Optimization Methods for Civil Engineering
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Module - 01
Lecture - 01
Introduction to Optimization

Hello student, welcome to the course on Optimization Methods for Civil Engineering. So, today, I would like to introduce you what is optimization and then, I will also discuss about necessary and sufficient conditions for optimality. So, that will that I will discuss after that, but before that let me introduce what is optimization.

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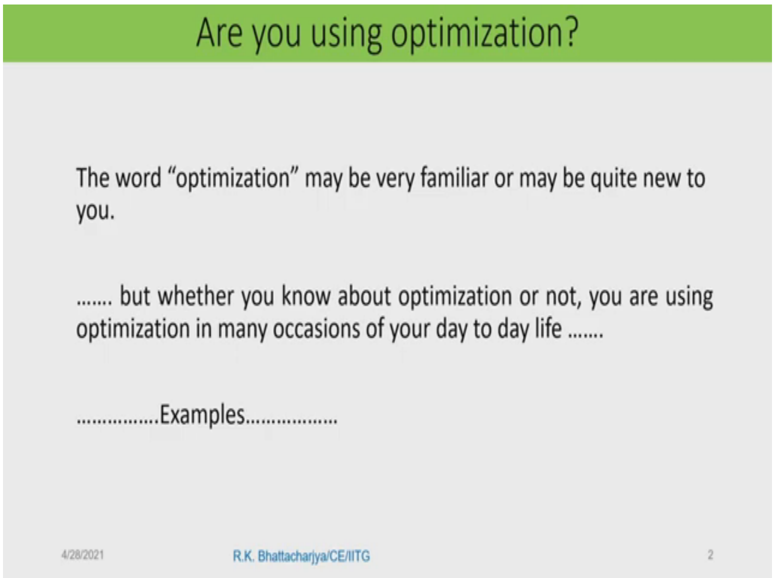

Now, if I ask you, are you using optimization? The answer is the word optimization may be very familiar or may be quite new to you; but whether you know about optimization or not, you are using optimization in many occasions of your day to day life. Let us see some example problem.

So, what I would like to say here that whether I whether a person is familiar with optimization or not familiar with optimization, suppose the layman, they are not familiar with the optimization; but still I can say that, they are solving optimization problem in their day to day life.

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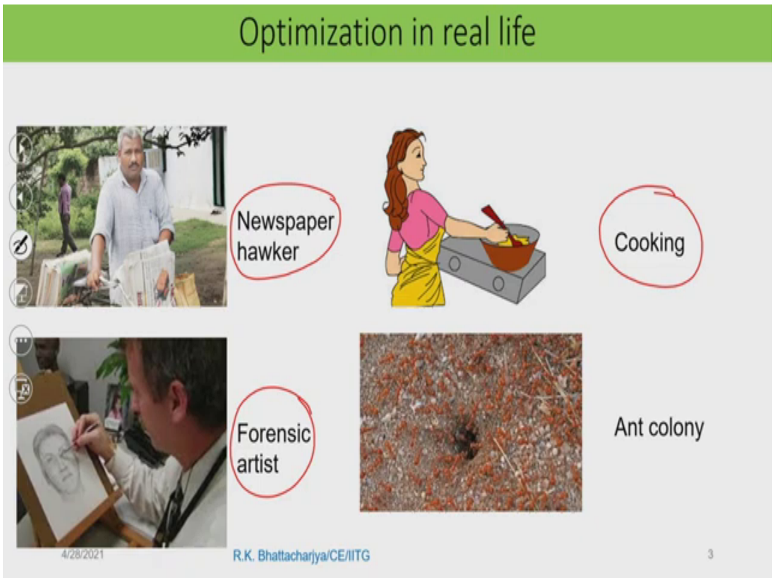

So, let us see some examples. I would like to give you few example. So, one of them is this person, he is a newspaper hawker. So, what he is doing basically? He is distributing newspaper in a colony. Now, question is that whether he is using optimization? So, I will discuss about that.

Then, second example is the lady is preparing some food or you can say that he she is preparing some recipe. As you can say that there is no link to optimization; but I can say during the process of preparation of food or you can say the during the process of your preparation of this particular recipe, she might be using some optimization technique or I will explain that one, how she is actually solving an optimization model.

Then, third example problem is forensic artist. So, what he is doing he is trying to construct a face of the criminal, based on the information he has received from the eye witnesses ok. So, eye witnesses has explained something that this is the nose, this is the shape of eye, this is the size of the eyes or something like that.

So, based on the information received from the eye witness, he is trying to construct the face of the criminal. So, now, I can explain I will explain how he is also solving an optimization model ok. Apart from that, suppose if you look at the ant ok. So, when they are moving or whatever social behavior they have, they are also using optimization.

Maybe we can have some other example problems, so when we will discuss about the metaheuristic optimization method. But today, I will take these four example problems and I would like to explain that how these people, the newspaper hawkers, during cooking, the forensic artist and ant colony how they are using optimization or by knowingly or by without knowing, they are using optimization as I said. So let me explain that one.

So, let me take the example of newspaper hawker ok. So, what he is trying to do? He is trying to distribute the newspaper in a colony ok. So, maybe there are different quarters, then suppose he is trying to distribute in a in an educational institute. So, he has to distribute this newspaper maybe in the residential complex, then to the hostel, to the admin block ok, to the girls hostel, boys hostel something like that.

Now, the first day, he does not have any information about that colony. So, what he will do? He may have some planning based on the information he has received. So, he will try to distribute the newspaper. Then, once he is distributing the newspaper on that day. So, he will have some experience that ok, if I go to this quarter first, then I will go to hostel, then I will go to admin building and probably, I can distribute this newspaper in a minimum time.

So, his objective is to distribute the paper within a shorter duration of time. So, what he is doing basically? He is trying to minimize the time of distribution here. So, you can say that he is trying to solve an optimization problem. So, what is the objective function here? The objective function is to minimize the distribution time.

So, you will surprise that after few days ok, after few days maybe first day he has distributed this, he has chosen some route. Then, second day, he will modify that one; then, he will try to minimize the time and you will see that after few days, maybe after 7 days or after 6 days or maybe before that. So, he will he has actually obtained an optimal route or that is the optimal solution of this particular problem.

So, what he is doing? He is trying to minimize the time. So, you can say that he did not solve any optimization problem mathematically. But with some experience or with some experience or he is basically trying to find out a solution so that he can distribute this newspaper within a shorter duration or short or he is trying to minimize the time.

Then, second example is suppose the lady is preparing some food or you can say the recipe. So, how we are preparing a recipe? So, it is a combination of some spice you are putting on it and the duration of heat basically or you are heating that recipe. So, that is one your parameter.

So, what will happen suppose the first day when you do not have any information. So, you are putting some spice based on your experience and then, what you will do? That you will taste it and after that, you will try to modify the recipe. Maybe combination of different spice, you will basically you will sense.

And after few days what will happen? Or you can say with experience, then you will actually find out an optimal combination, an optimal time basically; at what time you want to put this particular spice, so you can find out an optimal combination of that one. So, that way what you will have?

With some experience, so you are mathematically you are not solving an optimization model; but actually you are solving an optimization model. And you can say this is an iterative base, this is an iterative based optimization model and after few days, you will have basically a recipe.

So, what is the objective function here? The objective function is that I would like to maximize the taste ok. The taste here and what are the decision variables here that how much spice you are putting, at what time you are putting, what are the spice you are putting. So, these are the decision variable ok. So, that way she is also solving an optimization model.

Then, third example is a forensic artist. So, what he is doing basically? So, he has the information from the eye witness. So, eye witness will say that basically, so this is the eye witness will give you the description about the criminal and based on that, he will initially first he will construct a face and then, he will get the he will take the feedback from the eye witness ok.

So, whether his nose is similar to that criminal, then eye witness will give you no, this is something like that. So, then he will modify that one. And then he will make his second case, third one, fourth one, fifth one like that. Then, after some time, you will see that after some iteration, you will see that he has constructed the face of the criminal.

So, what is the objective function here? The objective function is to maximize the similarities between his drawing and the criminal face basically. So, and with some iteration, so he will be able to do that and this is basically he is also solving an optimization model. Then, the fourth one is an ant colony.

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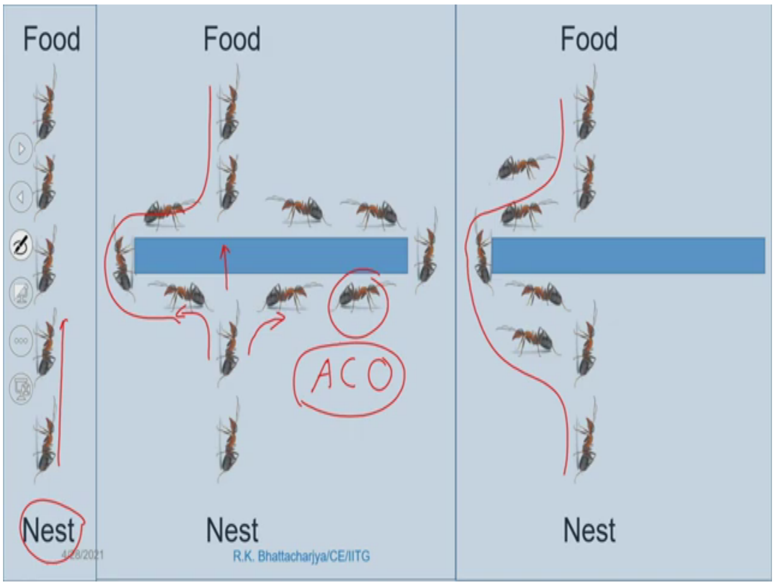

So, I will give an example of that one. So, let us see this ant. So, what they are doing? So, there is a nest and they are trying to search the food. You can also do this experiment and you will see that these ants are moving in a straight line. Basically what they are trying to do? This is the shortest distance basically. So, you can see suppose that the ant nest is somewhere here and they are moving towards the food and you can see that they are moving in a straight line, if there is no obstruction.

Now, let us see you can you can do these experiments. So, I am putting some obstruction here. Now, obstruction are that if you put the obstruction in such a way that suppose initially what ant will do that they will try to move because of these obstructs because of this obstruction, he the ant cannot move like that.

So, they will move some ant will move in this direction, some will move in this direction. And you will see after some time that because this is the shortest distance ok, this is the shortest distance and you can see that no one is there on this side and everybody is moving in this route because that is the shortest distance ok.

So, here also they are trying to solve an optimization model or they have actually solved an optimization problem; but without doing any mathematics ok. So, they have a different mechanism and you can say that whatever method they have used, based on that there is an metaheuristic optimization algorithm known as ant colony optimization.

So, ACO that is Ant Colony Optimization. So, this is basically based on the behavior of this ant basically. How they are trying to search food in a through a shortest distance and based on that actually, this algorithm has been developed and this is a metaheuristic optimization algorithm.

So, what I would like to say that whether we know optimization or we do not know optimization, so we are actually using optimization in our day to day life. So, you can say suppose when you are preparing for your exam, it is not that you are preparing arbitrarily ok.

So, with some experience, so what you will do? You will have some planning and when you will have an optimal planning because what you have to do that within that particular duration ok, so you have to prepare for a exam.

So, you should have some planning and there also actually you are trying to solve an optimization model without basically knowing the optimization. So, what I am telling? That we are using optimization in our day to day life without knowing about the optimization, without knowing about the optimization.

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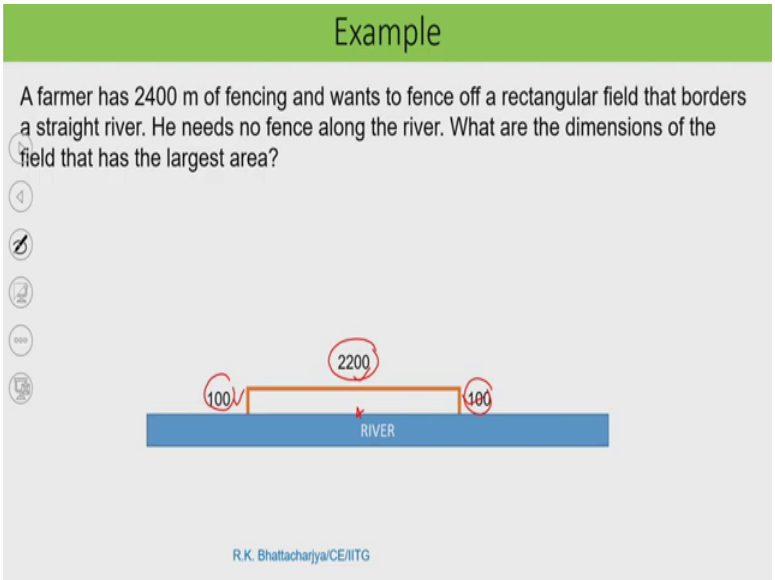

Now, let us take an example problem. So, this is an simple example problem. So, a farmer has 2400 meter of fencing ok and want to fence a rectangular field that borders a straight river. He needs no fence along the river. What are the dimension of the field that has the largest area? So, what is the problem here? So, problem here is the farmer has 2400 meter; that means, 2400 meter of fencing. So, that is the material he has and he want to fence a rectangular area that borders a straight river ok.

So, basically on the bank of the river and he needs no fencing along the river. So, on the river side, there is no fencing; on the other three side, you have to put the fencing. Now, question is that; what is the dimension of the field that has the largest area ok? Now, suppose this is the river and what I can do basically?

So, on the river side, you do not need fencing; this side you do not need fencing. So, you need fencing on the left side, on the right side and the on the other opposite side of the river. So, this is one this is also a solution that I can put 100 meter on this side and 100 meter on this side and this side this is 2200. This is the this is also a solution and I can find out the what is the area.

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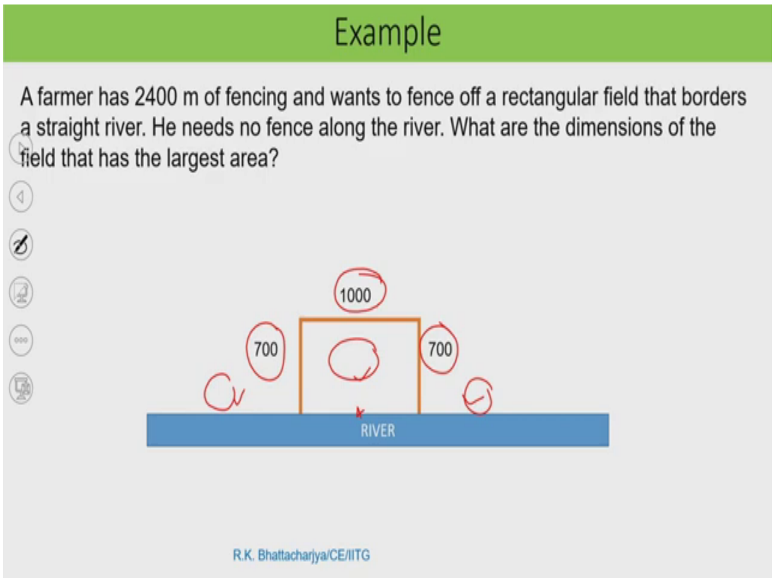

Now, second solution is maybe from 700 meter here, 700 meter here and 1000 here. So, this is also a solution.

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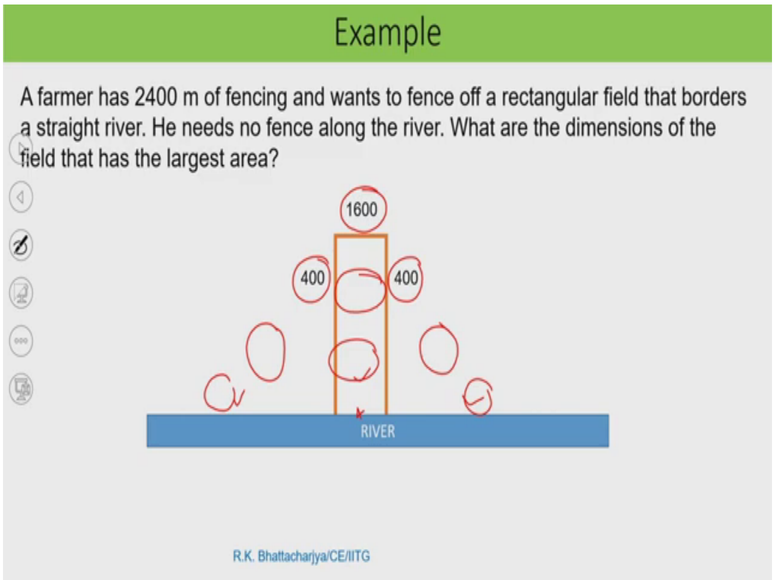

And the third one is I can also put 400 on this side, 400 on this side and 1600 on this side. So, these are possible solution of this particular problem. Now, what is we have to do basically? So, we need to find out we need to find out the dimension of this rectangle so that this 2400 meter of fencing means suppose I can use and the area will be the largest area. So, this is an optimization model. So, here what is your objective? The objective is to maximize the area of the maximize the area of this of the rectangle ok.

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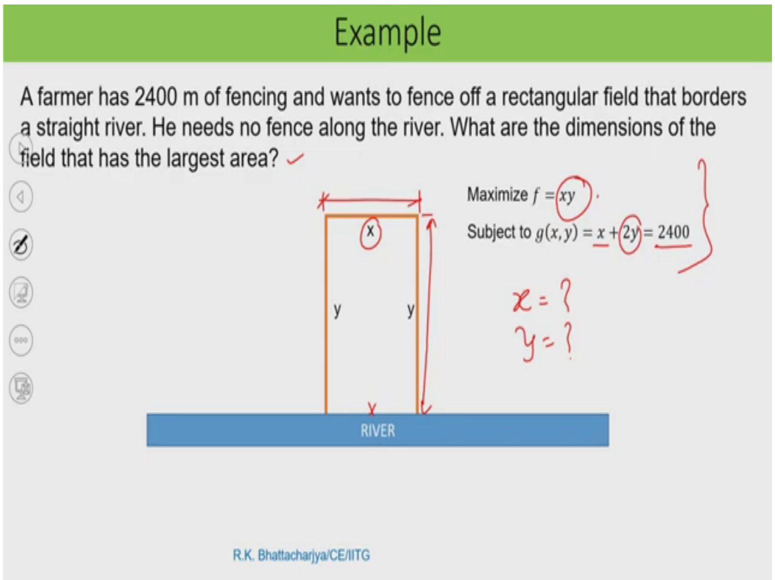

So, what I can do? That I can take this variable. So, this is your x. So, this is your this dimension is x and this is your y ok; this is your y. So, therefore, I would like to maximize the area. So, what is your area? x into y ok. So, this is my objective function; but there is a constraint. What is the constraint here?

The constraint is that x plus twice y ok. So, this x plus twice y; that means, the perimeter. So, riverside there is no fencing, there is no fencing on this side; on the other three side, we have fencing. So, that is x plus twice y should be equal to 2400. So, this is an optimization model.

So, here, we have the objective function that is maximize the area. So, maximize f equal to x y; x into y and subject to there is a constraint and constraint is that the total material availability is 2400. So, therefore, x plus twice y equal to 2400. So, the problem actually; so, whenever you are getting a problem, so you may not you will you may not get a problem in this mean suppose in mathematical form. So, somebody will explain that one that the in this case suppose the farmer has 2400 meter of fencing.

So, this is the this is the fencing he has and he wanted to fence an area. So, the problem will be described like this and then, we have to formulate the optimization model and once I solve this problem, so I will get the dimension what is the value of x and what is the value of y, I can find out. So, I will discuss about the solution part later on. But right now, I would like to discuss that how to formulate an optimization model ok.

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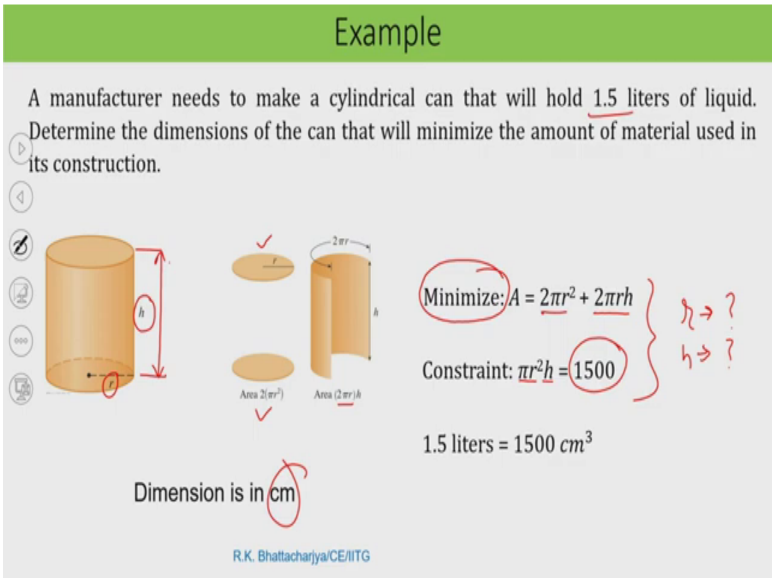

Let us take another example problem. A manufacturer need to make a cylindrical can that will hold 1.5 liters of liquid ok. So, this is design problem. I would like to design a cylindrical can here and the capacity of the can is 1.5 liter ok. Determine the dimension of the can that will minimize the amount of material use in its construction ok.

So, what you have to do here? So, here, we have to design a can. So, the capacity of the can is 1.5 liter ok. So, that is the capacity of the can and what you have to design? Design means I have to find out the dimension of this particular can, where the capacity of the can is 1.5 liter and the cost is minimum ok.

So, I have to find out the dimension of the can, so where cost is minimum; that means, I would like to minimize the cost here, at the same time the volume of this can should be equal to 1.5 liter. Let us see how we can formulate an optimization problem ok. Now, this is the can. It is a cylindrical can. So, therefore, I the dimension of this is the height of this can h and the radius is r ok. So, this is a cylindrical can. So, this is the height and this is the radius ok.

So, now, if I say the material cost, so that means, this is the surface area of this can. So, area is area is twice pi r into h. So, twice pi r into h and then, we have to take the material used at the top and at the bottom. So, that is pi r square and at top and bottom. So, this is twice pi r square.

So, therefore, what is the total material used? This is twice pi r square plus twice pi r h and what is your objective? I would like to minimize the material cost ok. So, I would like to minimize this one. So, this is minimize A equal to twice pi r square plus twice pi r h and there is a constraint here. What is the constraint? The volume. So, what is volume? That pi r square into h that should be equal to 1500.

So, it should be equal to 1.5 liter and all dimensions are in centimeter. So, here this r is in centimeter, h is in centimeter and therefore, the volume we have converted. If we convert 1.5 liter into centimeter cube, then it will be 1500 centimeter cube ok. So, now, this is your optimization a model and here, what we are doing?

We are trying to design a cylindrical can; with cylindrical can of capacity 1.5 liter and now, if I solve this problem, so I will get the value of r and h and that will be the optimal solution and if you are using that, so the cost of this or construction cost of this particular can will be minimum ok.

So, we will be we will find out what is the value of r and h. Anyway, so today, we are not solving this problem. So, we are just trying to show you how you can formulate an optimization problem.

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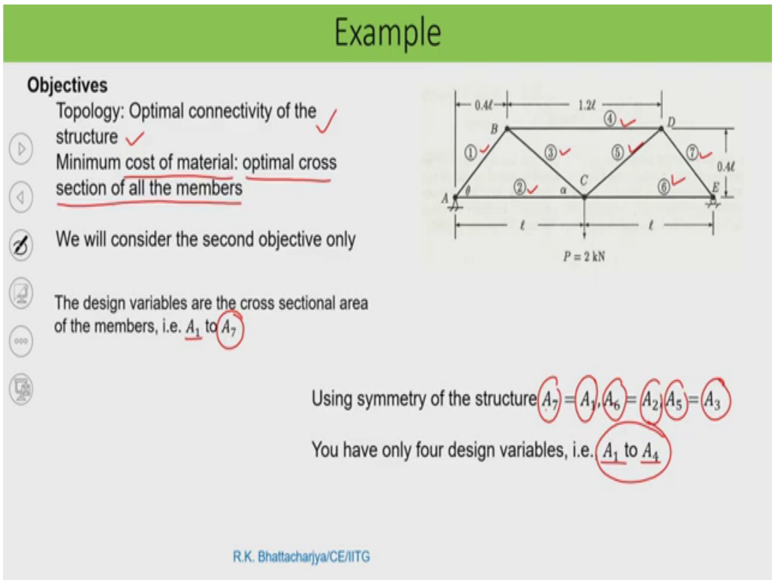

The third example problem is a truss design problem ok. Now, in case of truss, what is, what are the objective? So, objective is one is the topology optimization. I can say that optimal connectivity of the structure ok.

So, one objective maybe I would like to find out the minimum connectivity or I can say optimal connectivity of the structure, we call it topology optimization and second one is once you are getting the minimum connectivity or optimal connectivity, then I would like to minimize the cost of material ok.

So, minimum cost of material and we can say that optimal cross section of all members ok. So, this is another objective. So, I can have topology optimization; that means, I would like to find out optimal connectivity of the structure. So, what does it mean? Whether suppose there may be other connections here ok. So, if I go for the first objective, so what are the connection actually I need basically and that is I can say that optimal connectivity.

So, in this case, suppose this is not required; this is not required; this is not required. So, finally, what I am getting? So, I am getting this particular your structure ok. Now, in the second objective, what you have to do? We have to find out the optimal cross section of all the members so that the material cost is minimum. So, here objective is the minimization of the material cost.

So, we will consider the second objective here. The design variables are the cross sectional area of the members; that means, there are 7 members here. So, there are 7 members here; that means, A 1 to A 7.

So, this is A 1, A 2, A 3, A 4, A 5, A 6 and A 7. So, we have 7 member, so we should have seven design variable ok. But using symmetry, so if I say that A 1 equal to A 7 ok; so or A 7 equal to A 1 and similarly, A 6 equal to A 2; that means, A 6 equal to A 2 and A 5 equal to A 3.

So, therefore, if I use the symmetry of the structure in this case that particular problem; so, if I say that A 7 the cross sectional area of the seven member equal to cross sectional area of the first member, then A 6 equal to A 2 and A 5 equal to A 3. So, in that case, what I can only consider A 1 to A 4 because A 5 equal to A 3, then A 6 equal to A 2 and A 7 equal to A 1. So, I have actually reduced the design variable. So, this is also an important state in case of optimization model.

So, because when your number of variables are more, so in that case, solution will be little bit or solution will be complicated. So, therefore, you should always look at whether it is possible to reduce the number of design variable. So, in this case, by using symmetry, so we can actually reduce the number of design variable and as I said the earlier design variable was 7 and now, we have only 4 design variable.

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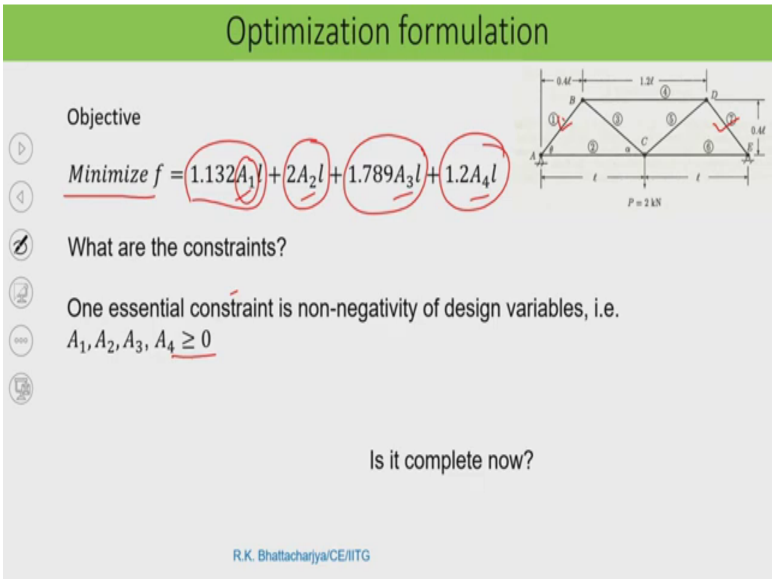

Now, let us see what is the objective function? So, objective function as I said that the minimization of the material cost ok. So, what is your material cost? So, material cost of all the structure. So, we have actually, so A 1 is this member and this member. So, total length of this A 1 member is 1.132 l into cross sectional area will give you the volume and basically, so this is the volume of the or you can say this is the material, you need for A 1 and A 7 member.

Similarly, for A 2 that is twice l into A 2 and similarly for A 3, I need 1.789 A 3 into l and for A 4. So, we have 1.2 A 4 into l. So, this is the total material used in this particular truss and I would like to minimize. So, here what I would like to find out? I would like to find out what is the value of A 1, A 2, A 3 and A 4 so that the material cost is minimum ok. Now, this is your objective function, then what are the constraint?

So, one essential constraint is the non-negativity of design variable. So, what does it mean? That this design variable, what are the design variable here? The A 1, A 2, A 3 and A 4, they are the design variables and these variables cannot be negative. So, this cross section cannot be negative. So, therefore, the non-negativity of the design variable is one of the essential your constraints. So, all always you should put that one and therefore, A 1, A 2, A 3, A 4 should be greater than equal to 0; it cannot be negative.

Now, is it complete now? It is not complete because if I solve this problem, so what I will get? I will get the value of A 1, A 2, A 3, A 4 all will be 0 and that will give you minimum material ok. But that will basically that that structure will not exist, if I say A 1, A 2, A 3, A 4 all are 0. So, therefore, this problem is not complete. So, what you have to put? You have to put the constraints ok. So, what are the constraints here?

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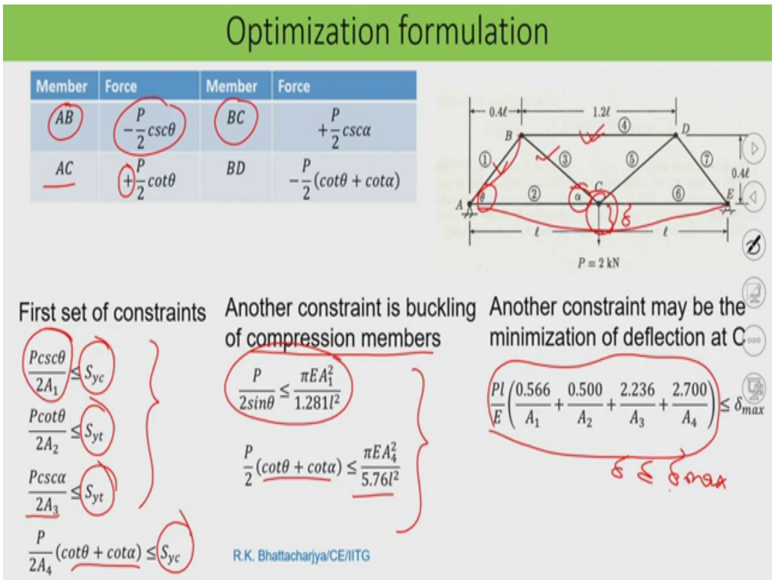

The constraints are that the member the cross section of the members should be sufficient enough to take the compressive that compressive force as well as tensile force. So, I can do the analyze here. So, what are the forces here in AB, the forces here is minus P by 2 cosec theta and the force is the compressive in nature. So, negative means compression and positive mean tension.

So, similarly, in member AC, so we have positive; positive means it is in tension that is P by 2 cot theta. In BC member, so BC, so this member; so BC, it is P by 2 cosec alpha. So, here, so this is theta and this is your alpha and member BD, the member BD, this is this member and that is again the force is compressive force and that is P by 2 within bracket cot theta plus cot alpha. So, these are the force.

So, now, whatever member you are you want to design, so the cross section should be sufficient enough to take this stress or to take this your compressive and tensile force. So, therefore, the first set of constraints are. So, this is the stress developed here that is P cosec theta by twice A 1 and this should be less than the permissible compressive stress of the material ok.

So, similarly that the second member that is AC member, the stress is P cot theta by twice A 2 and that is that should be less than the permissible tensile stress of this particular material ok. Similarly, for the member BC that is P cosec alpha by twice a less than S y t that is in tension and for the member BD that is P by twice A 4 within bracket cot theta plus cot alpha should be less than the permissible compressive stress of the material.

So, therefore, these are the these are first set of constraint; that means, if you are putting this constraint to your optimization model, then whatever cross section you will get, so this will ensure that the that the members are or cross sections are sufficient enough to take the tensile force as well as compressive force developed within the member ok.

So, what are the other constraints? The other constraints may be the compression members. So, what are the compression member here AB and BD ok. So, this is AB and BD ok; this member AB and BD. So, these two are your compression member and this member should not fail in buckling ok. So, it should not fail in buckling ok.

So, therefore, you have to put this constraint in order to prevent the buckling that is P by twice sin theta less than equal to pi E A 1 square divided by 1.281 l square and similarly, for a member BD that you have to put P by 2 within bracket cot theta plus cot alpha should be less than equal to pi E A 4 square divided by 5.761 square.

So, if you are putting these two constraint along with the first set of constraint, so then whatever cross section you are getting, those cross section will be safe enough for that to take the compressive as well as tensile stress and your member will also not fail and your member will also not fail in buckling.

There may be another constraint that the deflection; suppose the structure is safe, but there may be some deflection here ok. So, I can say that this deflection is delta. Now, what I can do basically? So, I can put another constraint. So, which will give you the deflection and deflection if we calculate at point C here.

So, deflection is P l by E within bracket 0.566 divided by A 1 plus 0.5 by A 2 plus 2.236 divided by A 3 plus 2.700 divided by A 4 and this is the deflection these will give you the deflection at point C ok. So, you can say delta and this delta should be less than equal to delta max ok.

So, I can put this constraint also. So, once you are putting that one that you are your truss will also be safe from deflection point of view and whatever you mean suppose whatever value of cross section you are getting, so in that will give you the deflection that will give you a structure where the deflection will be less than your delta max. So, I can define this your delta max.

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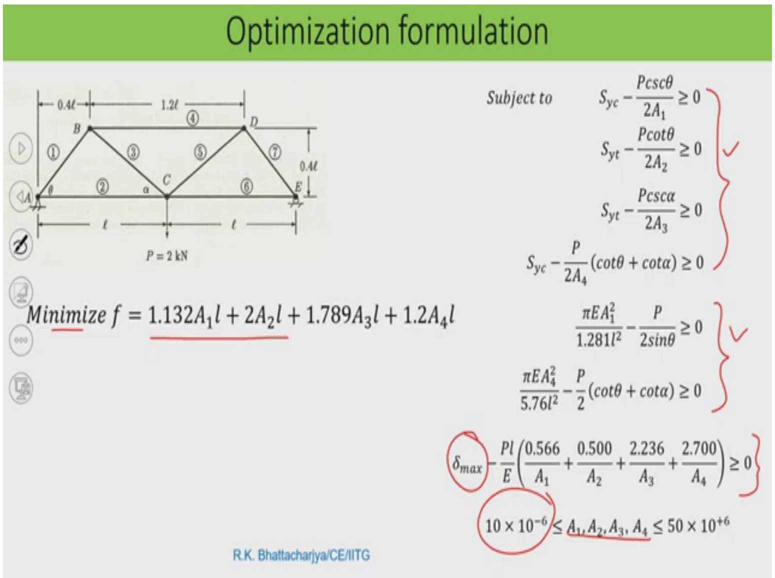

Finally, if I put all these constraints, then my optimization model will be something like that; that I would like to minimize the material cost ok. So, this is the material cost and these are the constraints and first four constraints are coming from the compressive forces and tensile forces; these two are coming from buckling; that means, these two constraint will ensure that the structure is safe from buckling.

And the third, the last one will ensure that the deflection is less than delta max and then, I can put what is the lower bound of the variable design variable that is A 1, A 2, A 3, A 4 should be within this particular range. So, I can define an overall. This will be my optimization model and if I solve this problem.

So, I will get the dimension of the truss that means cross section of the members and your truss will be safe from compression, from tension, then from buckling and as well as the deflection will be less than delta max ok. So, this is your another example problem.

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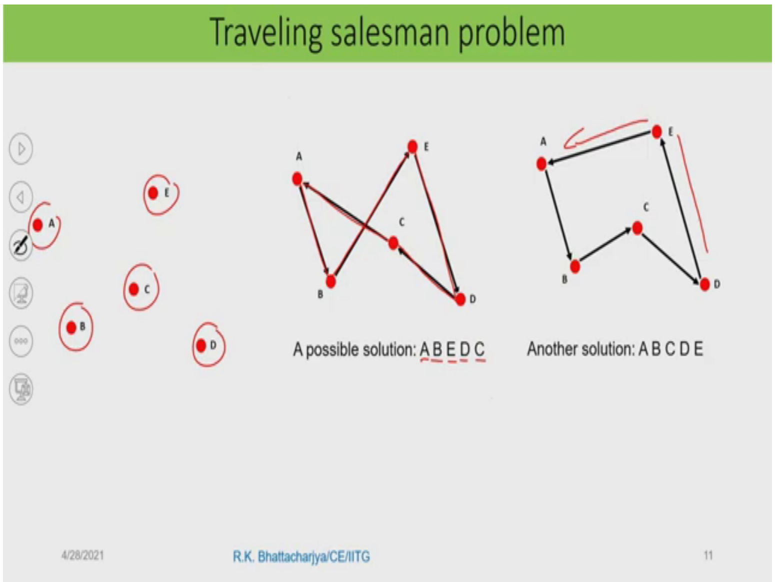

Now, let us take another example problem. So, we call it travelling salesman problem. So, here the problem is there are few cities ok. So, there are suppose city A, city B, C, E and D. So, now, a person would like to visit all these cities and he basically want to visit it within minimum time. Basically what is the objective here?

Just like the newspaper hawker, he is trying to distribute the newspaper. So, here the salesman is willing to visit all these cities within with short time ok, short duration. So, here the objective is minimization of time. So, one solution maybe this is one possible solution that you are from A to B. So, you are starting from A to B, then E, then D, then C basically.

So, A to B ok, then B to E ok, E to D, then D to C and C to finally, A. So, this is one solution and the second solution may be something like that. So, you are coming from A to B, B to C then C to D, D to E and finally, E to A. So, here what you want to find out? So, you want to find out the optimal route here ok. So, whether you are from A, you are coming to B or coming to C and from C, whether you are coming to B or coming to D, something like that.

So, you need to find out a network or you need to find out a route and which will give you or which will give you an optimal route basically and in this case, the you are minimizing the time. There may be some other constraint here because we can put different constraints like availability of the flight and other your transportation modes. So, that we can put it and based on that those constraint, so you need to find out an optimal route. So, that you can visit all the cities for your work and basically, you are trying to find here an optimal route ok.

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So, this may be an example problem. Suppose, a particular flight is trying to cover all these city ok suppose a flight. So, in that case, how a flight will basically start? So, maybe from Mumbai whether he will go to Bhopal first or he will go to Delhi first and like that. So, this is one route basically the flight is moving from Mumbai, Mumbai to Bhopal, Bhopal to Delhi, Delhi to Guwahati, Guwahati to Calcutta, Calcutta to Chennai, Chennai to Bangalore and finally, you are reaching again Mumbai.

So, this may be a flight route ok. Nowadays, you have seen that flights are actually giving connectivity to different your location. As you see that if you if somebody is trying to go from Mumbai to Delhi. So, maybe he can take this flight and, but when he is the flight is going from Mumbai to Guwahati.

So, maybe from Mumbai to initially it will Bhopal or Mumbai to Delhi, Delhi to Guwahati or something like that or you may have some optimal route ok. This is another example problem. We call it travelling salesman problem. So, we will try to solve some problem related to this aspect also, apart from our other civil engineering problems.

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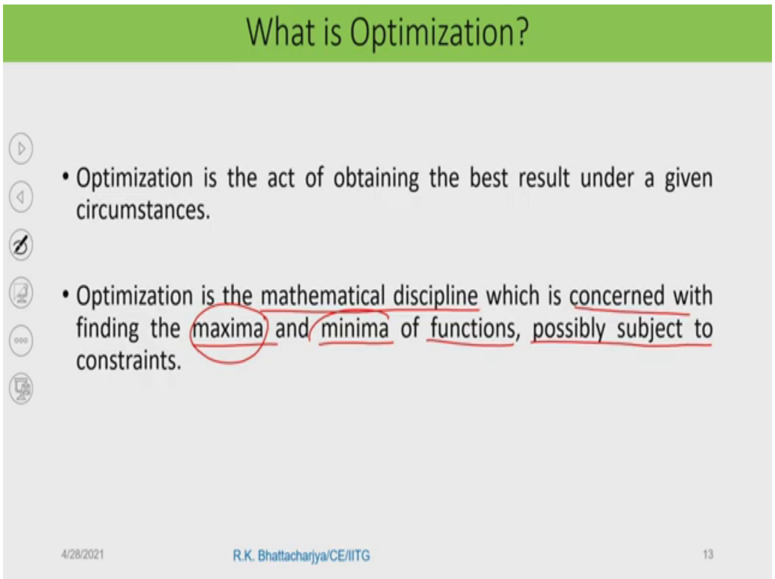

Now, what is optimization? So, with the example problem we have discussed, so let me define what is optimization. So, optimization is an act of obtaining the best result under a given circumstances. So, what is the optimization? So, optimization is an is the act of obtaining the best result under a given circumstances.

So, we have seen some example problem. So, under those circumstances, we would like to find out what is the best result and that is basically known as optimization ok. So, optimization is the mathematical discipline which is concerned with finding the maxima and minima of functions, possibly subject to constraint ok.

So, it is a mathematical discipline; it is a mathematical discipline which is concerned with finding the maxima and minima of a function, possibly subject to a constraint. I think in your school level, you have already solved these finding minima and finding maxima of a function. So, I think already you have you have done using calculus.

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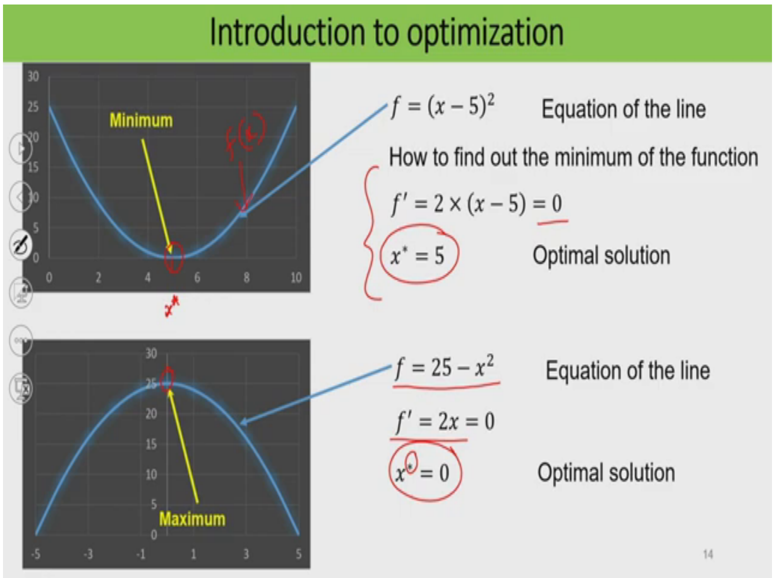

So, I will also explain that one. Suppose, this is a function this function and this is the minima of this particular function. So, if I say this is a function. This is a function and minima of this function is somewhere here and we generally, denote it by your x star ok. So, this is x star ok. So, now, this function is f equal to x minus 5 whole square.

So, this is the equation of this particular function and what we know basically? So, how to find out this minima or maxima? So, I will take the first derivative ok. So, I will take the first derivative. So, what is first derivative here? The twice x minus 5 and first derivative will be equal to 0 and if I solve it, so I will get minima of this particular function and that is x equal to 5.

So, this is the solution and this is the minimum point of this particular function and I would like to find out and that is basically known as finding minima or maxima of a function. Now, let us take another function. So, this is another function and here the maxima of this function is somewhere here ok. So, this is the maxima of this particular function and maxima is at 0.

So, in this case, the minima was at 5 and in this case maxima is at 0 and suppose, this is the function; the function is 25 minus x square. So, this is the function and I would like to find out the maximum point of this particular function. So, what I will do? I will take the first derivative; the first derivative is twice x equal to 0 and if I solve it, I will get the solution and that is x star.

So, as I said, so we put star to denote the optimal point; x star means that is an optimal point ok. So, either minima or maxima or maybe an inflection point, I will also discuss what is inflection point. So, in this case, the solution is x star equal to 0. So, I am getting this one and I think you have already solved this type of problem in your school level. So, this is called the finding minima or maxima of a function.

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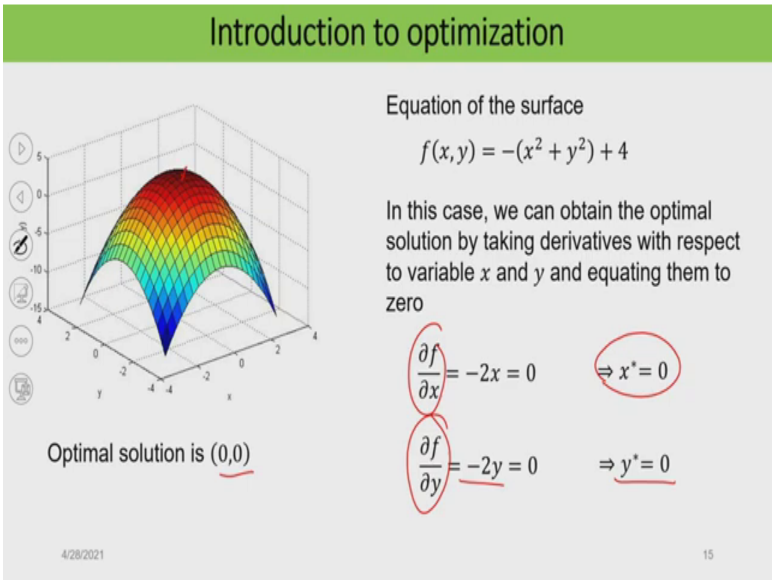

So, if I take a 2 variable problem. So, in this case, suppose this is the function. The function is minus x square plus y square plus 4. So, this is the function and I would like to find out what is the maxima of this function. So, maxima of this function is somewhere here.

So, what I will do? I will take derivative with respect to x and I will take derivative with respect to y, partial derivative with respect to x and partial derivative with respect to y and if I equate it to 0 and if I solve it, so I will get the optimal value or I can say the maximum your point of this particular function.

So, we are taking derivative. So, derivative is del f by del x equal to 0. So, if I do that. So, I will be getting minus twice x equal to 0 and x star equal to 0 and similarly, if I take derivative with respect to y, then twice y equal to 0 and then, y star equal to 0. So, therefore, the solution of this function is 0 0 ok or you can say optimal solution or you can say the maxima of this function is at 0 0 ok.

So, I think already you have solved this type of problem in your school level. So, what we are doing here? We are taking derivative and then, we are equating to 0; the derivative value is 0 at your optimal point ok.

Thank you very much.