the constraint here this is the constraint ok.

So, this is the constraint and this is function the argument is c and ceq and we are passing the x vector ok. So, here there is no equality type constraint. So, non-linear equality type constraint. So, I am putting null and then we have inequality type constraint that is c 1 and c 2 so, that I have defined here ok.

So, I have written the entire code here. So, if you are executing this particular M file ok. So, you will get the solution of this particular problem. So, that I will show you what is the solution of this particular problem. Let us see another example problem and this is also non-linear problem minimization problem with constraint.

(Refer Slide Time: 29:40)

Introduction to Optimization

Constrained multivariable

Example 3

Minimize $f(x) = e^{x_1}(4x_1^2 + 2x_2^2 + 4x_1x_2 + 2x_2 + 1)$

Subject to $1.5 + x_1x_2 - x_1 - x_2 \leq 0$

$-x_1x_2 \leq 10$

$x_1, x_2 \geq 0$

Use the starting point $x_0 = (-1, -1)^T$

So, this is example 3. So, here this is the objective function and we have 12 inequality type constraint, non-linear inequality type constraint and x_1 and x_2 are positive and starting point is minus 1 minus 1.

(Refer Slide Time: 29:58)

Introduction to Optimization

Constrained multivariable

Example 3

```

clc
clear variables
close all
objective = @(x)
exp(x(1))*(4*x(1)^2+2*x(2)^2+4*x(1)*x(2)+2*x(2)+1);
x0 = [-1 -1];
% nonlinear constraints
nonlincon = @confun;
% optimization calculation using fmincon
[x,fval] =
fmincon(objective,x0,[],[],[],[],[],[],[],[],[],[],nonlincon);

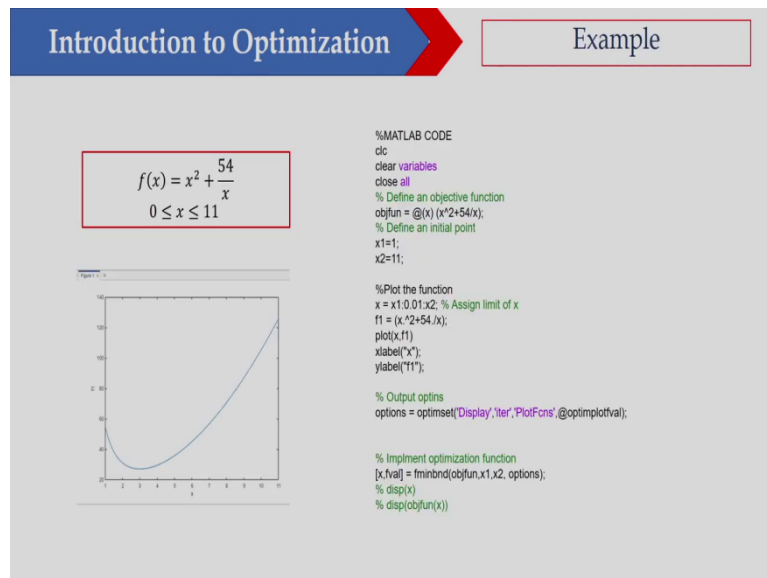
% constraint function
function [c, ceq] = confun(x)
c = [1.5*x(1)*x(2)-x(1)-x(2); -x(1)*x(2)-10];
ceq = [];
end

```

x fval

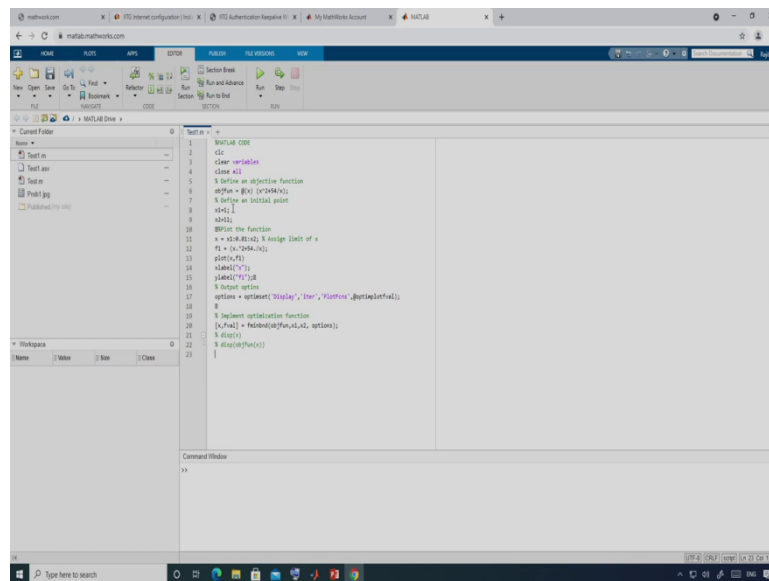
So, I have written the code here. So, this is the objective function, then this is the initial point, initial point is minus 1 minus 1 and I have defined here the constraint function. So, we have two constraint. So, this is inequality type constraint and second one is also inequality type constraint. So, I have defined this is your first constraint ok c_1 and this is your second constraint so, that way also you can define and we do not have equality type constraint. So, therefore, this is null and once you are defining and if you execute this particular line. So, then you will get the value of x and value of objective function value ok. So, that I have written here.

(Refer Slide Time: 30:49)



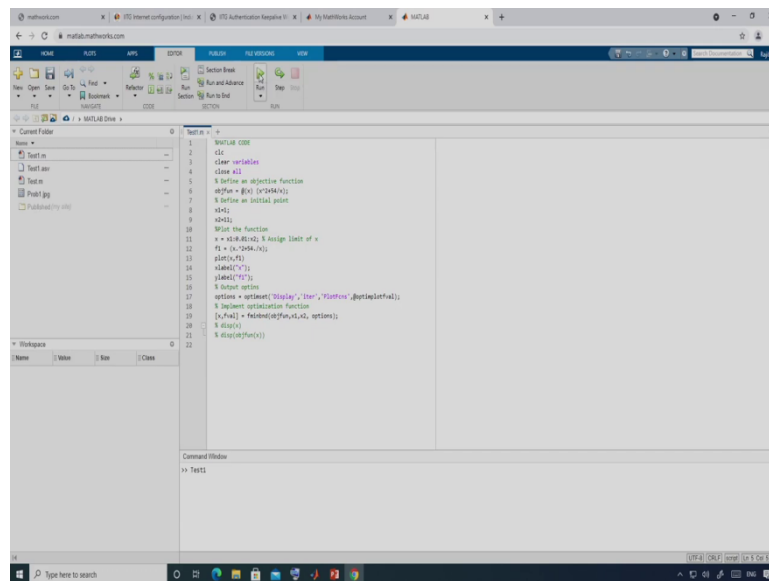
Now, let us go to MATLAB and try to solve some of the problem ok. So, I will use the online version of MATLAB. So, I have open the online version of the MATLAB online.

(Refer Slide Time: 31:07)



So, I am getting this particular interface. So, here I can write the code ok. So, in this particular window I can write the code and then I will try to solve the problem. So, let me copy this. So, I have written this MATLAB code and I will explain that one. So, let me copy this one. So, I have copied this MATLAB code ok.

(Refer Slide Time: 31:34)



The image shows a MATLAB IDE window with the following code in the editor:

```
1 % Test 1: f = x^2 + 54x  
2 clear  
3 clear variables  
4 class all  
5 % Define an objective function  
6 objfun = @(x) (x^2+54*x);  
7 % Define an initial point  
8 x0 = 1;  
9 % Define the function  
10 % Define the function  
11 x = x0:0.01:10; % Define limits of x  
12 f1 = (x.^2+54*x);  
13 plot(x,f1);  
14 hold on;  
15 xlabel('x');  
16 % Define options  
17 options = optimset('Display','iter','PlotFcn','@optimplotfval');  
18 % Define optimization function  
19 [x_opt,f_opt] = fminunc(objfun,x0,options);  
20 % Display results  
21 display(x_opt);  
22
```

The Command Window shows the output: `>> Test1`

So, here as I said that these are to clean the environment. So, I am cleaning and after that I have defined the objective function. So, objective function is that is x square plus 54 by x and then I have defined lower bound and upper bound.

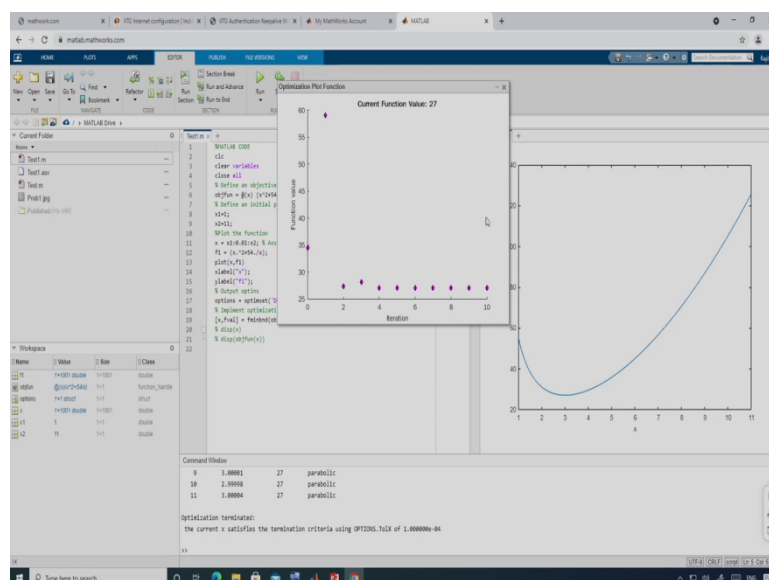
Now, suppose I would like to plot this particular function. So, I can see actually where optimal solution is there. So, for plotting that I am just defining x . So, x is a vector and this is starting from lower bound to upper bound and then the interval between two points is point 0.01 and then I am calculating the value of f the objective function value that is. So, I have used x dot to the power 2 then plus 54 dot by x ok.

And then, I am plotting using this particular function plot function. So, already we have learned how to plot it. So, I can plot this particular function and in the plot I would like to level the x axis as x and y axis as f 1 ok. So, these lines ok these 5 lines will plot this

particular function ok between x_1 and x_2 and after that after that so, I would like to give I would like to solve this problem. So, I have defined the option. So, often here; that means, I would like to display iteration and plot function and finally, I am using `fminbnd`.

So, I have defined the objective function and then x_1 x_2 and option ok. So, I can display what is the value of x and what is the value of your objective function that I can display, but without that also it is fine. So, let me check that one. So, you can see if I execute this particular MATLAB code using the run button. So, it is running now, in the command window you can check that one yeah.

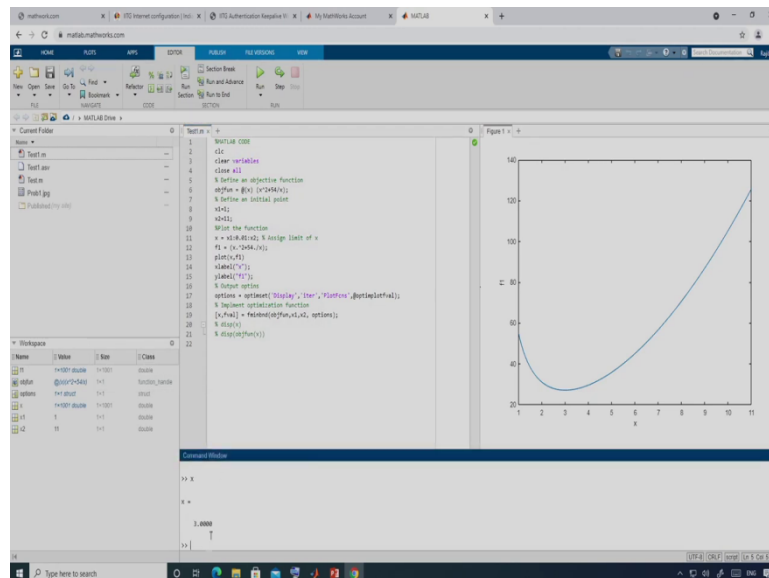
(Refer Slide Time: 33:38)



So, I got the solution here. So, you can see. So, I have plotted this particular function and this is between 1 and 11 and optimal solution is somewhere here 3. So, this is the your optimal solution of this particular problem and you can see the with iteration the objective function

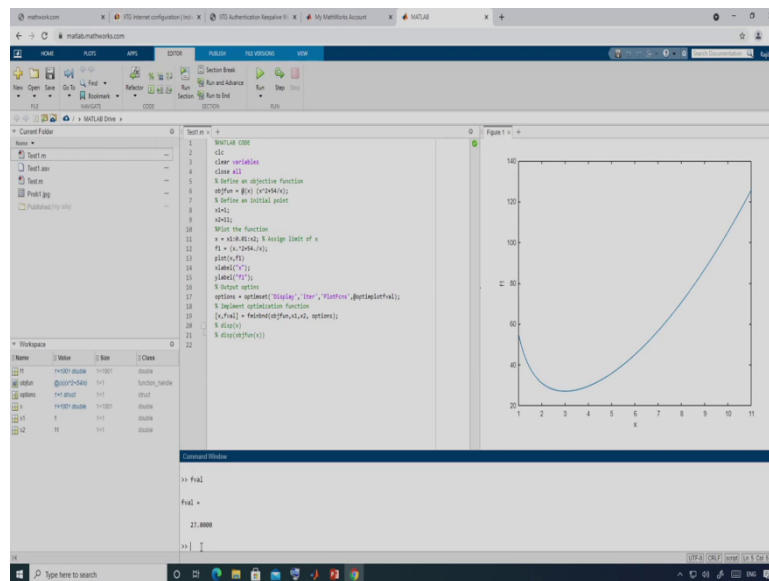
value has changed and finally, you are getting 3 and then solution is 3. So, I can see the solution here by typing what is the value of x.

(Refer Slide Time: 34:10)



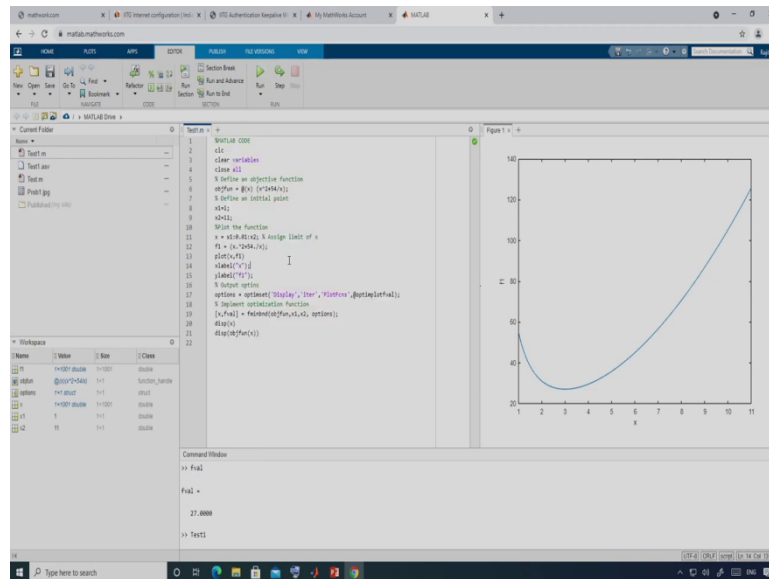
So, I can see the solution is 3.

(Refer Slide Time: 34:16)



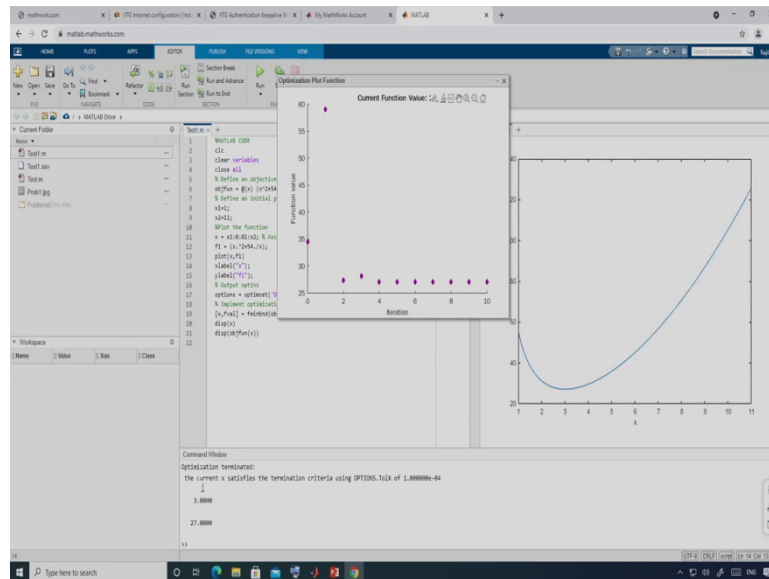
And I can also check what is the value of objective function value ok. So, this is 27.

(Refer Slide Time: 34:24)



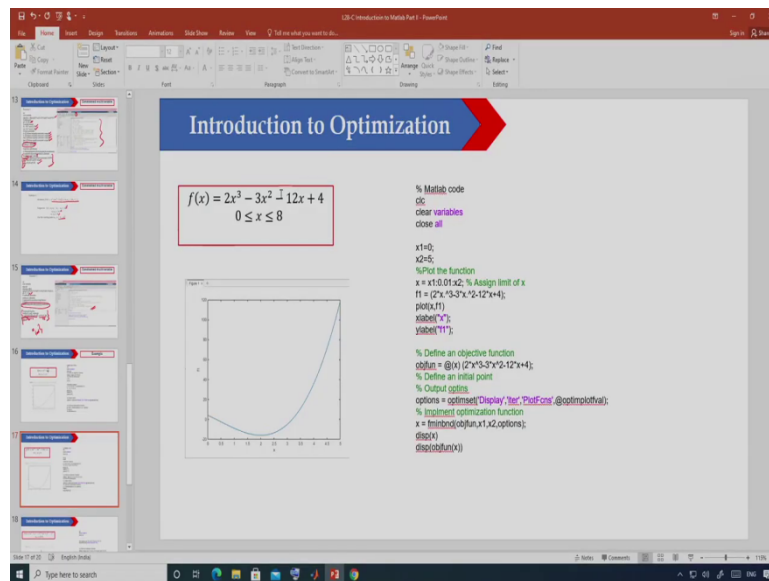
So, I can also use this one this is fine. So, if I execute these two line, then also I am getting I am getting the solution just see basically so, I can execute that one.

(Refer Slide Time: 34:33)



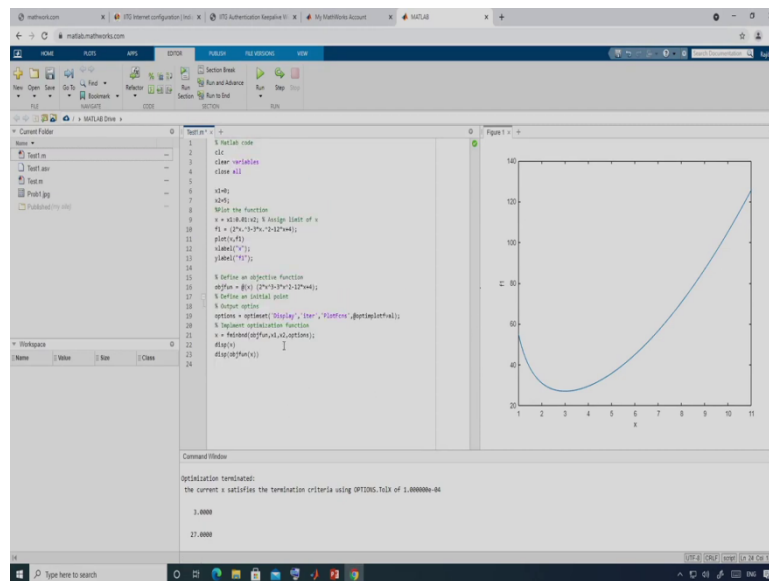
So, here you can see that optimization terminated and I am getting this particular solution; that means, x equal to 3 and objective function value at optimal solution is 27 ok. So, this is 27 I hope this is fine. So, let us solve another example problem.

(Refer Slide Time: 34:57)



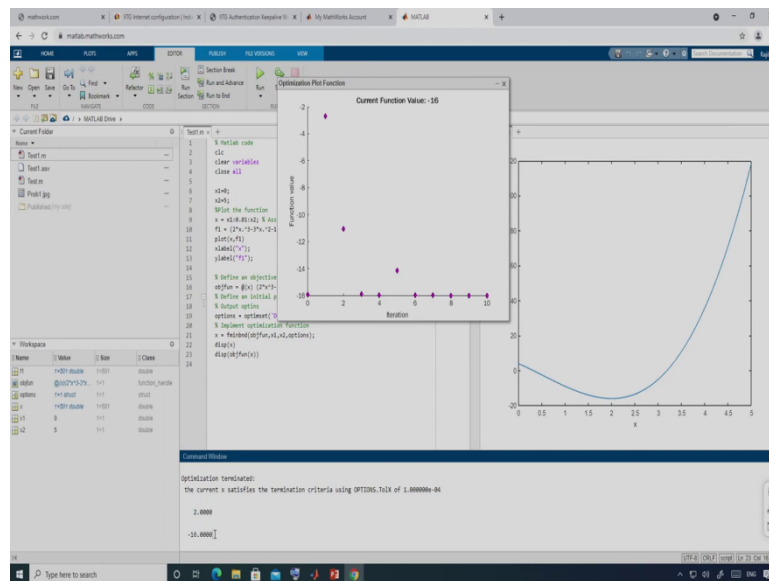
So, next I would like to solve this particular problem that is f of x equal to $2x^3$ minus thrice x square minus $12x$ plus 4 and this is between 0 and 8 ok and this is the plot of this particular function and solution is somewhere here ok. So, let me copy this MATLAB code.

(Refer Slide Time: 35:23)



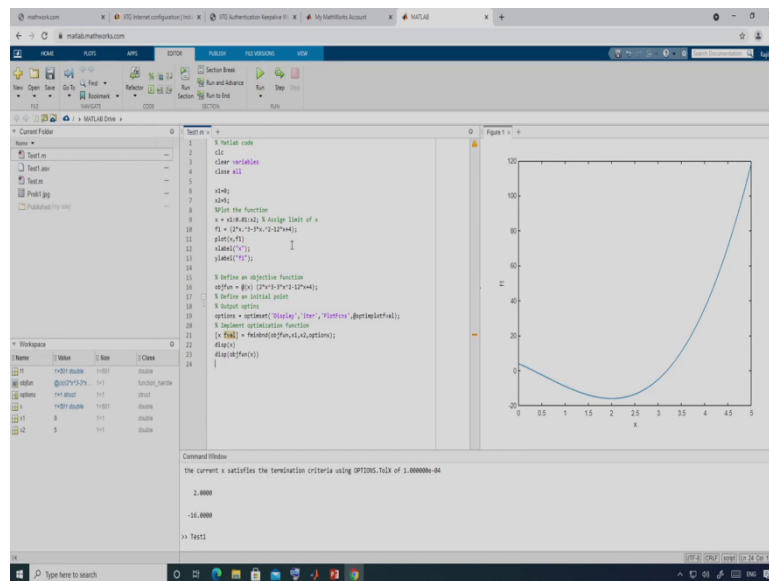
So, let me go there and ok. So, I have copied this code and here now this particular lines to plot this particular function and then I have defined the objective function and then the option and then I have used f b and d and I have to put objective function $x_1 x_2$ and option and finally, I can display the solution. So, just see.

(Refer Slide Time: 35:55)



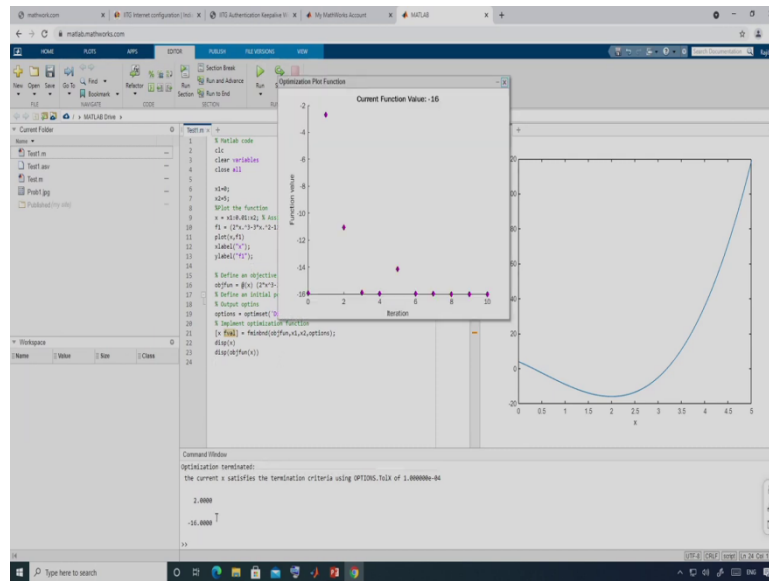
So, I can. So, I am getting the solution and the solution is x equal to 2 and optimal objective function value is minus 16 ok. And this is the plot this is the plot I am getting and then this is the plot between function value and iteration. So, this is the plot between function value and iteration..

(Refer Slide Time: 36:23)

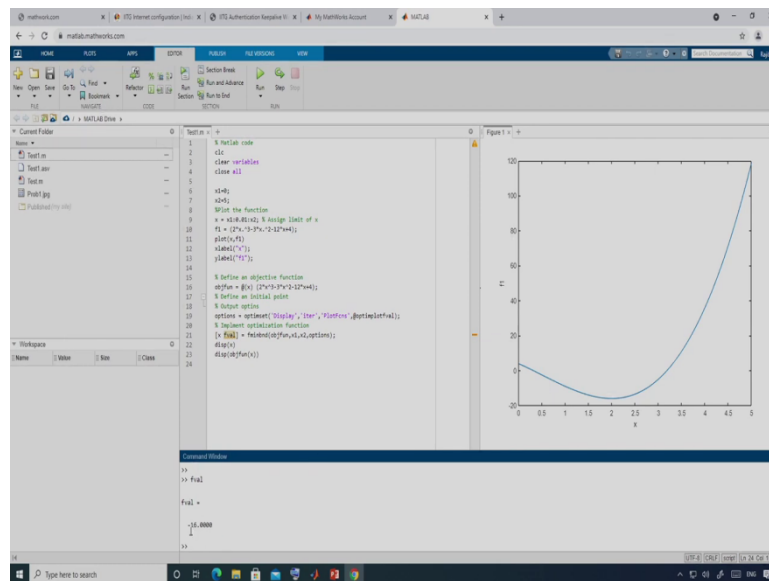


So, now, suppose if I write that I am getting x, but I would like to see what is the objective function value. So, what I will do fval ok. So, that there I can write may be then let me run this particular.

(Refer Slide Time: 36:36)

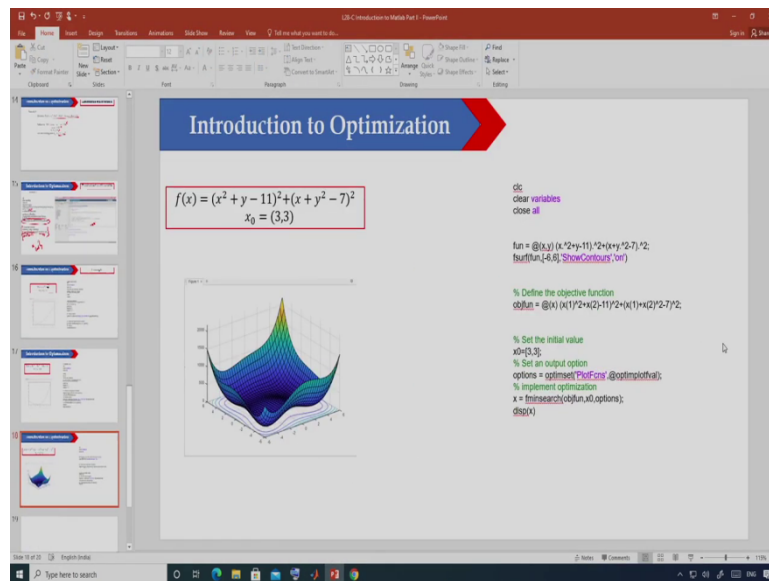


(Refer Slide Time: 36:39)



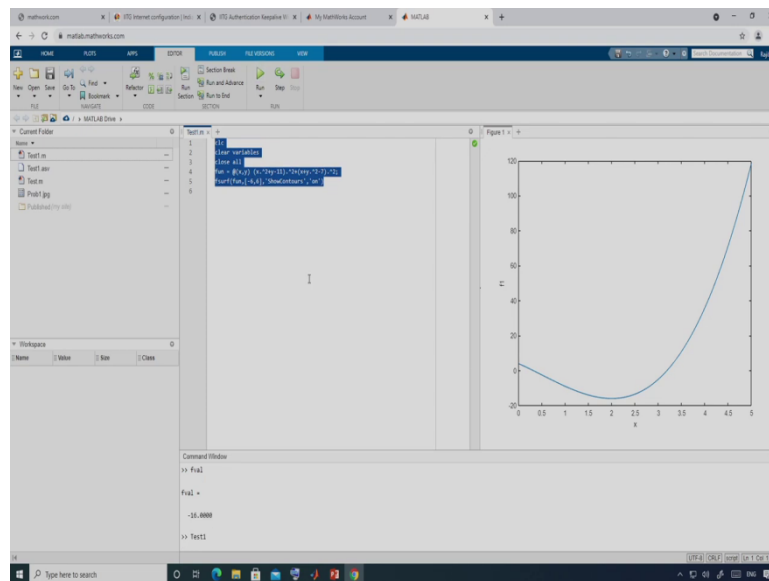
So, you can see what is the value of what is the value of optimal value of the function, optimal function value is minus 16. So, I am getting this particular solution let me see another function.

(Refer Slide Time: 36:55)



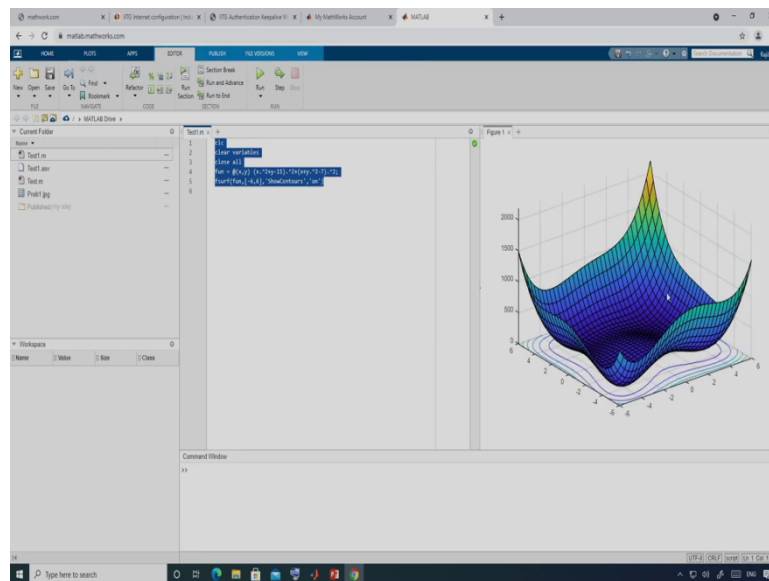
Suppose I would like to solve this particular problem and this problem is multi variable problem. So, here what I will use, I will use here I will use the fminsearch function.

(Refer Slide Time: 37:15)



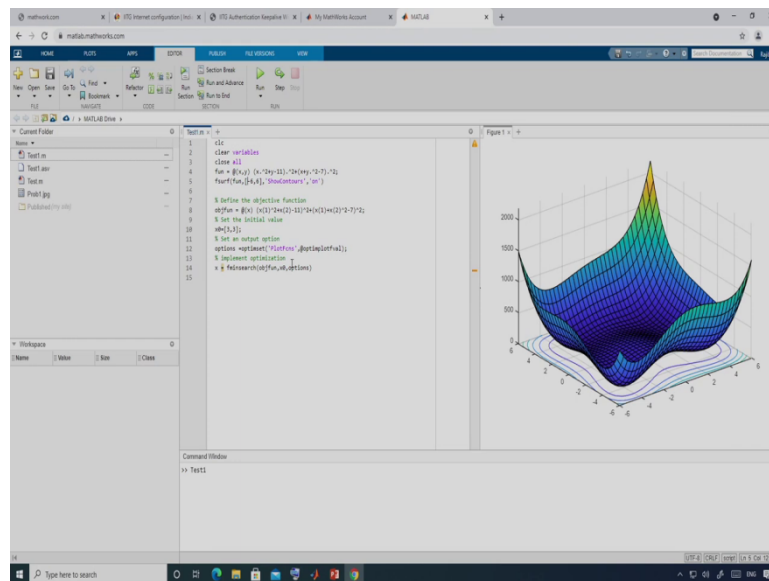
So, first let me copy this few lines. So, here what I am doing, I am trying to plot this particular function. So, initially I have cleared the environment and after at I am defining the function and function is $x^2 + (x-2)^2 + 11$ then whole square then x plus y square minus 7 whole square. So, I have defined this particular function here. So, in the last class I said actually so, how to write this expression and then I have used `fsearch` function to plot it and also show the control. So, if you are executing this few lines. So, you can get the surface plot of this particular function.

(Refer Slide Time: 37:57)



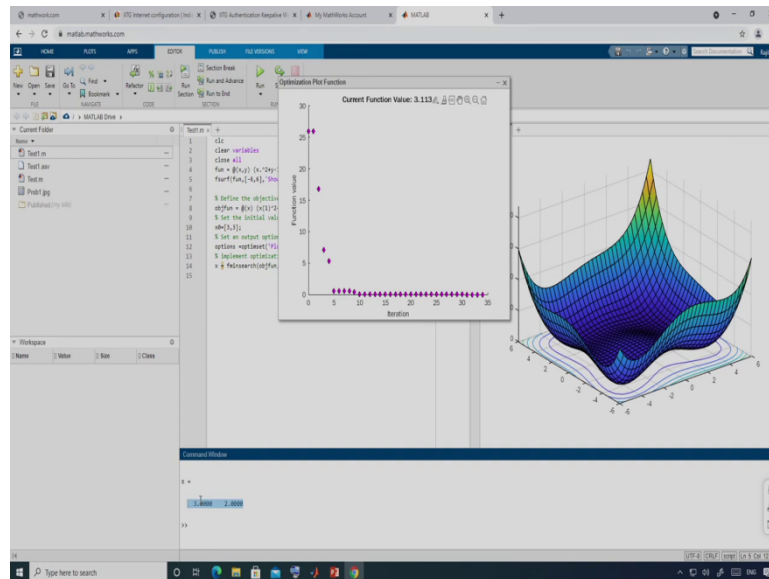
So, you are getting this surface plot. And this is between minus 6 and plus 6 . So, as you know there are 4 optimal solution here, but we will get one of that because we are using classical optimization technique ok. So, then if I am. So, then let us define this function and I am using here fminsearch ok.

(Refer Slide Time: 38:27)



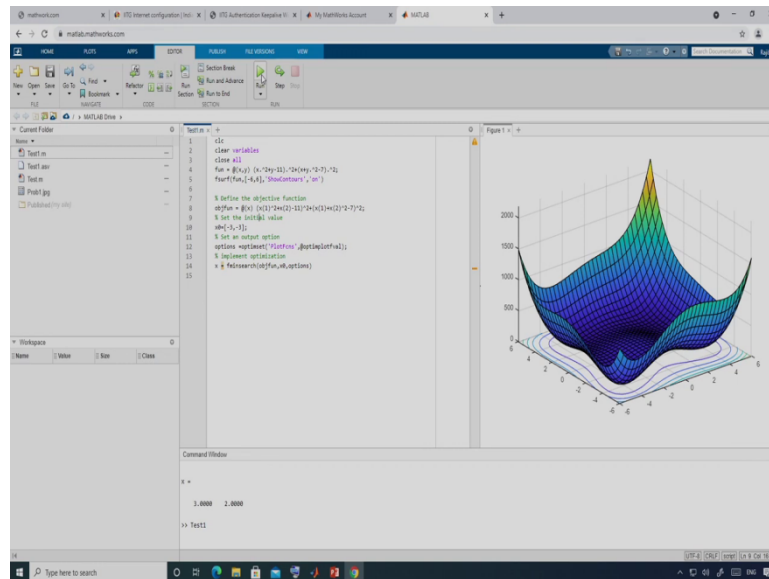
So, let us copy this few lines and here I am just copying ok. So, I will paste it here and finally, you are getting the this is the objective function I have defined ok. Then this is the initial point I have defined and then I have defined the option and finally, I am defining this. So, I would like to see the value of x ok, if I remove this semicolon I can see the value of x ok.

(Refer Slide Time: 38:58)



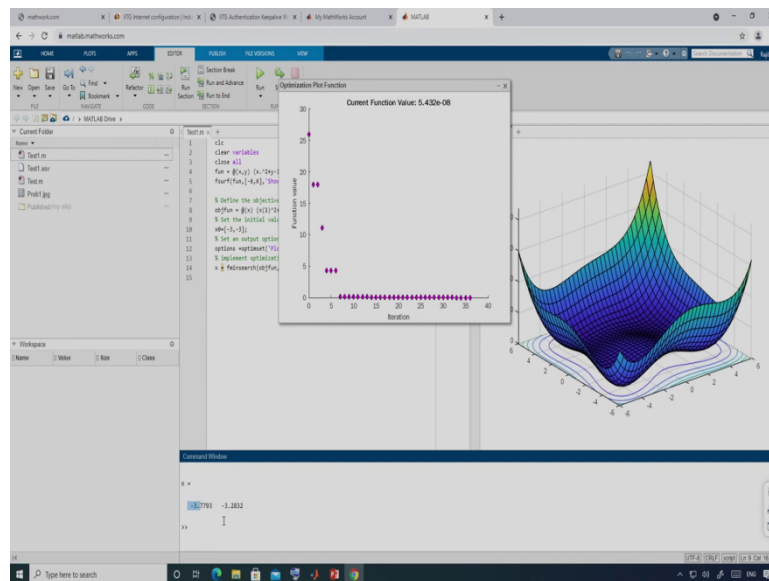
So, let us see. So, let me run this ok. So, you just see I am getting 3, 2 ok. So, I am getting 3, 2. So, that is the solution of one of the solution of this particular problem. So, I am getting that one and objective function value is 0 here ok. Objective function value is 0 and this is the plot of this your particular function.

(Refer Slide Time: 39:28)



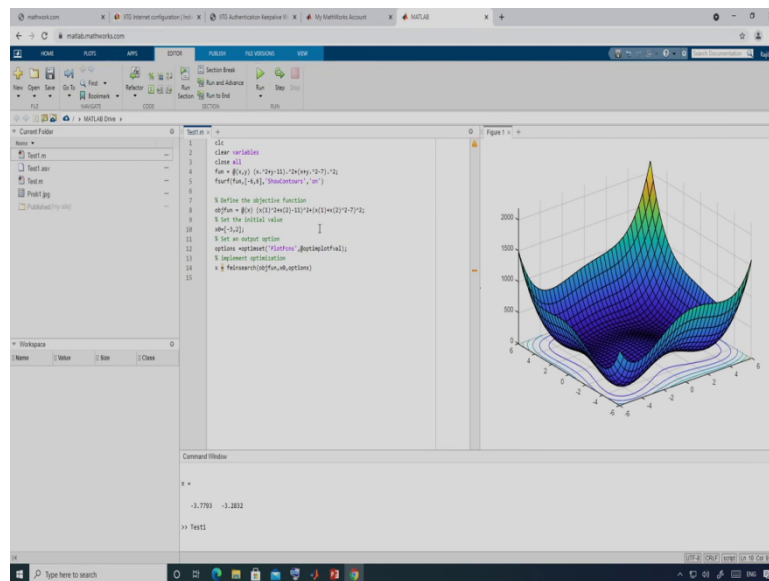
Now, if I change the initial point so, suppose I am taking minus 3 minus 3 ok. So, minus 3 minus 3 then what will happen you just see, let me run it.

(Refer Slide Time: 39:39)



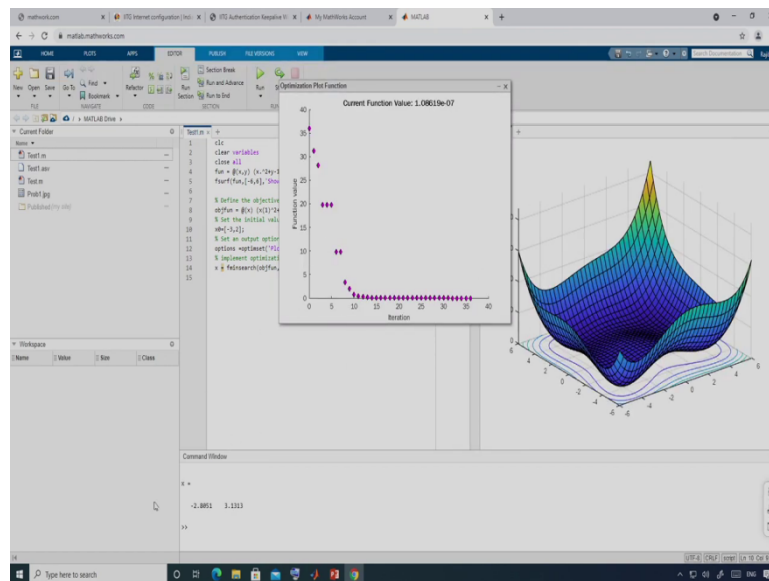
Yeah. So, I am getting another solution and this solution is minus 3.7793 and minus 3.2832. So, I am getting a different solution. I am getting a different solution once I have change the initial point .

(Refer Slide Time: 40:02)



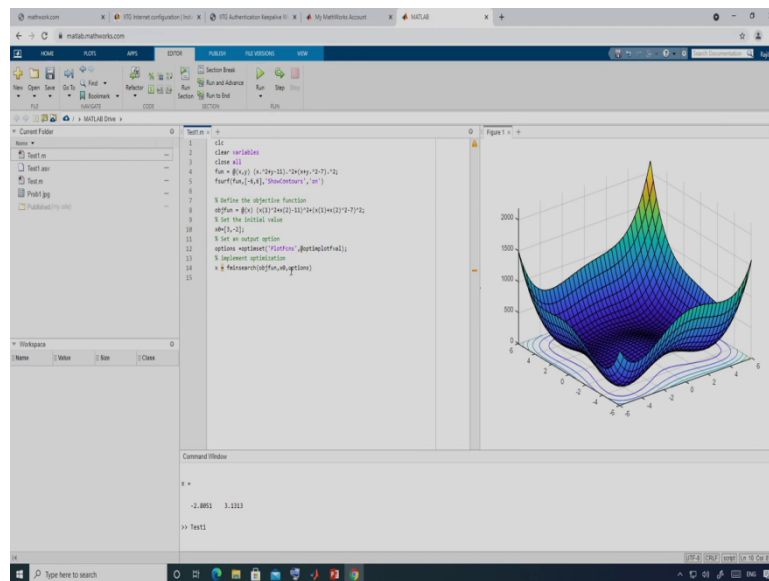
Suppose I am changing an initial point supposed to minus 3, 2 then let us see.

(Refer Slide Time: 40:08)



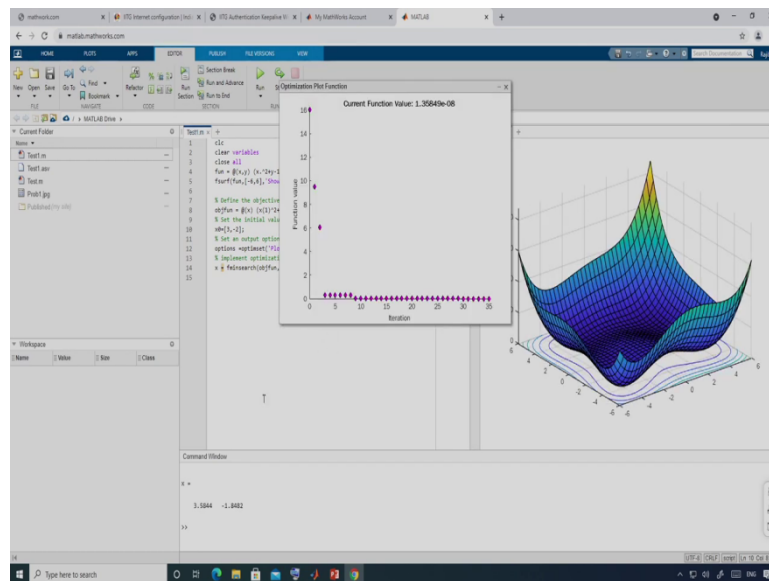
So, this time also I am getting a solution here and this solution is minus 2.8051 and this is 3.1313. So, I am getting a different solution. So, if you are changing your initial solution. So, if you are changing your initial solution then you are getting a different solution, because in this particular function we have total four solutions and all of them are alternate optimal solution because objective function value is same that is 0, but solution is different. So, once you are changing your initial solution the classical optimization problem is finding the other solution ok different solution.

(Refer Slide Time: 40:59)



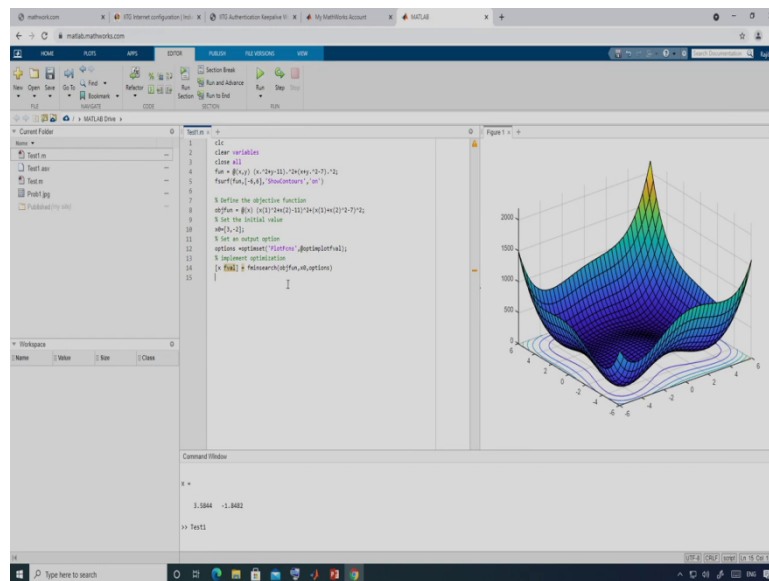
So, let us change the initial solution suppose if I take minus if I take 3 minus 2 then what will happen ok.

(Refer Slide Time: 41:04)



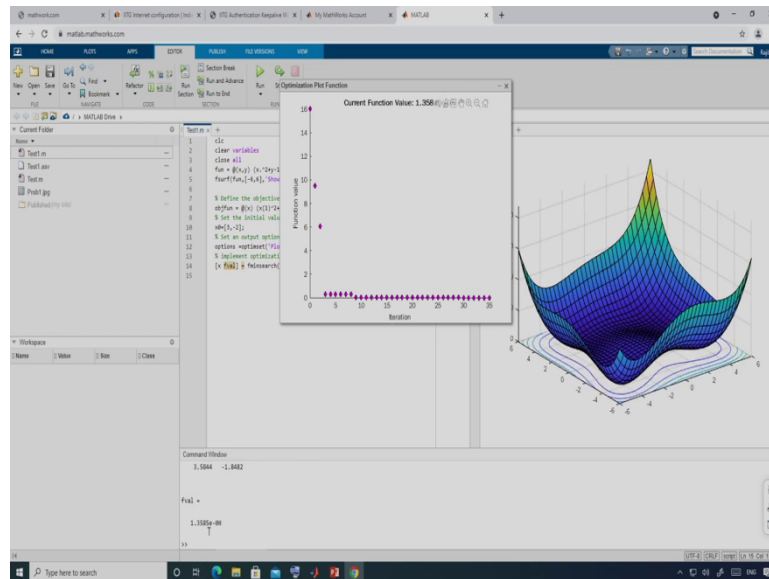
So, I am getting a solution and that is a different solution and that solution is 3.5844 ok 3.5844.

(Refer Slide Time: 41:21)



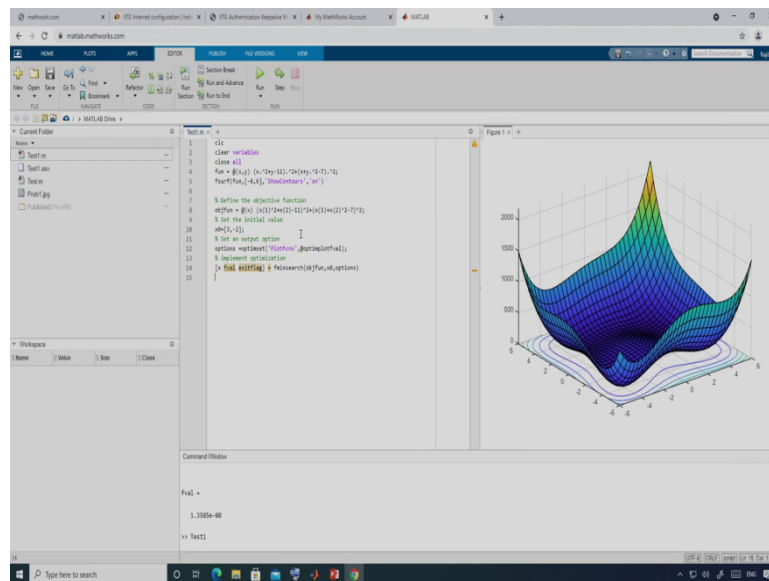
So, you can see here. So, I can also write x ok. So, I can write I can see what is the objective function value.

(Refer Slide Time: 41:33)



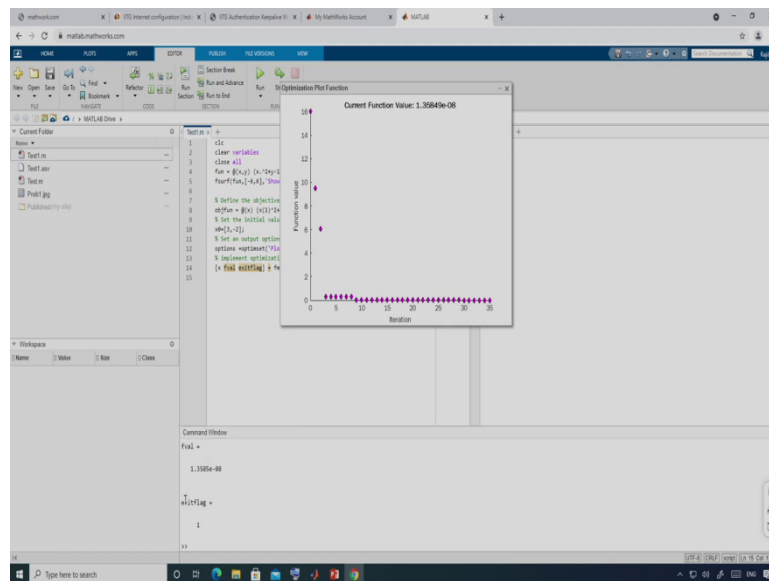
You just see the objective function value is almost 0; that means, 1.35×10^{-8} ok. So, this is almost 0 and you can see that I can also get the other information like let me check here. So, I can also get the exit flag ok. So, this exit flag I can see.

(Refer Slide Time: 41:59)



So, you go here and if you are writing exit flag ok. So, what will happen you just see.

(Refer Slide Time: 42:07)



Yeah exit flag is 1 ok. So, exit flag is 1. So, this is actually showing how why this is terminated ok. So, that you can you can check what is the value of this exit flag so, that you can check.

(Refer Slide Time: 42:23)

Introduction to Optimization

Minimize $f(x) = e^{x_1 x_2 x_3 x_4 x_5} - 0.5(x_1^3 + x_2^3 + 1)^2$

Subject to $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$
 $x_2 x_3 - 5 x_4 x_5 = 0$
 $x_1^3 + x_2^3 + 1 = 0$

Use the starting point $x_0 = (-1.71, 1.59, 1.82, -0.763, -0.763)^T$

```

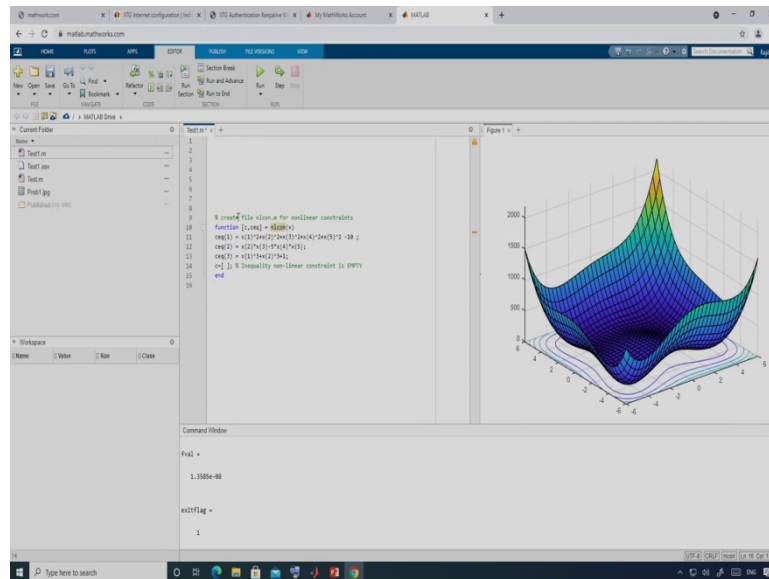
clear variable
close all

objective = @(x) exp(x(1)*x(2)*x(3)*x(4)*x(5))-0.5*(x(1)^3+x(2)^3+1)^2;
% initial guess
x0 = [-1.71,1.59,1.82,-0.763,-0.763];
% variable bounds
lb = -5.0*ones(1,5);
ub = 5.0*ones(1,5);
% show initial objective
disp(['Initial Objective: ', num2str(objective(x0))])
% linear constraints
A = []; % linear inequality constraint is empty
b = []; % linear inequality constraint is empty
Aeq = []; % linear equality constraint is empty
beq = []; % linear equality constraint is empty
% nonlinear constraints
nonlcon = @nlcon; % call function from step-1
% optimize with fmincon
x = fmincon(objective,x0,A,b,Aeq,beq,lb,ub,nonlcon);
% show final objective
disp(['Final Objective: ', num2str(objective(x))])

% create file nlcon.m for nonlinear constraints
function [c,ceq] = nlcon(x)
ceq(1) = x(1)^2+x(2)^2+x(3)^2+x(4)^2+x(5)^2-10;
ceq(2) = x(2)*x(3)-5*x(4)*x(5);
ceq(3) = x(1)^3+x(2)^3+1;
c=[]; % inequality non-linear constraint is EMPTY
end
  
```

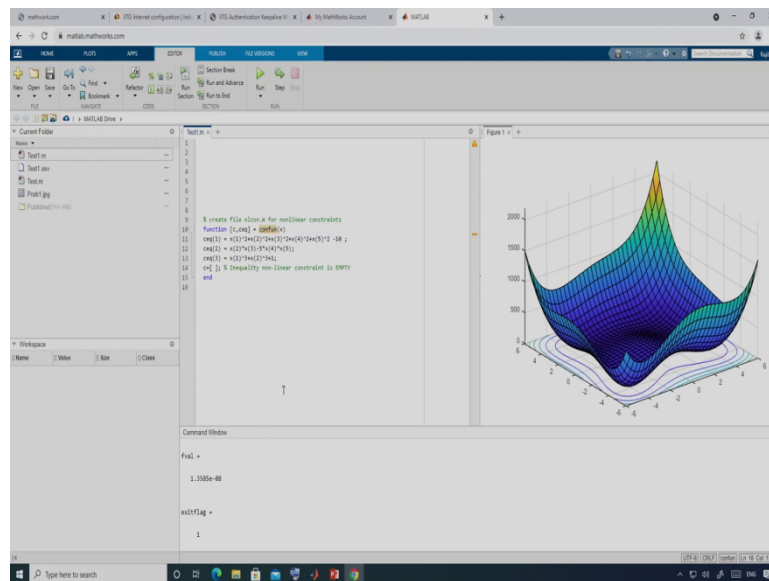
Now, let us solve this constrained optimization problem and the problem here is minimization of $f(x)$ equal to $e^{x_1 x_2 x_3 x_4 x_5}$ minus $0.5 x_1^3 + x_2^3 + 1$ whole square and we have total three constraint and all of them are equality constraint. So, let me copy this thing. So, what I have to define; first I have to define the constraint function. So, I am just copying this portion and I will paste it here.

(Refer Slide Time: 42:59)



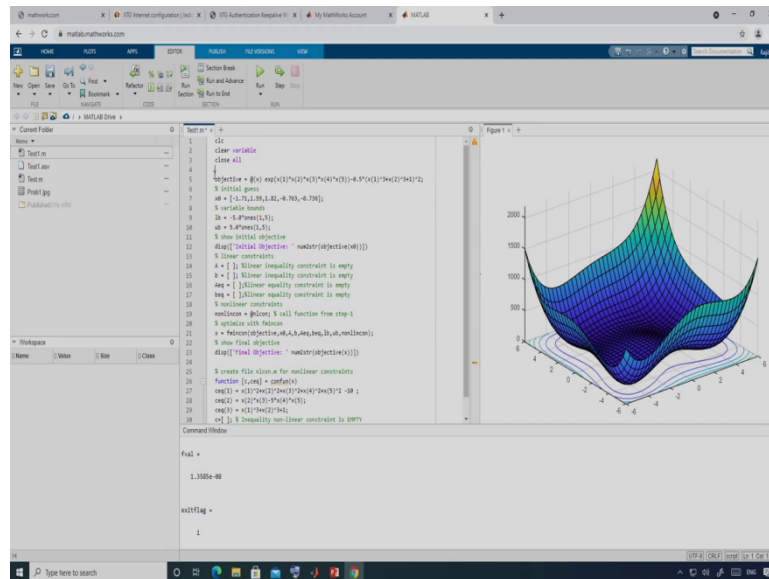
So, here I am defining this constraint. So, you can see I have defined a function and that is you can give a different name.

(Refer Slide Time: 43:09)



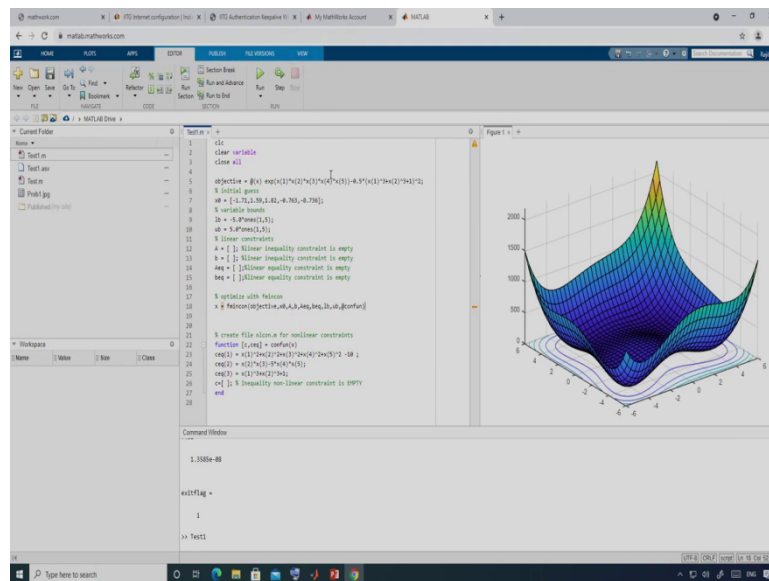
Suppose this is I can give you a confun ok. So, confun and this is x, x is a vector and then I have defined the first constraint, second constraint and third constraint ok. So, I have defined this constraint and then and there is no inequality type constraint. So, this is null ok. So, I have defined the constraint function. So, what you can do, you can define it like that or you can save it as a M file and you can call it ok so, but here I am writing in the same M file. So, now, I need to get this one ok.

(Refer Slide Time: 43:59)



So, here I have defined the objective function and then the initial solution and then the lower bound and upper bound.

(Refer Slide Time: 44:13)



And then this is not required actually to show the initial objective function. So, you can see that one, but otherwise it is fine ok. Now, here I have defined that A equal to null, b equal to null, A equality is null, b equality is null and then this is non-linear constraint. So, this also I can directly define here and here I can define that this is your this is your confun ok. So, this is your confun I have defined and then I need not display this thing. So, I will directly see the value ok..

So, you just see what I have defined, I have defined the objective ok objective function, then initial solution, then A, b, A equal to, b equal to so, that is not there A equal, b equal is not there. So, this is null then lower bound and upper bound I have defined and then confun ok n confun also I have defined. So, I can see the solution just see let me run it.

(Refer Slide Time: 45:18)

The image shows the MATLAB R2020a interface. The main window displays a script for solving a linear programming problem. The script defines the objective function, constraints, and solves the problem using the `fmincon` function. The workspace window shows the variables defined in the script, including the optimal solution `x` and the Lagrange multipliers `lambda` and `mu`.

```
1 %  
2 clear variables  
3 close all  
4  
5 objective = @(x) exp(x(1)*x(2)^2*x(3)^2)-8*x(1)+x(1)^2*x(2)^2*x(3)^2;  
6 % initial guess  
7 x0 = [-1.75, 1.99, 1.42, -8.798, -8.798];  
8 % variable bounds  
9 lb = -5; ub = 5;  
10  
11 % linear constraints  
12 A = []; % Linear inequality constraint is empty  
13 b = []; % Linear inequality constraint is empty  
14 Aeq = []; % Linear equality constraint is empty  
15 beq = []; % Linear equality constraint is empty  
16  
17 % optimize with fmincon  
18 x = fmincon(objective, x0, lb, ub, Aeq, beq, lb, ub, [], [], [], []);  
19  
20  
21  
22 % create file xcon.m for nonlinear constraints  
23 function [c, con] = confun(x)  
24 c(1) = x(1)^2*x(2)^2*x(3)^2-8*x(1)+x(1)^2*x(2)^2*x(3)^2-18;  
25 c(2) = x(2)^2*x(3)^2*x(4)^2*x(5)^2;  
26 c(3) = x(1)^2*x(2)^2*x(3)^2;  
27 end  
28
```

The workspace window shows the following variables:

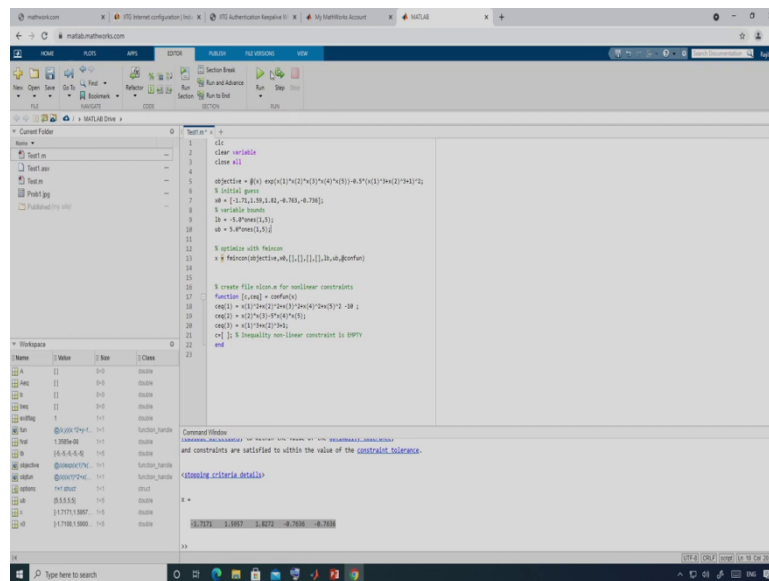
Name	Value	Size	Class
A	0x0 double	0x0	double
Aeq	0x0 double	0x0	double
b	0x0 double	0x0	double
beq	0x0 double	0x0	double
lb	-5x5 double	5x5	double
ub	5x5 double	5x5	double
lambda	1x5 double	1x5	double
mu	1x5 double	1x5	double
options	1x1 struct	1x1	struct
x	1x5 double	1x5	double
x0	1x5 double	1x5	double

The Command Window shows the following message:

```
Optimization terminated: constraints are satisfied to within the value of the constraint tolerance.  
<optimization>  
x =  
-1.7511 1.9987 1.4272 -8.7938 -8.7938
```

So, I am getting this particular solution ok now, here I have defined that A equal to null b equal to null.

(Refer Slide Time: 45:29)



```
1 clear
2 clear variables
3 close all
4
5 objective = @(x) exp(x(1)*7/2)/(x(1)*x(2)^3)-8.5*(x(1)+34*(2/34)^2);
6 % initial guess
7 x0 = [-1.75, 1.39, 1.42, -8.78];
8 % variable bounds
9 lb = -5.8*ones(1,3);
10 ub = 5.8*ones(1,3);
11
12 % optimize with fmincon
13 x = fmincon(objective,x0,[],[],[],[],lb,ub,confun);
14
15
16 % create file xcon.m for nonlinear constraints
17 function [c,con] = confun(x)
18 con(1) = x(1)^2*(2+3*x(1)^2+4*x(2)^2)/2 - 18 ;
19 con(2) = x(2)^2*(3+5*x(1)^2+6*x(2)^2);
20 con(3) = x(1)^2*(2/34)^2;
21 % [c] is % inequality non-linear constraint is 0/0/1
22 end
23
```

Command Window

Optimization terminated: the function value is at a minimum and constraints are satisfied to within the value of the constraint tolerance.

<missing criteria details>

x =

-1.7511 -1.3907 -1.4272 -8.7808

So, here I can directly define that one. So, what I will do, I will I will define it here that A equal to null, b equal to null, then there is no inequality constraint equality constraint. So, I am defining this thing ok. So, this null and this is not required ok. So, these lines are not required. So, I am directly defining here..

(Refer Slide Time: 46:02)

The screenshot displays the MATLAB R2018a environment. The Editor window shows a script named 'Test.m' with the following code:

```

1 % Test.m
2 clear variable
3 close all
4
5 objective = @(x) exp((x(1)^2*(x(2)^3*(x(3)^4*(x(4)^5))-4.5*(x(1)+3*(x(2)+1/2))
6
7 % Initial guess
8 x0 = [-1.75,1.39,0.42,-0.703,-0.709];
9
10 % Variable bounds
11 lb = -5.*ones(1,5);
12 ub = 5.*ones(1,5);
13
14
15 % Update with fmincon
16 x = fmincon(objective,x0,[],[],[],[],lb,ub,@nonfun)
17
18
19 % create file names for nonlinear constraints
20 function [c,ineq] = nonfun(x)
21 eqn1 = x(1)^2*(x(2)^3*(x(3)^4*(x(4)^5))-38 ;
22 eqn2 = x(2)^3*(x(3)^4*(x(4)^5)
23 eq3 = x(1)^2*(x(2)^3);
24 % Is 3-Equality non-linear constraint is empty
25 end

```

The Workspace window shows the following variables:

Name	Value	Size	Class
lb	[]	1x5 double	
ub	[]	1x5 double	
x0	[]	1x5 double	
eqn1	[]	1x1 double	
eqn2	1	1x1 double	
eqn3	0.000000000000000	1x1 double	
nonfun	@(x)[x(1)^2*(x(2)^3*(x(3)^4*(x(4)^5))-38;x(2)^3*(x(3)^4*(x(4)^5);x(1)^2*(x(2)^3)]	1x1 function_handle	
objective	@(x)exp((x(1)^2*(x(2)^3*(x(3)^4*(x(4)^5))-4.5*(x(1)+3*(x(2)+1/2))	1x1 function_handle	
x	[-1.750000000000000 1.390000000000000 0.420000000000000 -0.703000000000000 -0.709000000000000]	1x5 double	
ineq	0.000000000000000	1x1 double	
lb	[-5 -5 -5 -5 -5]	1x5 double	
ub	[5 5 5 5 5]	1x5 double	
x0	[-1.750000000000000 1.390000000000000 0.420000000000000 -0.703000000000000 -0.709000000000000]	1x5 double	

The Command Window shows the following error message:

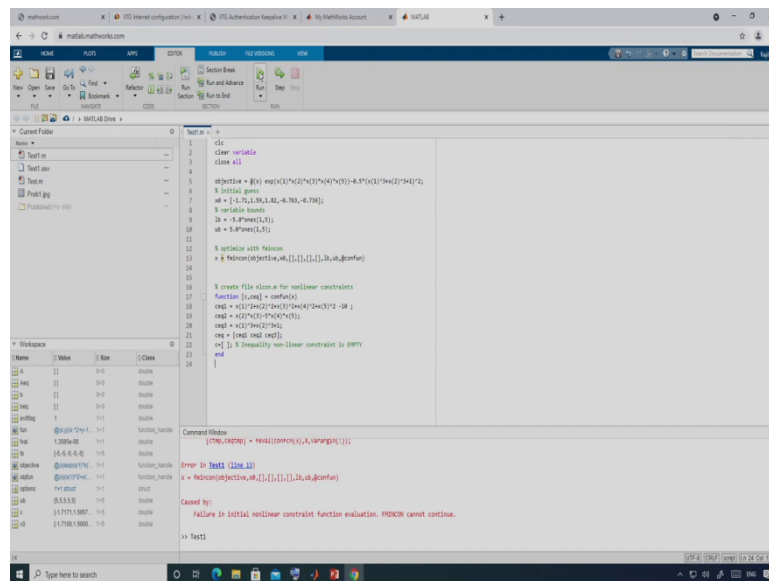
```

Error in Test (line 16)
x = fmincon(objective,x0,[],[],[],[],lb,ub,@nonfun)
Failure in initial nonlinear constraint function evaluation. FMINCON cannot continue.

```

So, now, if I run it just see yeah, I am getting the same solution.

(Refer Slide Time: 46:09)



```
1 % Test 1
2 clear variables
3 close all
4
5 objective = @(x) exp(x(1)^2*(2*(x(2)^2*(x(3)^2)-8.5*(x(1)^3*(2*(x(2)^2)+3))
6 % Initial guess
7 x0 = [-1.79, 1.39, 1.42, -6.789, -8.738];
8 % variable bounds
9 lb = -5.8*ones(1,5);
10 ub = 5.8*ones(1,5);
11
12 % optimize with fmincon
13 x = fmincon(objective, x0, [], [], [], [], lb, ub, @confun)
14
15
16 % create file xfun.m for nonlinear constraints
17 function [c,ineq] = confun(x)
18 ceq1 = x(1)^2*(2*(x(2)^2*(x(3)^2)-8.5*(x(1)^3*(2*(x(2)^2)+3))-10;
19 ceq2 = x(2)^2*(1-x(3)^2*(x(1)^2));
20 ceq3 = x(1)^2*(2*(x(2)^2)+3);
21 ceq = [ceq1;ceq2;ceq3];
22 % Inequality non-linear constraint is empty
23
24 end
25
26
Command Window
[ceq1;ceq2] = newfmincon(x0,[],[],[],[],[],lb,ub,@confun);
Error in Test1 (line 13)
x = fmincon(objective, x0, [], [], [], [], lb, ub, @confun)
Caused by:
Failure in initial nonlinear constraint function evaluation. FMINCON cannot continue.
>> Test1
```

So, what we can do suppose if you are defining this is this is equality type constraint1 and this is 2 and this is 3 ok. So, this is 3 and then I can define it that $c \leq q$ equal to $c \leq q_1$, $c \leq q_2$ and $c \leq q_3$ ok. So, I can also define like this and if you are executing this one so, I am getting the solution ok.

(Refer Slide Time: 46:57)

```

1 % Test m = 1
2 clear variable
3 close all
4
5 objective = @(x) exp(x(1)^2*(2*(x(2)^2*(x(3)^2)-8.5*(x(1)^3*(2*(x(2)^2)+1))
6 % Initial guess
7 x0 = [-1.79, 1.39, 1.42, -8.798, -8.798];
8 % variable bounds
9 lb = 5.*ones(1,5);
10 ub = 5.*ones(1,5);
11
12 % optimize with fmincon
13 x = fmincon(objective,x0,[],[],[],[],lb,ub,confun)
14
15
16 % create file xcon.m for nonlinear constraints
17 function [c,ceq] = confun(x)
18 ceq1 = x(1)^2*(2*(x(2)^2*(x(3)^2)-8.5*(x(1)^3*(2*(x(2)^2)+1))-10;
19 ceq2 = x(2)^2*(1-x(3)^2*(x(3)^2));
20 ceq3 = x(3)^2*(2*(x(2)^2)-1);
21 ceq = [ceq1;ceq2;ceq3];
22 % Inequality non-linear constraint is empty
23 end
24

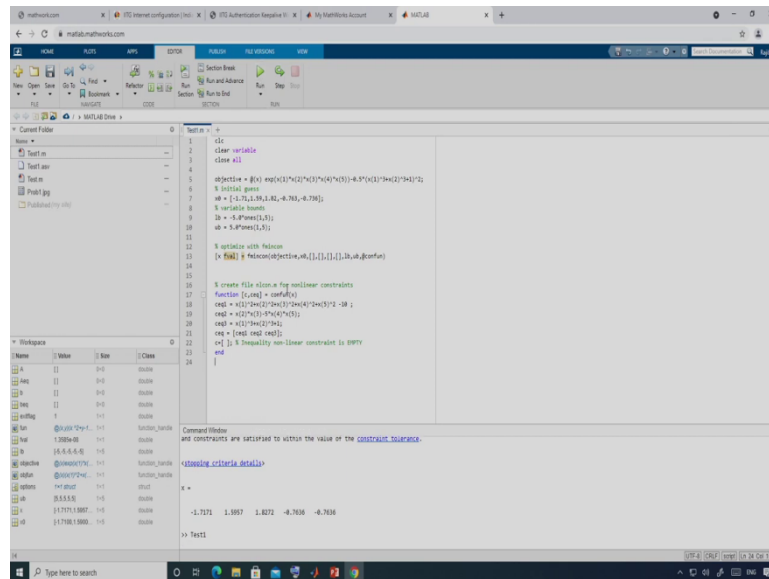
```

Name	Value	Size	Class
A	[]	0-by-0	double
ceq	[]	0-by-0	double
b	[]	0-by-0	double
ceq1	[]	0-by-0	double
ceq2	[]	0-by-0	double
ceq3	[]	0-by-0	double
con	@(x)[x(1)^2*(2*(x(2)^2*(x(3)^2)-8.5*(x(1)^3*(2*(x(2)^2)+1))-10;x(2)^2*(1-x(3)^2*(x(3)^2));x(3)^2*(2*(x(2)^2)-1)]	1-by-1	function_handle
confun	@(x)[x(1)^2*(2*(x(2)^2*(x(3)^2)-8.5*(x(1)^3*(2*(x(2)^2)+1))-10;x(2)^2*(1-x(3)^2*(x(3)^2));x(3)^2*(2*(x(2)^2)-1)]	1-by-1	function_handle
constr	@(x)[x(1)^2*(2*(x(2)^2*(x(3)^2)-8.5*(x(1)^3*(2*(x(2)^2)+1))-10;x(2)^2*(1-x(3)^2*(x(3)^2));x(3)^2*(2*(x(2)^2)-1)]	1-by-1	function_handle
options	fminconoptions	1-by-1	struct
ub	[5.0000 5.0000 5.0000 5.0000 5.0000]	1-by-5	double
x	[1.7911 -1.3907 -1.4272 -8.7980 -8.7980]	1-by-5	double
lb	[5.0000 5.0000 5.0000 5.0000 5.0000]	1-by-5	double

Command Window
 Optimization completed because the objective function is non-increasing and constraints are satisfied to within the value of the constraint tolerance.
 <string criteria details>
 x =
 1.7911 -1.3907 -1.4272 -8.7980 -8.7980

So, either you can define it like that or you can also define that $c \in q_1$, $c \in q_2$ and $c \in q_3$ that way you can define. Now I can also see what is the objective function value ok at optimal solution. So, I can write it that this is x and this is fval.

(Refer Slide Time: 47:24)



```
1 clear
2 clear variables
3 close all
4
5 objective = @(x) exp(x(1)^2*(x(2)^2*(x(3)^2))-8.5*(x(1)^3)+x(2)^3+1);
6 % Initial guess
7 x0 = [-1.75, 1.99, 1.42, -8.769, -8.769];
8 % variable bounds
9 lb = -5.8*ones(1,5);
10 ub = 5.8*ones(1,5);
11
12 % optimize with fmincon
13 [x,fval] = fmincon(objective,x0,[],[],[],[],lb,ub,@confun)
14
15
16 % create file xfun.m for nonlinear constraints
17 function [c,ineq] = confun(x)
18 ceq1 = x(1)^2+2*(x(2)^2)+5*(x(3)^2)-20 ;
19 ceq2 = x(2)^2*(1-x(3)^2)*(x(3)^2);
20 ceq3 = x(1)^2+2*(x(2)^2);
21 ceq = [ceq1;ceq2;ceq3];
22 cv[]; % Inequality non-linear constraint is empty
23 end
24
```

Command Window

and constraints are satisfied to within the value of the constraint tolerance.

<Showing criteria details>

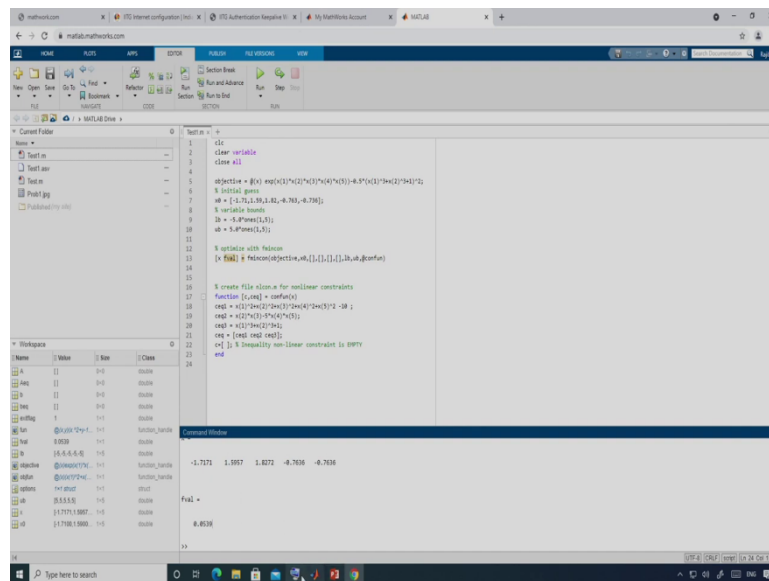
```
x =
-1.7571    1.9957    1.4272   -8.7696   -8.7696
```

>> Test1

Name	Value	Min	Max	Class
x	[1.7571, 1.9957, 1.4272, -8.7696, -8.7696]	-5.8	5.8	double
fval	1.2024e-05	-5.8	5.8	double
ceq1	1.2024e-05	-5.8	5.8	double
ceq2	1.2024e-05	-5.8	5.8	double
ceq3	1.2024e-05	-5.8	5.8	double
cv	1.2024e-05	-5.8	5.8	double
options	1.2024e-05	-5.8	5.8	double
lb	1.2024e-05	-5.8	5.8	double
ub	1.2024e-05	-5.8	5.8	double
lb	1.2024e-05	-5.8	5.8	double
ub	1.2024e-05	-5.8	5.8	double

And I can see that one. So, if you are running that one.

(Refer Slide Time: 47:32)



```
1 % Test = 1.4
2 clear variables
3 close all
4
5 objective = @(x) exp(x(1)^2*(2*(x(2)^4*(x(3))) - 8.5*(x(1)^3*(2*(3*x(1)^2)
6 % Initial guess
7 x0 = [-1.7171, 1.5887, -1.6272, -0.7036, -0.7036];
8 % variable bounds
9 lb = -5.0*ones(1,5);
10 ub = 5.0*ones(1,5);
11
12 % optimize with fmincon
13 [x,fval] = fmincon(objective,x0,[],[],[],[],lb,ub,@confun)
14
15
16 % create file xfun.m for nonlinear constraints
17 function [c,ineq] = confun(x)
18 ceq1 = x(1)^2*(2*(2*(x(3)^5*(x(4)^2*(x(5)^2) - 20 ;
19 ceq2 = x(2)^2*(1 - 5*(x(4)^2*(x(5)))
20 ceq3 = x(3)^2*(2*(3*x(1)^2)
21 ceq = [ceq1;ceq2;ceq3];
22 % Inequality non-linear constraint is empty
23
24
25
```

Name	Value	Size	Class
x	0.0539	1x1	double
fval	1.4	1x1	double
lb	-5.0000	1x5	double
ub	5.0000	1x5	double
options	[1.7171, 1.5887, -1.6272, -0.7036, -0.7036]	1x5	double
ceq1	0.0539	1x1	double
ceq2	0.0539	1x1	double
ceq3	0.0539	1x1	double
ceq	[0.0539; 0.0539; 0.0539]	1x3	double

Final = 1.4

So, optimal value of this particular objective function is 0.0539.

(Refer Slide Time: 47:51)

The image shows the MATLAB R2019a interface with a script named 'Test1.m' open. The script defines a nonlinear optimization problem. The objective function is $f(x) = x(1)^2 + x(2)^2 + x(3)^2 - 8 \cdot x(1) + x(1)^3 + x(2)^3 + x(3)^3$. The initial guess is $x_0 = [-1.777, 1.5957, 1.6272, -8.7636, -8.7636]$. The variable bounds are $lb = 5 \cdot \text{ones}(1,3)$ and $ub = 5 \cdot \text{ones}(1,3)$. The problem is solved using the 'fmincon' function. The results are displayed in the Command Window and the Workspace.

```
1 % Test1.m
2 clear variables
3 close all
4
5 objective = @(x) x(1)^2 + x(2)^2 + x(3)^2 - 8 * x(1) + x(1)^3 + x(2)^3 + x(3)^3;
6 % Initial guess
7 x0 = [-1.777, 1.5957, 1.6272, -8.7636, -8.7636];
8 % variable bounds
9 lb = 5 * ones(1,3);
10 ub = 5 * ones(1,3);
11
12 % optimize with fmincon
13 [x,fval,exitflag,output,lambda,grad,hessian] = fmincon(objective,x0,[1,1,1],[-1,1,1],lb,ub,@confun);
14
15
16 % create file xfun.m for nonlinear constraints
17 function [c,ceq] = confun(x)
18 ceq = x(1)^2 + x(2)^2 + x(3)^2 - 5 * x(1)^2 - 5 * x(2)^2 - 5 * x(3)^2 - 10;
19 ceq = x(1)^2 + x(2)^2 + x(3)^2 - 10;
20 ceq = x(1)^2 + x(2)^2 + x(3)^2 - 10;
21 ceq = [ceq; ceq; ceq];
22 % Inequality non-linear constraint is empty
23 c = [];
24 end
```

Command Window:

```
>> Test1
-1.7771  1.5957  1.6272  -8.7636  -8.7636

fval =
    8.6539
```

Workspace:

Name	Value	Size	Class
A	11	0x0	double
ceq	11	0x0	double
c	11	0x0	double
ceq	11	0x0	double
c	1	0x0	double
exitflag	@(x) fmincon(x,f)	1x1	function_handle
grad	0.0039	1x1	double
h	[5.6, 5.6, 5.6]	1x3	double
lambda	@(x) fmincon(x,f)	1x1	function_handle
lambda	@(x) fmincon(x,f)	1x1	function_handle
options	rstfplot	1x1	struct
ub	[5.5, 5.5]	1x2	double
x	[1.7771, 1.5957, 1.6272]	1x3	double
lb	[1.7771, 1.5957, 1.6272]	1x3	double

So, I can also see the other parameters. So, you can see. So, if I copy this thing.

(Refer Slide Time: 47:59)

The screenshot shows the MATLAB R2020a environment. The script file 'test.m' contains the following code:

```

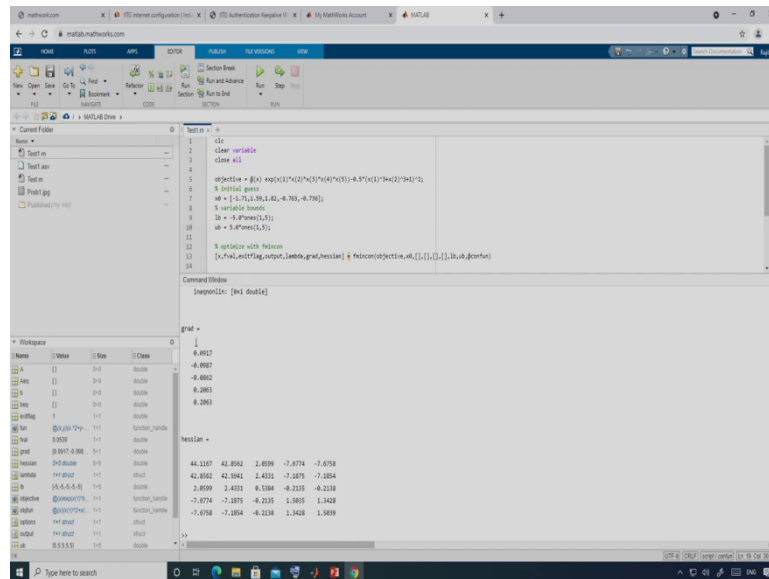
1 clc
2 clear variable
3 clear all
4
5 objective = @(x) exp(x(1)^2*(x(2)^4*(x(3))^4-8*(x(1)^3*(x(2)^3*(x(3)^2))))
6
7 % initial guess
8
9 x0 = [-1.2,1.39,3.48,-8.793,-8.793];
10
11 % variable bounds
12 lb = 5.*ones(3,1);
13 ub = 5.*ones(3,1);
14
15
16 % optimizer with fmincon
17 [x_opt,exitflag,output,lambda,grad,hessian] = fmincon(objective,x0,[1,1,1],lb,ub,gradient)
18
19
20 % Create file x0.m as for nonlinear constraints
21
22 function [c,eq] = conf(x)
23 ceq = x(1)^2*(x(2)^4*(x(3))^4-8*(x(1)^3*(x(2)^3*(x(3)^2))) - 10 ;
24 ceq = x(2)^4*(x(3)^4*(x(3)^4))
25 ceq = x(1)^2*(x(2)^3*(x(3)^2))
26 ceq = [ceq;ceq;ceq];
27
28 c[] = 0; % Inequality non-linear constraint is empty
29
30 end
  
```

The Command Window displays the results of the optimization:

```

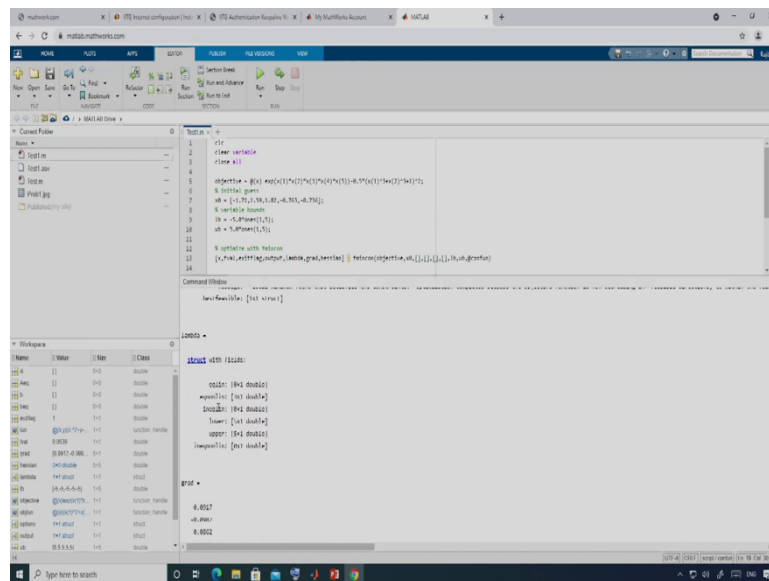
hessian =
44.1187    42.8562    2.8099   -7.8774   -7.8758
42.8562    42.8944    2.4332   -7.1875   -7.1854
2.8099    2.4331    8.5384   -0.2235   -0.2238
-7.8774   -7.1875   -0.2235   1.0485   1.0428
-7.8758   -7.1854   -0.2238   1.0428   1.0489
  
```

(Refer Slide Time: 48:02)



So, you can see the other informations are also available now ok.

(Refer Slide Time: 48:05)



```
1 % fmincon
2 clear variables
3 close all
4
5 % Objective function
6 objfun = @(x) 4*x(1)^2 + 2*x(2)^2 + 3*x(3)^2 - 8*x(1) + 12*x(2) + 14*x(3);
7 % Initial guess
8 x0 = [-1.75; 1.75; 1.75];
9 % Variable bounds
10 lb = -5; ub = 5;
11 % Nonlinear constraints
12 % Optimize with fmincon
13 [x_opt, fval, exitflag, output, lambda, grad, hessian] = fmincon(objfun, x0, [], [], [], lb, ub, [], [], []);
14
15 % Command Window
16 format long
17 disp('Optimal value:');
18 disp(fval);
19
20 % Display the optimal values of the variables
21 disp('Optimal values of the variables:');
22 disp(x_opt);
23
24 % Display the values of the constraints
25 disp('Values of the constraints:');
26 disp(lambda);
27
28 % Display the values of the objective function and its gradient
29 disp('Objective function value and its gradient:');
30 disp([fval, grad]);
31
32 % Display the values of the Hessian matrix
33 disp('Hessian matrix:');
34 disp(hessian);
```

The gradient information is available and then your equality type constraint, inequality type constraint so lower bound, upper bound so, everything is available, the gradient is available and hessian matrix is also available.

(Refer Slide Time: 48:21)

The image shows the MATLAB R2019a interface. The main window displays a script for an optimization problem. The script defines an objective function, constraints, and initial values. The optimization is performed using the 'fmincon' function. The output window shows the results of the optimization, including the number of iterations, function count, constraint violation, step size, algorithm used, first-order optimality, and the final message.

```
1 % Test = 1
2 clear variables
3 close all
4
5 objective = @(x) exp(x(1)*x(2)^2*x(3)^2*x(4)^2)-8.5*x(1)+34*(2+3*x(1)^2);
6 % Initial guess
7 x0 = [-1.75, 1.39, 1.42, -6.798];
8 % variable bounds
9 lb = -5.8*ones(1,4);
10 ub = 5.8*ones(1,4);
11
12 % optimize with fmincon
13 [x,fval,exitflag,output,lambda,grad,hessian] = fmincon(objective,x0,[],[],[],[],lb,ub,@nonfun);
14
```

Output:

```
iterations: 8
function: 54
constraint violation: 3.5606e-39
stepsize: 1.1983e-45
algorithm: 'interior-point'
firstorder optimality: 9.8832e-47
optimality: 0
message: 'Local minimum found that satisfies the constraints.--Optimization completed because the objective function is non-decreasing in --feasible directions, to within the value bestfval=1.01e+01'
bestfval: 1.01e+01
```

So, you can also get this information. So, you can see actually how many how many iterations you did actually. So, there are total 8 iteration, then function count is 54, constraint violation is this ok, then step size is this, then what algorithm we have used, we have used interior point algorithm ok. So, this information you can get. Then structure you will get, the gradient information you will get and hessian information you will also get ok.

(Refer Slide Time: 48:52)

[illegible]

So, here this is the optimal solution and this is the objective function value at optimal solution. So, you can also get this information if you want or otherwise I will just define it only the optimal solution and objective function value. So, you can keep this thing ok.

(Refer Slide Time: 49:14)

[illegible]

So, if you are running it you are only getting the optimal solution and optimal objective function value. So, that is all about today's class. So, in today's class we have solved few non-linear optimization problem with constraint and without constraint using classical optimization technique. So, here we have mainly use three function MATLAB function and that is fminbnd. So, that is for single variable optimization problem without constraint. So, I can use if your problem is a single variable optimization problem without constraint. So, in that case I can use fminbnd.

Problem without constraint then you can use `fminsearch` ok so you can use `fminsearch` and if your problem is a multivariable problem with constraint linear constraint, non-linear constraint, equality type constraint, inequality type constraint then you can use `fmincon`.

So, fmincon is one of the very popular MATLAB function which you can use for solving a non-linear optimization problem having linear constraint, non-linear constraint, equality type constraint and inequality type constraint with lower bound and upper bound. So, in that case you can use fmincon function it is a very powerful function..

So, I have shown you how you can use fmincon function. So, you are also getting lot of information apart from the optimal solution value and optimal your objective function value ok optimal solution optimal objective function value, you are also getting the other information like hessian matrix, gradient information. So, that also you can get ok so.

Thank you very much. See you in the next class, thank you very much.