Optimization Methods for Civil Engineering
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Module - 02
Lecture - 01
Introduction to Linear Problem

Hello student, in this class we will discuss about Linear Problem. So, let me explain what is linear problem. So, in the last class we have discussed about optimization problem. So, what is the necessary condition for optimality, what is the sufficient condition for optimality both for a single variable problem as well as multi variable problem ok. So, today we will discuss linear problem. So, let me explain what is linear problem.

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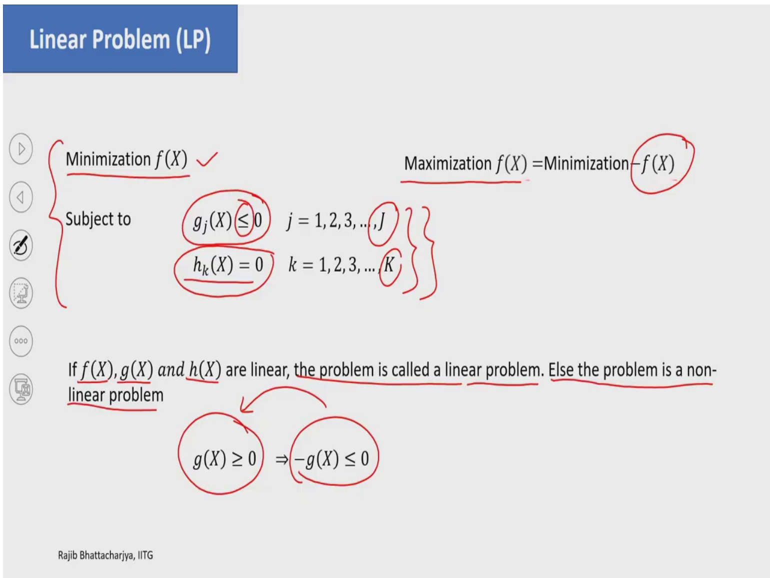

Now, an optimization problem can be represented as a minimization problems and there may be some constraints like we may have inequality constraint that g j X is less than equal to 0. So, we have total J number of constraint and these constraints are inequality constraint. So, in this case this is less than equality type constraint. So, you may have greater inequality type constraints suppose some time you may have this type of constraint.

So, in this case I can convert this problem to a less than equality type constraint by multiplying minus 1. So, if you have greater inequality type constraints. So, I can convert it to less than equality type constraint by multiplying minus 1 or other way also I can do if you have less than equality type constraint. So, you can convert it to greater than equality type constraint ok.

So, let us discuss this problem. So, we may have in equality constraint that g j X less than equality type or we may have equality constraint like h k X equal to 0 and suppose we have total K constraint. So, altogether we can say that this is an optimization model. So, what we have in an optimization model?

One is objective function. Objective function can be a minimization type, but we may have maximization type objective function suppose maximization of f X. So, in that case I can also convert a maximization problem to a minimization problem by multiplying minus 1. So, a maximization of f X is equal to minimization of minus of f X.

So, I can convert that problem like a minimization problem or I can also convert a minimization problem to a maximization problem ok. So, altogether a problem will have an objective function. So, this is objective function and we will have constraint. So, we will have constrain this constraint may be in equality type constraint like g j X less than equality equal to 0 or we may have equality constraint h k X equal to 0. Now if f of X g X and h X are linear. So, all of them are linear function. So, in that case the problem will be called a linear problem ok.

So, I hope this is clear. If the objective function that is f X the constraint that is g X and h X are linear. So, in that case we will say that the problem is a linear problem otherwise the problem is a non-linear problem. So, we will discuss both linear and non-linear problems, but in this particular class today we will discuss about linear problem.

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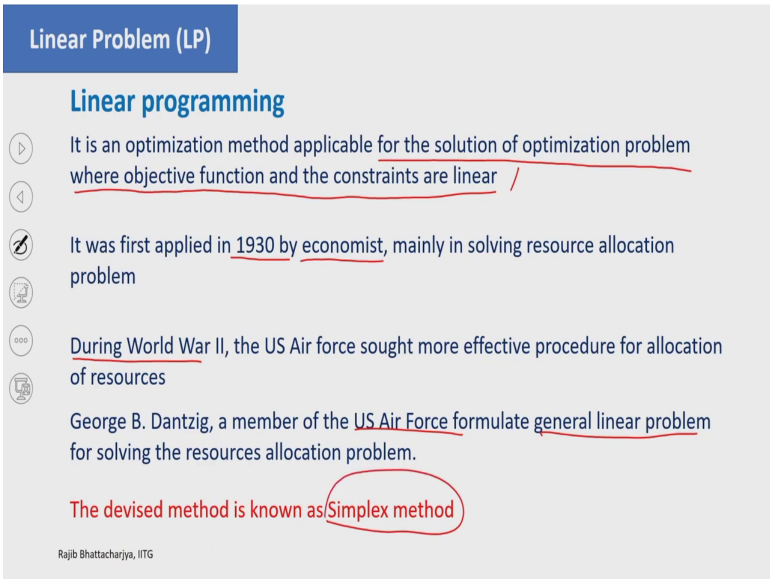

Now, what is linear programming? So, linear programming is an optimization method ok. So, this is an optimization methods applicable for the solution of optimization problem where the objective function and the constraints are linear. So, what is linear programming? So, it is an optimization methods.

So, it is an optimization method applicable for solution of an optimization problem where objective function and the constraints are linear ok. So, as I said the both objective function and a constraints should be linear. So, in that case the problem is known as linear problem, but the solution procedure or solution or you can say the optimization method is called linear programming.

It was first applied in 1930 by economists, mainly for solving resource allocation problem. Suppose you have limited resources and you want to allocate optimally. So, in that case this optimization linear problem or linear programming method can be applied and people have used basically for resource allocation ok. So, it was as I said. So, it was first applied in 1930 mainly for solving resource allocation problem.

During World War II, when this linear programming was used. So, during World War II the US Air force sought more effective procedure for allocation of resources and they have used a linear your programming ok for allocation of resource then George B. Dantzig a member of US Air force formulated general linear problem for solving the resource allocation problem ok.

So, he was a member of US Air force and he actually device a method for allocation of resources and it is a linear programming method and the device method is known as simplex method.

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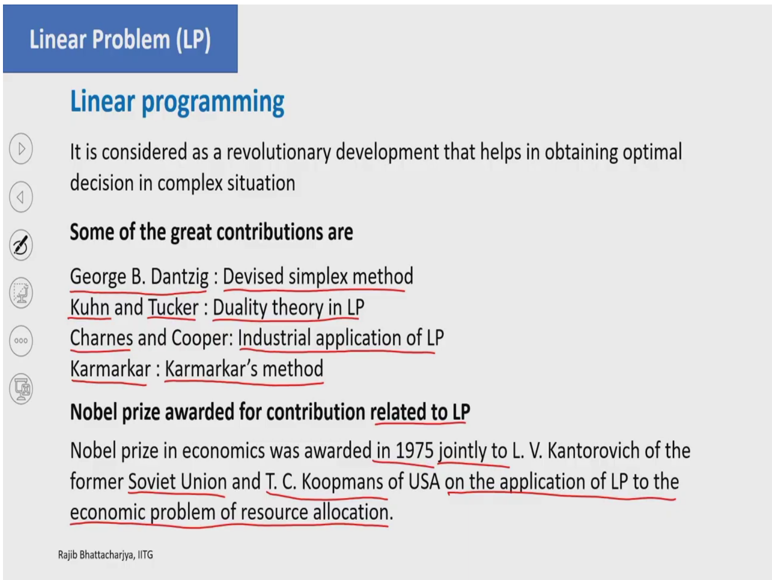

So, we will discuss we will also discuss about this method in this class. So, we will discuss. So, it was given by George B. Dantzig. Now it is considered as a revolutionary development, that help in obtaining optimal decision in complex situation. The linear programming is considered as a revolutionary development that helps in obtaining optimal decision in complex situation. Some of the great contributions are as I said that George B. Dantzig.

So, he devised simplex method; devised simplex method. So, we will discuss actually. So, this is a method for solving a linear problem. So, we will discuss that one then Kuhn and Tucker, he they have given Duality theory in LP; duality theory in LP, then Charnes and Cooper they have work on Industrial application of LP then Karmarkar. So, he has given Karmarkar's method. So, this is also a method for solving a linear problem and Nobel Prize was also awarded for contributions related to LP.

Nobel Prize in economics was awarded in 1975 jointly to L.V Kantorovich of the former Soviet Union and T.C. Koopmans of USA on the application of LP to economic problem of resource allocation ok. So, Nobel Prize was awarded for their work and their work was basically applied to resource allocation and they got Nobel Prize in 1975. So, this is some of the history related to linear programming.

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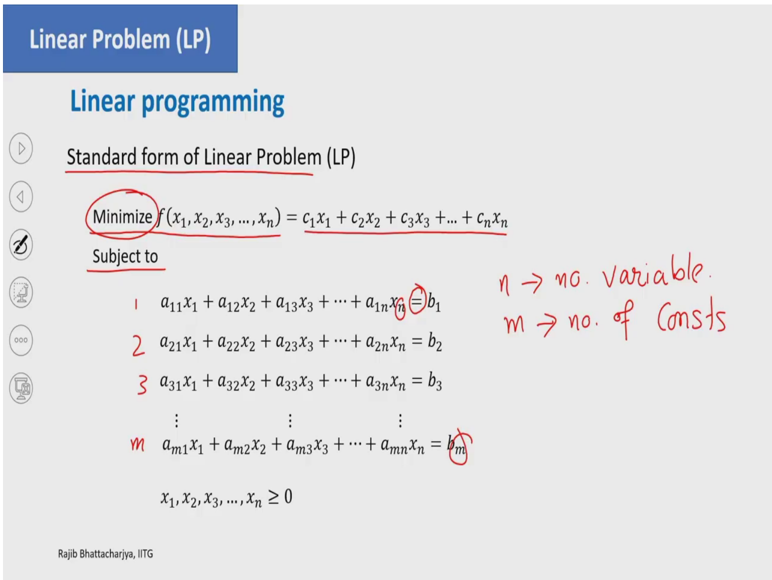

So, let us go to the linear problem. Now standard form of linear problem. A linear problem as I said that I can define as a minimization problem. So, here minimization of f x. So, now, I may have n variable x 1, x 2, x 3, x n variable. So, this is a linear function. So, therefore, the function could be c 1 x 1 plus c 2 x 2 plus c 3 x 3 plus c n x n. So, this is your objective function and subject to the linear constraints ok. So, this is the standard form. So, in the standard form this must be equality sign ok.

So, this must be equally it cannot be less than equality type or it cannot be greater than equality type in case of standard form of linear problem. So, we may have total m constraints, so in this case m number of constraints. So, this is number 1 constraint, this is 2, this is 3 like this is your m constraint m constraint. So, we have m constraint and how many variable we have? So, we have total n variable ok. So, n is the variable number of variable ok and m that number of constraints ok.

So, in this case, so this is a standard form as I said. So, it is a minimization type problem, but you need not worry if your problem is a maximization type. So, you can convert your maximization problem to a minimization problem and the constraints should be equality type constraints. So, it is, it must be equality type constraint, but if you have in equality type constraints. So, you have to convert your problem to a equality type constraint. So, that also we will discuss.

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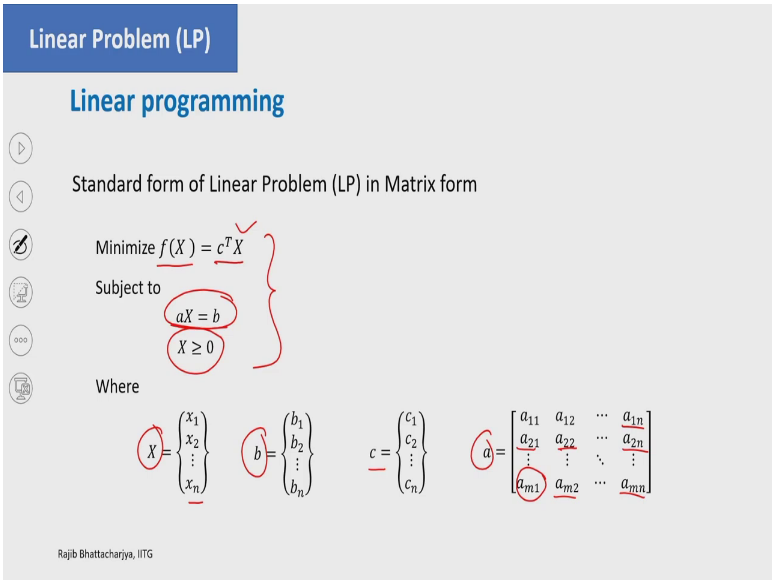

So, in a matrix form. So, I can write this standard form of linear problem like this. So, it is a minimization of f X. So, this is c transpose X and subject to aX equal to b and the all variable X is should be greater than equal to 0 ok. So, this is in matrix form. So, this is the objective function and this is the objective function and these are your constraint and this constraint should be of equality type constraint and the all variable all distinct variable should be positive; that means, it should be greater than equal to 0.

So, where X is a vector. So, X equal to x 1 x 2 x n then b is the right hand side that is b 1 b 2 b n c is equal to c is also a vector. So, that is c 1 c 2 up to c n and a is a matrix. So, this matrix can be written a 11 a 12 a 1n then a 21 a 22 a 2n and we have total m constraint. So, therefore, a m1 a m2 a mn. So, this matrix this is a matrix ok.

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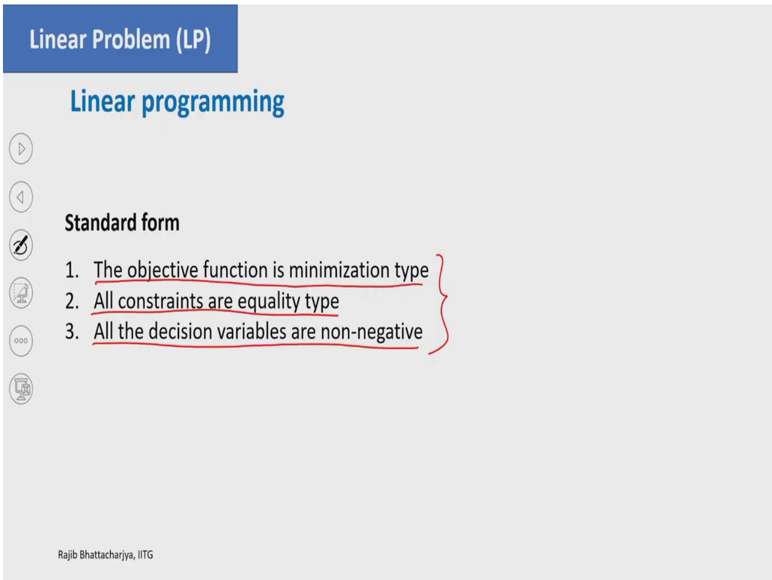

So, I can write a linear problem in this matrix form or otherwise also I can write in algebraic form. Now the standard format is saying that the objective function is a minimization type ok. So, it is a minimization type. All constraints are equality type; that means, all constraint should be of equality type ok and all the decision variables are non negative. So, these are the three conditions of a standard form. So, if your problem satisfied these three conditions, then I will say that you that the linear problem is in standard form ok.

Now, question is that you may have that your objective function is a maximization type function then what you have to do? You have to convert in order to convert to the standard form you have to convert the maximization problem to a minimization problem. Similarly, if your constraints are not equality type constraint. Then what you have to do?

You have to convert it to equality type and similarly if all the decision variables are not positive ok is not non negative. So, you have to have some method so, that all these n variables are positive ok. So, now, let us discuss if your problem is not in a standard form, then how can you convert your problem to standard form.

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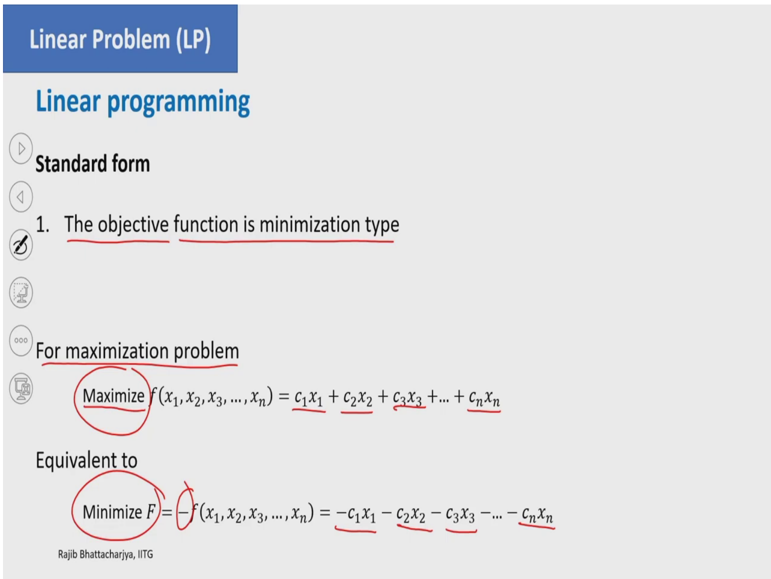

So, the first condition is the objective function should be of minimization type ok. Now what you will do? So, for maximization problem suppose your problem is a maximization type problem that is maximize c 1 x 1, c 2 x 2, c 3 x 3, c n x n. So, what you have to do? You have to convert your objective function to a minimization type function. So, to do that what I will do? I will multiply it by minus 1 ok and then your objective function will be converted to a minimization type problem.

So, now, if you multiply it by minus 1 then what you will get? Minus c 1 x 1, minus c 2 x 2, minus c 3 x 3, minus c n x n. So, therefore, to convert your problem to a standard form if your objective function is a maximization type function just you multiply it by minus 1 and that will convert your problem to a minimization type problem.

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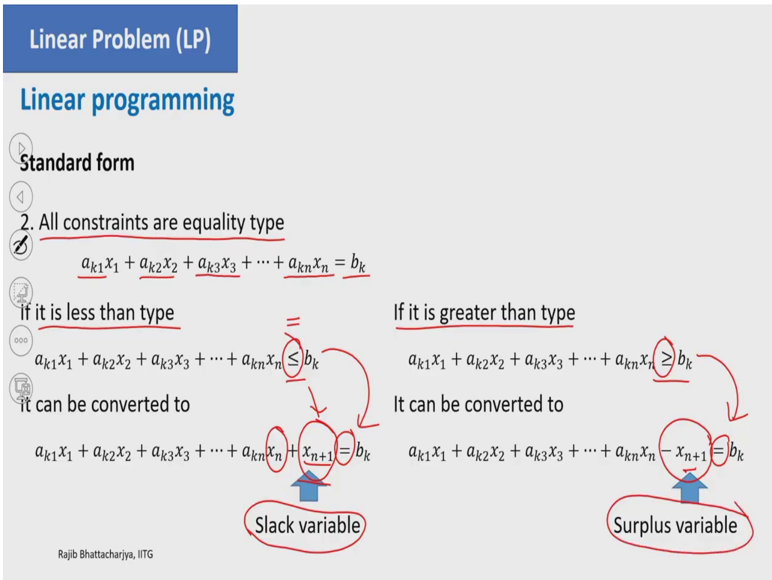

The next condition is the all constraints are equality type ok. So; that means, a k 1. So, this is a particular kth constraint a k 1 x 1, a k 2 x 2, a k 3 x 3, a k n x n which is equal to b k, but if it is less than equality type constraint that your constraint is not equality type constraint, but it is a less than equality type constraints.

So, what do you have to do in order to convert it to standard form? So, you have to make this equality sign. So, how you will do that? So, it can be converted by adding a variable here there is a extra variable I am adding here and this extra variable. So, is n plus 1. So, we have total n variable upto to x n. So, I am adding another variable in order to make it equal ok. So, equality sign now you just see this constraint is a equality type constraint.

So, earlier it is a less than equality type constraint. So, by adding x n plus 1 by adding an extra variable here. So, we can convert this particular constraint to equality type constrains ok. Similarly, if you have greater than equality type constraint ok. So, this is the sign now it is not less than equality type, but it is a greater than equality type constraint. So, what I can do basically? So, I can subtract another variable that is x n plus 1 and in order to convert these particular constraint to a equality type constraint.

So, what you will do basically? If it is less than equality type, then I will add a variable and if it is greater than equality type then I will subtract a variable to make it equality type constraint ok. So, this variable is known as when you are adding. So, when you are adding a variable this variable is known as slack variable and when you are subtracting a variable then this variable is known as surplus variable.

So, if it is a less than equality type you are adding a variable and name of the variable is slack variable and if it is greater than equality type constraint. So, you are subtracting a variable. So, in that case we will call that variable as a surplus variable ok. So, by using a surplus variable or a slack variable, so I can convert a non-equality type constraint to equality type constraint.

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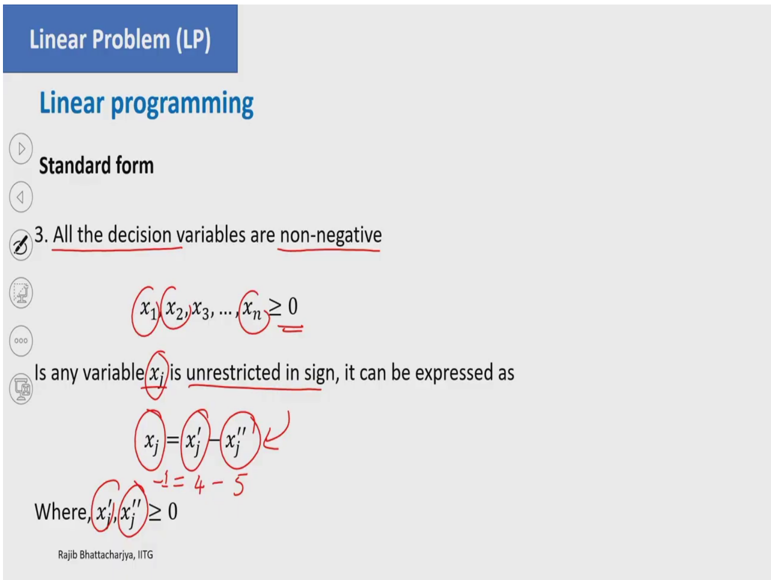

Now, the third condition that the all the variable should be non negative; that means, should be positive ok so, but you may have some negative variable ok. So, in some cases you may have negative variable. So, it means that x 1 x 2 up to x n should be you positive, but when you have a negative variable suppose x j is unrestricted in sign; that means, x j can be negative also or it can be positive also.

There may be some problem sometime that a particular variable can go or can have a negative values ok. So, in this case suppose x j is unrestricted in sign; that means, the variable can have a negative value in that case what I will do? I can write x j equal to x j dash minus x double j dash. So, by doing that where x j dash and x j double dash. So, that is actually positive ok.

So, that is these two are positive, but x j can be negative also depending upon what are the value of x j and x j double dash. Suppose, I would like to have a negative value of x j. So, what I can do? This is suppose 4 minus 5. So, x j will be equal to minus 1 ok. So, I can have a negative value something like that, but in LP problem. So, what I will have? In place of x j I will have 2 variables and the both the variables x j dash and x j double dash should be positive ok. So, should be positive. So, in that case. So, if you have a variable which is unrestricted in sign.

So, I can convert that particular variable or I can write that variable like this and I can have actually two other variable which are positive and, but results will be negative x j will be negative ok. So, by using this three rules ok we can convert a linear problem to its standard form. So, we can convert the linear problem if it is not in standard form. So, we can convert it to standard form.

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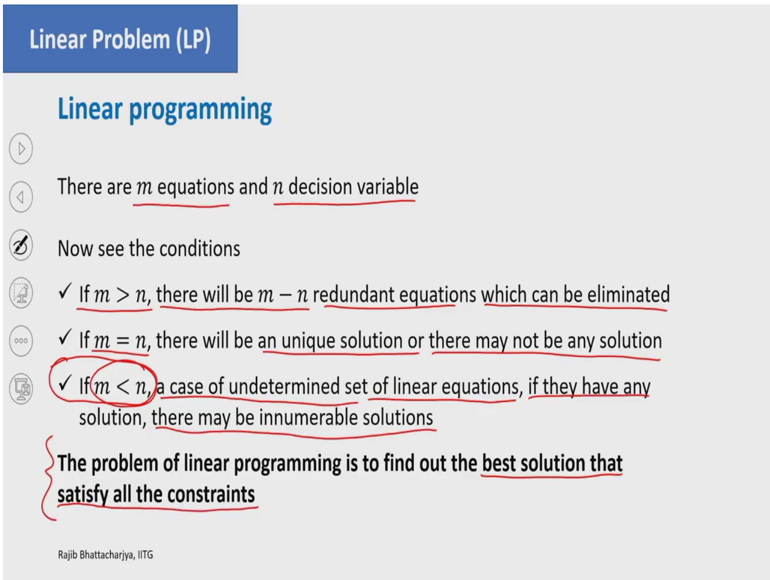

So, let us take a set of linear equations ok. So, suppose there are m equations and n decision variables ok. So, we have total n decision variables and total m equation. Now see the conditions. The first condition is that m is greater than n. So, what does it mean? There are m minus n redundant equations which can be eliminated ok. So, in this case what is happening? The number of equations is more than the number of decision variable.

So, in that case we will have m minus n redundant equations and which can be eliminated. So, this is the first condition. Now second condition is that if m equal to n; that means, the number of equation is equal to the number of variables ok. So, there will be an unique solution in this case and there may not be any solution ok. So, we may have an unique solution in this case if number of equation is equal to number of variables. So, in that case we may have a unique solution or there may not be any solution ok.

So, this is the second condition and the third condition is that if number of equation is less than the number of variable number of equation in this case is less than number of variable. So, this is a case of undetermined set of linear equations if they have any solution there may be innumerable solutions ok. So, in this case if there is a solution or if they have any solution in that case there will be innumerable solution. If the number of equations is less than the number of variable.

So, in that case if there is any solution there will be or there may be innumerable solutions. So, the problem of linear programming is to find out the best solution that satisfy all the constraints ok. So, the linear programming. So, that we are discussing here. So, if this is the case if this is the third case basically I am talking about the third case, when the number of equations is less than the number of variable so, in that case if there is a solution as I said.

So, we may have innumerable solution then the problem of linear programming is to find out the best solution because you have more than one solution you have multiple solution in this case and the problem of linear programming is to find out the best solution that satisfy all the constraint ok.

So, that is the function of linear programming. So, what we want to do? So, we have multiple solution, we may have innumerable solution and out of this solution we would like to find out the best solution and that is the function of the linear programming or you can say the objective of linear programming. So, we would like to find out the best solution.

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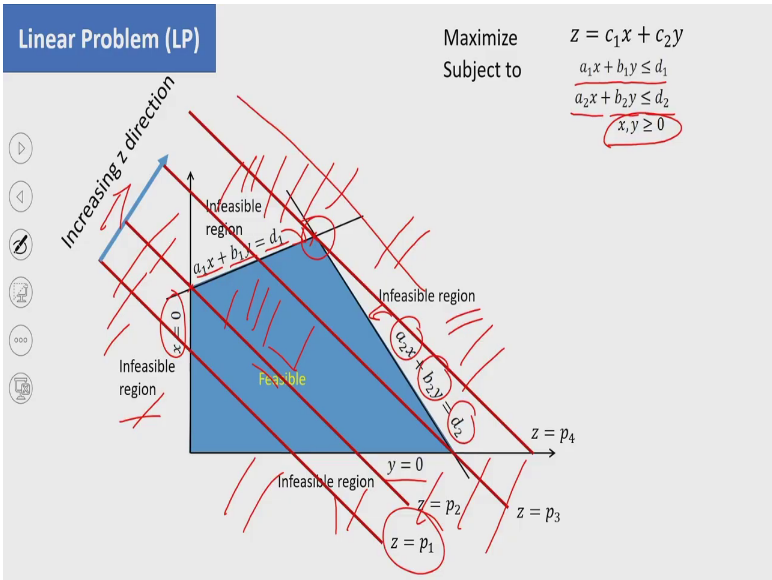

Now, let us see this linear problem. So, here this is a maximization type objective function maximize z equal to c 1 x plus c 2 y. So, I have taken a two variable problem in order to draw it ok and this is a maximization type problem as I said z equal to c 1 x plus c 2 y subject to a 1 x plus b 1 y less than d 1 and similarly a 2 x plus b 2 y less than d 2 and x and y this should be positive ok; this should be positive.

So, let us plot it. So, this is y equal to 0 and this line is x equal to 0 and this part is infeasible region ok because the x and y should be positive. So, if it is in this side the x value will be negative and in this side y value will be negative. So, therefore, this is infeasible this is infeasible region ok. So, this is x equal to 0 and this is y equal to 0. Now if I plot the constraints. So, the first constraint is a 1 x plus b 1 y equal to d 1. So, this is the constraint and this is a less than equality type constraints.

So, therefore, this region is infeasible region ok. Then second one is a 2 x plus b 2 y equal to d 2 and this is also less than equality type. So, this is the line a 1 x plus b 1 y equal to d 1 and this is the region which says a 1 x plus b 1 y greater than d 1 and this the side. And these are the side where it is less than d 1 and similarly this is the line which is a 2 x plus b 2 y equal to d 2 and this is your infeasible region, this is your in feasible region and this part is feasible region ok so this is feasible region.

So, now if I say that if I take any value of z ok. So, suppose z equal to p 1 and then if I draw this particular line. So, I am getting z equal to p 1. Now if I increase the z value that mean this is the increasing direction then maybe this may be one line and I may get this solution.

So, this is one solution you may get or there may be other solution also now because our objective is to maximize the value of z. So, in this direction z value is increasing and finally, I am getting a solution here. So, this is the solution which is the best solution because after that if I draw any line, then this will be in infeasible region.

So, whatever solution you are getting it will be in infeasible region. So, therefore, this is a particular solution which is basically the maximum point you can say this will maximize this particular function, I will get the best solution somewhere here then similarly if my z is something like that.

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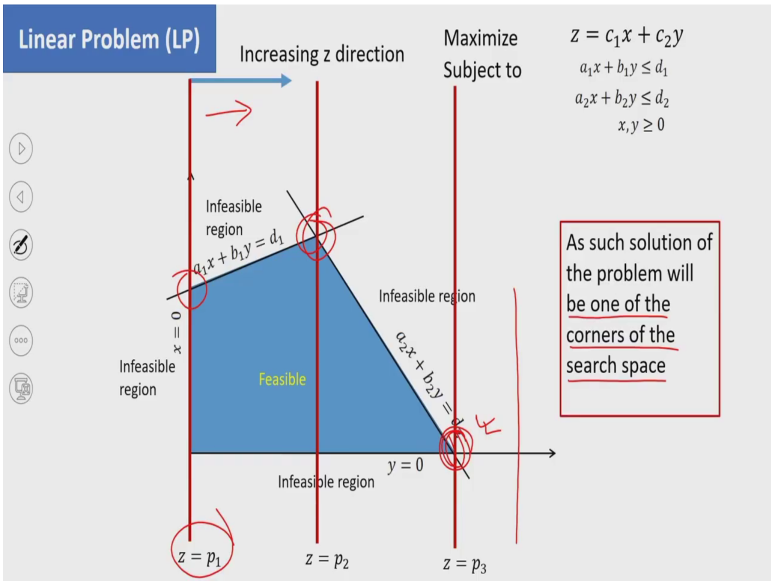

So, this is the z equal to p 1 this is the increasing direction in this case, this is the increasing direction I may have one solution here, I may have one solution here. So, maybe this is basically this may be the best solution in this case and if I have if I increase this value. So, then it will be on the infeasible region.

So, therefore, this is one of the solution of this particular problem and probably the best solution. So, as such the solution of the problem will be one of the corners of the search space. So, what we are getting here. So, one of the solution will be either here or here or here depending upon the objective function ok. So, solution will be one of the corners.

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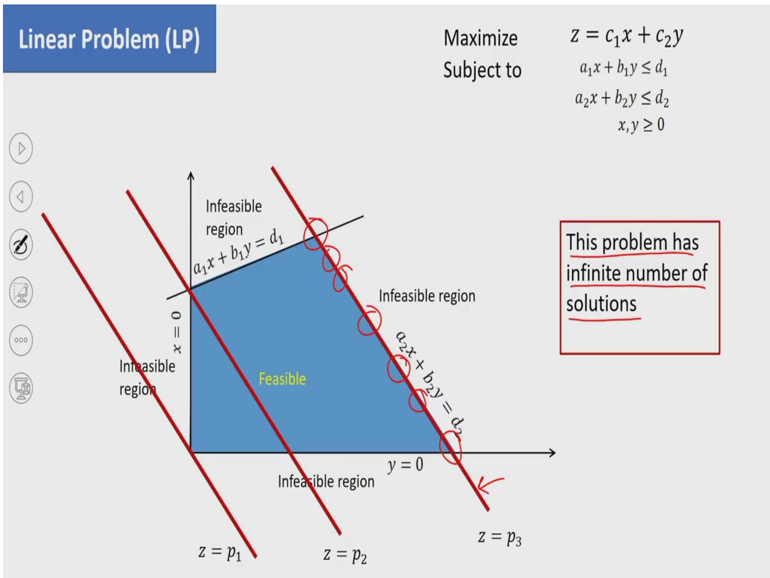

Now suppose if your z function is like this, then what will happen? So, finally, this is the maximum value of z I am getting. So, this is the maximum value of z I am getting. So, in this case this is also one solution, this is also one solution and there are several solution in between.

So, in this case this problem has infinite number of solution. So, I will have infinite number of solution; that means any solution on this particular line feasible solution on this particular line is the maximum solution. So, in that case you can say that this particular problem has infinite number of solution.

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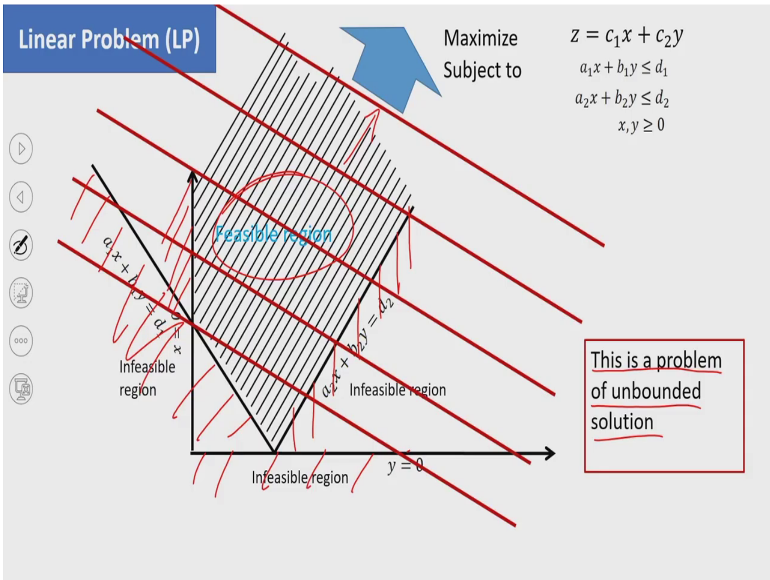

Let us see this problem. So, here this is the infeasible side. So, this is the infeasible side and this is the infeasible side ok and in this case. So, this entire search space is basically the feasible search space ok. So, this is the feasible region. This is also infeasible part because x and y should be positive. So, anyway, so this is also infeasible something like that. So, now, in this case this is the feasible region. Now if I take any z then basically we can continue ok.

So, there is no limit I can go up to infinity in this case. So, therefore, this particular problem is unbounded problem and this problem this is the problem of unbounded solution; that means, the solution will be infinity. So, you can go along that direction and there is no bounding. So, we can say that this particular problem is an unbounded problem. Now let us discuss search space ok.

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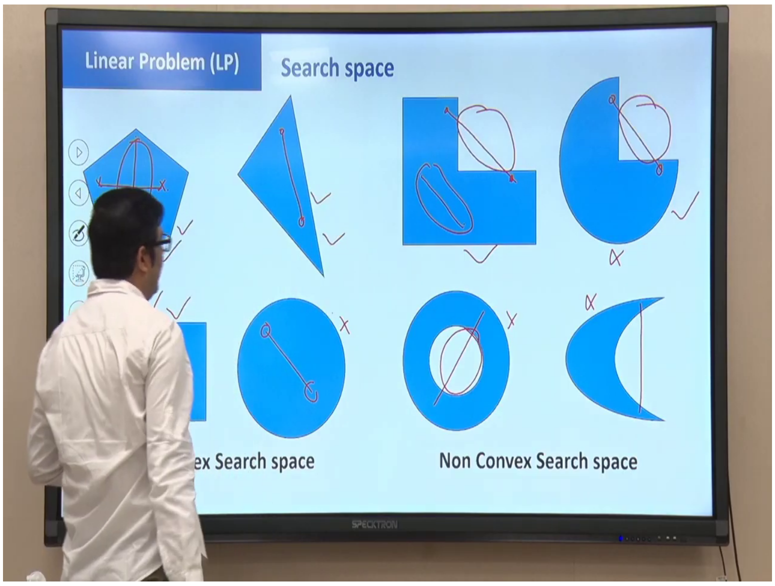

So, I will explain what is convex search space and what is non convex search space. So, this is a convex search space and this is also a convex search space. Now if I see that this search space is not convex similarly this is also not convex let me explain why these two are non-convex search space and this is convex search space ok. So, for this particular search space if I take any two point suppose if I take any two point and if I draw a line, then whatever points on this line will be within that search space.

So, you take any two point within maybe I can take this two point and all the points are within the search space. Similarly, if I take any two point then all the points or this line is within that search space. Now question is in this case if I take these two points then if I draw this line, these line is within this search space, but I can also take this two point when I draw this line is not within the search space.

So, therefore, this is a non-convex search space similarly if I take these two points this particular line is not within the search space and these search space is a non-convex search space ok. And anyway. So, this is not a linear search space, but this is a linear search space and this is also a convex search space. Because I can take any two points and if I draw this particular line will be within that search space.

Similarly, in this case also this if you take any two points and if you join these two point and that particular line will be within that search space, but this is also a non-convex search space because the line will be not within the search space similarly here also this is a non-convex search space.

So, I will discuss the mathematical definition of what is convex search space and what is non convex search space later on. So, as I have explained you take any two points and you draw a line and if the line is within the search space. So, in that case the search space is called a convex search space and, but if suppose if you take any two points and you join that two points and you whatever line you are getting, if that line is out of that search space. So, in that case the search space is a non-convex search space.

So, I will discuss it and I will give you the mathematical definition of convex search space. Now if you look at all these search space, then this is a linear search space, this is also a linear, this is also linear and this is a non-linear search space, this is linear this is non-linear, this is non-linear and this is non-linear. So, when you are talking about a linear problem.

So, your search space. So, whatever the equation ok. So, these equations will be a linear equation. So, your search space may be like this, maybe like this, maybe like this ok.

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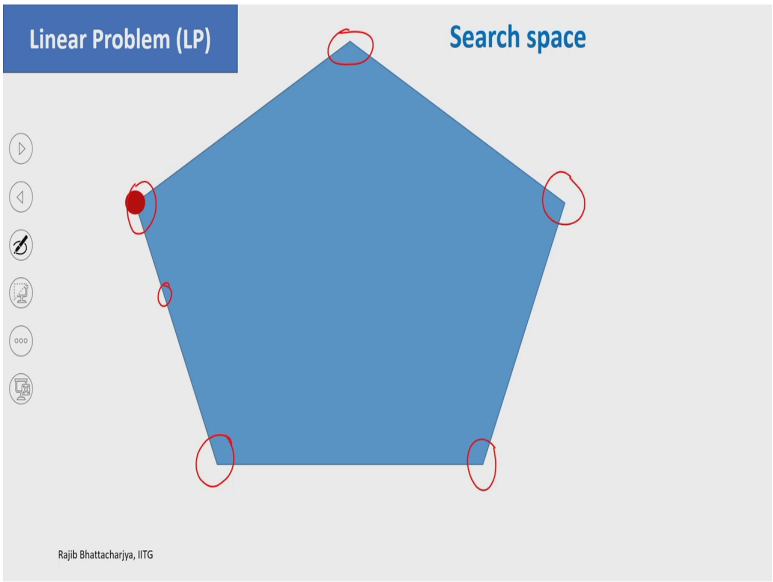

Now, as I have said or I have explained you already that the solution will be. So, whatever optimal solution will be in one of the corners ok. So, either in this corner or in this corner or in this corner or in this corner or in this corner ok. So, therefore, whatever method a you want to develop or you want to use basically that I would like to see the corners of the search space because solution will be either in one of these corners ok.

So, the methods should be developed in such a way that you are not actually looking at any solution on the other part except these corners ok. So, this is you can say one property of linear problem that your solution will be in one of the corners of the convex search space.

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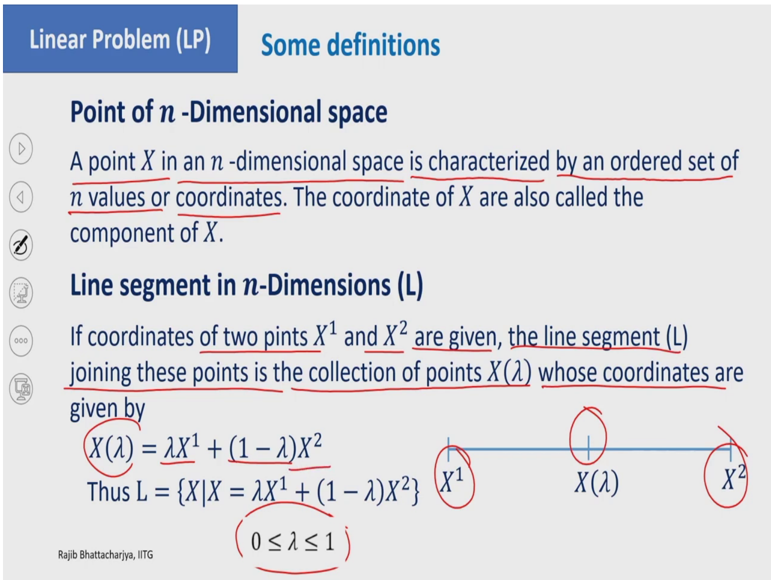

Now let us have some definitions use in a linear problem. So, first one is the point of n dimensional space ok. A point X in an n dimensional space is characterized by an ordered set of n values or coordinates ok. The coordinates of X are also called the component of X. So, what a point of n dimensional space? A point X in an n dimensional space is characterized by an ordered set of n values or I can say that n coordinates.

The coordinates of X are also called the component of X ok. So, line segment in an n dimensions if coordinates of two points x 1 and x 2 are given, the line segment L joining these two points is the collection of points X lambda whose coordinates are given by this equation. So, X lambda is lambda X 1 plus 1 minus lambda X 2. So, lambda is between 0 and 1. So, if you are putting 0 if I put 0 lambda equal to 0.

So, I will get this particular point X 2 and if I put lambda equal to 1 I will get this particular point and if I take lambda equal to 0.5. So, I should get the midpoint ok. So, I should get the midpoint. So, therefore, by varying the value of lambda. So, I can get any points on this particular line ok. So, line segment in n dimensions. So, if coordinates of two points X 1 and X 2 are given the lines segment L joining these two points is the collection of points X lambda whose coordinates are given by this particular equation ok.

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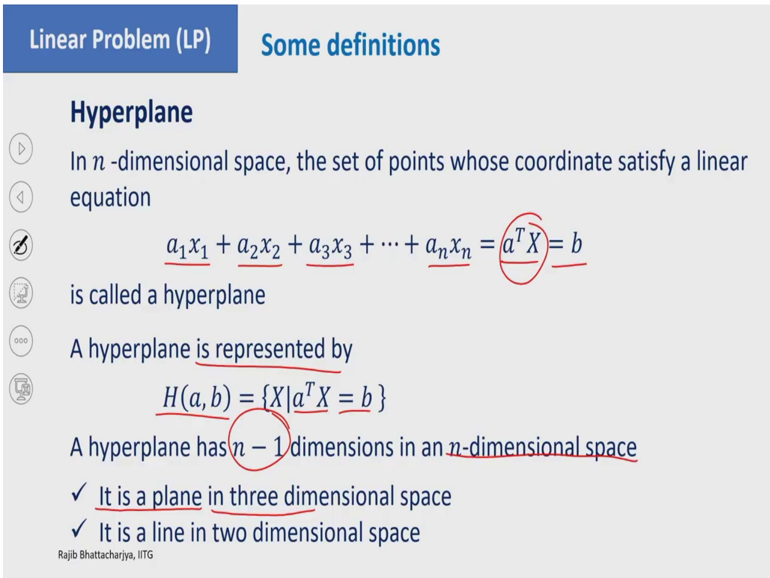

So, I hope this is clear to you now what is hyper plane? In n dimensional space the set of points whose coordinate satisfy a linear equation a 1 x 1 plus a 2 x 2 plus a 3 x 3 plus a n x n equal to a transpose X equal to b or I can write this is a transpose X equal to b. If these points satisfy this particular equation and then we will call it as a hyper plane ok. A hyper plane is represented by H a, b ok.

So, it is represent by H a, b which is a transpose X equal to b. A hyper plane has n minus 1 dimension where n is the number of variable ok. So, a hyper plane has n minus 1 dimension in an n dimensional space ok so; that means, it is a plan in three dimensional space and it is a line in two dimensional space ok. So, this is your hyper plane.

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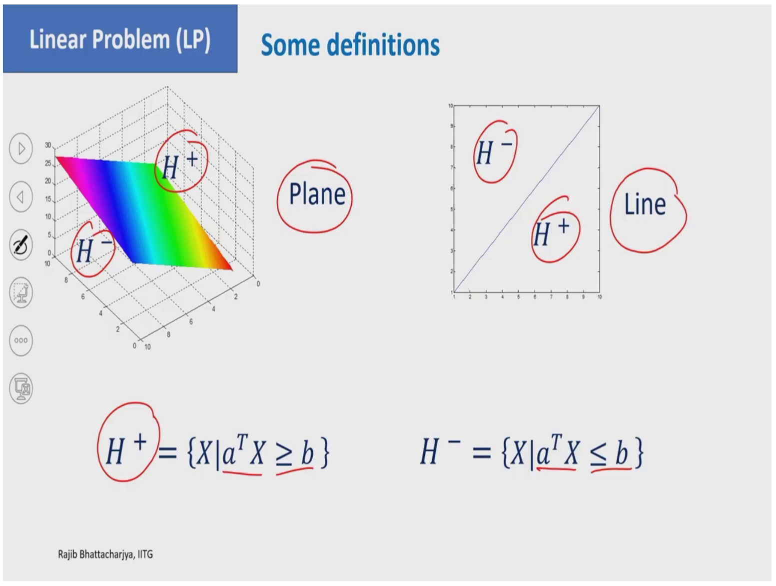

Now I have shown a hyper plane in three dimensional space. So, this is a plane as I said and I can say that this is H positive and this is H negative. So, H positive is that a transpose X greater than b and H negative is a transpose X less than your b and this is in case of line. So, this is H negative and this is H positive ok.

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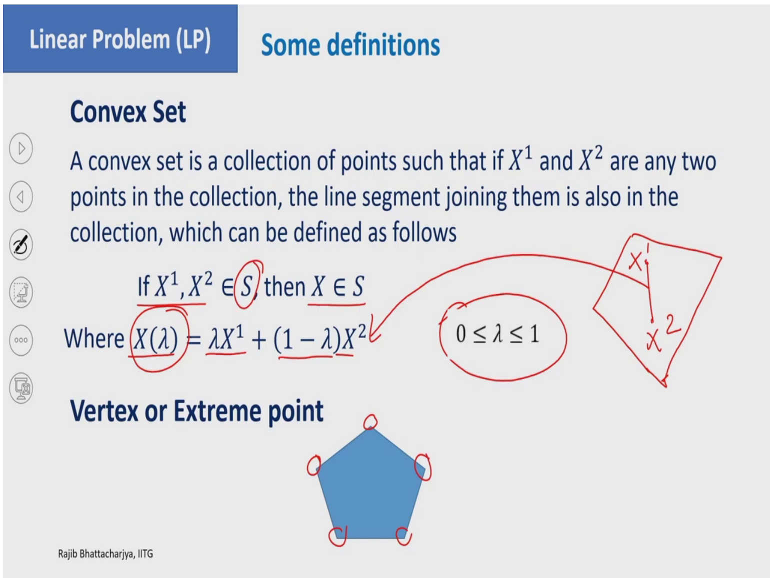

Now what is convex set? So, I have already defined what is convex search space and let us see what is convex set. A convex set is a collection of points such that if X 1 and X 2 are any two points in the collection, the line segment joining them is also in the collection ok. So, which can be defined as that if X 1 and X 2 is in the collection of S, then X is also in S. So, where X lambda; that means, I am joining these two points, that is lambda X 1 plus 1 minus lambda X 2 are also in that particular collection.

So, where this is basically lambda is between 0 and 1. So, what I am saying that if any two points suppose I am telling about the convex search space. So, this is basically a convex search space and if you take any two point this is the X 1 and this is X 2 and suppose this is X 1 and this is X 2 and you are joining these two line and this line I can represent by this particular equation.

So, whatever value of this X lambda it should be within that your collection ok. So, in that case I will call that this set is a convex set. Now vertex or extreme point. So, these are the vertex or extreme point of the search space.

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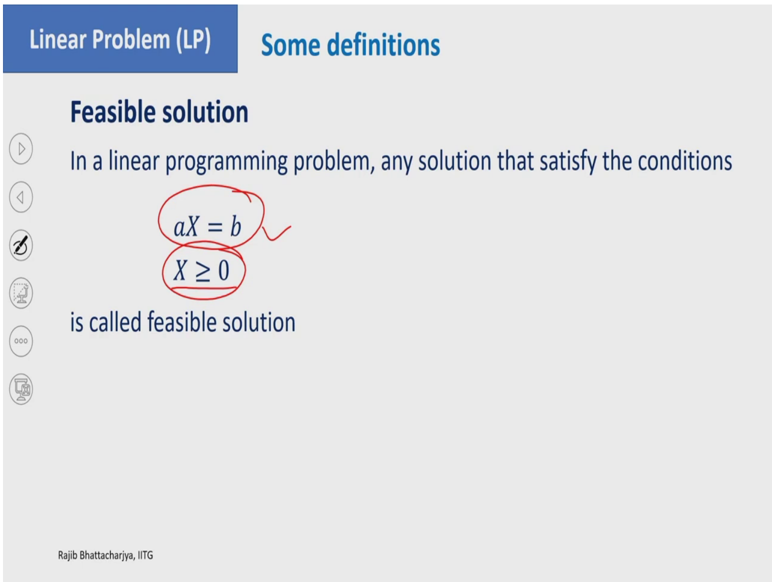

Now what is feasible solution? In a linear programming problem any solution that satisfy the conditions that condition means the constraints ok. So, aX equal to b and X greater than 0. So, is called feasible solution. So, what is feasible solution? In a linear programming problem any solution that satisfy the constraints and non-negativity this is also constraint is called feasible solution.

So, I will say a particular solution if that particular solution is satisfying the constraints including the non-negativity constraint. So, in that case that solution is a feasible solution ok.

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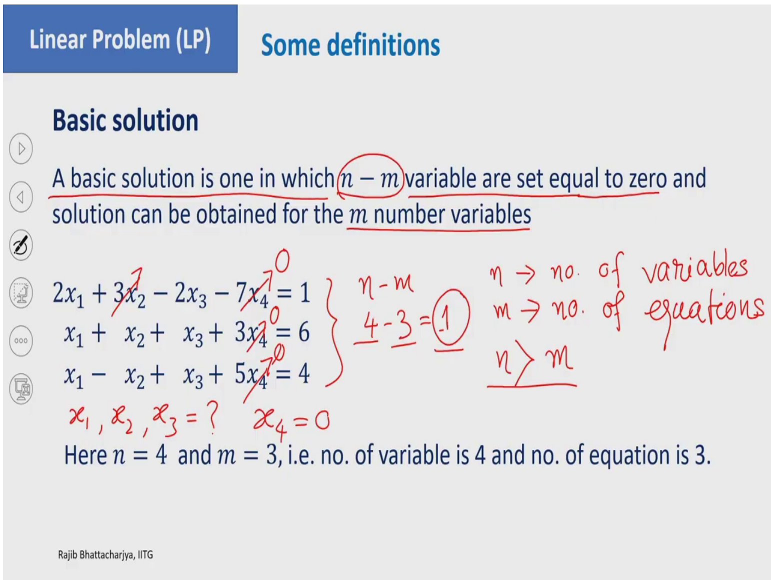

Now what is basic solution? Now a basic solution is one in which n minus m ok. So, here n is the number of variables and m is the number of equation. So, what we are doing? A basic solution is one in which n minus m variables are set equal to zero and solution can be obtained for m number of variables ok.

So, what you will do? Suppose n is the number of variable number of variables and m number of equation number of equations ok. Now in this case what is happening that n is greater than m. That means, the number of variables is more than number of equation that third condition or we have discussed already.

So, let us take this example problem suppose there are 3 equations; there are 3 equations here and number of variables is 4; that means, 4 variable that is x 1 x 2 x 3 and x 4 and we have 3 equations. So, what I will do that n minus m. So, in this case that 4 minus 3 equal to 1. So, one variable is set equal to 0 and we will try to find out the solution of the problem.

Suppose, one variable you may take any variable suppose if I say that x 4 equal to 0 ok. So, x 4 equal to 0. So, then now I have x 1, x 2 and x 3. So, I can find out what is the value of x 1, x 2, x 3 ok. So, that I can find out an x 4 is equal to 0. So, in that case I will say that this solution whatever solution I am getting that this solution is a basic solution I hope this is clear.

So, in this case what we are doing that n minus m variables are set equal to 0 in this case. So, in this particular case we have 4 variables and 3 equation therefore, one variable will set equal to 0. So, one variable means any variable suppose I have taken x 4, but you can also take x 2 ok. So, in that case whatever solution you are getting this solution we can say it is a basic solution.

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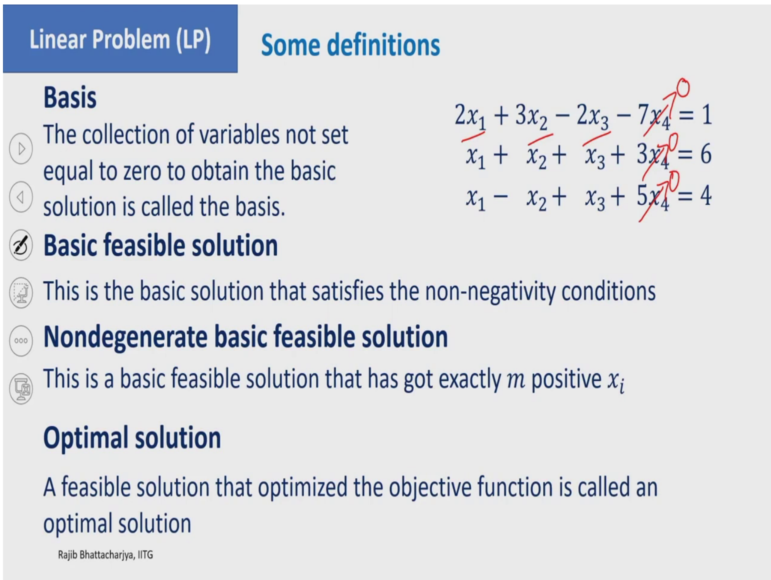

Now what is basis? The collection of variables not set equal to zero to obtain the basic solution is called the basis ok. The collection of variable not set equal to zero. So, I can say that the example problem we have taken suppose x 4 we are putting 0; x 4 we are putting 0 and x 1 x 2 x 3 r not 0.

So, what the definition? The collection of variable not set equal to 0. So, in this example problem what are variable? This is x 1 x 2 and x 3 they are not set equal to 0. So, they are not zero So, I am not putting them considering them as 0. So, this collection is known as the basis ok. So, here x 1, x 2 and x 3 are in basis ok I hope this is clear now next is. So, we may also take like that suppose if I say that I would like to make x 1 equal to 0.

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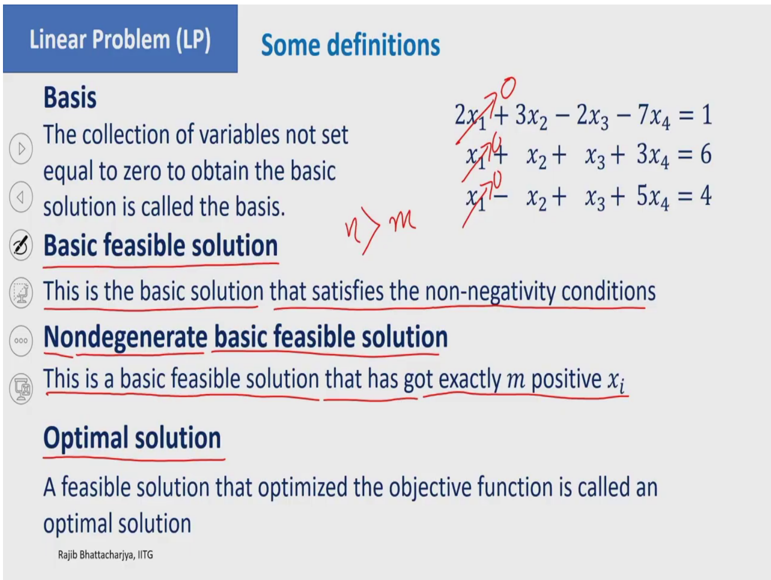

So, if I say that x 1 equal to 0, x 1 equal to 0. So, in that case x 2, x 3, x 4 a we are not setting equal to 0. So, they are not 0. So, therefore, x 2, x 3 and x 4 will be on the basis ok. So, the collection of variable not set equal to 0. So, in this case x 2, x 3 and x 4 in earlier case it was x 1, x 2 and x 3.

So, they are in basis or we can say this is the basic variables now next is what is basic feasible solution. So, this is the basic solution. Basic solution already you know what is basic solution. So, this is the basic solution that satisfies the non-negativity conditions ok. So, non-negativity conditions that is your basic feasible solution. So, feasible solution also should satisfy the non-negativity conditions. So, then we will call basic feasible solution next is non degenerate basic feasible solution.

So, this is a basic feasible solution that has got exactly m positive x i. So, in that case we will call it non degenerate basic feasible solution. We will discuss this when we will solve the problems and finally, what is optimal solution? A feasible solution that optimize the objective function is called an optimal solution ok. So, the feasible solution a feasible solution because as I said.

So, when n number of variables is greater than number of equation. So, in that case we may have innumerable solution then a feasible solution that optimize the objective function is called the optimal solution ok. So, that will be the optimal solution and basically our objective is to find out the optimal solution of this problem and that solution must be a feasible solution.

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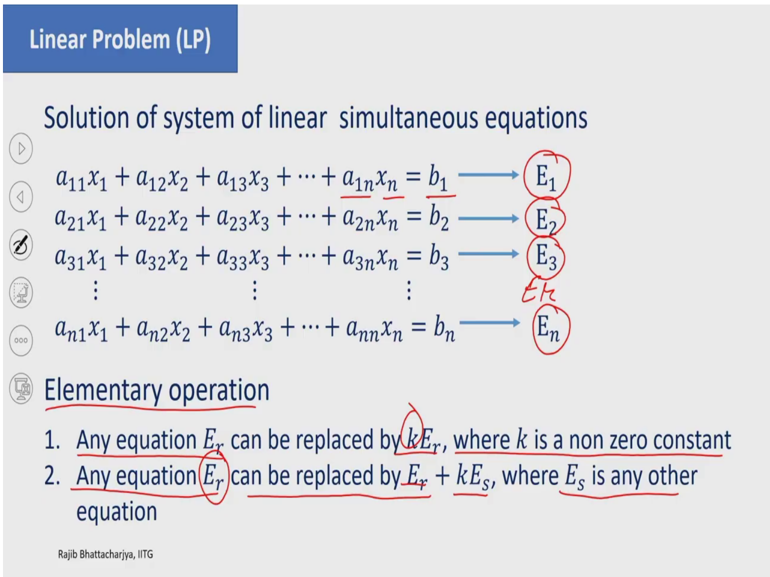

Now let us take a system of linear simultaneous equation ok. So, suppose I have this equation that is a 12 x 1 plus a 12 x 2 plus a 1 3 x 3 plus a 1 n x n equal to b 1 and I have name it as E 1 and similarly I have another equation that is E 2 that is E 3 and total equation is n equation. So, in this particular problem we have n variable and n number of equation that is number of variable is n and number of equation is also n ok.

So, what we can do basically? So, I would like to solve I would like to find out the value of x 1 x 2 x 3 ok. So, I would like to solve this linear equations ok. So, what I can do? I can apply the elementary operation I think all of you know that we have already did it. So, what I can do? Any equation E r. So, any equation suppose E r can be replaced by k E r ok. So, I can multiply equation by a constant. So, that I can do and where k is non zero constant ok.

So, the equation will not change if I multiply particular equation by a constant similarly what I can do any equation E r can be replaced by E r plus k E s, where E s is any other equation in that case also the solution will not change ok and this two is known as elementary operation I can do that. So, first one is that I can multiply a equation by a constant then solution will not change ok.

So, this is 1 and second one is I can replace a equation E r by E r plus k E s ok. So, then also solution will not change. So, by using these two operations I can solve this problem.

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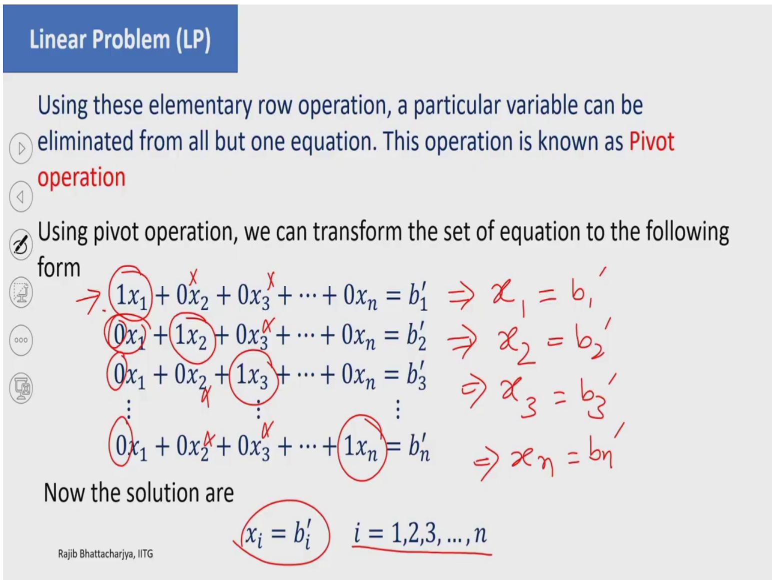

So, let us see using this elementary row operation we call a particular variable can be eliminated from all, but one equation this operation is known as pivot operation. So, what I can do, I can eliminate a particular variable from all the equation, but one. So, in one equation I will keep and I would like to eliminate that variable from the other equation and using these two operations whatever row operation we have discussed and this operation is known as pivot operation. So, idea is that, so in this particular case. So, what I am doing that I am eliminating this x 1 variable from the other equation ok.

So, you just see here it is 0, here it is 0, here it is 0 my objective is to eliminate this x 1 variable from the other equation. So, I am keeping only on the first equation and I am eliminating from the other equation similarly the x 2 variable I am keeping in the second equation I am eliminating from the other equation. Similarly, x 3 variable I am keeping at the third equation and eliminating from the other equation and finally, I am keeping x n variable in the last equation and eliminating from the other equation.

So, as I said that it is not that a you have to keep the variable x 1 in the first equation it is not like that. So, the variable x 1 has to be kept at the first equation. So, you can also keep in the second equation you can take any order, but here I have shown you that you are keeping the variable 1 at the first equation, variable 2 in the second equation 3 in the third equation and nth variable in the nth equation. So, in this order I have said, but it can be in a different order also.

So, now if I can transform this set of linear equation, if I can transform in this form then what is the solution? You just see that in the first equation we have only one variable that is x 1 x 2 the coefficient of x 2 is 0, x 3 is 0, x n is 0. So, therefore, from the first equation I will get that x 1 equal to b 1 dash. Similarly, from the second equation I will get you just see the variable of x 1 x 3 up to x n is 0 only we have x 2 variable. So, x 2 will be b 2 dash.

Similarly, from this I can find out x 3 equal to b 3 dash and from the last equation I can find out that x n equal to b n dash. So, what is the solution? Solution is x i equal to b i dash for i equal to 1, 2, 3 up to n ok. So, using this row operation. So, I can eliminate a particular variable from all the equation, but keeping at only one equation and if we can transform our equation like that the way we are doing here, I can easily find out the value of x 1 what is the value of x 1 x 2 x 3 and x n.

So, I can do that I think this operation as I said. So, this operation is known as pivot operation. Pivot operation means I am keeping a variable in one equation and for other equation the coefficient of that particular variable is equal to 0.

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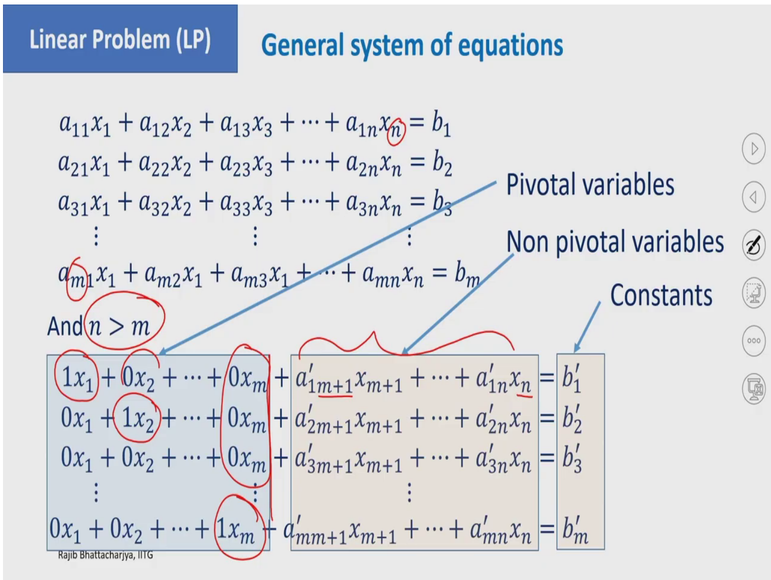

Now if the number of variable is more than number of equation ok. In the earlier problem the number of variable is equal to number of equations, but in this case the number of variables is more than number of equation. So, we have now n variable and we have m equation.

So, what I can do? So, in this case the n is greater than m. So, here what we are doing? We are applying the row operations and up to the number of variable m we are doing the pivoting; that means, what we are doing here for the x 1 variable the coefficient is 1 and for others coefficient are 0 similarly for x 2 only in the second equation this is 1 and other equation that is 0 up to x m. So, x m the coefficient is 1 and for others the coefficient are 0 ok.

And we can say that this is the basic variable and these are the non-basic variable. So, for non-basic variable that is from m plus 1 to n ok. So, m plus 1 to m we call it non basic variable and what you will do for the non-basic variable? We will set equal to or this variable will set equal to 0 ok. So, one possible solution. So, these are pivotal variable and these are non-pivotal variable and these are the constants ok.

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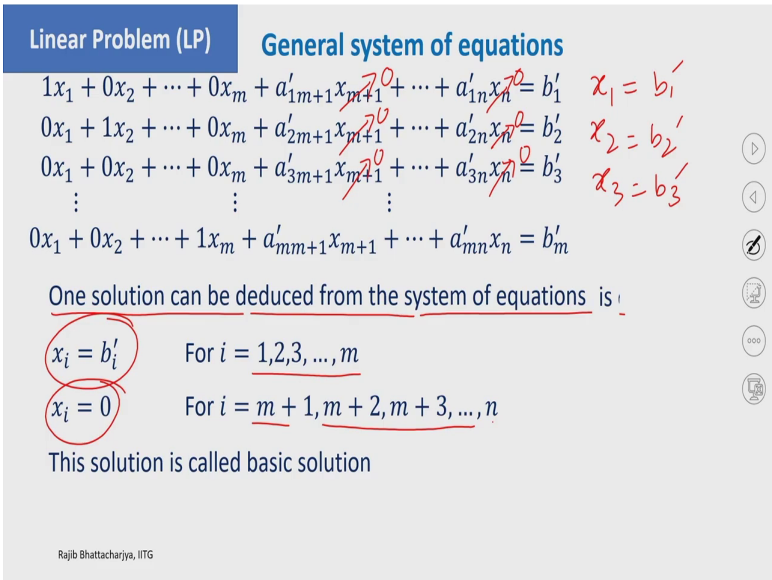

So, therefore, for this particular problem one solution. So, one solution can be determined from the system of equation is that x i equal to b i dash. Suppose if I say that this particular variable is equal to 0, this particular variable is equal to 0. So, therefore, I can say that x 1 equal to b 1 dash similarly if I say that this is 0 and this is 0. So, x 2 equal to b 2 dash and similarly if I say that this is 0, then x 3 equal to b 3 dash ok.

So, one solution is that, x i equal to b i for i equal to 1 to m and x i equal to 0 for i equal to m plus 1 up to n ok so; that means, for the basic variable I am finding the x i equal do b i and for the non-basic variable ok. So, for the non-basic variable x i equal to 0. So, this is one possible solution.

So, this solution is called basic solution. So, what is basic solution? Already we have discussed that this solution is called basic solution; that means, for the non-basic variable we are setting it equal to 0 and for the basic variable that x i equal to b i. Then let us stop here. So, we will continue our discussion in the next class.

Thank you.