Optimization Methods for Civil Engineering
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Lecture - 19
Genetic Algorithms

Hello student, welcome back to this course on Optimization Method. So, in the last class we discuss about Genetic Algorithm.

So, last two classes we discuss about genetic algorithm and I have introduced you what is the genetic algorithm and then I also give you the flow chart the of simple genetic algorithm.

And then I have discussed the genetic operators that is your selection operator, crossover operator, mutation operators.

Then I have discussed the different genetic operators. So, mainly selection operator, then crossover operator, then mutation operator so this I have discussed in the last class.

Today, I will show you one hand calculation and I will explain the different process of genetic algorithm through hand calculation.

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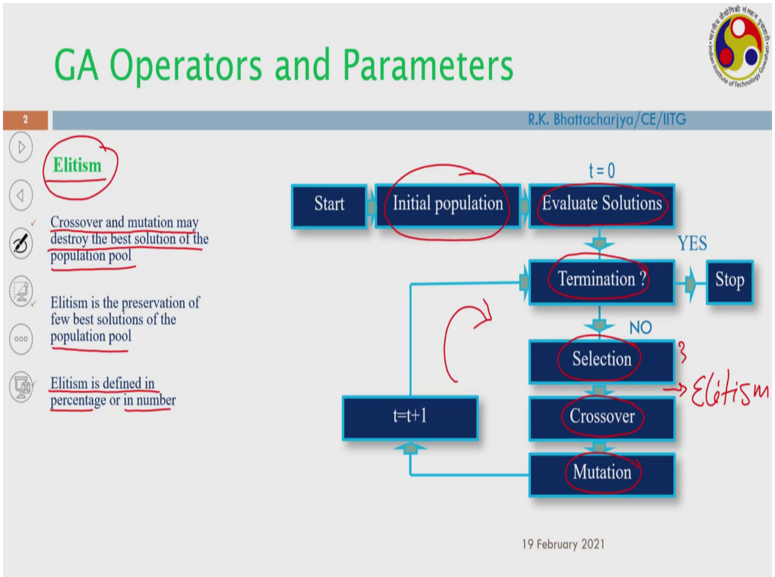

So, for whatever we have discussed that I did not discuss how to generate initial solution or how to create initial solution. So, today I will discuss that one and then we should evaluate all these solutions ok and these solutions are evaluated using a fitness function.

So, I also explained that one and then I explained the termination criteria as I said I can take maximum generation as a termination criteria or if the function value is not changing for some generation maybe 20 generation may be 50 generations ok.

So, if these values are not changing then also I can terminate this loop; that means, already I got the optimal solution like the classical method.

So, where I we generally evaluate the gradient if gradient is near to 0 then we stop this iteration, but here we are not using any gradient information if the fitness value of the best solution is not changing with time; that means, you can conclude that you have already reached a optimal solution or you are not getting any improve solution and therefore you can stop the iteration.

Then selection operator as I said the main function of the selection operator is to create multiple copies of the better individual ok and then you are eliminating the inferior solution. The worst solution anyway it will be eliminated and some of the inferior solutions will also be eliminated here and then we have discussed about crossover operator.

So, what crossover operator is doing? So, it is creating new solution. So, as I have explained that crossover operator is creating two new solutions near the parents ok. So, you are not you are basically doing the local search using crossover and then you are applying mutation and mutation is also creating new solutions, but it is trying to explore the other region.

So, what crossover is doing basically it is trying to converse to an optimal solution and the mutation is basically trying to diverge it and going out of the optima and trying to explore the other region basically.

So, this is the function of selection crossover and mutation and then you will continue this ah iteration or generation till you are not meeting the termination criteria ok.

So, this is the simple genetic algorithm.

Now, what may happen that when you are applying crossover and mutation. So, at that time what may happen that some of the solution, suppose even the best solution or better individual may be destroyed.

So, what may happen? So, I have written here the crossover and mutation may destroy the best solution of the population pool.

So, sometime it may happen it is not that crossover and mutation will always create a better solution than their parents it, but sometime what may happen they may create some inferior solution, but at that time you need not worry because when this solution will go through the selection operator again this solution will be eliminated by the selection operator..

So, if any inferior solution is created by crossover and mutation then what will happen that will be eliminated by the selection operator in the next iteration.

So, what I want. That my better individual which is the generation best individual that is not destroyed by crossover and mutation and in order to preserve those things. So, I can actually use elitism.

So, what is elitism? Elitism is basically you are preserving the best solution the better solution you are preserving that is basically called elitation.

So, I want that the some of the better solution they will not go through the crossover and mutation they will directly go to the next generation. So, that we can preserve it and that process is known as elitism ok.

So, I have defined here what is the elitism; elitism is the preservation of the best solution of the population pool basically I am preserving those solution and I can implement this elitism by percentage or in number basically.

So, what I can do that some percentage I can preserve maybe 5 percent maybe 2 percent of the best individual. So, that can be preserved and they will directly go to the next generation or I can also put a number that ok I would like to preserve 10 individual the best 10 individuals ok. So, they will go to the next generation without participating in crossover and mutation because the crossover and mutation may destroy that solution.

So, apart from selection operator then crossover operator and mutation operator we should also use the elitism. So, just to preserve the better individuals and so that they are not destroyed by the crossover and mutation operator.

So, I hope this is clear. So, with this actually so, I will also put one yeah one operator and that is elitism somewhere ok. So, I will put elitism ok. So, elitism I will put it and this is another operator. So, where we are just preserving the best individuals.

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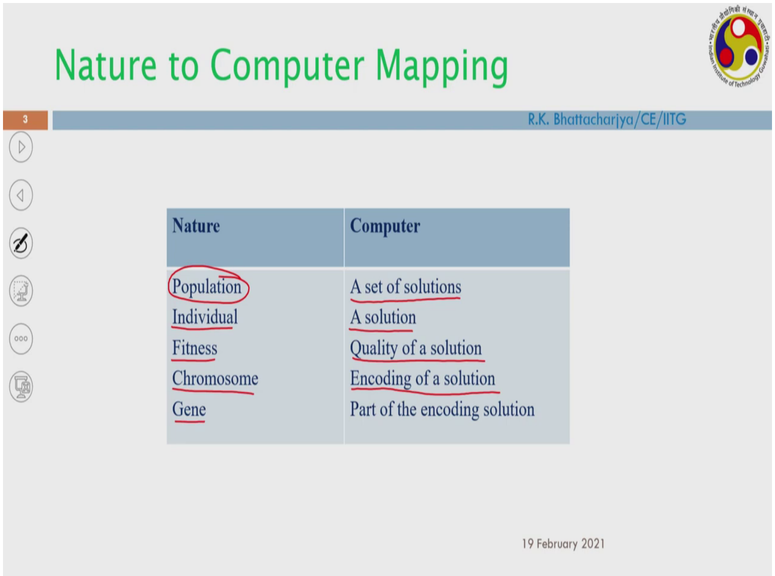

Nature	Computer
Population	A set of solutions
Individual	A solution
Fitness	Quality of a solution
Chromosome	Encoding of a solution
Gene	Part of the encoding solution

If I compare the nature to the computer mapping. So, in nature so we are using this nomenclature suppose we are using population.

What is population? Population is nothing but a set of solution.

Then individual this is a solution ok. So, in set of solutions. So, this is a solution and we call it individual here and in computer we are call a solution, then fitness.

So, we call it fitness it is nothing but the quality of a solution that is your fitness and we call here chromosome and here this is encoding of a solution.

So, you are actually converting in case of binary coded GA you are converting it to a binary bit. And what is gene? We call it gene and here it is a part of the encoding solution. So, we are using this nomenclature.

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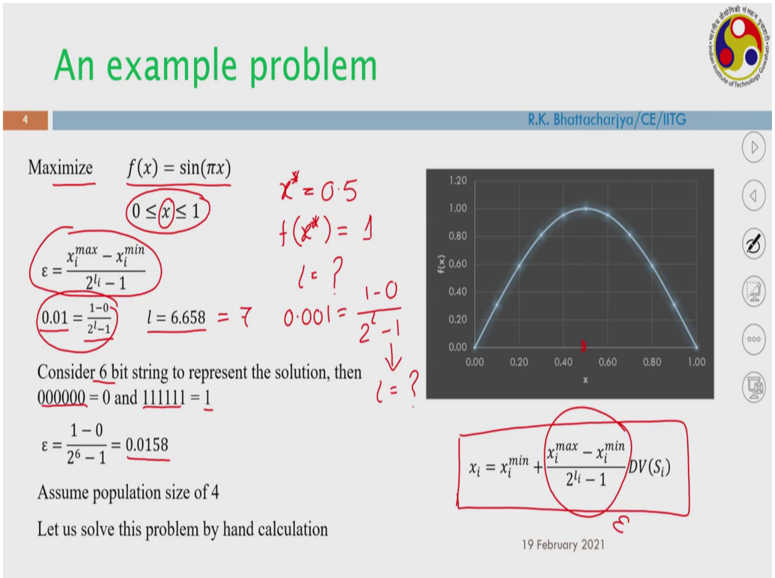

Now, let us solve one example problem. This is a very simple example problem we have only one optimal solution of this particular problem. And so, I will solve it here. So, with using hand calculation only and then I will explain the different process of genetic algorithm.

So, the problem is it is a maximization problem and it is a maximization problem and the function is f x equal to sin pi x and x is between 0 and 1 ok. So, x is between 0 and 1 and if I plot this the solution is 0.5 somewhere here. So, this is 0.5 that is the solution and optimum function value is 1 ok.

So, solution if I solve it. So, I should get x star is equal to 0.5 and then function value at x star is 1 ok. So, this is the solution now we will try to get this solution and basically I will complete one iteration of genetic algorithm to solve this particular problem.

Now, before that I have to take couple of decisions here or now. So, I have to convert this continuous variable x to binary bit ok. So, for that I have to take a decision what should be the bit length ok. So, what should be the value of l; this I have to define. The string length I have to define.

Now, how to define that? So, I have given this equation. So, this is the mapping we are using basically. So, to convert a real value to binary value.

So, I can use this mapping and I said that. So, this is showing the precision ok. So, what type of precision you want basically.

Now, if I say this is my precision epsilon is by precision here and now I need suppose I need the precision of 0.01. So, in that case. So, what is i max, i max is 1 and then i minimum is 0 and 2 to the power l minus 1 and if I solve it for l. So, I am getting l equal to 6.658.

So, that means, to get a precision of 0.01 the string length should be 6.65. So, you cannot use 6.65. So, you have to use 7. So, in that case you are getting your precision will be little bit more than 0.01. But here what I am doing because this is a hand calculation if number your string length is more then it will be little bit complicated.

So, what I have done. I have consider your a 6 bit string ok the 6 bit string I have consider for this particular problem, but you can also consider 7 as I said or if you want to have more precision then what you will do you will increase this the string length ok 7, 8, 10. So, you can find out. So, what may happen? Suppose, if you are using 0.001. Then, what should be the string length.

So, I can calculate it here 1 minus 0 and this is 2 to the power l minus 1. So, if I solve it and I will get the required string length. So, from here I can find out what should be the value of l. Now what will happen certainly it will be more than 7 now. So, ok so, I need your more bits in order to increase the precision. So, here what I am doing I as I said. So, I have used 6 bits ok.

So, 6 bit and 0 0 0 all 0 means this is 0 and all 1 means it should be 1 this is the mapping I am using this equation..\

So, here what precision I am getting. So, I am not getting 0.001, but I am getting a precision of 0.0158. So, this is the precision I will get ok.

So, then this is the first decision you have to take; then the second decision is the population size. So, what should be the population size.

Now, question is that suppose, your string length is 6. So, you can use around 8 to 10 times as population size. So, maybe around your 40 to 60 something like that you can use.

So, now, what will happen if you are using less number of population; suppose you are using if your string length is your 6, but you are using 4 or 5; then what will happen mixing will not happen.

So, crossover will not be able to find new solution after some time. So, therefore, population should be little bit large.

Now, question is that if it is a large population, you are using it is a 100 population. Then what will happen? The computational time will increase because 100 population means 100 function evaluation in each iteration the 100.

So, if you are doing it for 100 population and 100 iteration you just see 100 into 100 this mass function evaluation you are doing basically. So, your computational complexity will increase. So, therefore, it should not be too large ok and it should not be too small also.

So, depending upon the complexity of the problem if your problem is highly non-linear and then you have multiple optimal solution and in that case you should use more population ok. So, population size should be more, but this is a hand calculation. So, therefore, I have use only 4 population. So, in this case I use population size of 4.

So, this is the hand calculation. So, therefore, I have just used population size of 4; population size of 4.

Now, let us solve this problem using genetic algorithm ok by hand calculation.

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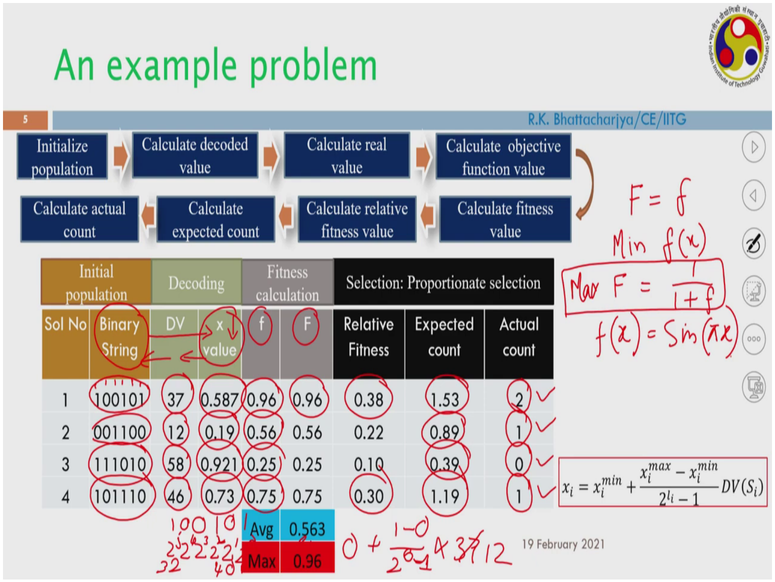

So, what I have to do now.

The first step is initialize population. So, you have to initialize the population. So, you can do in two ways you can initialize the population.

The first one I am just giving the algorithm. So, how you can do; you can write a computer program to do that, but the concept is something like this.

Suppose, I would like to generate this string randomly. So, means I want that this is a random generation of the string. So, what I will do I will do. So, I have to generate 6 bit and I have only two options that the bit may be either 0 or 1. So, right now you are at the first bit ok.

Then what you will do; you will just take a coin and toss it if it is head then you put 1; if it is tail you put 0. So, in this case suppose head I am getting 1, then you go to next bit you toss the coin in this case it is tail so, you are getting 0.

Then go to the next one; you are getting tail this is 0, then head this is 1, then this is tail you are getting 0 and this is head you are getting 1 ok.

So, this way I can generate a random string; you can also write a computer code. Suppose, what I can do. I can generate a random number between 0 and 1; if the number is more than 0.5 then I will put 1 and if it is less than 0.5 then I will put 0. So, I can generate this string randomly.

So, similar way I will generate the second one also ok this is first solution, second solution, then third solution and then fourth solution. So, this is one way to generate the binary string. Now, I there is another way I will explain that one also.

Now, next is what you will do; you will find out the decoded value ok. So, you will calculate the decoded value and decoded value is 37 here and for this is 12 and then for this is 58 and this is your 46. So, how we are getting thirty 37 here; this is your 1 0 ok 0 then this is 1 0 1.

So, this is 2 to the power 0, then 2 to the power 1, then 2 to the power 2 ok 2 to the power 3, then 2 to the power 4 and 2 to the power 5. So, all are 0.

So, here 2 to the power 0 means this is 1 and then anyway this is 0 ok because it is 0, then this is 4, this is 4 and this is your these two are 0 and this is 32 ok. So, you are getting 32 plus 4; 36 plus 1. So, we are getting 37.

So, similarly you can also calculate the decoded value for solution two that is coming 12, then for 3 this is 58 and for 4 this is 46; then you calculate the real value.

So, real value I have used this mapping here and so, I can find it out. So, how I am getting 0.58 0.587. So, this is x i which is equal to x minimum. So, x minima is 0, then plus that x maxima maximum value of x is 1 and then 0 and this is 2 to the power 6 minus 1 into decoded value. So, decoded value is 37. So, then if you calculate this thing. So, you should get 0.587.

Similarly, for if you put decoded value of 12 decoded value of 12; then you should get 0.19, then this is 0.921 and this is 0.73 ok. So, you are getting the real value.

So, now this is one way to initialize the population. So, you initialize the population as a binary string then you get the decoded value and then you find out the x value I can also do in a reverse order.

So, what I can do, I can generate this x value randomly. So, what is my upper bound and lower bound I know that lower bound is 0 and upper bound is 1. So, I can generate the four solutions between 0 and 1 and this solution I can do and then using the equation using this mapping equation.

So, I can find out the decoded value and once you are getting the decoded value. So, corresponding binary string you can find out ok. So, you can find out. So, you generate this randomly now.

So, what we are doing here we have generated the binary string and then we are doing the decoded value and then x value or otherwise what you can do you generate this x value randomly and then decoded value and then you calculate what is the binary you find out the binary string or you calculate the binary string ok.

So, there are two ways I can do. So, either binary string randomly I can generate or I can generate the x value randomly and then corresponding binary string I can generate ok. So, this is the two ways I can generate the initial population.

Now, next is the calculation of objective function; calculate objective function value. So, objective function value you just calculate it.

So, what is your function? The function is that f x is equal to sin pi x ok. So, this is your this is your function getting my.

So, if you put the x equal to 0.587; then you should get 0.96 for 19 you should get 0.56, 0.921 you should get 0.25, then 0.75 and then you calculate the fitness function that is this is the objective function and this is the this is small f is objective function and capital F is fitness function.

So, if you recall that this problem is a maximization problem. So, therefore, I can take that F equal to small f; that means, the same function I can use as an fitness function. So, here what will happen the if the capital F value is more in that case the solution is better, but if you have a minimization problem.

Suppose, as I said that minimization of f x. So, what you can do you can write this is maximization of F which is equal to 1 by 1 plus small f. So, I can also use I can also use this function ok.

So, but in this case I will not convert it because this is a maximization problem. So, therefore, I will just use I will just use that small f or equal to capital F ok fitness function is equal to capital F and which has actually same value ok.

Now, the next step is selection operator. So, you apply selection operator. So, in this case I am just doing one iteration. So, I am not telling about the termination criteria, but you can check whether you are getting the optimal solution or not. So, in that case you can terminate, but here I am just showing one iteration.

So, therefore, you just go to the selection operator.

So, in this particular problem I have use proportionate selection ok. So, you can also apply tournament selection or you can also apply rank selection or you can also apply your roulette wheel selection ok here I am using proportionate selection.

So, what you are doing. So, you calculate the relative fitness. So, what is relative fitness? So, relative fitness is that sum of all this fitness ok. So, you divide this by sum of all this fitness.

So, therefore, the relative fitness you are getting 0.38, then 0.22, then point 0.1 and 0.30 ok. So, this is the relative fitness you are calculating.

So, if you recall this relative fitness is equal to the area occupied under roulette wheel ok. So, this is the area this particular solution will occupy on the roulette wheel. So, this is 38 percent, then 22 percent, then 10 percent and 30 percent.

So, now the expected count will be selected. So, what is the expected count? That means, 30 38 percent of 4 will go to the next generation or expected count that is; so, 38 percent into 4. So, 0.38 into 4. So, you should get 1.53, then 0.22 into 4. So, you are getting 0.89 and this is 3.9 or 0.4 and then 1.19.

So, this is the expected count and then you calculate what is actual count. So, actual count as I said that this is the expected count, but you have to have a integer value. So, is for that. So, actual count will be. So, for 1.5 is so, I just round this to the higher one. So, this is 0.2. So, actual count will be 2 then here it is 1 and here it is 0 and then 1 ok.

So, what you are getting here. The first solution that is the best solution and this solution will have 2 copies and then the, this one will have 1 copies and third one is the worst solution. So, it will have 0 copies and the fourth one will have 1 copies.

Now, if you recall what is the function of selection operator. Selection operator is creating multiple copies of the best solution or better solution and the worst solution has been eliminated at the same time we are also maintaining the total population size population size was 4 and now also you are getting total 4 population; that is 2 copies of the first solution, then second solution 1 copy and third fourth solution 1 copy. So, you are getting 4 solution.

So, that way we are implementing the selection operator. So, as I said I have done it using proportionate selection you can also apply rank one or you can also apply tournament selection.

The next step is the crossover operator. So, you have to apply the crossover on the solution on the matting pool.

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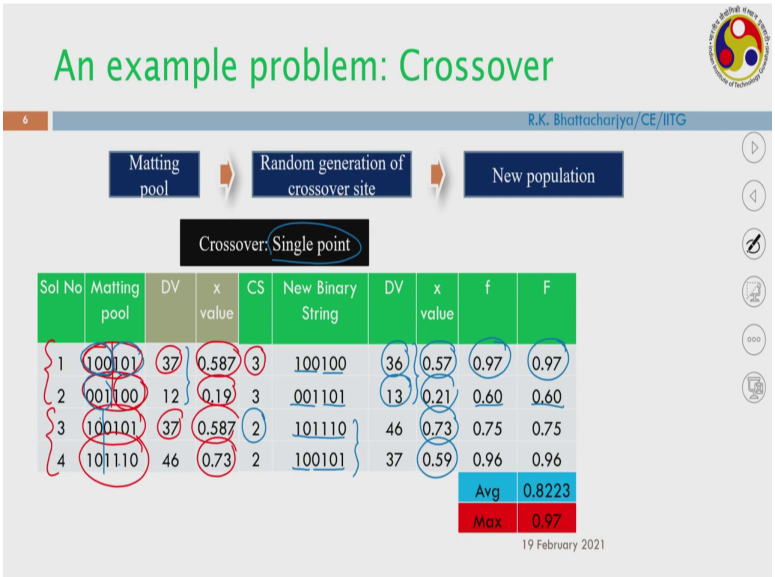

So, if you look at what is the matting pool is whatever you are getting after the selection operator. So, now, I have 2 copies of the first solution this is one and this is one 2 copies.

Then I have 1 copies of this solution and I have 1 copies of this solution and I am just writing what is the decoded value.

So, this is 37 and this is 37 and x value is 0.58 for here and 58 for here and this is 0.19 and 0.73. So, this is the this is the solution on the matting pool.

Now, how to apply crossover. So, you take two solution from the matting pool and then you apply crossover operator. So, that has to be done randomly; that means, you have to select the solution randomly from the matting pool; then they will go for crossover with a crossover probability of pc.

As I said, that pc I can take 0.65 to 0.95 or sometime 100 percent, but in this case I am taking 100 percent; that means, all this solution will go for crossover and then you have to do it randomly in this case I have selected that the 1st solution will go for crossover with the 2nd one and similarly 3 and 4 will go for crossover.

So, in order to implement crossover. So, what you have to do that you have to select a crossover site ok. So, you just select a crossover site and this crossover site to be selected randomly and between 1 to l minus 1. So, in this case between 1 and 5 ok. So, you select a site. So, I got it randomly this is 3 and for this; this is 2. So, this is the crossover site ok after 3.

So, therefore, the new population will be that first part of the first solution this part and then the second part of the other solution. So, what I am getting the first part of this solution that is 100 and then 100. So, you can see that 100 and 100 you are getting; and then first part of the second solution and second part of the first solution that is 001 101 001 101.

Now, when you will go for crossover of the 3rd and 4th solution. Then crossover site is 2 ok. So, this is the crossover site now. So, what will happen. So, this is 10 and this is 10.

So, therefore, once you are swapping up. So, you are not getting any new solution sometime it may happen like that, but anyway. So, here I am getting the crossover site of 2. So, therefore, first solution will be this is 101110.

So, 101110 and the second solution is 100101. So, 100101. So, in this case you are not getting any new solution. So, whatever the earlier solution you have the same solution you are getting after crossover. So, sometime it may happen also.

So, therefore, what we want basically. So, we want to maintain diversity in the population pool. So, I explain about the selection pressure.

So, what will happen if suppose, if you are increasing the tournament size. So, what will happen your population will be saturated by the best solution then best solution.

So, what will happen? So, if you are picking two solution for crossover then there is a probability that you may get the same solution ok. So, then if you are applying crossover. So, you are not generating any new solution..

So, therefore, I want to maintain diversity in the population pool. So, therefore, you should not use higher tournament size two three is fine if your tournament size is more. So, in that case your population will be saturated by the best solution.

So, whatever we have done. So, this is a single point crossover ok. So, this is a single point crossover. So, you can also apply multi point crossover. So, multi site crossover you can use, but in this case I have done single point crossover.

Now, let us see whether we are getting better solution after crossover. So, here this is the decoded value 36, then 0.57 and f value is 0.97 and capital F is also 0.97 and then this is 13 and then this is 21, then 0.60 and 0.60 anyway. So, you are not getting new solution here. So, you are getting the same solution that is your 0.73 and 0.587 or 0.59.

So, you are not getting any new solution from your 3rd and 4th solution, but from 1st and 2nd solution. So, you are getting two new solution.

Now, if you look at this particular solution. So, you just see the decoded value of this parents was 37 and 12. 37 and 12 was the decoded value and here you are getting a decoded value of 36 and 13 so; that means, the crossover is creating a solution near to their parent.

So, you are not getting a solution from 37 you are not getting 58 or from 12 you are not getting your 5 or 6 something like that. So, what you are getting you are getting a solution near to the parents ok.

So, therefore, what crossover is doing? So, crossover is creating a solution near the parents. So, slowly it is going towards the optima now the next step is to implement mutation ok. So, crossover operation has been done.

So, let us apply mutation.

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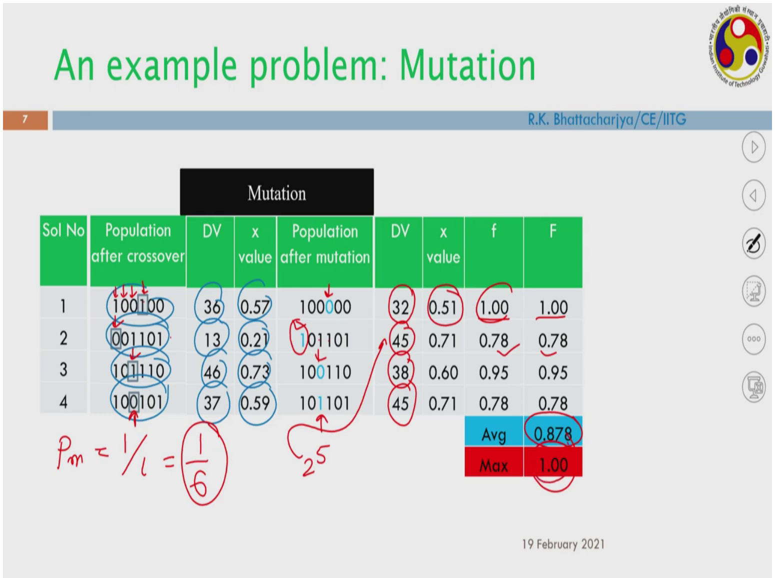

Now, this is the population after crossover. So, I got four solutions ok. So, these are the four solution after crossover and this is the decoded value of this crossover and this is the x value of this crossover real value of this crossover. Now, what we will do basically. So, how to apply crossover operator that you have to take a crossover probability P m.

So, as I said the P m should be very very your low value it is a very low value. So, generally what I use what we use that 1 by l. So, in this case suppose I am using 1 by 6 ok. So, this is the P m probability mutation probability I am using here and then what you will do basically.

So, first you will go to this particular bit the first bit and then you generate a random number if the random number is less than 1 by 6; then you mutate it ok mutate means use a sense if it is 1 then you make it 0 and if it is 0 you make it 1 or otherwise you just keep it.

So, that way then you go to next one, then go to next one and then go to next one like that. So, in this case what is happening. So, in this case what is happening for the four bit that random number is less than 1.6; so, this bit will be mutated.

So, I have used 1 by 6 it means that out of 6 trial so, 1 will be mutated ok. So, out of 6 trial 1 will be mutated. So, in this case the fourth one means suppose will be mutated.

So, similarly so, fourth one. So, fourth one will be mutated. So, here it is 1. So, it become 0 and for the second case this one will be mutated. So, this is 0 this will be 1 and for the third one this will be mutated. So, therefore, this is 1. So, it is 0 and the fourth one this will be mutated. So, this is 0 and this is 1. So, you are getting a new solution.

So, you can see basically just see. Now, this is the decoded value of the mutated solution. So, 32, 45, 38 and 45. So, if you look at for the second solution this bit has been mutated. So, this is this been mutated means you are adding 2 to the power 5 here.

So, in this case the decoded value is increased by 2 to the power 5. So, therefore, it was 13. Now, you are getting 45 ok so; that means, 32 you are adding.

So, here you just see it is not creating near the near a parents basically. So, if I say that this is the solution before mutation and after mutation you may get a completely different solution and may be at a at different place. So, therefore, what mutation is doing. So, mutation is trying to explore the unexplored region in the search space ok.

So, here it is not creating a solution near to the earlier solution. So, now, you just look at that. So, I am getting 32 here. So, this is 0.51 and then f value is 1 and this is 1 and if you look at for other solution also I have calculated the small f and capital F anyway capital F and small f are equal here and your average value it is increasing to 0.878 and maximum value ok.

So, the solution is 0.50, but anyway for 51 also you are getting approximately your the function value of 1; that means, you are almost near the optima ok.

So, if you look at that after crossover and mutation the average function value is increasing. So, and maximum function value is also increasing after crossover and mutation we have solved one problem.

So, I can also do one more iteration. So, with this solution in the population. So, now, you can apply selection operator again. So, if there is any inferior solution that will be eliminated by the selection operator and then you can apply crossover and mutation again and probably after some iteration your solution will improve or you are marching towards the optimal solution.

So, today I have explained Genetic Algorithm using hand calculation. So, I hope the process is clear to you how we can apply the crossover operator, the mutation operator, then the selection operator. We have to use some programming language to solve a problem using Genetic Algorithm.