Plates and Shells

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Lecture - 34
Simplified Bending Theory of Cylindrical Shell – Beam and Arch Theories

Hello everybody, today I am starting module 12. This is the last module of the course, and here we will try to discuss about the bending theory of the cylindrical shells. So, we have learned about the membrane analysis of surface of revolution then we have studied this membrane analysis of cylindrical shell. The membrane analysis has several advantages and it can give you quick results and reliable results, except for the boundaries.

Now, it is seen that near the edges or if the shell is long. Then, sometimes the bending action may become a governing criteria. So, in that case the bending analysis is to be carried out and different theories are involved for analysing the shell using the bending theory. Now, here we will discuss some of the theories including bending moment in the shell, but with certain simplifying assumptions.

So, today our topic of discussion will be simplified bending theory of cylindrical shell beam, shell beam and arch actions considering beam and arch theories. So, in that case the cylindrical shell is analysed assuming or idealizing the shell as a beam and incorporating the action of beam and arches.

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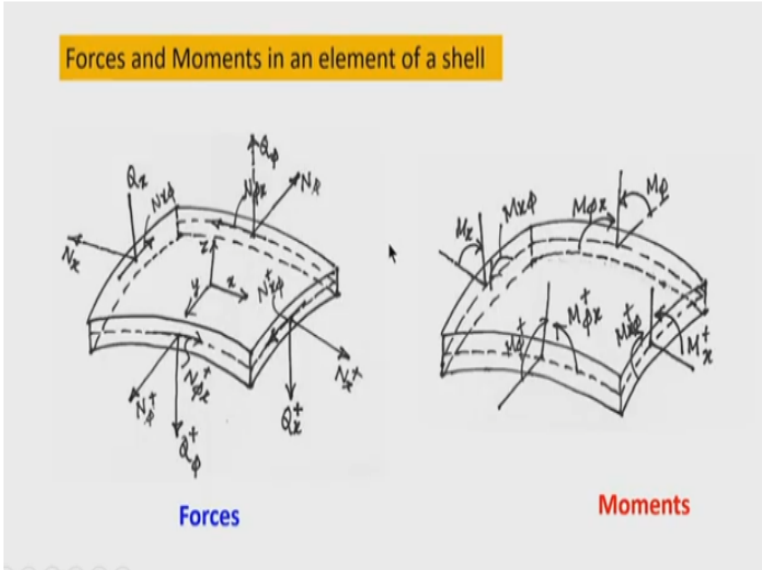

Now, let us see in general, what are the forces and moments that are possible in the shell element? Here you are seeing an element of the shell. And if you look at this picture, which is giving the forces acting on the element, you can see that this edge whose length is say $R \times d\phi$, this is the direction of x, so, this length we have taken dx. So, in this edge the longitudinal force N_x, the membrane force per unit length, it is acting here.

The vertical shear Q_x is acting here and here plus sign is given that, it is accompanied by increment caused in this shell from the opposite edges. So, in the opposite edges you are seeing that the membrane forces are N_x, that is the direct force you can tell along the x direction in plane forces, and Q_x is the vertical shear in this edge per unit length and $N_{x\phi}$ is the membrane shear force. On the opposite edges, these forces are accompanied by some increment.

So, the increment will be say $N_x + \frac{\partial N_x}{\partial x} dx$. So, I have written as plus, so N_x+ indicate this N_x + some increment. This N_x quantity here is increased by some increment. Similarly, in the other edges you can see that, the membrane forces are denoted here and the vertical shear Q_x and Q_ϕ are also given in the edges. Membrane shear force $N_{\phi x}$ and $N_{x\phi}$ are written here.

And these are shown here as different, but actually if the distance from the middle surface, that is z is very, very less compared to the radius of curvature of the shell. Then the $N_{x\phi} = N_{\phi x}$, so similarly $M_{x\phi}$ will be $M_{\phi x}$. Now, come to the moment which will completely neglected in case of membrane analysis. So, if we come for moment analysis, then we can see along the x axis, the moment is M_x on one edge, on the opposite edge it will be M_x + some increment.

So, this increment of course will be $\frac{\partial M_x}{\partial x} dx$. Similarly, on the transverse direction, if I see the moment, then it will be M_ϕ and on the opposite edges it will be M_ϕ +, and twisting moment $M_{\phi x}$ and $M_{x\phi}$ are also shown. So, these moments that you are seeing here, we completely neglected in the membrane analysis and the analysis of the shell was simplified in the absence of these quantities.

But it is seen also that near the edges suppose a cylindrical shell, which is supported at the ends on the traverse and at the edges the shell is treated as free or if it is joined with the edge beam, then the shell behaves in a different manner. So, if the shell is free at the edges or whether it is supported at the edges by the beam or any edge member, then also we have seen that the membrane shear force or other stresses resultants is not 0 at the edges.

But at the free edge that should be 0. So, this is the drawback of the membrane theory that the stress conditions are not fully or completely realized near the edges, where it is joints with the beam or at the cut outs or in the vicinity of concentrated loop. So, therefore, there is a need of incorporating the bending theory in the shell analysis.

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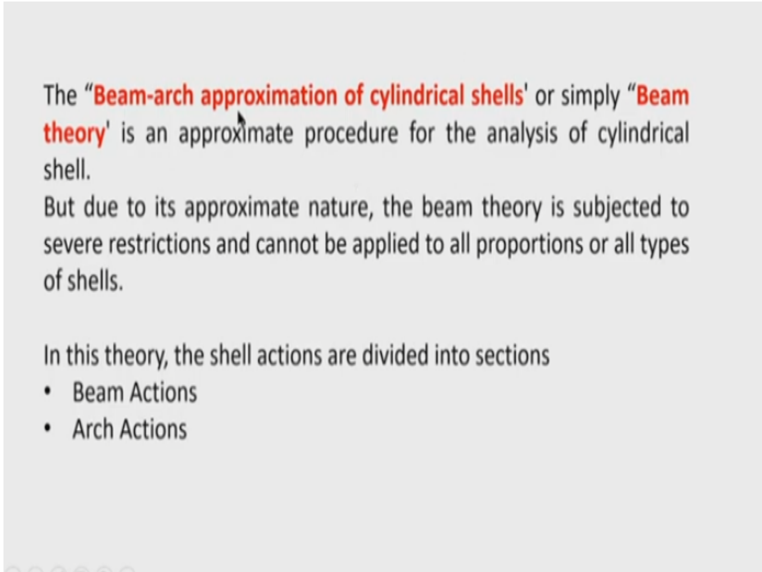

The beam theory or simplified beam theory for cylindrical shells actually takes into account of the beam and arch action, that we already used in our strength of material courses or we are familiar with this beam an arch action in an analysis of structure. So, these beam and arch actions are brought into picture, in case of simplified beam theory of cylindrical shell. Now, here two actions have to be taken, one is beam actions and other arch actions.

And due to this approximate nature of the theory, the analysis become very much attractive that, it gives a quick solution and sometimes it becomes solution, which is on the conservative side and can be used safely in the design practice. But these are also subjected to severe restrictions, because in many cases for all proportions of shell that means length by radius ratio, this theory may not be satisfied.

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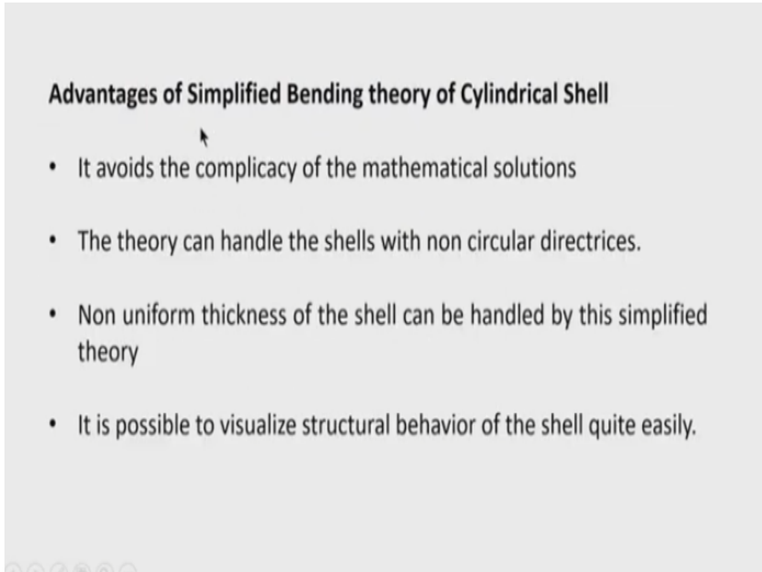

So, let us see what are the advantages of simplified bending theory of cylindrical shell? The bending theory of cylindrical shell actually avoids the complicacy of mathematical solutions. So, if I consider the rigorous analysis for bending moment and twisting moment in the shell, the order of differential equation increases to eight. So, this simplified theory which takes into account of the beam and arch action of the strength the material approach, that avoids the complicacy of the solution of the differential equations. And therefore, it can be used very easily for preliminary design or even sometimes conservative designs can be obtained with the help of this simplified theory. The theory can handle these shells with non-circular directrices. So, that is also another advantages, because in rigorous bending theory that is developed by various authors, there the shell of particular directrices only has to be considered otherwise, the solution can be very much complicated and this becomes unattractive. That means, nobody will prefer to use this type of solution. So, here the simplified theory can incorporate the non-circular directrices and non-uniform thickness that is another advantage of this theory. The thickness can be varied along the ϕ direction, that is meridional direction.

So, this variation of thickness can be easily incorporated in this theory, but if you consider a differential equation for the solution of bending moment and displacements, then the non-uniform thickness that will produce the non-uniform flexural rigidity of the shell. That will yield a differential equation, governing differential equation with variable coefficients. So, naturally the solution will be too much complicated for that kind of situation.

It is possible to visualize structural behaviour of the shell quite easily. Instead of going after this solution of complicated differential equation, this theory, bending theory of the shell which takes into account of beam and arch action can help one to visualize the structural behaviour of the shell very easily. So, these are some of the advantages of the beam theory of this cylindrical shell.

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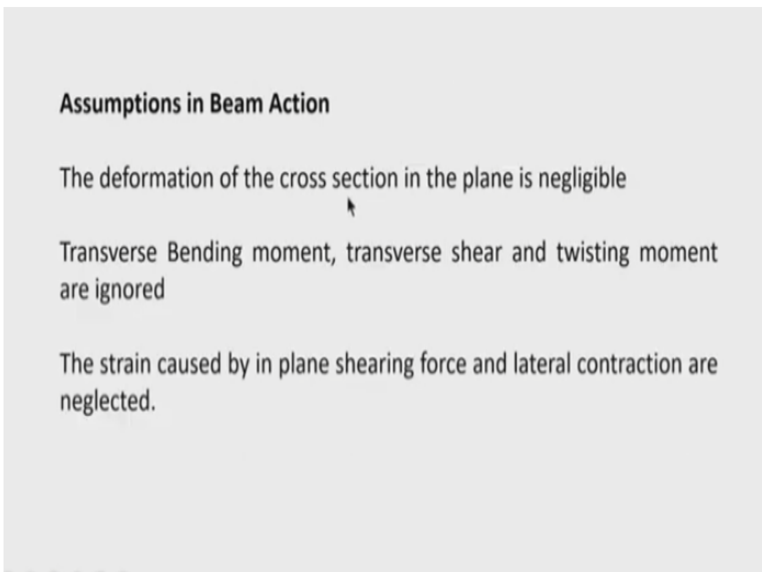

Now, let us see what are the assumptions that are made in the beam theory? The deformation of the cross section of the plane in the plane is negligible. So, cross section of the shell, here it is a curved cross section and deformation in the cross section of the plane of the shell is neglected. So, that is the first assumption. Then transverse bending moment, transverse shear and twisting moment are ignored in beam theory.

So, these are the assumption. Then, strain caused by in plane shearing forces and lateral contractions are neglected. So, three assumptions are mainly taken into account to solve the cylindrical shell problem using the beam action.

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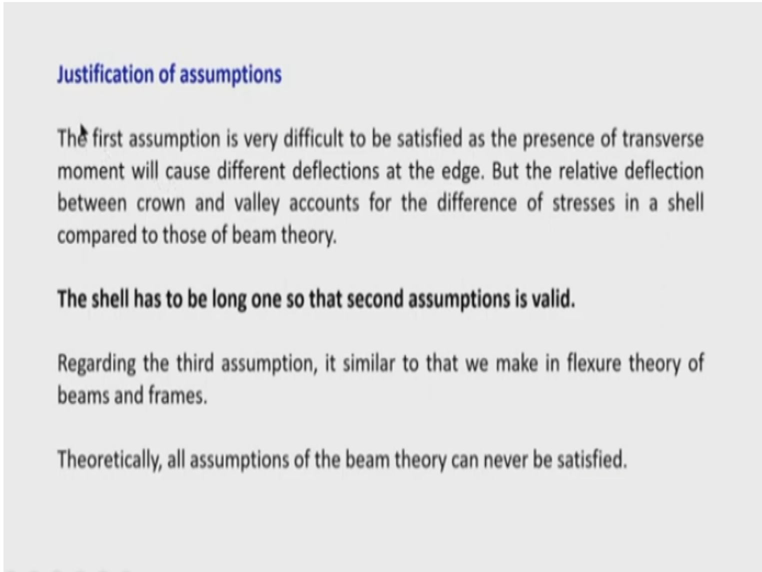

Now, let us see what are the justifications of the assumption? The first assumption tells that deformation in the plane of the shell is neglected. The first assumption is very difficult to be satisfied in the real practice; this theory fails to predict the deformation in the cross section of the shell. However, the difference of this displacement from edges to the crown actually causes the change of stresses.

So, change of stresses can be accounted due to this relative deformation of the shell between crown to valley or edge. So, this is the drawback of this first assumption. The second assumption that we neglected the transverse bending moment, transverse shear and twisting moments, that are true or is idealized in practice, if the shell is long one, so, that these quantities that is transverse bending moment, then transverse shear and twisting moment are neglected.

So, this is very important assumption for beam theory of the cylindrical shell, the long shell. Long shell can all only be analysed using the beam theory. Regarding the third assumption that is third assumption says that, shear strain caused by this membrane shear is neglected and the axial deformations are neglected. So, that is also made in the flexure theory of beams and frames.

So, theoretically we can tell that the assumptions can never be satisfied. So, these assumptions are made only to obtain a quick result within a reasonable accuracy.

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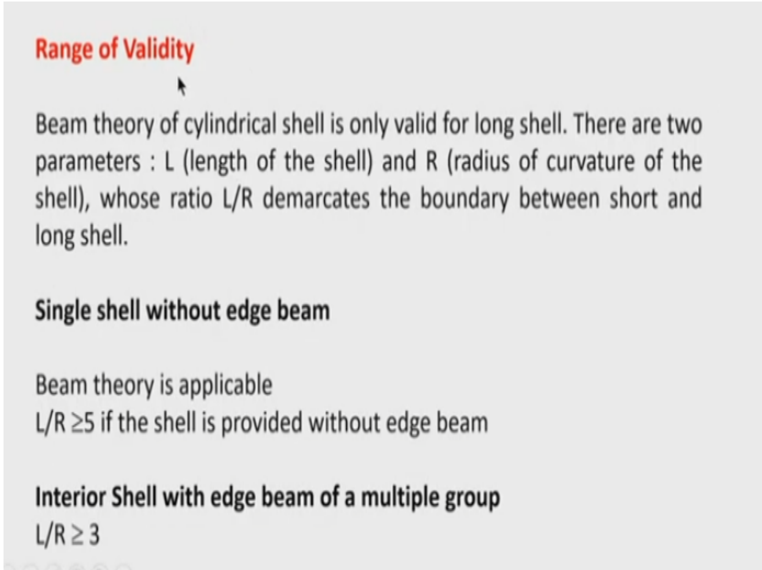

Now rang of validity, as the theory says that, the beam theory or beam analysis if applicable for long shell, now this parameter L/R that is L is the length of the shell and R is the radius of the curvature of the shell. Now, radius of the curvature may be variable, so here the radius we take the radius of the crown. So, the ratio L/R demarcates the boundary between short and long shell.

Now, how we can say that shell is long or shell is short? If we take a single cylindrical shell without edge beam, then beam theory is applicable if $L/R \geq 5$. The shell may be with different condition that, edges may be joined with the edge beam or edge member, shell is an integral part of the edge member or it may be free at the edges. So, if the shell is free at the edges or shell maybe of single span, single barrel or multiple barrel type.

So, in this single barrel type, the theory is applicable if $L/R \geq 5$. If the shell is having no edge beam, but if we take a multiple barrel shell then the interior shell that can be analysed where the edge beam is also present in a multiple group, then $L/R \geq 3$. So, these two ratios are very important for application of beam theory.

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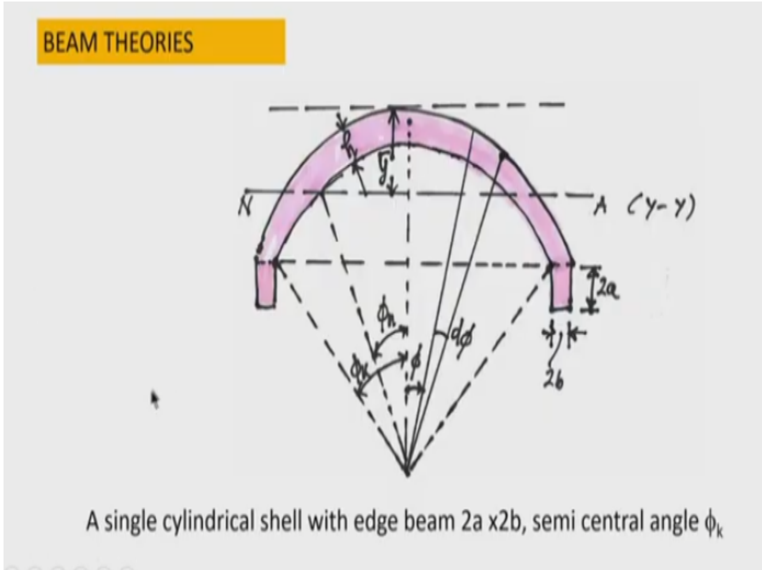

Now, let us see what the theory says? Now, theory is actually bringing the beam and arch action in the shell. Now, shell is here considered as a beam long beam of obviously $L/R \geq 5$ for beam without edge beam, for shell without edge beam $L/R \geq 3$, if the shell is provided with the edge beam. Now, here we are seeing a cross section of the shell and generally cross section of this shell is circular cross section.

This shell is provided with an edge beam. Now, edge beam dimension here is the depth is $2a$ and the width is $2b$. Now, if we see the cross section, this is the crown, this is the location of the crown and this is the edge beam, the top of the edge beam, which you can call as a springing. So, here the depth of the neutral axis of the curve cross section is denoted by $\overline{y}$ from the crown. So, $\overline{y}$ denotes the depth of the neutral axis from the crown.

This is the neutral axis where the stresses are 0. Now, in the cross section, you can also see that the neutral axis is located by an angle measured from the crown which is denoted by ϕ_n. Now, if I take a small element of the shell here, then area of this element will be $Rd\phi$ and this element this angle subtended by this elemental arc is $d\phi$. So, the length of this element is $Rd\phi$ and the thickness of the shell is h.

So, naturally the cross section will be $Rd\phi$. Now, this element is located at an angle ϕ. So, these are the symbol and semi central angle is ϕ_k. So, total angle that subtended by the shell cross section, that is you can see it looks like an arch, that is $2\phi_k$.

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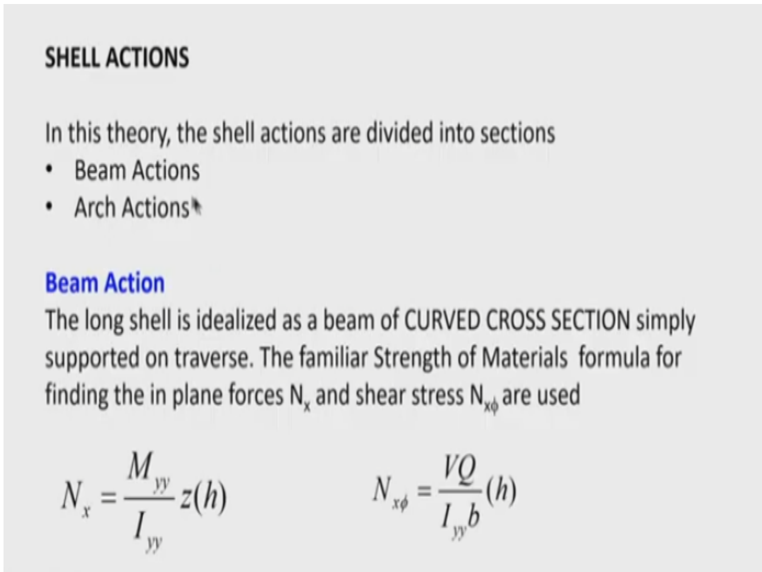

So, first we will analyse the beam actions, then we will go to find out the stress resultant with the help of arch action. Now, in the beam action the shell is idealized as a beam of curved cross section. If the shell is having a circular directrices, then the cross section of the shell is the part of a circular arc with certain width, and if other types of directrices are there say cycloid type of directrices or parabolic type of directrices or it may be also catenary type of directrices, then the cross section will be different.

The familiar strength of materials formula for finding the in-plane forces N_x and shear stress $N_{x\varphi}$ are used. So, what is N_x? N_x is $\frac{M_{yy}}{I_{yy}} z(h)$. Now, let us explain what are the meanings of this symbol? The meanings of these symbols are understood from this figure. This is y-y axis. So, bending moment about y-y axis is your M_{yy}. Then I_{yy} is the moment of inertia of the curved cross section about the neutral axis that is y-y axis. So, that is I_{yy}.

z is the vertical distance of any point on the arch from the neutral axis. So, z is maximum you can see at the crown at the extreme fibre, where the maximum compressive stress is developed under the vertical load. Then, other symbol appearing in this equation is h; h is the thickness of the shell. So, we have explained all these symbols here, then the membrane shear force the $N_{x\varphi}$ will be determined by familiar expression in the strength of material, that is $\frac{VQ}{I_{yy}b}(h)$. V is the vertical shear force at this section and Q is the first moment of the area at any level about the neutral axis. So, this is the meaning of the symbol Q. Then, h is the thickness of the shell and this b the width of the shell.

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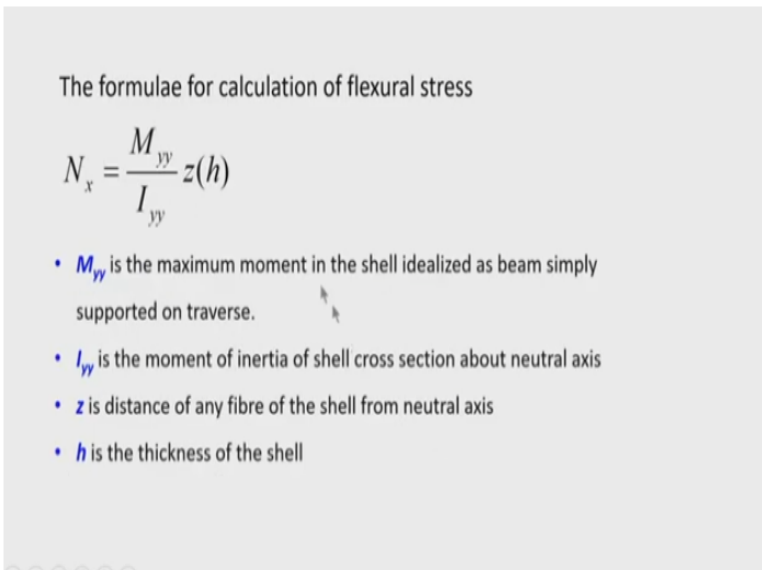

Now, the formula for calculation of flexural stress is N_x; N_x is the direct force that is the membrane force we call it. But when we find out the flexural stresses, then we divided by h so, it becomes $\frac{M_{yy}}{I_{yy}} z(h)$. So, flexural stress says σ_x will be $\frac{M_{yy}}{I_{yy}} z$. And multiplication with this stress by h will yield the membrane in-plane force that is N_x. The meaning of the symbols is told you and also it is written here.

The maximum moment we generally take and if the shell is simply supported at the traverse, then it behaves like a simply supported beam and maximum bending moment will occur at the centre of the span. And if say the vertical load at per meter length on the shell is said w and L is the length of the shell. Then maximum bending moment in the shell at the centre of this span will be wL^2/h. Maximum shear force will be at the traverse and this value will be $wL/2$, so V is the maximum shear force. So, meanings of the symbols are now explained.

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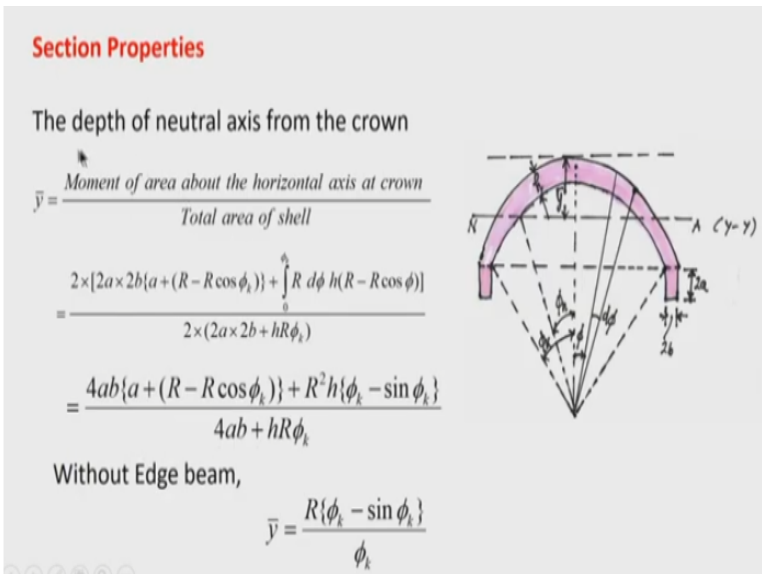

So, the important thing in the analysis of the shell is the determination of these section properties. Now, seeing the cross section of the beam, that is beam I call here because the shell is idealized as a beam of curved cross section. So, cross section of the beam is here curved. So, therefore neutral axis and the moment of inertia to be found from the first principle, because no readymade formula is available here just like a rectangular section, we take it 1/12 bd^3.

So, this type of formula is not available here. So, we have to work it from the first principle. So, let us see, that $\bar{y}$ is the distance of the neutral axis from the crown. So, $\bar{y}$ will be given by moment of the area about the horizontal axis at the crown divided by total area of the shell. So, whatever moment of area you are seeing, the moment of this area, we have to take about the crown. So, crown is the datum from which we are measuring the depth of the neutral axis.

So, let us take the moment of the section of the shell. Now, here we isolated one is curved cross section and another is this edge beam. So, two edge beams are there and this seems the arch is symmetrical, we can only take in the formula one half and then we multiply it by 2. So, because of symmetry these advantages we can take. So, now it is calculated like that, so 2 is a common multiplication factor because it is symmetrical. Then we come to the edge beam, you can see this $2a$ is the depth of edge beam and $2b$ is the width of the edge beam. So, $2a \times 2b$ and the edge beam cg is at the centre, so it is at distance a from the top of the edge beam and its distance from the crown will be $a+(R-R\cos\phi_k)$, where ϕ_k is the semi central angle, that can be easily verified from this triangle. So, if this is R, this is distance $R\cos\phi_k$ and this distance is again R.

So, $R-R\cos\phi_k$ will give this distance. So, therefore you can find this $a+(R-R\cos\phi_k)$ is the distance of the centroid of the edge beam from the crown. Then we come to the curved cross section, take an element of the curved cross section, that I told you earlier also the area of the element is $R\,d\phi\,h$. So, $R\,d\phi\,h$ is the area of the element and its distance the centre of the area is at a distance of $(R-R\cos\phi)$ from the crown.

So, from the crown the distance of the centroid of this element will be $(R-R\cos\phi)$. So, here we are writing the first moment of the area of this element of the curved cross section about the crown. And since we are taking only one element here, then for the full arch to give this effect, we have to integrate it from 0 to ϕ_k. So, the expression is integrated with the limit from 0 to ϕ_k and since it is symmetrical, we have already multiplied the expression by 2.

Now, the result of this integration is shown in the next step and this quantity is divided by the total area of the cross section. So, total area of the cross section is $2a \times 2b$ is the area of one edge beam plus the area of the curved cross section. So, area of the curved cross section; h is the uniform thickness of the shell. In that case we have taken a uniform thickness, then $hR\phi_k$, where ϕ_k is the semi central angle.

Of course, we have multiplied it by 2 to get the total area of the shell. Now, this expression can be simplified now, the area of this edge beam, we have now got it 4*ab* and then, you can see this is the distance of this centroid of the edge beam from the crown and after integration this is the result $R^2h\{\phi_k - \sin\phi_k\}$ divided by total area of the shell. Now, if edge beams are neglected that means, suppose there is no edge beam on the shell; shell is free at the edges.

Then, we can put $a = 0$ $b = 0$ and the expression is simplified as $\frac{R\{\phi_k - \sin\phi_k\}}{\phi_k}$.

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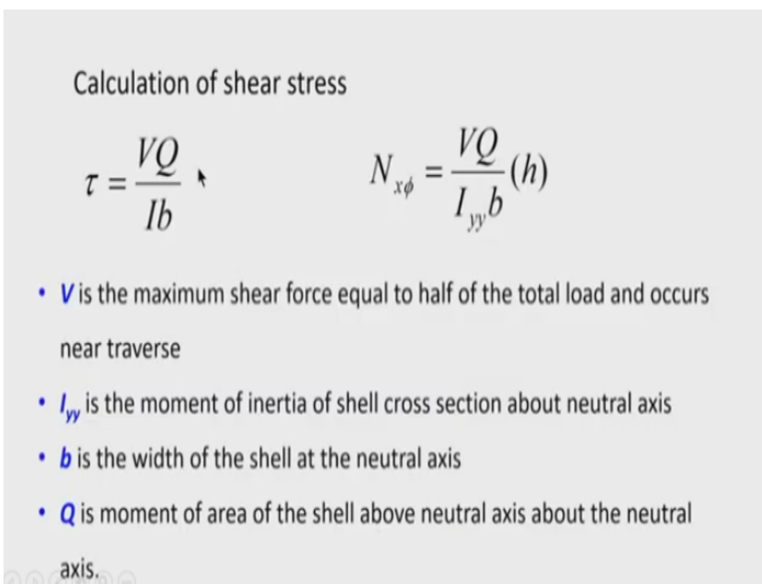

Now, we come to the calculation of shear stress. Shear stress is calculated as $\tau = \frac{VQ}{Ib}$, where the $N_{x\phi}$ is nothing but τh. So, this quantity is multiplied by h to give the membrane shear force

$N_{x\phi}$. The meanings of this symbol has been already discussed in our earlier slide, only new symbols that Q is appearing here is the moment of area of the shell about the neutral axis, so that we have to take into account.

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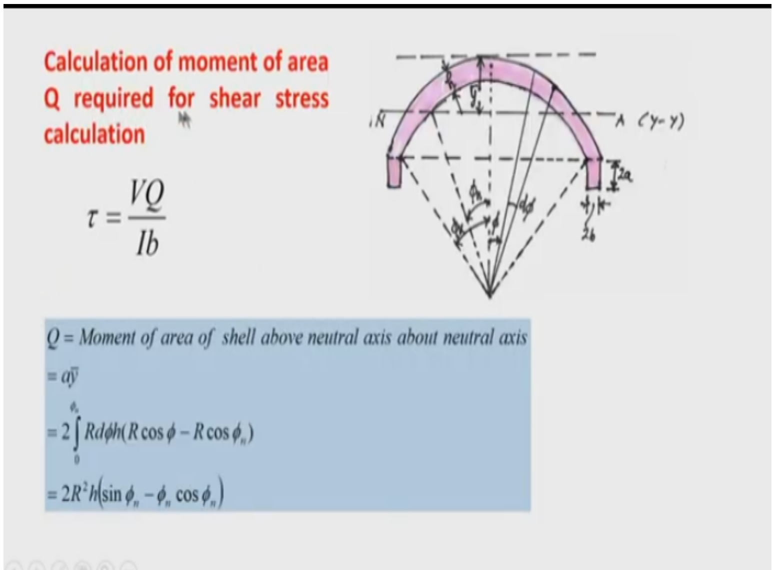

Now, calculation of the moment of area Q required for shear stress calculation. So, $\tau = \frac{VQ}{Ib}$. Moment of area of the shell about neutral axis if we considered here, so $Rd\phi h$ is the area of the element and its distance from the neutral axis is $R\cos\phi - R\cos\phi_n$, where ϕ_n is the location of the neutral axis from the crown; angular location, so integrating this we get that $Q = 2R^2h(\sin\phi_n - \phi_n\cos\phi_n)$.

So, this is if the moment of area of the shell taken about the neutral axis. So, you remember it very carefully that moment of area is taken about this axis and that is the neutral axis. But, for any other axis also it can be taken just by changing of variable, in that case you have to consider this integration limit as general of ϕ. So, at any angle ϕ you can change the limit of the integral and then write down the expression.

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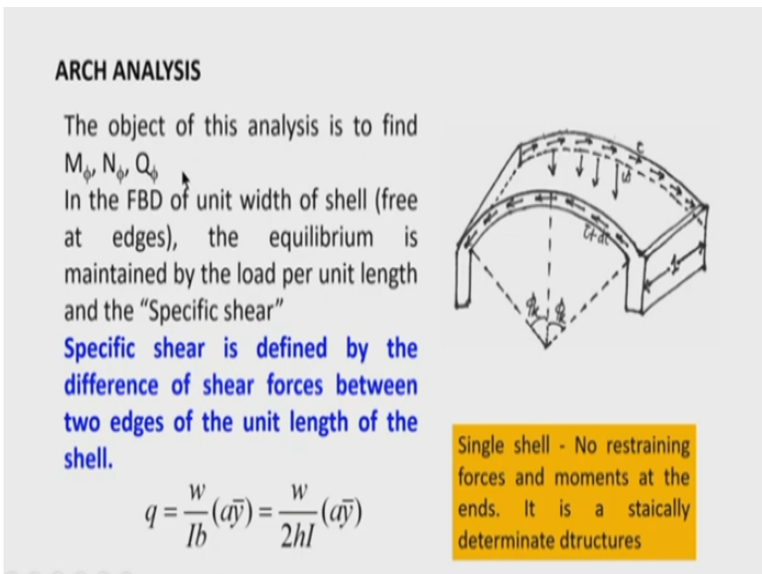

Now, after doing the beam analysis the results are obtained as N_x and $N_{x\phi}$. Next, we go for arch analysis, the object of the arch analysis is to find M_φ, N_φ and Q_φ. But, in a shell which has no edge beam, so shell is free at the edge beam. So, therefore shell is treated as a determinate system. So, in that case no restraining moment will be produced at the edge beam. However, the crown bending moment will be produced and this is due to a quantity, which is known as specific shear.

So, in that case you can see the equilibrium of this shell element, if I take an element of the shell of unit width, then equilibrium is maintained by the vertical load and specific shear that is developed at the edges. Now, what is specific shear? Specific shear is defined as the difference of shear forces between two edges of unit length of the shell. So, we have taken a shell and edges are one unit apart.

So, this specific share is defined as $q = \frac{w}{Ib}(a\bar{y})$, so this is the moment of the area of the shell above the plane considered about the neutral axis. But you can see that at the neutral axis that $b = 2h$, so we substitute $b = 2h$, so the expression can be simplified here. Single shell no restraining forces and moments at the ends, so it is a statically determinate structure.

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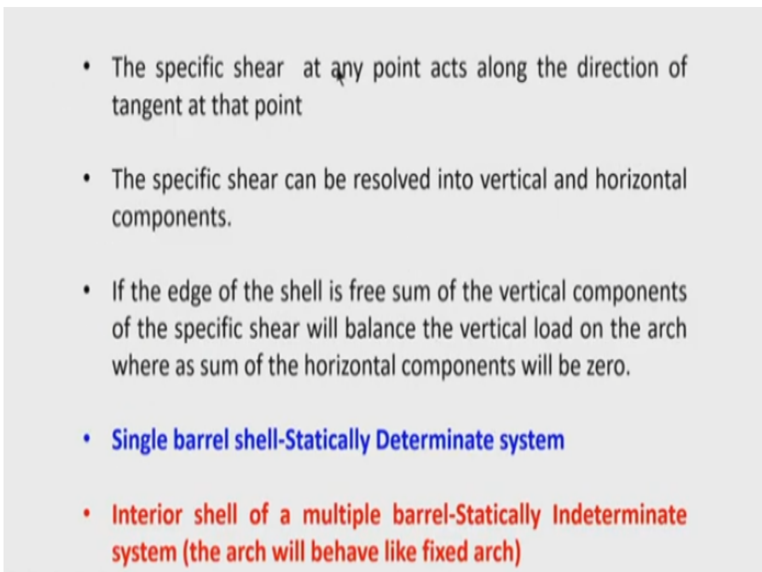

The specific shear at any points acts along the direction of the tangent at that point. So, it is clear here this is the specific shear that is q, we have denoted by q and w is the vertical load. So, the specific shear at any point x along the direction of the tangent at that point and specific shear can be resolved into vertical and horizontal components, that is obvious, and if the edge of the shell is free, some of the vertical components of the specific shear will balance the vertical load on the arch. Whereas sum of the horizontal components will be 0. Single barrel shell statically determinate system, interior shell of a multiple barrel statically indeterminate system, the arch will behave like a fixed arch, and fixed arch analysis by using Castigliano's theorem or the column analogy method or elastics centre method can be used and single shell barrel shell, it can be analysed as a statically determinate system.

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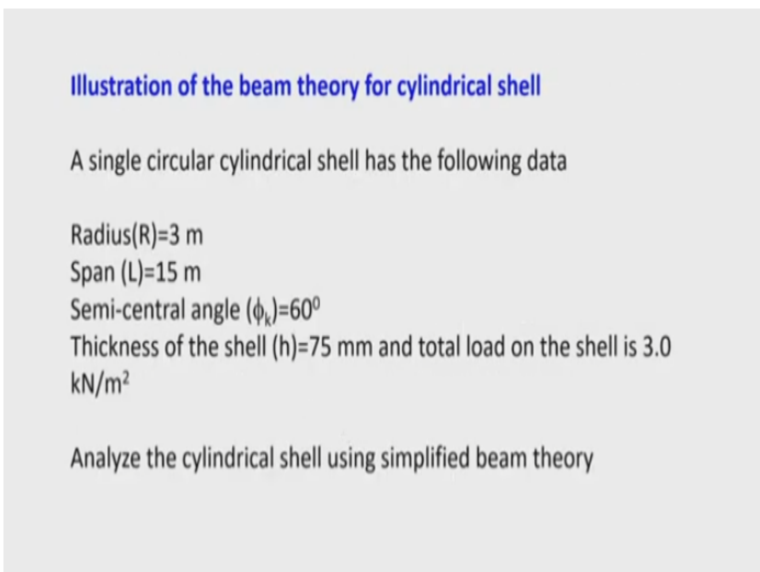

Let us illustrate the theory with the help of an example. A single circular cylindrical shell is taken here to illustrate the process or approach. Now, following data are assumed, radius of the shell is 3, span L along the longitudinal direction is 15 meter. Now, here you can see that L/R ratio is 5, so we can use the beam simplified beam theory of the shell. So, that ratio you have to see, whether the theory will be valid, now semi central angle for the shell is 60°.

So, total angle subtended by the arc is 120°, thickness of the shell we have taken 75 mm and total load on the shell which includes say self-weight and other types of loads, that is finishing or the live load etcetera is 3 kN/m^2. Let us analyse this shell with the help of simplified beam theory.

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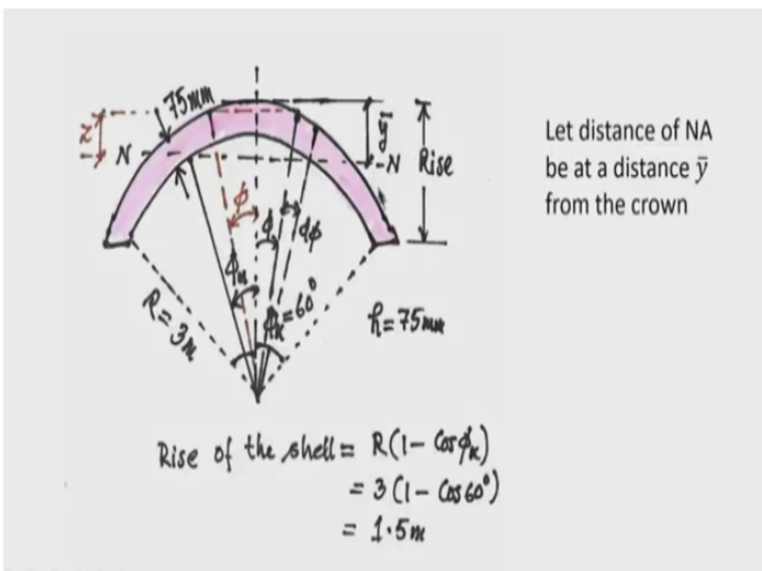

Say this is the shell cross section thickness is uniform everywhere it is 75 mm and first we have to determine this quantity say $\bar{y}$, then ϕ_n that is angular location of the neutral axis. Then other quantities say moment of inertia about the neutral axis that is also required. So, let us calculate one by one. So, first see rise of the arch, so rise of the arch is the vertical distance from the springing to the crown.

So, that is determined here $R(1-\cos\phi_k)$. So, ϕ_k is 60°, so therefore the rise of the arch is here 1.5 meters

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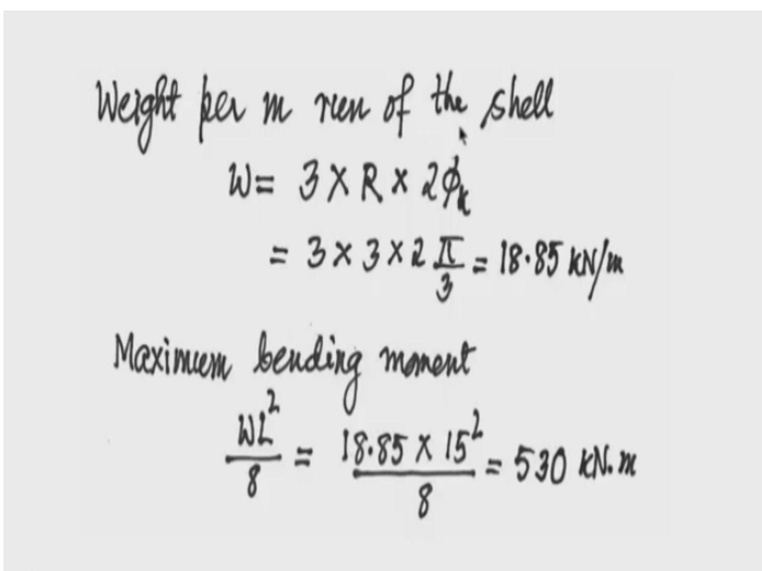

Weight per meter run of the shell; because the load that is given is kN/m^2. So, per meter run let us determine, the cross section of the shell is R is the radius of the shell, that is constant R and 2 ϕ_k is the total angle subtended by the R. So, therefore the cross sectional area is $R \times 2\phi_k$. So, substituting these ϕ_k as $\pi/3$ in radian, we get this $w = 18.85$ kN/m.

So, maximum bending moment will be $wL^2/8$, $18.85 \times 15^2/8$, and if we calculate it we get 530 kNm.

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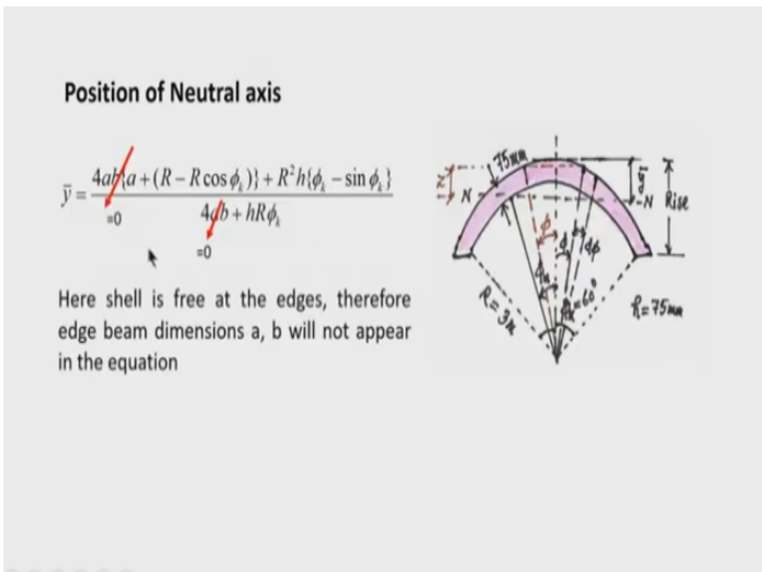

Next is the determination of neutral axis. We have already derived the formula for finding out the neutral axis of the shell cross section with edge beam. Now, here edge beam is absent, so therefore this quantity that 4ab was there in the formula earlier we have derived, we can put here as 0. So, similarly in the numerator this in the absence of edge beam this quantity is 0. So, we are left with only this quantity $\dfrac{R^2h\{\phi_k - \sin\phi_k\}}{hR\phi_k}$.

That also you can see R will get cancelled here, so the numerator only R will be there and h will be also cancelled.

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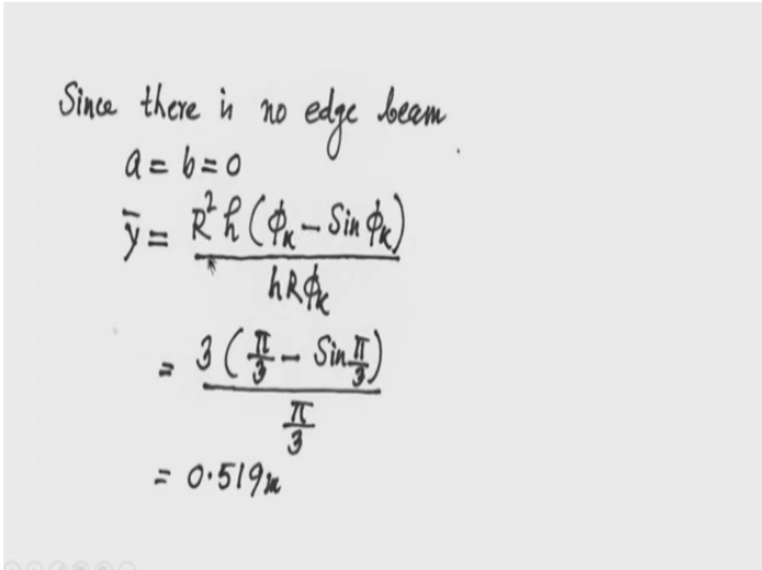

So, the expression will be here his $R^2 h\{\phi_k - \sin\phi_k\}$ and h you can cancel it and R^2 can be divided by R, so in the numerator only R will be there. So, after substituting the numerical value of the quantity we get now, $\bar{y}$ = 0.519 meter. So, this is a very important parameter for the analysis that is the distance of the neutral axis from the crown is now found as 0.519 meters that is 519 millimetres.

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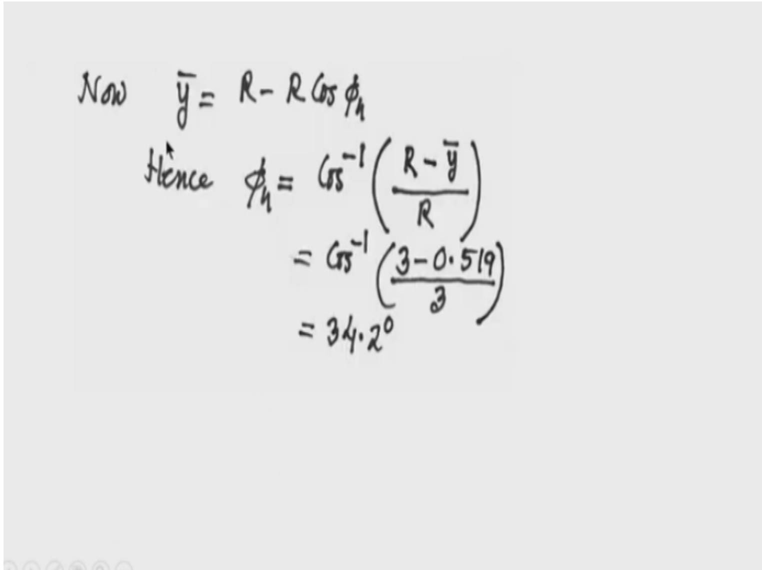

So, since $\bar{y}$ is known, so we can now determine the angular distance of the neutral axis from the crown. So, $\bar{y}$ is again given as $R - R\cos\phi_n$, so using the data that we have already obtained, we can now obtain ϕ_n is $\cos^{-1}\left(\frac{R-\bar{y}}{R}\right)$. Because, $\bar{y}$ is now known, so $\bar{y}$ is 0.519 and R is 3. So, this quantity now is calculated and ϕ_n becomes 34.2°.

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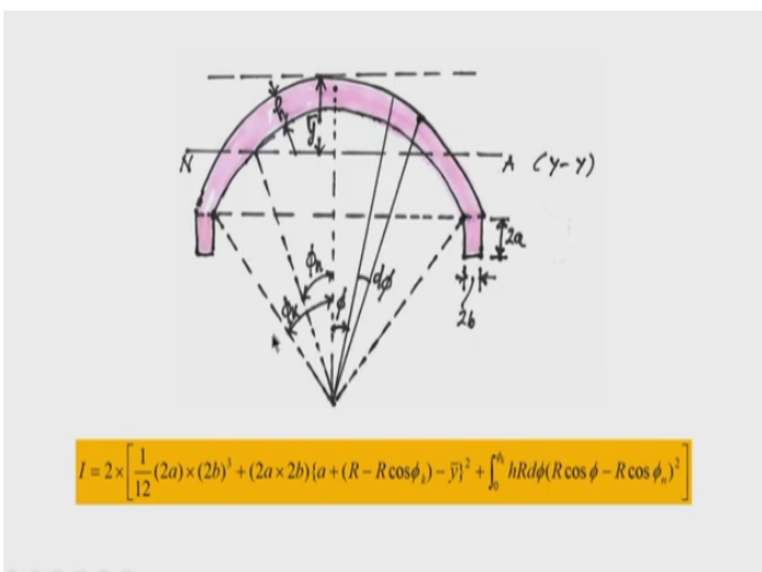

Moment of inertia is now having to be determined. Since there is no edge beam, so this quantity may be neglected.

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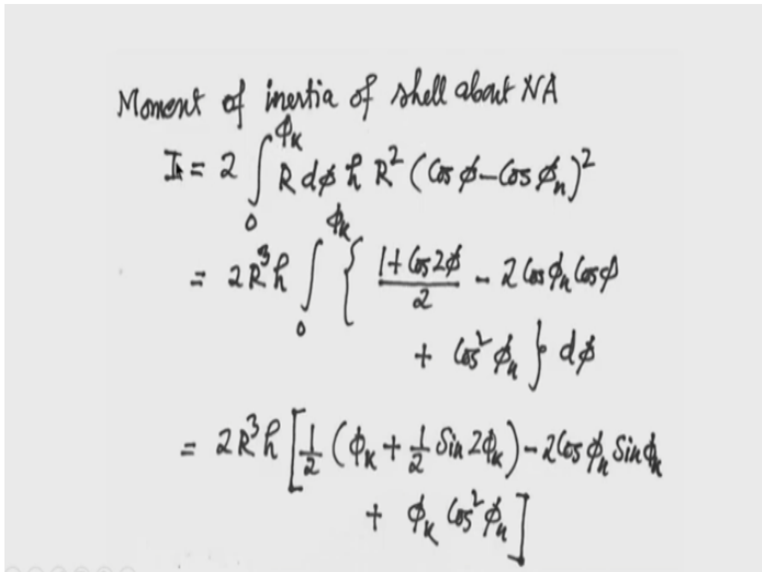

So, we will get here, the moment of inertia of this shell about the neutral axis is now $I = 2$ times, because we are considering only one part, so we are multiplying it by 2. So, 0 to ϕ_k is the limit of the integration, so $Rd\phi h$ this is the area and this is the distance square of the centre of the area about the neutral axis, so $R^2(\cos\phi - \cos\phi_n)^2$. So, this term is expanded here.

And using the trigonometrical formula $\cos^2\phi$ can be written here $\frac{1+\cos 2\phi}{2}$ and there are three terms. So, the term by term integration is carried out and limit is substituted. So, ultimately, we get the moment of inertia of the shell cross section about the neutral axis is $2R^3h\left[\frac{1}{2}\left(\phi_k + \frac{1}{2}\sin 2\phi_k\right) - 2\cos\phi_n \sin\phi_k + \phi_k\cos^2\phi_n\right]$.

And ϕ_n is the angular location of the neutral axis from the crown and ϕ_k is the semi central angle which is given as 60° and ϕ_n already we have determined it is 34.2°. *R* is 3 meter and *h* is 75 millimetre, if you put all the quantities in meter, then the *I* will be coming out as meter to the power 4. So, in that case *h* has to be substitute as 0.075 meter.

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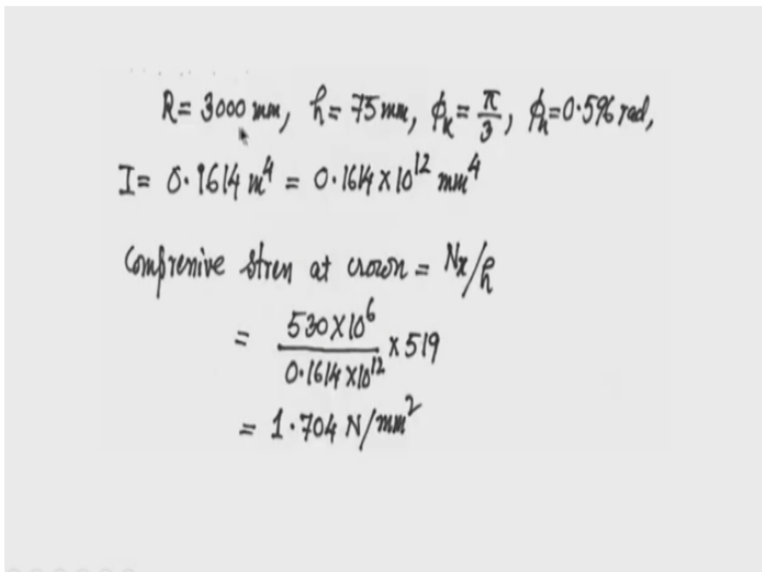

After putting numerical values, we get $I = 0.1614$ m^4 or in millimetre we can express this 0.1614 × 10^{12} mm^4. So, compressive stress at the crown that will be the maximum we can determine it now, 530 × 10^6, it is the bending moment in N mm. Previously we have determined the bending moment in kN m.

Now bending moment is converted into N mm. So, 530 × 10^6 is the bending moment in Newton millimetre and this is the distance of the extreme fibre in the compression that is at the crown measured from the neutral axis. So, this was determined as $\bar{y}$ and its value is 519 divided by moment of inertia that quantity we have substituted here and ultimately, we get the compressive stress at the crown is 1.704 N/mm^2.

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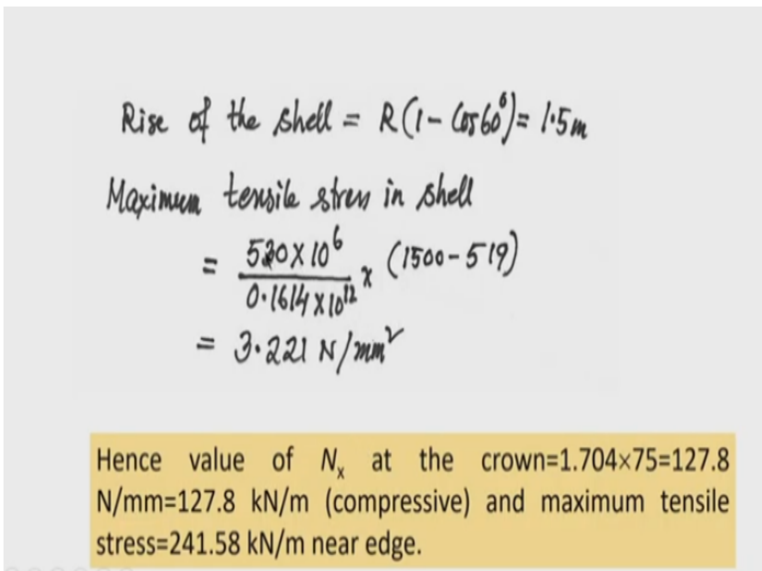

So, now maximum tensile stress will be occurring near the edges and its value is again found as 530×10^6 is the bending moment and this is the moment of inertia 0.1614×10^{12} into the distance of the shell point that is near the edge measured from the neutral axis. So, it is 1500 - 519, because 1500 was the rise of the shell. So, this value is coming out as 3.221 N/mm^2.

So, value of N_x at the crown is 127.8 and value of the N_x at the maximum tensile stress near the edge is 241.58 kN/m.

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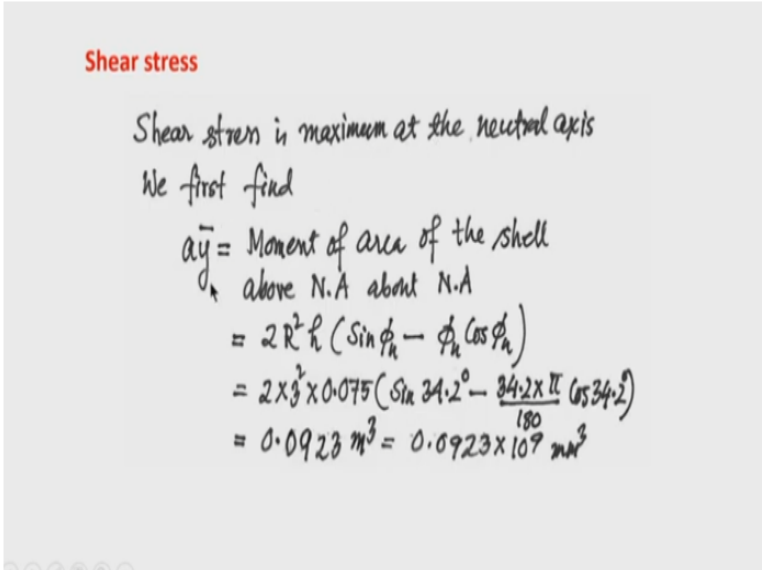

Shear stress, so shear stress is maximum at the neutral axis and, first let us finds what is Q? That is $a\bar{y}$? $a\bar{y}$ is the moment of area of the shell about the neutral axis. So, that relationship we have derived earlier, so this $a\bar{y}$ is nothing but $2R^2h(\sin\phi_n - \phi_n\cos\phi_n)$. After substituting the numerical value of the parameters say R = 3, h = 0.075, ϕ_n is here substituted in radian, so 34.2 × π /180 × cos 34.2 degree.

So, after calculating this we get $a\bar{y}$ about the neutral axis is 0.0923 m^3 or 0.0923 × 10^9 mm^3.

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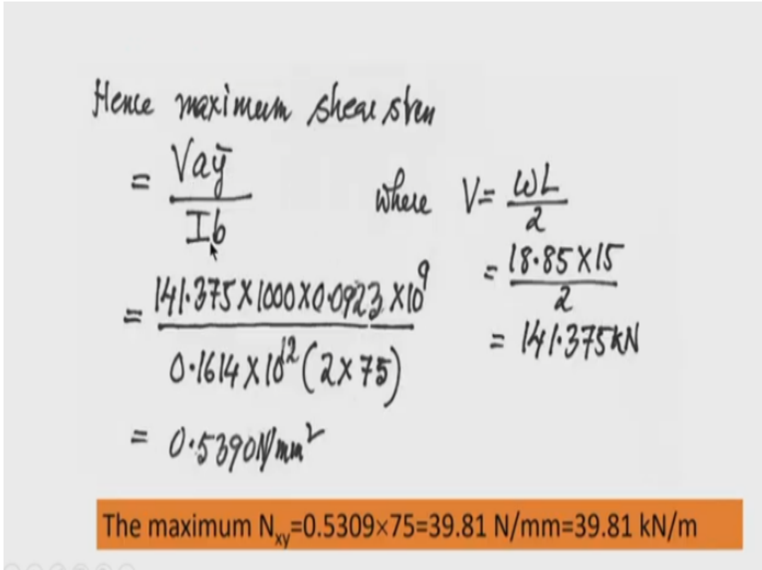

So, maximum shear stress is $\frac{Va\overline{y}}{Ib}$ and V is the maximum shear force that occurs at the traverse and its value is $wL/2$, after substituting the value of w as 18.85 and L is 15/2. So, we get maximum shear force as 141.375 kN. And it is converted here into newton by multiplying with 1000 and then we substituted $a\overline{y}$ as 0.0923×10^9, I is 0.1614×10^{12} and at the neutral axis b is $2h$, so 2×75.

So, therefore after substituting we get the maximum shear stress is 0.5390 N/mm^2. So, maximum $N_{x\phi}$ or N_{xy} whatever you call will be 0.5309×75, that is 39.81 N/mm or it is 39.81 kN/m.

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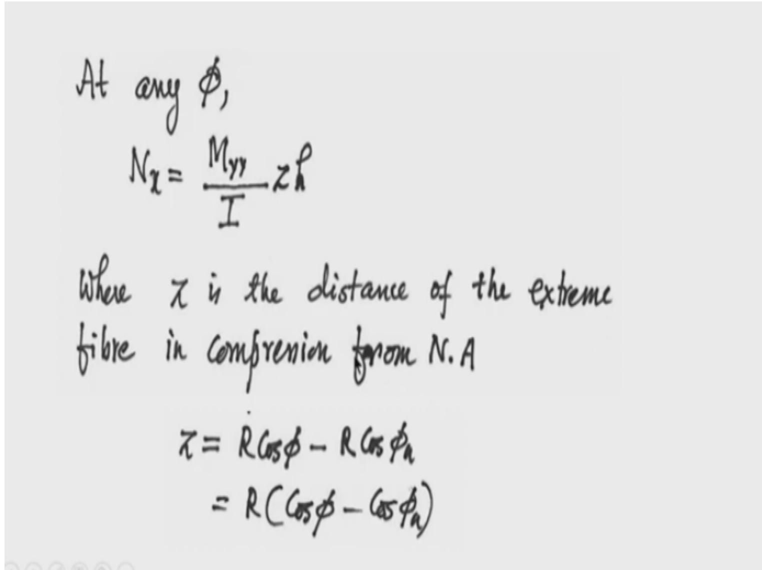

Now, we have determined the maximum value, but let us see how the N_x and $N_{x\phi}$ varies with ϕ. So, that quantity is determined by putting z at any ϕ. So, the relation between z and ϕ is obtained here, z = R cos phi - R cos phi n, that is very much a bias from the figure and with the geometry you can find the relation between z and phi. So, $z = R(\cos\phi - \cos\phi_n)$, h is the thickness of the shell and now, we can find out the variation of N_x with any ϕ.

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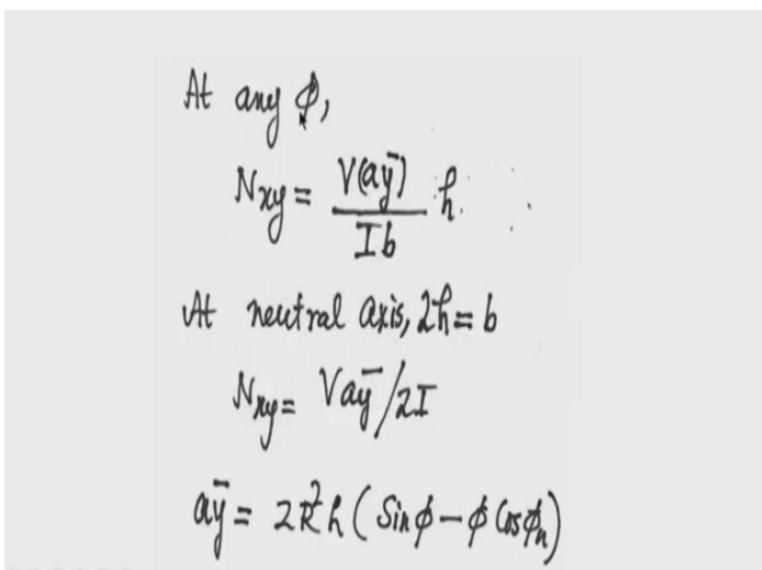

Similarly, the variation of $N_{x\phi}$ or N_{xy} that is the membrane shear force, ϕ is analogous to y direction; tangential direction. So, therefore the $N_{x\phi}$ is nothing but or N_{xy} is nothing but $\frac{V(a\bar{y})}{Ib}h$.

b is $2h$, so substituting b is $2h$, we get $N_{x\phi}$ or N_{xy} = $\frac{V(a\bar{y})}{2I}$, $a\bar{y}$ at any ϕ is determined as $2R^2h(\sin\phi - \phi\cos\phi_n)$. So, here is the ϕ is the angle measured from the crown and ϕ_n is the location of the neutral axis from the crown.

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Calculation of N_x and $N_{x\phi}$ at different values of ϕ

ϕ (degree)	ϕ (radian)	Z (m)	$a\bar{y}$ m³	N_x (kN/m)	N_{xy} (kN/m)
5	0.0872	0.5073	0.2022	124.90	8.856574
15	0.2618	0.4165	0.5709	102.60	25.00398
25	0.4363	0.2377	0.8334	58.50	36.50172
35	0.6109	−0.0238	0.9226	-5.85	40.40717
45	0.7854	−0.3599	0.7764	-88.64	34.00837
55	0.9599	−0.7605	0.3403	-187.30	14.90647

These calculations are done in a tabular form, and it is displayed here ϕ is the degree, angle that is measured from the crown, it is written in degrees, that is the shell is divided into 12 parts and each part will be 10°, because it is 120° that total angle subtended by the arch, subtended by the circular arc. So, each division will subtend an angle of 10°. So, we are considering a point in the middle of the division so, it is denoted by ϕ.

Next division, at the centre the angle will be 15, 5 + 10. In the next it will be 15 + 10, 25 and then 25 + 10, 35 like that we can go here up to 60°, 60° is the maximum. So, ϕ is written in here in degrees and then it is converted into radians. So, in radian it is converted simply by multiplying it with π/180. So, we will get this in radians here and then z is calculated, because z we have found out in terms of ϕ.

So, z is calculated at any ϕ as this, $a\overline{y}$ is calculated at any ϕ. So, now N_x that is the in-plane forces, direct forces and it is due to bending action of the beam, it is now calculated at the ϕ is near the crown, because the angle is measured from the crown and you can see the maximum value is obtained at the crown. So, but it changes its nature, because the compressive stress is reversed after neutral axis.

So, the maximum tension is obtained near the edges. The shear stresses are calculated and you can see this maximum shear will be obtained near the neutral axis. And neutral axis is located somewhere in the 34.2°, so here we are getting the maximum value.

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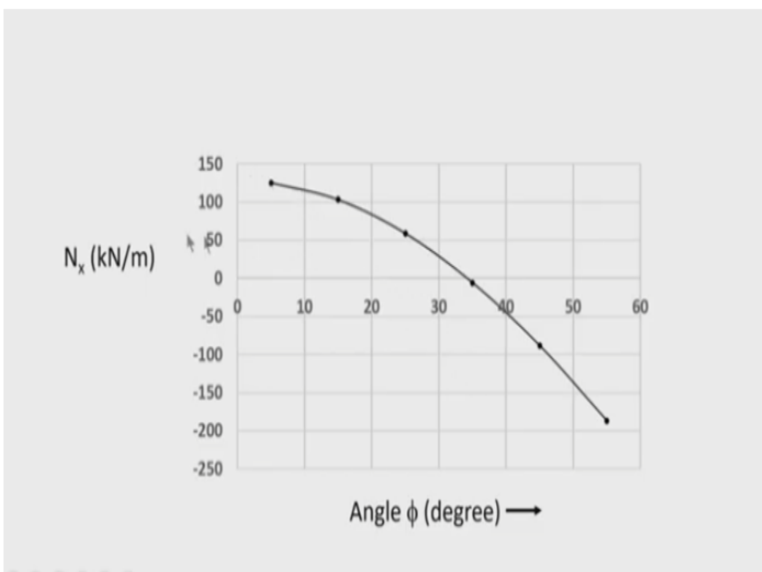

These quantities are plotted and you can see the variation of N_x with ϕ, how it varies? First it is compressive and then it is, this ϕ is measured from the crown. So, this region is crown region, so this is the crown point. So, here the maximum flexural stress is obtained N_x here is 124.7. Similarly, maximum tensile stress is obtained near the edges and its value is obtained here as this 187.3, as shown here.

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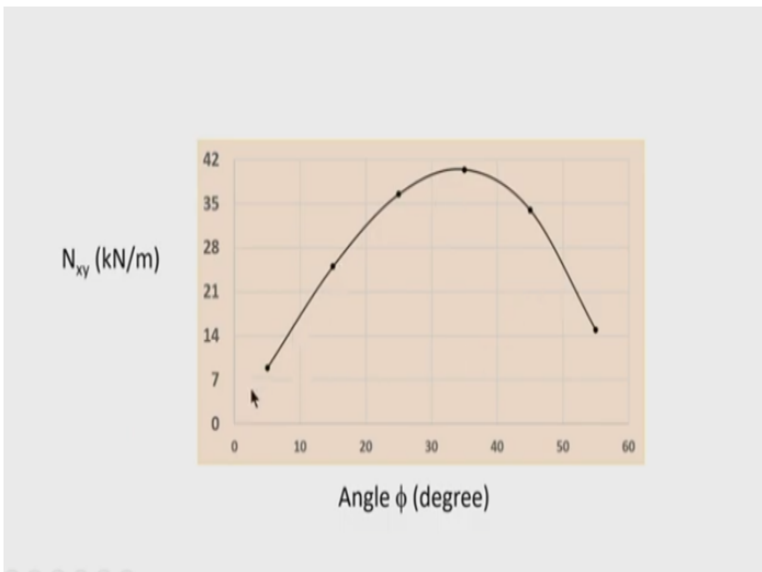

Similarly, the membrane shear force is plotted with the different ϕ and its variation is shown like that. Maximum shear, membrane shear is obtained at the crown.

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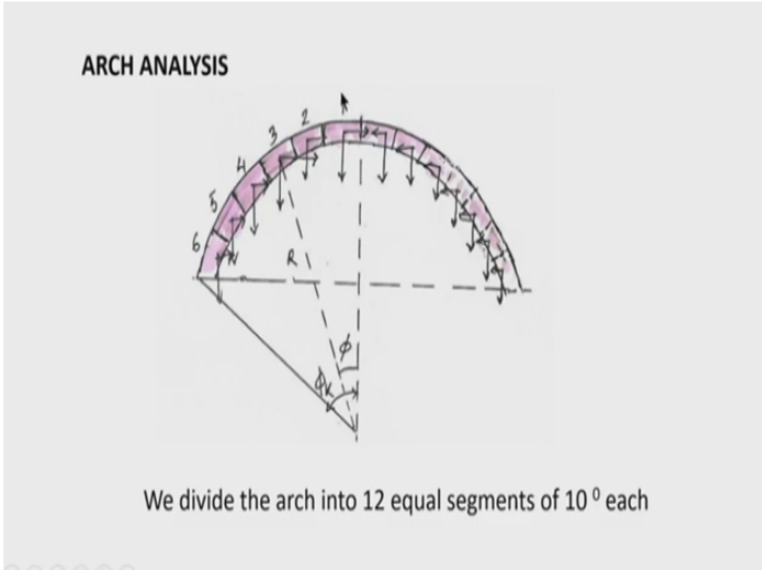

Now, we are bringing the arch analysis to find out this bending moment M_ϕ and Q_ϕ. N_ϕ of course will not be significant here, because this shell is not restrained at the edges. So, since there is no restraint at the edges, so do not consider this arch the indeterminate system and we analyse it as a determinate arch. So, we divide the arch into 12 equal segments of 10° edge, earlier we have shown you.

And each division this centre of its division or is located by angular distance. So, therefore, we start from 5°, 15°, 25°, 35° like that will go up to these n divisions.

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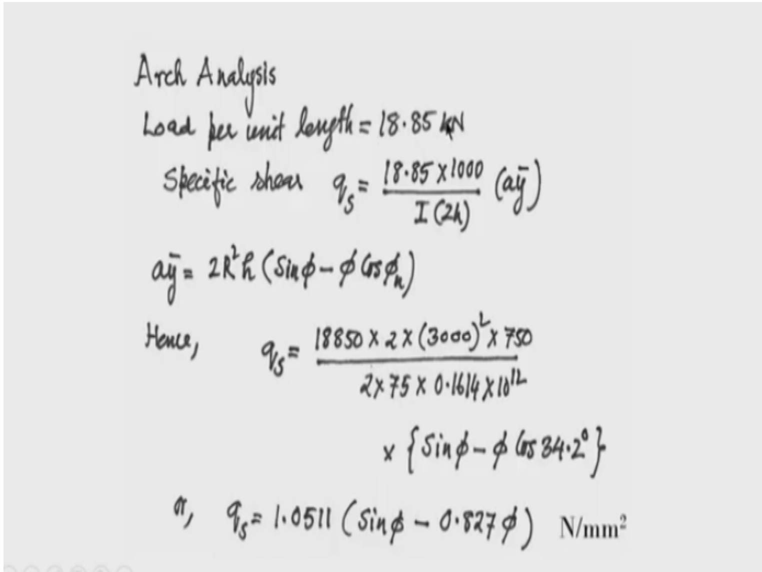

Load per unit length will we know it is 18.85 kN. So, specific shear is the most important quantity in the analysis of the cylindrical shell using the arch theory. So, arch section is only relevant if we can find the q_s. So, q_s is found out 18.85 that is the w, it is converted into newton. Then I is the moment of inertia about the neutral axis and then $2h$ is the b, that is the width of the shell at the neutral axis and this is $a\bar{y}$.

$a\bar{y}$ is calculated at any angular distance ϕ, so, this formula is derived earlier also and it is shown here. So, the quantity q_s is now found as 1.0511 × (sinϕ - 0.827 ϕ). So, that means this q_s that is calculated from this quantity, $a\bar{y}$ is substituted here and this other quantity I we have already found and h is known. So, these are the quantities here, this represents the moment of inertia and this is the thickness and this quantity is your $a\bar{y}$.

Of course, this R etcetera R^2 up here. So, q_s is now 1.0511 × (sinϕ - 0.827 ϕ), remember that ϕ should be here in radian.

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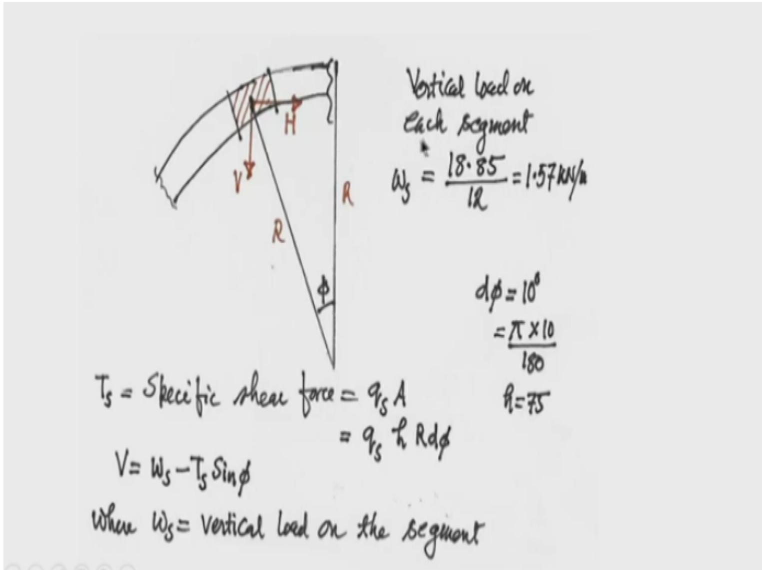

So, this vertical load on a segment is w_s, because there are 12 segments, so total load per unit length is 18.85 divided by 12. So, in each segment vertical load is 18.85 divided by 12, so this is 1.57 kN/m. And the angle subtended by a segment is 10°, so 10° in radian, we can now find out 10 × π/180, h = 75, q_s is a specific shear stress. So, specific shear force T_s capital T_s is now found out $q_s \times A$.

So, q_sA is h × $Rd\phi$, so this quantity can be substituted here to calculate the numerical value of T s T_s So, net vertical forces, net vertical force shown here is w_s - T_s sinϕ. So, that is the vertical component of this T_s.

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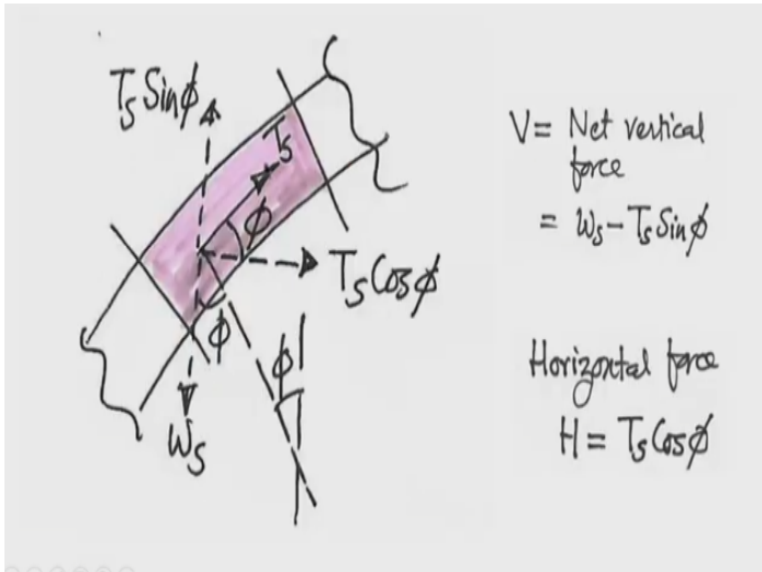

That is obvious from this figure, if this is the T_s, the specific shear force and its vertical component is T_s sinϕ, horizontal components T_s cosϕ. So, net vertical force will be w_s - T_s sinϕ and horizontal forces $H = T_s$ cosϕ.

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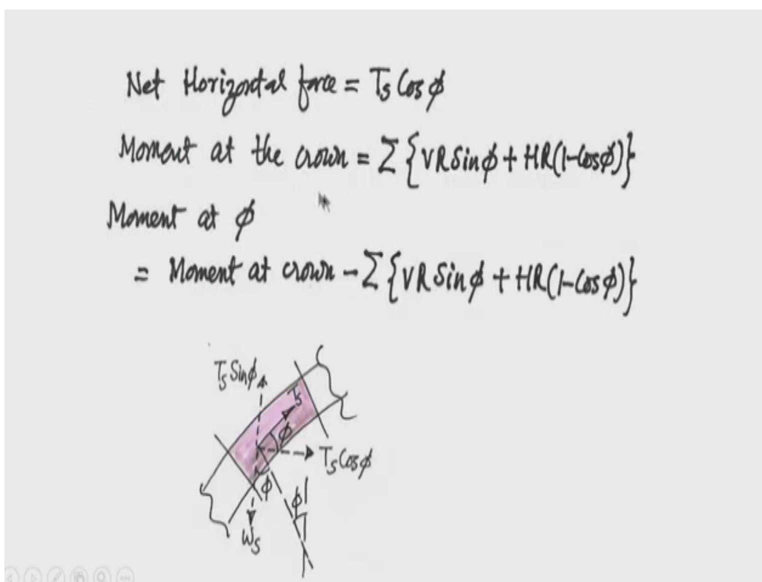

So, moment at the crown will be, if I take the moment of this forces about the crown T_s cosϕ and T_s sinϕ, we can now write it as the net vertical forces w_s - T_s sinϕ, that is V. So, if we take the moment of the net vertical force and horizontal force about the crown, then we can get it $VR\sin\phi + HR(1 - \cos\phi)$ and it is summed up over all the elements. Then, moment at ϕ at any location will be found from the equilibrium of the moments. So, moment at crown - summation of the moment $VR\sin\phi + HR(1 - \cos\phi)$.

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Calculation of Arch Bending moment

ϕ (degree)	ϕ (radian)	q_s(N/mm^2)	T_s (N)	V(N)	H(N)	$VR\sin\phi + HR(1-\cos\phi)$ N.m	M (N.m)
5	0.0872	0.0157	618.58	1516.08	616.23	403.44	7249.21
10	0.2618	0.0445	1746.45	1117.98	1686.94	1040.51	6208.70
25	0.4363	0.0649	2549.71	492.44	2310.82	1273.86	4934.84
35	0.6109	0.0718	2822.94	-49.17	2312.42	1169.94	3764.86
45	0.7854	0.0605	2376.81	-110.65	1680.65	1242.02	2522.85
55	0.9599	0.0266	1043.84	714.93	598.72	2522.85	0

$\sum colum\ 7 = 7652.62$

So, these calculations are again presented in a table, so, ϕ is written here it is 5, 10, 25 like that. It will be 10 with a 10° interval; you have to write the angle. So, its radian is this 0.0872, 0.2618 these two columns are already displayed in earlier table that is here, same two columns are written here also. Then q_s the specific shear is written here, the T_s the specific shear force is written here.

The net vertical forces written in this column, horizontal forces written in this column and then summation of the moment of the vertical forces in each segment and horizontal forces in each segment have to be calculated. So, these are written in this column and then it is summed up. So, the elements of this column are summed up added and it is found as 7652.62. So, this summation minus this will be the moment at this element.

Then this moment minus this moment, this moment will be this, like this will calculate the moment at all individual elements. And then we can get the maximum moment at the crown and theoretically since the edge is free, the moment is coming is 0 here near the free edges.

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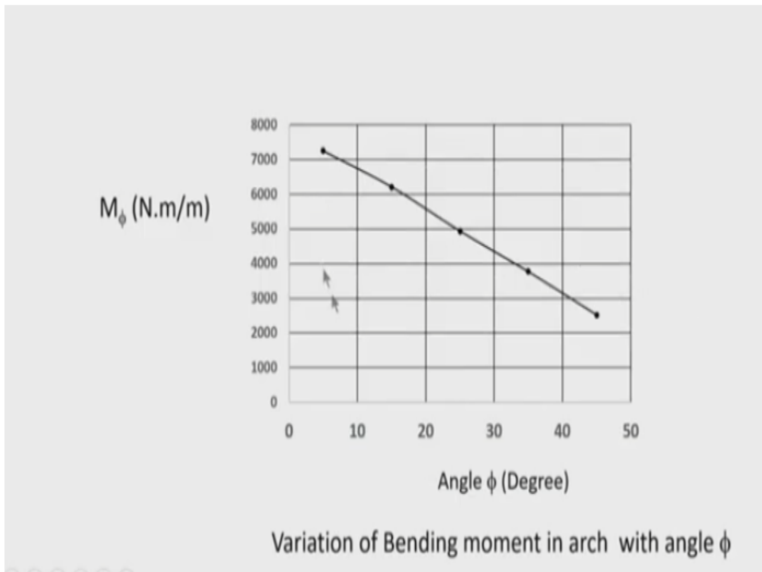

So, this moment is plotted along the ϕ and you can see the variation of this moment from crown to springing and maximum moment M_ϕ that is obtained at the crown is shown here and it is varying like that and becoming very low value near 0, near the edges.

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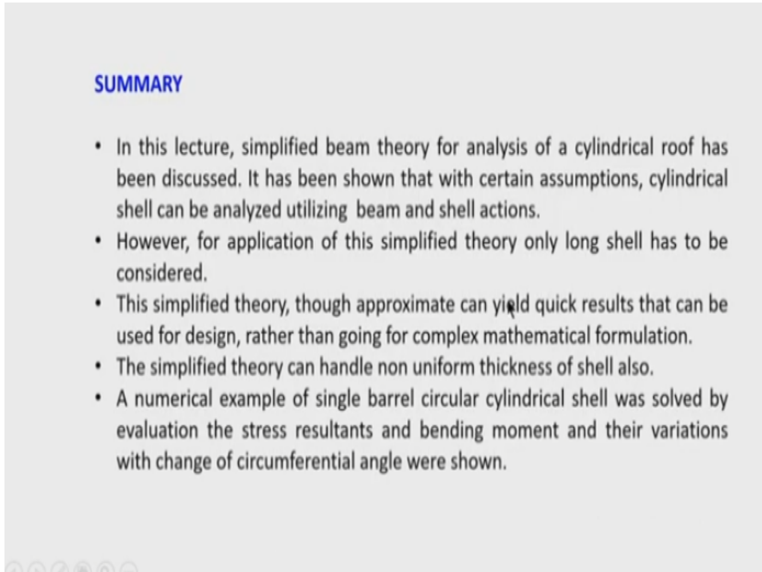

So, let us summarize today's lecture in this lecture is simplified beam theory for the analysis of cylindrical roof has been discussed. And it has been shown that with certain assumptions cylindrical shell can be analysed utilizing beam in shell actions. The main assumption of this theory is that, L/R ratio should be within some limits. So, if the shell is without edge beam, then $L/R \geq 5$.

And if the shell is provided with edge beam, then $L/R \geq 3$. So, L/R parameter is very important, which demarcates the boundary between the short shell and long shell. And beam theory is only applicable if the shell is long, otherwise the assumption of beam for the analysis of shell is not valid. The simplified theory though approximate, we have seen that it can yield quick results that is membrane forces are obtained as well as the bending moment is also obtained.

So, we can see whether the bending moment is underestimated or not or the reinforcement, especially the reinforcement in a cylindrical RCC shell provided based on this direct tensile force can be checked with the reinforcement that needs for the bending moment generated in the shell. So, generally we provide the higher quantity of reinforcement. The simplified theory can handle non uniform thickness of the shell.

That statement is very important, because we have seen that ultimately, we have found the variation of the stresses with the different ϕ. So, in that process we have divided the shell cross section into different subdivisions. So, even the shell is non-uniform, each division can be of different thickness and weight of the load acting on this division will change, because cross sectional area will change.

So, it can be easily computed for non-uniform thickness also by discretizing the shell into different subdivisions. So, that is the beauty of this method. In other theory, bending theory where the simplification is done with a uniform thickness, because the differential equation with variable coefficient becomes very complicated and it loses the attraction of the analysis, because of so much complicacy.

So, therefore, simplification is done in the differential equation for uniform thickness shell. But in this theory, since it can be discretized into different elements, an element by element computation can be done and it is summed up. So, the non-uniform thickness can also be incorporated. And lastly, I have shown a numerical example of a single barrel circular cylindrical shell subjected to vertical load.

And we have analysed this using the beam action to find out this N_x and $N_{x\phi}$ or N_{xy}. And, then also using the arch action, we have calculated the bending moment at the crown. Thank you very much.