Plates and Shells
Prof. Sudip Talukdar
Department of Civil Engineering
Indian Institute of Technology-Guwahati

Module-02
Lecture-06
Simply Supported Plate Subjected to Concentrated Load and Couple

Hello everyone, today I am starting lecture 2 of module 2. So, taking reference of the earlier classes, that is the last class that I discussed especially the rectangular plate simply supported along all edges, formulation was done. That is a general formulation for any type of load was given. And then for a particular 2 cases that we have seen, one is uniformly distributed load about the entire area of the plate and second one is a batch load that is uniform load, but it is distributed over a certain rectangular area which is less than the plate area.

So, in 2 cases, we have seen the formulations, we derive the equation of deflection surface, and from there, we could arrive at the expression for bending moment, shear force, edge shear, etcetera. Then we also discuss the corner reactions specially it happens for simply supported plates when the corners are not restrained, it has a tendency to lift up the corners, and these happen due to twisting moments along the two adjacent edges, which introduces as an upward reaction at the corners.

So, that case we have discussed and the expression for corner reaction is also derived based on the simple mechanics. And then, we discussed one numerical problem to find out this deflection. Now I want to proceed; the partially distributed load formula is known to you; from that, I want to proceed to find out the deflection surface due to concentrated load. So, let us see what we will cover in this lecture.

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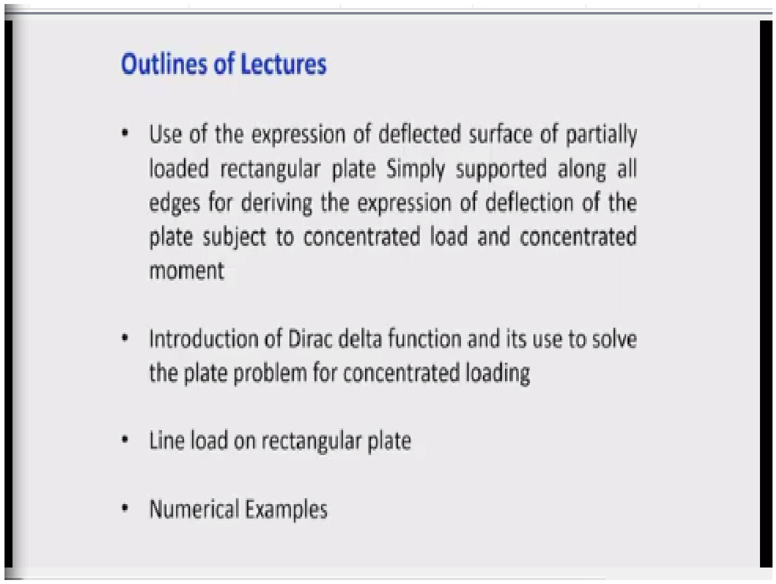

We shall use the expression of deflected surface of a partially loaded rectangular plate. Simply supported along all edges, that formulation which was ready to us will be used for deriving the expression of deflection of the plate, rectangular plate, simply supported along all edges subjected to concentrated load and concentrated couple. So, concentrated load is very common in bridge actually when a bridge deck which can be modelled as a plate vehicle wheels which are moving on the bridge.

And as a result of this vehicle movement, the wheel loads are transferred on the bridge deck as a concentrated load. So, concentrated load formulation we want to show today and code has given actually if you refer the reinforced concrete design code, then you will find that effective (()) (03:49) method is given for concentrated load. But here, we will derive the exact expression with the help of plate theory.

So, this is our first objective in the lecture. Secondly, we will discuss a special method by using the direct delta function. Dirac delta function is a special mathematical function or operator you can call, which simplifies the calculated effort significantly. So, that function how it can be used for discrete load this concentrated load? That will be discussed in today's class. Then line load, this is a strip type of loading acting on the plate.

The practical examples are seen in the case of a brick wall constructed over a slab. So, brick wall along the length or along the breadth of the slab will impose a load which can be considered as distributed in a line distributed like a strip loading. So, that loading imposed due to brick wall construction or any other manner on the plate can pose some difficulty.

Because of the distributed load, actually, it is not a distributed load but distributed along only one direction. So, we shall see how the line load or strip loading can be tackled or handled using our known expression or theory. Then we shall solve some numerical problems. So, let us see recapitulate the earlier expression that we have derived.

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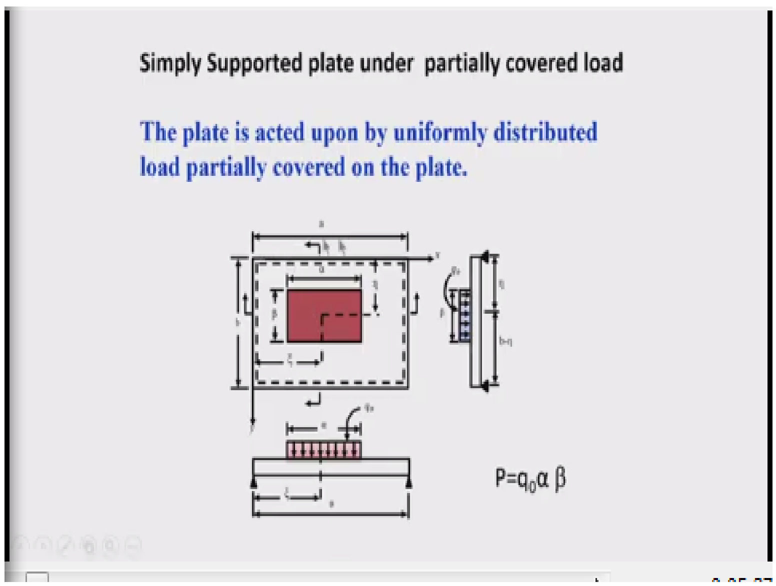

We have shown you earlier that a plate, a rectangular plate of length a and width b subjected to a batch loading of the rectangular area, uniform intensity and the load covers the area α cross β. So, α is the length of the load, and β is the width of the load. So, α is parallel to the longer direction of the plate, and β is parallel to the shorter direction of the plate. Now here other two parameters are important to locate the centre of gravity of the load.

The centroid of the load here is coordinate of the centre of the load is ξ, β with respect to the origin taken at the top left-hand corner. So, with respect to this point, we have the x-coordinate as ξ and y coordinate as η, so these are the parameters that we know. So, it is not necessary that the load will be symmetrical alloys; it may be eccentric also. Then you will find that $\xi \neq \frac{a}{2}$

If $\xi = \frac{a}{2}$, then the centre of the load is exactly at the centre, and you will get this symmetrical load distribution. If ξ is $\frac{a}{2}$ and η is $\frac{b}{2}$, but this may not happen always. So, the load may be eccentric over the bridge deck; specification is there, say wheel load must have a certain clearance from the edge of the footpath. So, if this minimum clearance is maintained, then you will find that load is not always symmetrical with respect to the x and y-axis of the plate or not symmetrical in the plate.

So, due to unsymmetrical distribution again, you will find the twisting moment is very becomes predominant. But with this formulation that we derived today, you can handle symmetrical as well as unsymmetrical distribution of the load without any difficulty. So, for that loading case, we found the deflected surface of the rectangular plate.

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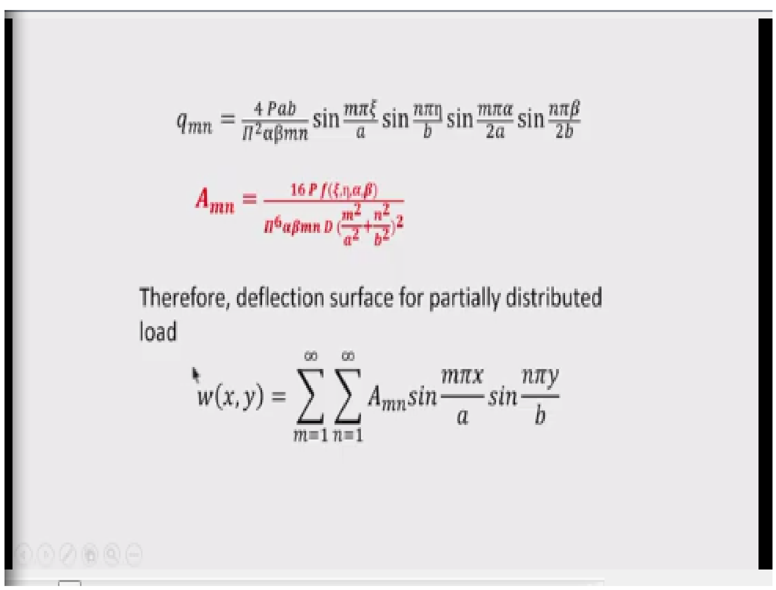

And the deflected surface, the general formulation requires the Navier series. In Navier series is given by double summation $A_{mn} sin\frac{m\pi x}{a} sin\frac{n\pi y}{b}$. So, this is actually the Navier series. So, in the

Navier series, A_{mn} is given by 16 P and some function of ξ, η, α, β, what are these parameters? Let us see ξ and η, these are the coordinates of the centre of the load and α and β is the length and width of the loaded area.

So, these are the parameters which give a function f, which is dependent on ξ, η, α, β, then P is the total load acting on the plate. So, if the density of the load or intensity of the load is uniform, say q_0 then P will be $q_0\alpha\beta$, so that is P total load. Then these other parameters m and n are the harmonic numbers, or actually, these are integers. So, the half-wave number you can tell if m = 1, you will get one half-wave along the x-direction.

If m = 2, you will get two half-waves along the x-direction; if m = 3, you will get three half-waves in the x-direction. Similarly, if n = 1, you will get one half-wave in the y-direction; if n = 2, you will get two half-waves in the y-direction and so on. And these are the parameters involved in A_{mn}, coefficient of deflected curve. Now q_{mn} actually is found from these A_{mn} is actually dependent on q_{mn} in original formulation if you see.

But q_{mn} for that case is found as $\frac{4\,Pab}{\Pi^2\alpha\beta mn}\sin sin\,\frac{m\pi\xi}{a}\ \sin sin\,\frac{n\pi\eta}{b}\ \sin sin\,\frac{m\pi\alpha}{2a}\ \sin sin\,\frac{n\pi\beta}{2b}$. You can easily observe that ξ and η are the coordinates of the centroid of the load, and α and β are the length and width of the load. So, knowing this quantity q_{mn}, we can find A_{mn}, and once A_{mn} is found, we can find the deflection. But question arises what is $f\,(\xi,\ \eta,\ \alpha,\ \beta)$?

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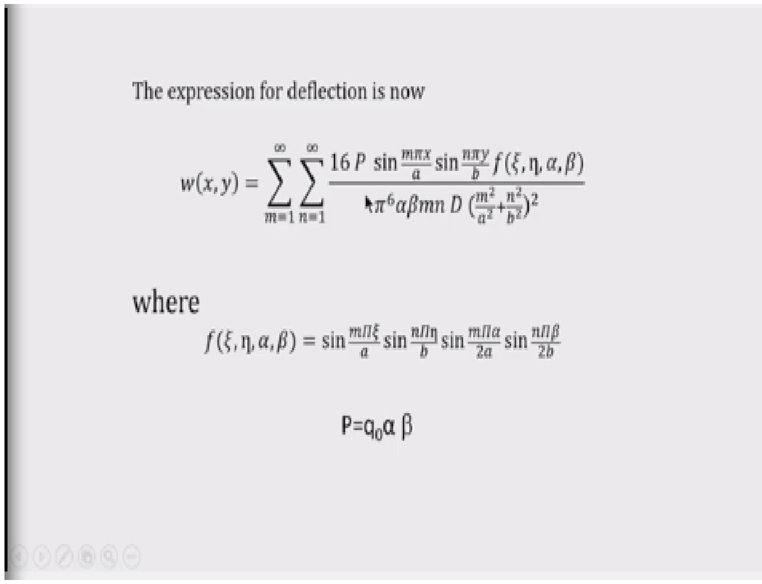

Now let us see the expression for deflection again I have written here, substituting mn in this place, and we get this is the expression for deflection w(x, y), where f function of ξ, η, α, β is nothing but sin $sin\,\frac{m\Pi\xi}{a}$ sin $sin\,\frac{n\Pi\eta}{b}$ sin $sin\,\frac{m\Pi\alpha}{2a}$ sin $sin\,\frac{n\Pi\beta}{2b}$. So, these are the functions of this f(ξ, η, α, β) which is found from this expression. Now, let us see how we can obtain the case of concentrated load? How we can formulate the expression of $w(x, y)$ for concentrated load starting from this expression?

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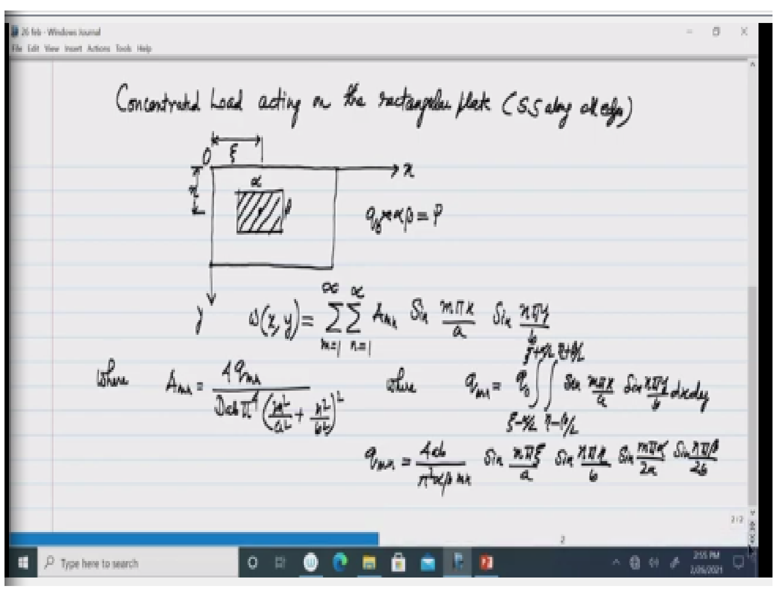

So, the expression that we have earlier derived, from that will start the influence of concentrated load acting on the rectangular plate simply supported, SS means simply supported in short along all edges. So, that is our boundary condition. So, if I see the plate, we have this as the x-axis and this is y-axis, and this can be taken as the origin. Now we have a patch loading of uniform intensity; the area of the patch loading is α × β.

And centre of gravity of this patch loading is here, the x-coordinate is ξ, and y coordinate is η. So, these are the parameter involves, and intensity of the loading is q_0. Such that q_0 α β will give you the total load P. Now, with the help of this expression, we have derived earlier the expression of deflected surface. The expression of deflected surface was that

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} sin\frac{m\pi x}{a} sin\frac{n\pi y}{b}$$

So, these are original expressions for deflected surface, where $A_{mn} = \frac{16\, P\, f(\xi,\eta,\alpha,\beta)}{\Pi^6 \alpha\beta mn\, D\, (\frac{m^2}{a^2}+\frac{n^2}{b^2})^2}$. Where the q mn, q mn is here actually, the patch loading is there, patch loading is q_0 which is of constant intensity. And then, if we integrate between the limit $\xi - \frac{\alpha}{2}$ and $\xi + \frac{\alpha}{2}$ in x-direction and $\eta - \frac{\beta}{2}$ lower limit and $\eta - \frac{\beta}{2}$ upper limit of this function $sin\frac{m\pi x}{a} sin\frac{n\pi y}{b}$.

We get an expression which will be now using. We will be using to find the deflected surface for concentrated load. So, this expression is given by this $\frac{4\, Pab}{\Pi^2 \alpha\beta mn} \sin sin\, \frac{m\pi\xi}{a}\ \sin sin\, \frac{n\pi\eta}{b}\ \sin sin\, \frac{m\pi\alpha}{2a}\ \sin sin\, \frac{n\pi\beta}{2b}$, so this is our q_{mn}. So, once the q_{mn} is known, then A_{mn} is known, and after knowing A_{mn} we can find completely the deflected surface.

Now to find the effect of concentrated load, we shall assume that the area α × β is very small. So, that is possible when the limit in the limit α and β goes to 0, then the total load $q_0 \alpha\beta$ is concentrated at a point. This point is defined as the centroid of the load. So, this indicates that we have to take the limit of this function only q_{mn} is of concern because if we know q_{mn}, q_{mn} carries the variable, which is actually the location of the load and the extent of the load.

So, if I take the limit of q_{mn} as α and β tend to 0, then we will be able to find the appropriate value of q_{mn} and hence A_{mn} that have to be substituted in the expression for w to get the deflected surface of the plate. So, now let us see how we can find the limit.

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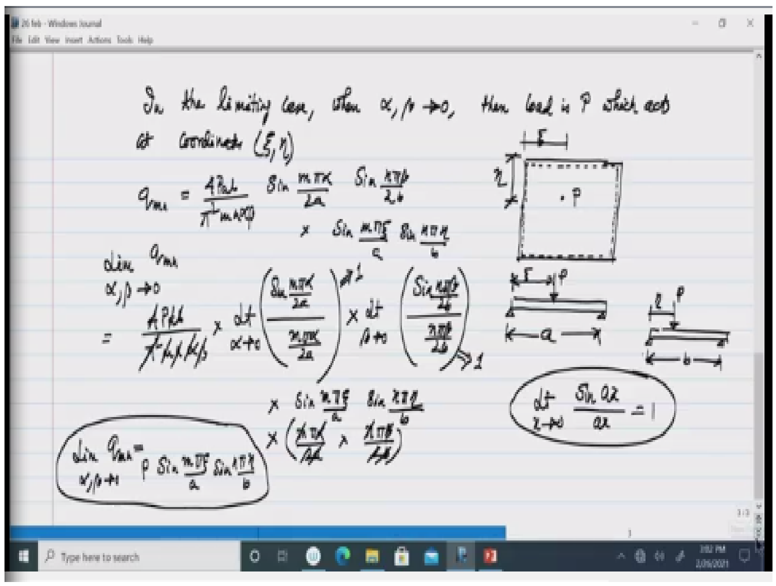

So, in the limiting case, when α and β tends to 0, then load is a point load, load is P which acts at coordinate. Previously we had the distributed load; now instead of distributed load we have the concentrated load acting here, for example. So, P is acting here, and this location of P is defined by the coordinates ξ and η. That means if you see a section and this is simply supported slab, of course**.**

If I see this section, then you will find, let me see, the section of the plate along the load; you will find the load is acting here P if this section is shown along the longer direction. If this section is shown in the shorter direction, then you will find this length is a; if this section is shown in the shorter direction, you will find, this is η, and the length of the plate in the shorter direction is b.

So, we have the expression for q_{mn}, $q_{mn} = \frac{4\, Pab}{\Pi^2 \alpha\beta mn} \sin sin\, \frac{m\pi\xi}{a}\ \sin sin\, \frac{n\pi\eta}{b}\ \sin sin\, \frac{m\pi\alpha}{2a}\ \sin sin\, \frac{n\pi\beta}{2b}$. Now take the limit of q_{mn} as α and β goes to 0. So, if we take the limit, we know that very important formulation in our differential calculus.

This sine say ax limit of say ax as x tends to 0 = 1, so that formulation is known to us. Based on that limit, now let us take the limit of this function you will get that. α and β goes to 0, that means if I take $\frac{4\, Pab}{\Pi^2 \alpha\beta mn}$ then limit α goes to 0, then I write $\sin sin\, \frac{m\pi\alpha}{2a}$, in the denominator again, I will write $\frac{m\pi\alpha}{2a}$. That means I have to multiply this function by $\frac{m\pi\alpha}{2a}$, so that I will show you later; let me write the limiting value.

So, again limit β tends to 0; we will get this $\sin sin\, \frac{n\pi\beta}{2b}$ and here $\frac{n\pi\beta}{2b}$. So, that limit we have written and then $\sin sin\, \frac{m\pi\xi}{a}\ \sin sin\, \frac{n\pi\eta}{b}$. So, we have actually divided this function by $\frac{m\pi\alpha}{2a}$, and here $\frac{n\pi\beta}{2b}$. So, to keep the equality of this expression, we have to multiply this function by $\frac{m\pi\alpha}{2a}$ and $\frac{n\pi\beta}{2b}$..

So, you can see now, from this a, b will get cancelled here, then 2, 2 is 4, that 4 will be also cancelled, α β will be cancelled here, m, n will also be cancelled, and π, π, π^2 will also be cancelled. And these value is actually 1, these value is also 1, so ultimately we get the limit of q_{mn} as α β goes to 0 is nothing but P, P is there and $\sin sin\, \frac{m\pi\xi}{a}\ \sin sin\, \frac{n\pi\eta}{b}$, so that is the limit of this function q_{mn}. Once the limit is found, then we can find the A_{mn}, so expression A_{mn} is now found out with the help of this limiting value.

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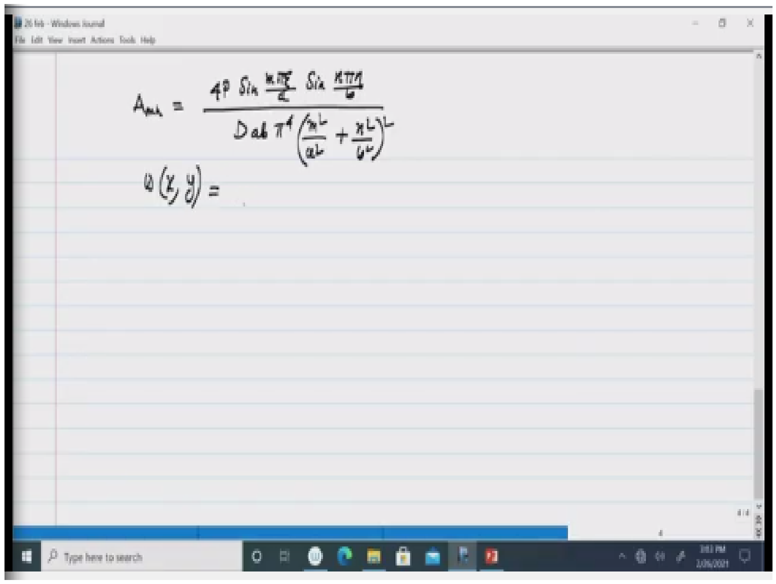

So, A_{mn} will now turns out to be $\frac{4P\sin sin\,\frac{m\pi\xi}{a}\sin sin\,\frac{n\pi\eta}{b}}{Dab\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}$, D is the flexural rigidity of the plate. So, this is the value of A_{mn}; once you know the A_{mn} then deflection surface is completely known.

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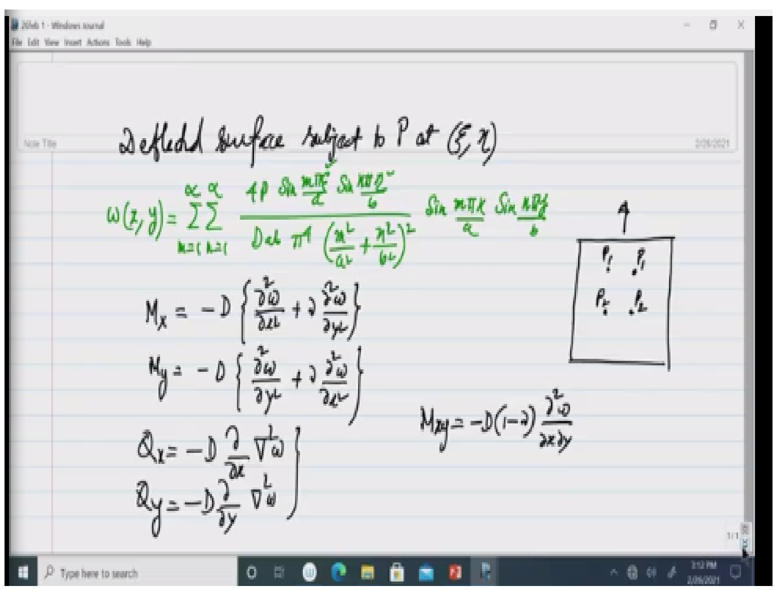

So, deflection surface is now, for a concentrated load situated at a distance of ξ from x-axis and η from the y-axis, we have the expression for concentrated load. So, let us see what are the expressions for concentrated load? So, deflection now becomes w x, y and limit we have found out earlier. So, 4P we can substitute and $\frac{4P\sin sin\frac{m\pi\xi}{a}\sin sin\frac{n\pi\eta}{b}}{Dab\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\sin sin\frac{m\pi x}{a}\ \sin sin\frac{n\pi y}{b}$

So, we have derived the expression for deflection surface of the plate rectangular plate simply supported along all edges subjected to a concentrated load. The location of the concentrated load is governed by or defined by 2 variables ξ and η. So, in a plate, actually, there may be a number of concentrated loads. For example, a bridge deck, a deck panel of a bridge subjected to vehicle loading; there are maybe two axles in general.

So, 2 axles on the both sides, right and left, you will get this wheel, so wheel loads are imposed. And you can see if this is, say P_1 in the same axial; of course, the load is same. Then P_2, P_2, so if this is the direction of motion, you will see that the load imposed on the plate is concentrated load and it is a moving load. So, concentrated load formulation is very important specially for bridge deck analysis when the wheel loads are treated as a concentrated load.

The expression for w is found out and based on which we can find out the expression for M_x, -D, an expression for M_y. So, this is the deflected surface of plate subjected to concentrated load, P at. From that expression, we can arrive that M_x, M_y and if required, other values can also be found out. Then Q_x, the shear force Q_x will be -D and Q_y, -D. So, all the stress resultants are found out after finding the deflection.

Deflection is the basic quantity from where we can differentiate, and we can find the quantities of interest, the bending moment even the twisting moment also you can find. Twisting moment = $-D(1-\nu)$ cross derivative that is the twist curvature will come into picture.

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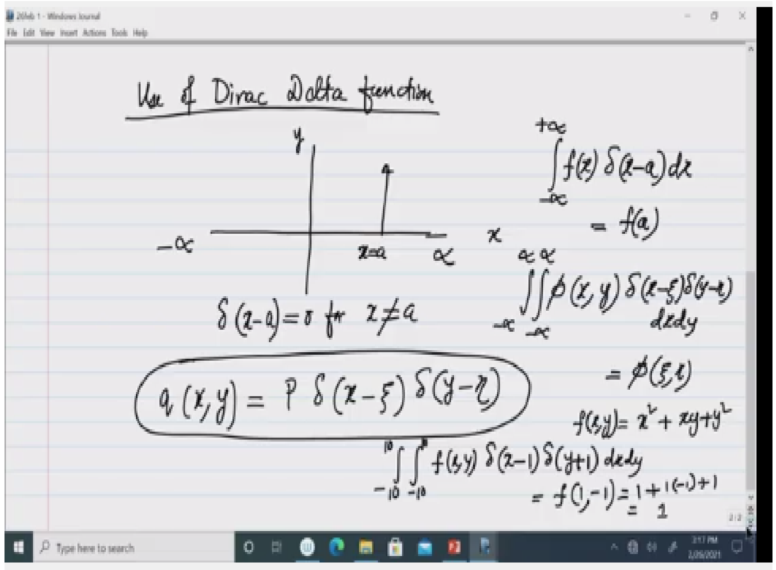

Now, let us introduce a very interesting method to tackle the problem of concentrated load that is by use of Dirac-delta function. Now in many situations, the forcing function, whether it is a plate problem or even any other problem, may not be continuous. For certain point it is significant or certain location it is significant; for other locations or other places, there is no importance of that load. So, that happens in many physical problems.

So, in that situation, suppose a real axis is taken which extends from minus infinity to plus infinity and on the axis at certain point sudden impulse is there. So, this impulse is defined only at this point, say x = a, if this is x-axis and this is y-axis. But at all other points except a there is no value of or there is no function does not exist. So, that means this type of function impulse can be defined by a Dirac delta function which has the significant here x - a is 0 for $x \neq a$.

But x = a, it still remains undefined, and the magnitude is very high. But restricting this limitation, we will now use this direct delta function to represent the concentrated load on the plate. So, equivalent distributed load in the plate subjected to the concentrated load we can write as, suppose the load is acting at the coordinate here x-coordinate is ξ, and y-coordinate is η. So, the q x is now expressed here like that.

The direct delta function has very important property, use of which the calculations are simplified. So, the properties of Dirac-delta function that is used to simplify the calculation is this. So, direct delta function defined at point x = a and multiplied with a function say f_x any function, and it is integrated in the real line between 2 limits minus infinity, minus infinity any limit, then the integral will be just the value of the function evaluated at this point x = a.

So, in case of concentrated load acting on the plate, if I write like that, say any function which is a function of x, y in two-dimensional problems, integration is carried out along the x-axis and along the y-axis in infinite domain. Then we get say a Dirac delta function is situated here, so double integration will result just the value of the function evaluated at this coordinate ξ and η.

So, if a function, say a function is defined f(x, y) is equal to say $x^2 + xy + y^2$, say a function is defined. Now, if I want to integrate this function with the direct delta function, say x - 1 and y + 1, for example. In limit say limit you can take any limit, say -10 to +10, finite limit or infinite limit and here say again -10 to +10. Just to explain this use of Dirac-delta function, what will be the integral?

Integral will be the value of the function evaluated at the point where the direct delta function is significant. So, here it will be evaluated as x is replaced by 1 and y is replaced by -1. So, if the function is given with this function, a function is defined as $x^2 + xy + y^2$. The value of the integral will be $1 + 1\times(-1) + 1$, so the answer is 1. So, now you have understood the use of Dirac-delta function.

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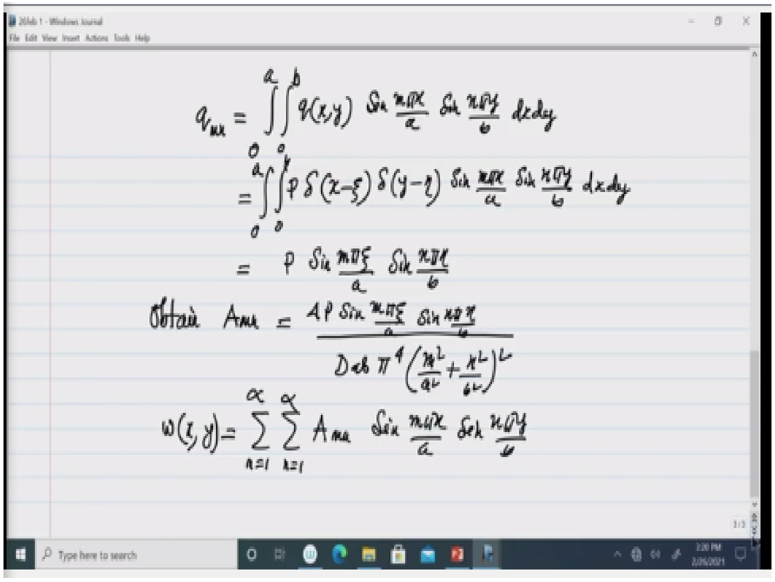

So, if I use this in the evaluation of q_{mn} in the plate problem, so q_{mn} is defined as, $q_{mn} = \int_0^a \int_0^b q(x, y) \sin sin\, \frac{m\pi x}{a}\ \sin sin\, \frac{n\pi y}{b}\, dxdy$. Now if a concentrated load x at a coordinate ξ and η, then we will write P double integration 0 to a, 0 to b. Then it is acting at coordinate x = ξ and y = η, then $\sin sin\, \frac{m\pi x}{a}\ \sin sin\, \frac{n\pi y}{b}\, dxdy$.

So, the q_{mn} is now for the plate problem is very simple now, it will become P into the function is given by $\sin sin\, \frac{m\pi x}{a}\ \sin sin\, \frac{n\pi y}{b}$, and it is evaluated at x = ξ and y = η. So, integral become $\sin sin\, \frac{m\pi\xi}{a} \sin sin\, \frac{n\pi\eta}{b}$. So, this is same what we obtained earlier in the limiting process. Now once the q_{mn} is obtained, obtain A_{mn}, A_{mn} is given by this as $\frac{4P\sin sin\, \frac{m\pi\xi}{a} \sin sin\, \frac{n\pi\eta}{b}}{Dab\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}$

After getting A_{mn}, we obtain the deflection function as $w(x,\ y)$, $w(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} sin\frac{m\pi x}{a} sin\frac{n\pi y}{b}$, and you put this index for summation. So, this is the procedure for handling the concentrated load in the plate. Now let us see what happens in case of line load. So, if a line load is acting in the plate instead of your concentrated load, then also it is possible to obtain this q_{mn}.

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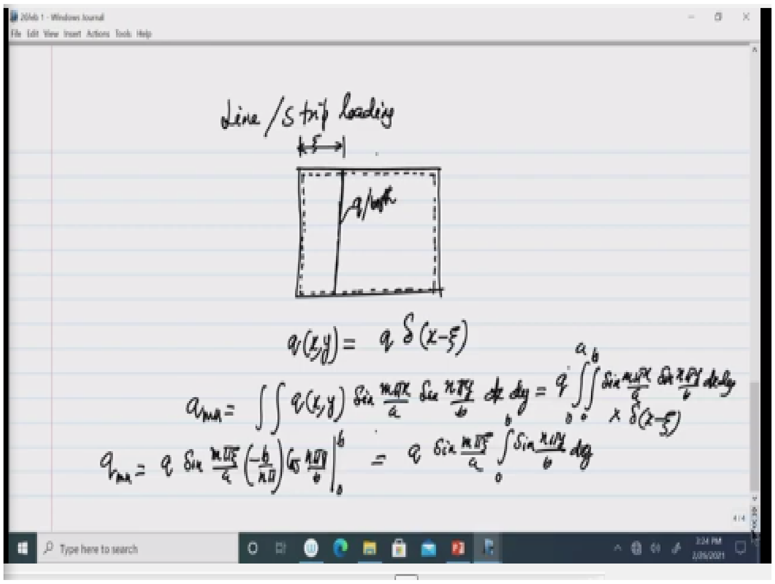

For example, we have the plate line or strip loading. Suppose a rectangular plate which is simply supported along all edges. And for example, we have this loading a brick wall is constructed, or any other load which imposes a strip loading, whose intensity is say q per unit length and this line load is imposed at x-coordinate is ξ, and it is running full along the y-coordinate.

So, in such cases, the load $q(x,\ y)$ is defined as q, then Dirac delta function, $\delta(x - \xi)$, where the q represents the intensity along the line. So, this is the load representing a strip loading on the plate. So, with the help of this representation, now we can obtain q_{mn} as this $sin\frac{m\pi x}{a}sin\frac{n\pi y}{b}\ dx\ dy$. Now replace this $q(x,\ y)$ by q, q is a constant, so we can take it out.

Then we have this value $sin\frac{m\pi x}{a}sin\frac{n\pi y}{b}\ dx\ dy$ multiplied by Dirac delta function, $\delta(x - \xi)$. Now integration with respect to x will result a value say $q\times sin\frac{m\pi\xi}{a}$, then the integration with respect to y remains. Because of the property of Dirac delta function, we have found this integral with respect to x, very easily by replacing x by ξ. This integral can be carried out, and finally, q_{mn} can be obtained as $q\times sin\frac{m\pi\xi}{a}$, and this is -b by n pi cos n pi y by b $-\frac{b}{n\pi}\cos cos\ \frac{n\pi y}{b}$, then you put the limit 0 to b.

So, after simplifying these, you can get completely q_{mn} and then you proceed to find A_{mn} and then deflected surface A_{mn}. Now let us discuss what happens if a concentrated couple is acting instead of a concentrated load.

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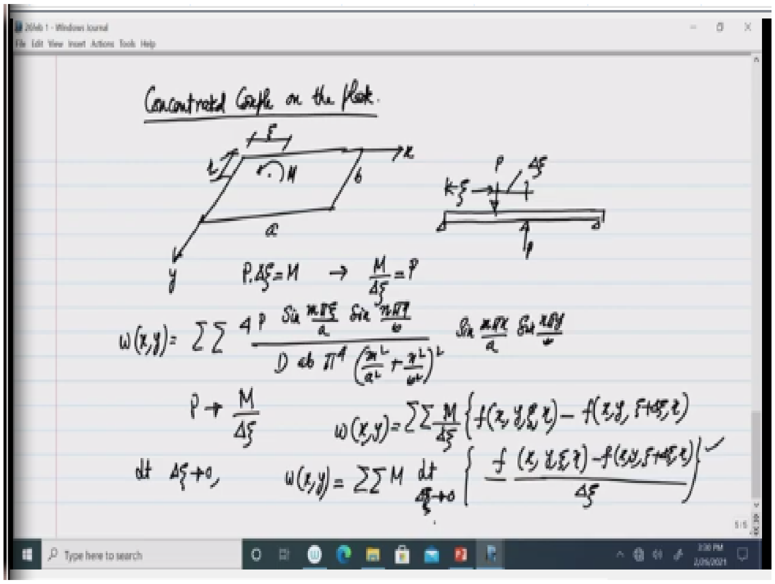

So, let us now discuss the case of concentrated couple. Say rectangular plate is shown here, this x-axis and this is y-axis a concentrated couple see M is acting here. The location of the couple is also defined by the coordinate ξ and η. So, at this point, the couple is acting, and the length of the plate is a and the width of the plate is b. Now to handle such problem, what do we do actually? The couple is acting along the x-direction, so what do we do?

Actually, we replace, suppose I am doing a section of the plate where a couple is acting, we replace this couple by two equal, and opposite forces say P acting at a distance Δξ, such that $P\Delta\xi = M$. Now what is done? The external moment is replaced by 2, equivalent opposite forces acting d ξ apart, as shown here. Now, if we decrease $d\xi$ to a very small value, say if we decrease $d\xi$ to a very small value while maintaining the original value of the M, $P\Delta\xi$.

We must proportionately increase P; this procedure is identical with replacing P by $\frac{M}{d\xi}$. So, P can be replaced by $\frac{M}{d\xi}$. Now here at this point, you are getting a downward load P, and at this point, you are getting an upward load P. So, at the point ξ or at the location ξ downward load is P at the location ξ + Δξ the load is upward, and its value is P. Now we know for the concentrated load P, the expression for deflection which is given by this earlier expression that was $\frac{4P \sin sin \frac{m\pi\xi}{a} \sin sin \frac{n\pi\eta}{b}}{Dab\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}$. Then a function of x and y which is nothing but $\sin sin \frac{m\pi x}{a} \sin sin \frac{n\pi y}{b}$.

So, in one case, in downward loading case will replace P by $\frac{M}{d\xi}$ and the location of this ξ will replace it as a ξ and η remain as it is. But when we come to the neighboring load at a distance of dξ, dξ apart from ξ or location define by ξ then, we replace ξ by ξ + delta ξ and η remains as it is. Because it is acting along the x-direction, so it is defined as this.

Now, if I see this is the function of say whole function if I see, so P is replaced by $\frac{M}{\Delta\xi}$ and then we can write this deflected surface as you're the $\frac{M}{\Delta\xi} f(x, y, \xi, \eta)$ for downward load and for upward load a function $f(x, y, \xi + \Delta\xi, \eta)$. And the summation process is there; of course, the summation process you cannot ignore this. Then in the limit, when dξ turns to 0, we get the effect of couple.

So, in that case, it is written as M, and if I see the limit of this as Δξ turns to 0, so what is this? From the definition of derivative, we can write this is the derivative of the $f(x, y, \xi, \eta)$.

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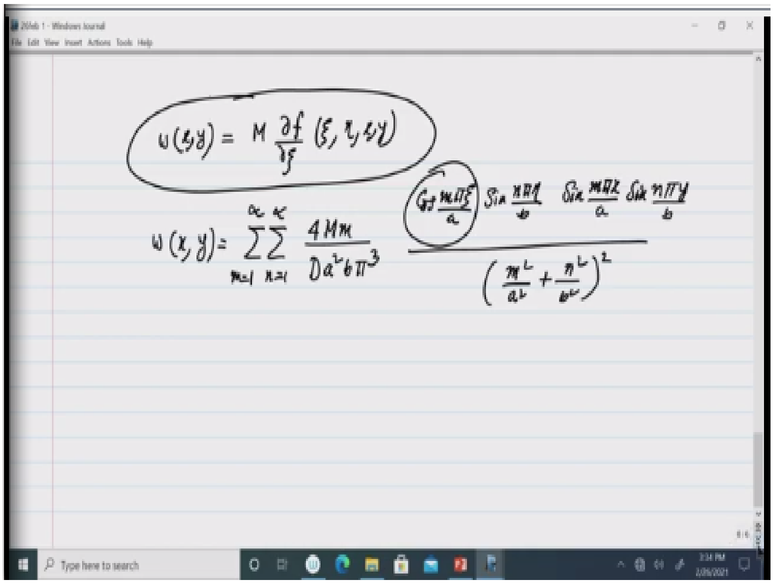

So, that means for concentrated load and concentrated moment, the expressions that are derived for concentrated moment is due to limiting process of the expression derived earlier for concentrated load. So, now w(x, y) can be written as $M\frac{df}{d\xi}$. So, for concentrated load, this is the expression for the deflection, where M is the couple that is acting at a certain location ξ, and η and x y are the functions of the location where we want the deflection.

And total function, so $f(x,\ y,\ \xi,\ \eta)$ has to be differentiated with respect to ξ with respect to location. That means first you differentiate with respect to x and then put the value of ξ. So, that is the process of finding this problem of or solving the problem of concentrated load. That means the next function form we can write w(x, y) is 4 M substituting the earlier expression we will get.

And because of differentiation of your sin $sin\,\frac{m\pi\xi}{a}$, you will find the cos function will be appearing, so cos $cos\,\frac{m\pi\xi}{a}$. Then you are differentiating with respect to ξ because the moment is acting along the x-axis. So, sin $sin\,\frac{n\pi\eta}{b}$ will remain unaffected and other function with variable x and y will appear as it is. So, due to differentiation of this function, earlier it was sin $sin\,\frac{n\pi\xi}{a}$, so it is changed to cos.

And the coefficient that is coming out of the differentiation is divided or multiplied with the coefficient existing with this expression. Then other important parameter that is wave number square that was also be existing in this. So, this is the full expression for the deflected surface of the plate subjected to a concentrated moment at location ξ and η. Now let us give an example of this.

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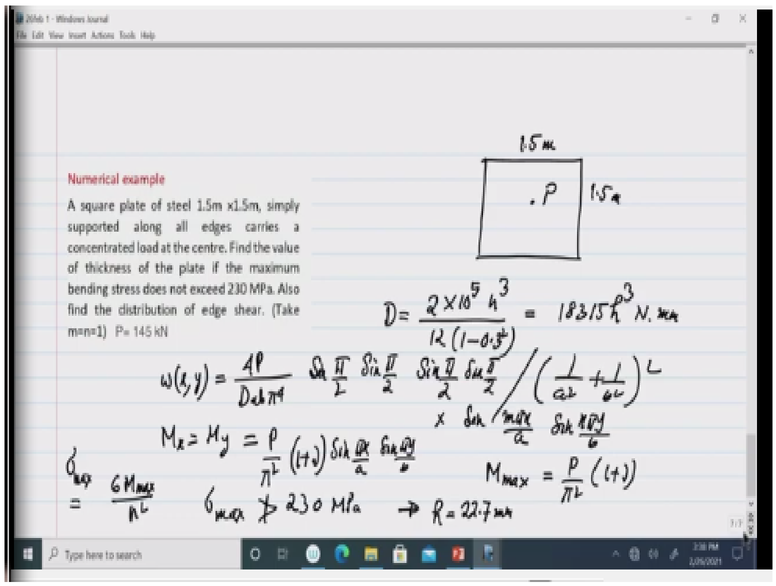

So, here a square plate of size 1.5 meter by 1.5 meter, simply supported along all edges and carries a concentrated load at the centre. The magnitude of the load is given, and the maximum bending stairs not to be exceeded 230 MPa, this condition was given. So, we have to find the shear force distribution, the edge shear distribution as well as this plate thickness.

So, first, you calculate D, the flexural rigidity of the plate, substituting the value of the quantity if it is a steel plate, then E is 2×10^5 and h remains in millimeter h^3. Then $12(1-\nu^2)$, ν is 0.3; we have taken 4 steel, this value is $18315\times h^3$, the unit is Newton millimeter, so D is found. Now, w(x, y), because it is a concentrated load problem, so it takes only one term, m = 1 and n = 1.

So, there is no need for summation, so we write it because it is acting at the centre, so $\frac{\xi\pi}{2a}$ there is $\frac{\pi}{2}$, so sin $sin\frac{\pi}{2}$ is 1. Again along the y-direction, you are getting also sin $sin\frac{\pi}{2}$. So, because of the centre location you are getting all sine function as $\frac{\sin sin \frac{\pi}{2}}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}$ because we have taken only one terms. Now since it is placed at the centre, so $M_x = M_y$ and after substituting these of course here the sine function will be there, location $\sin sin \frac{m\pi x}{a} \sin sin \frac{n\pi y}{b}$.

So, $M_x = M_y$, and you will get these values as $\frac{P}{\pi^2}(1 + \nu) \sin sin \frac{\pi x}{a} \sin sin \frac{\pi y}{b}$. Again, at the centre, $x \, and \, y = \frac{a}{2}$, so you are getting the maximum bending moment is $\frac{P}{\pi^2}(1 + \nu)$. So, hence stress that is produced due to bending moment is maximum at the centre, and it is given by 6 M max divided by a square. Now given condition is that delta max should not exceed 230 MPa. So, based on that, you can find out the thickness h; the thickness h here comes out to be 22.7 millimeter. For this problem and given that thickness comes out to be this.

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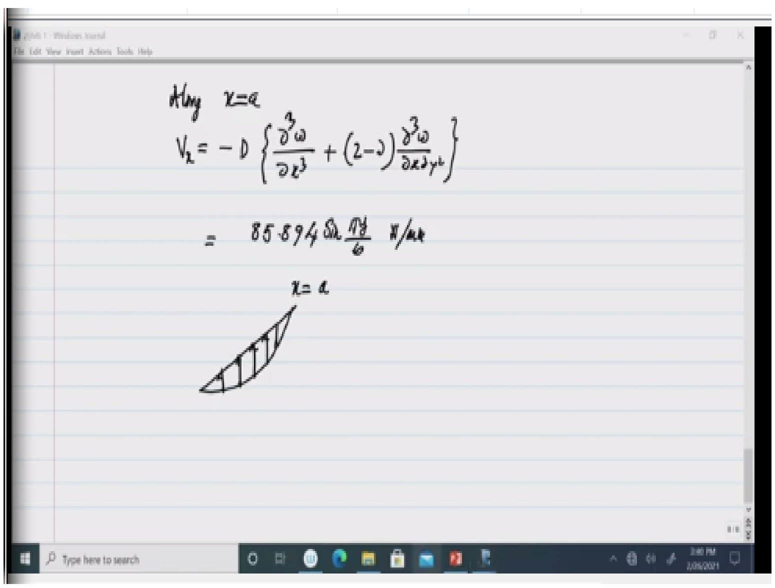

Now, if we want to calculate this shear force distribution, that is the edge shear. The expression for edge shear along x = a because it is symmetrical it will be same equal to -D. After substituting the value of derivative, you will finally get the result as edge shear value as $85.894 \sin sin \frac{\pi y}{b}$; this unit will be Newton per mm. So, if I see the distribution along 1 as this edge, I have taken as x = a.

So, distribution is sinusoidal; you will get the edge shear distribution like that. This is the distribution of edge shear along x = a. But since it is symmetrical, you will get similar distribution along the other edges. So, the problem is solved because we know the deflected function of the plate due to concentrated load. Using these deflected functions and taking derivative up to third derivatives, we can then obtain the edge shear along any edges you can find out. So, thank you very much.