Plates and Shells
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Module-11
Lecture-33
Circular Cylindrical Shell for Fourier Loading in a Membrane State of Stress

Hello everybody, today I am starting the lecture 3 of module 11. And I was discussing about the membrane theory of cylindrical shell roof specially that is used as a roof structure. And we utilize the membrane hypothesis to obtain the stress resultants. The beauty of the membrane analysis is that the in-plane forces are all acting in the shell only and there is no bending moment.

So, a moment less state is considered in this membrane theory and we have obtained the stress resultant for the shells. Now in this lecture I want to demonstrate how the stress resultants in cylindrical roof can be obtained using the Fourier loading of course with the help of membrane analysis? So, we will discuss a case of circular cylindrical shell but this method can be applied to other shell as well.

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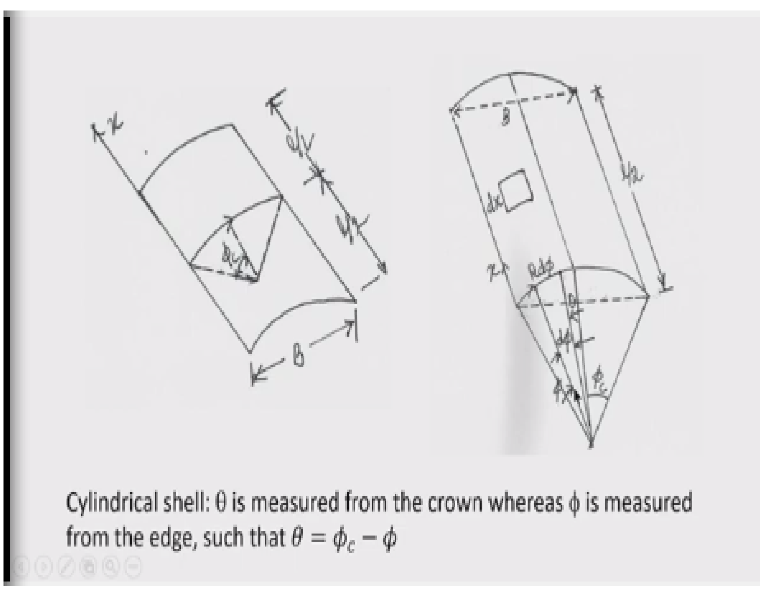

Now, we have seen the cylindrical shell roof maybe a single span or maybe a multiple barrel type. But we are now considering for simplicity a single span shell, where the length of the shell is or span you can see here it is l and width of the shell is B. We take the origin at the centre, so here we have taken half portion of the shell to analyze this stresses. So, here we have taken an element dx of the shell and which is located at an angular distance measured from this springing level which is φ .

But however in our earlier analysis you have noted that we have measured the angle from the crown and we denoted it as θ . So, there is a distinct relation between θ and φ , you can see that θ can be replaced by $\varphi_c - \varphi$, where φ_c is the semi central angle subtended by the arc of the cylindrical shell. That is here the directrix is a circular curve but other directrix can be also taken when analyze the shell.

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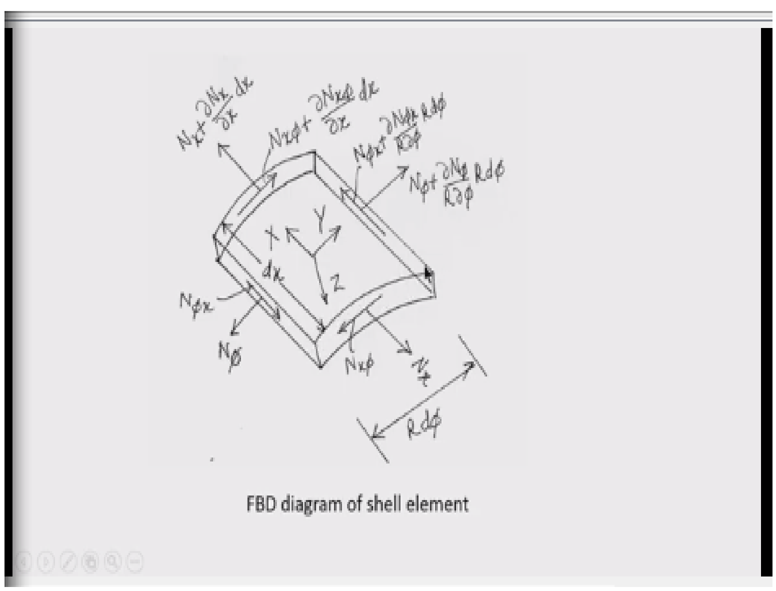

So, if we see a equilibrium of an element this length is $Rd\varphi$, if $d\varphi$ is the small angle subtended by this arc and this length of the side parallel to x axis is dx. So, we can denote the stress resultant in phases which have in this phase here you can see, this is N_φ that is the meridional stress N_φ. And on the opposite edge there will be N_φ plus some increment, so this increment is along the length $Rd\varphi$.

So, here you are seeing that $Rd\varphi$ is appearing in the denominator. Now, membrane shear force $N_{\varphi x}$ is acting at this edge, on the opposite edge the $N_{\varphi x}$ is accompanied by a incremental quantity. And this incremental quantity is $\partial N_{\varphi x}$ by $Rd\varphi \times Rd\varphi$, $R \times d\varphi$ is the length. So, $Rd\varphi$ is due to this length of the arc and you can earlier also I have explained that this term has come from Taylor series expansion of the stresses and we keep the first term only. So, in the other phase we can see there the stress is N_x and on the opposite phase here in the longer general reaction $N_x + \frac{\partial N_x}{\partial x} dx$.

So, here we are taking the partial derivative quantity because the N_x is function of x as well as phi. So, 2 variables are in involved here x and φ. Now other quantities X, Y, Z are the components of the load in x, y and z direction, y direction is tangential direction at any point that we call the meridional direction. And X direction is the longitudinal direction, Z is the radial direction. So, Z is the line along the z axis will pass through the centre of curvature. So, this is the free body diagram of the shell element.

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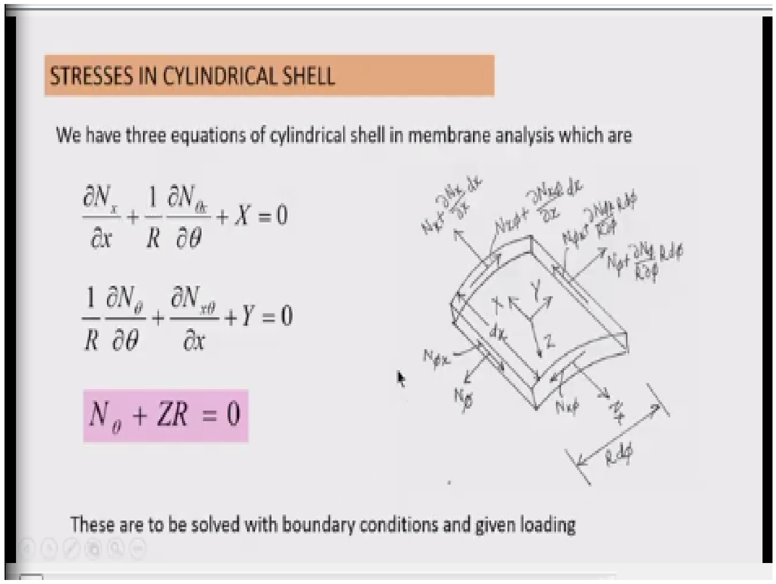

Now we have established the equilibrium equation of the shell element and we have seen that 1 equation fortunately it is an algebraic equation. So, meridional stress that is N_θ is directly obtained as $N_\theta = -ZR$, where Z is the component of the load along the radial direction, R is the radius of curvature at a particular location. So, to find the meridional stress one need not solve any differential equation, directly from an algebraic equation one can find the quantity N_θ .

Now after finding the N_θ , other 2 differential equations which involves the longitudinal stress N_x and membrane shear stress $N_{\theta x}$ that has to be found out by utilizing the value of N_θ . So, if I substitute the N_θ here, then after integration and substituting the condition at the edges we can obtain $N_{x\theta}$. So, when $N_{x\theta}$ is obtained and substituting $N_{x\theta}$ here, finally we can obtain this N_x after integration of the differential equation and applying appropriate boundary conditions.

So, you can see that this equilibrium equation has been solved to find the membrane stresses only the bending moment in the shell have been neglected in the theory. And why we neglect the bending moment because the shell behaves as a stress scheme, so here the bending rigidity can be neglected. However, it is seen from advance theory and after research the bending moment has some important role near the edges of the shell.

So, there we cannot neglect the bending moment and therefore the theories involving bending moment has been also developed. But while analyzing the shell using the bending theory, one needs to utilize the results of the membrane theory. So, now we discuss why the Fourier loading is taken? Fourier series is taken to express the loading in case of membrane theory. Here you will see the utility of this Fourier loading.

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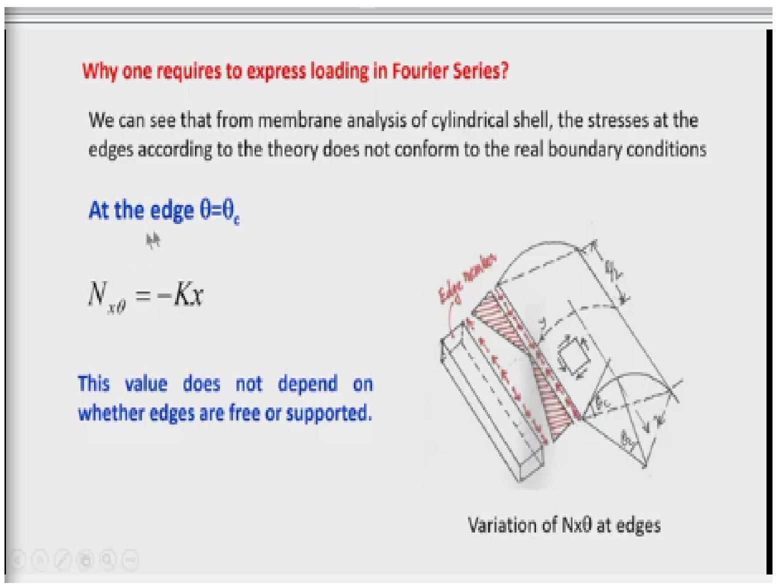

Now you can see in this shell at the edges, because we have seen that in earlier classes, we have derived that $N_{x\theta}$ that membrane sharing force = -Kx where K is a constant which does not depend on x. So, the variation of $N_{x\theta}$ that may membrane shear stress is linear, it is obvious from this express and diagrammatically it is represented here. So, the confusion arises if the shell is free then also there is a stress that is not realistic. So, therefore membrane theory fails here to predict the correct state of stress at the edges.

Now, if I use the corrective line load that is to correct this edge stresses suppose it is the shell is supported at the edge. So, then the membrane shear stress at the edges will exist. But suppose the shell is free at the edges, then there is no membrane shearing stress the free edge will be completely free of stress, there will be no stress acting at the free edge. So, therefore a corrective line load has to be applied at the edges and this load will produce the bending of the shell and alters the state of membrane stresses.

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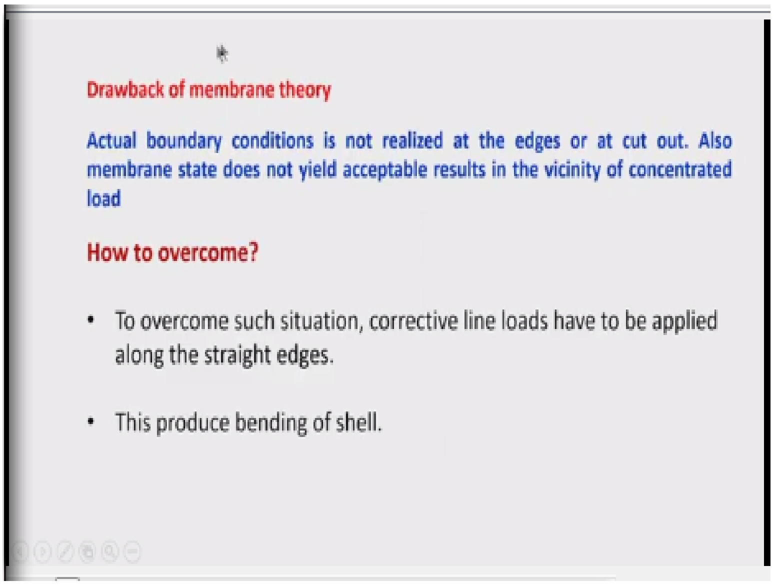

So, drawback of the membrane theory we have noted clearly the actual boundary conditions are not utilized at the edges or at cut out. Even at the cut out suppose airplane fuselage, this is also a cell structures but windows are there these are cut out. Or also in case of spherical dome, there is some cut out somewhere to allow the light to pass. So, therefore at the cut out, the edges of the cut out or hole there should not be any stresses.

But if we analyze this stress as a whole using the membrane theory, then the stresses are obvious at the free edges, so that has to be corrected by applying the corrective line load. The corrective line load actually produced the bending of the shell and this bending moment causes the change of membrane stress in the vicinity of the edges or support or at the cutout. So, how to overcome this difficulty? That I have discussed the corrective line load is one thing and then to consider the bending of the shell.

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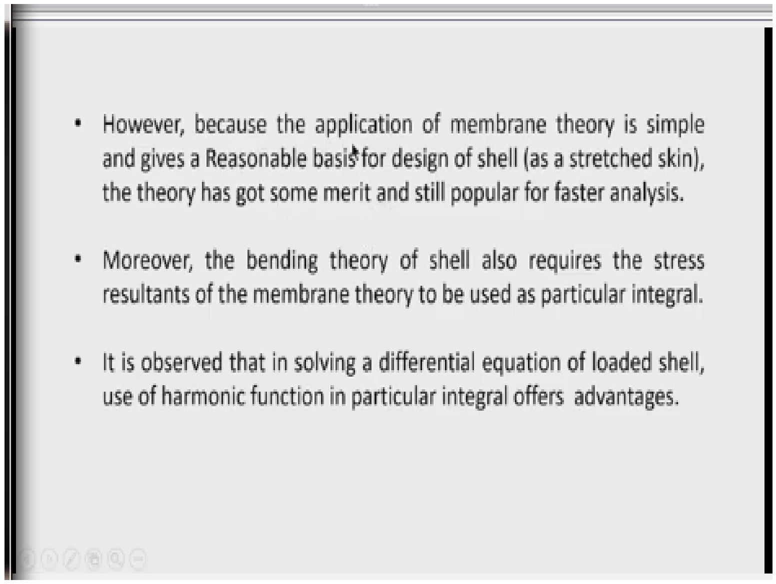

However, because the application of membrane theory is simple and gives a reasonable basis for design of shell, nobody can question on the membrane theory of the shell for the design of such structure in a rapid manner. So, what does it mean? That means away from the edges, the other region will reflect the correct stresses if we use the membrane theory that is also validated by several authors with the experiments.

And therefore this membrane theory has become popular in the design office and we cannot neglect this. Moreover, the bending theory of the shell requires the stress resultants of the membrane theory to be used as particular integral. Now when we establish the equation for bending of loaded shell, then the particular integral that have to be solved will contain the nature of the load that is applied on the shell.

So, if the load is expressed or transformed into a Fourier series, then finding the particular integral becomes easier. Because the standard solution for harmonic functions are available in the integral calculus or the tables of integral, there are handbooks there we can use the particular integral easily if this function conforms to a particular harmonic function. So, therefore this is the utility of using the load into a Fourier series.

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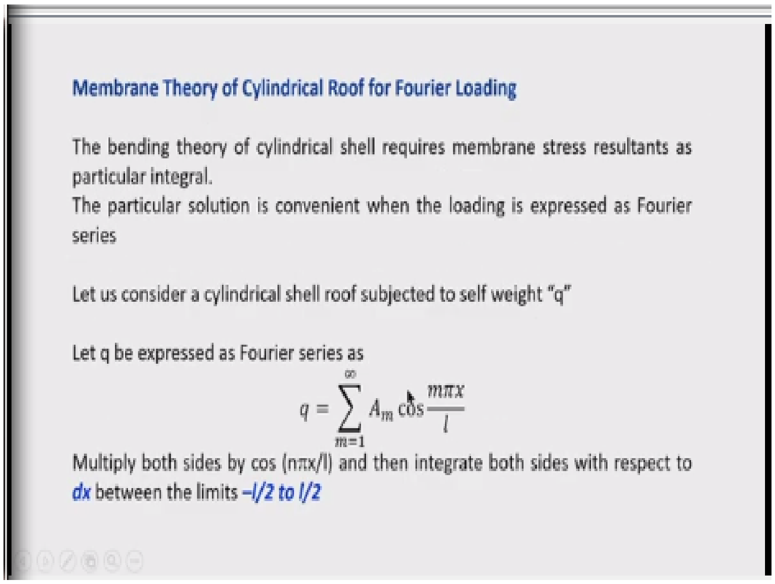

Now let us see how we can apply the Fourier loading in the shell? We have already seen that q is the gravity load acting on the shell, we are taking here the gravity load on the shell which consists of self weight, snow load or maybe this live load for some purposes. Wind load of course here we are not considering because this will give an unsymmetrical state of stress and we are so far considering the symmetrical condition of stresses.

Now, let q be the load which may be a constant or which may be a function of x.

$$q = \sum_{m=1}^{\infty} A_m \cos\frac{m\pi x}{l}$$

Now q is expressed as a series and series is given by this cosine series that is a harmonic function and it is an infinite series the summation includes m = 1 up to infinity. So, $A_m \cos\frac{m\pi x}{l}$. So, this is the general term m_{th} term of the series and by substituting m = 1, 2, 3 etcetera, you will get number of terms and if you sum up you will get the series.

Now question arises, what is the coefficient A_m? If we know the coefficient A_m then we can fully express the series because m is integer and it will vary 1, 2, 3 like that up to infinity, because this is an infinite series. However, for practical calculations the infinite series cannot be used. So, there we truncate the series to a limited number of terms. Now to convert this into a form in which we can solve the differential equation, let us express or let us determine the coefficients A m. Now to do this see the steps how we can obtain the coefficient A m? Multiply both sides by $\cos\frac{n\pi x}{l}$.

$\cos\frac{n\pi x}{l}$ that I have taken n is also integer, m is a integer, n is also integer. So, I am multiplied both sides by $\cos\frac{n\pi x}{l}$ and then I integrate both sides with respect to dx between the limits $-\frac{l}{2}$ to $+\frac{l}{2}$. Because we have taken the origin of the shell, origin of the coordinate system for the analysis of cylindrical shell at the centre of this span, so therefore our limit of x is $-\frac{l}{2}$ to $+\frac{l}{2}$.

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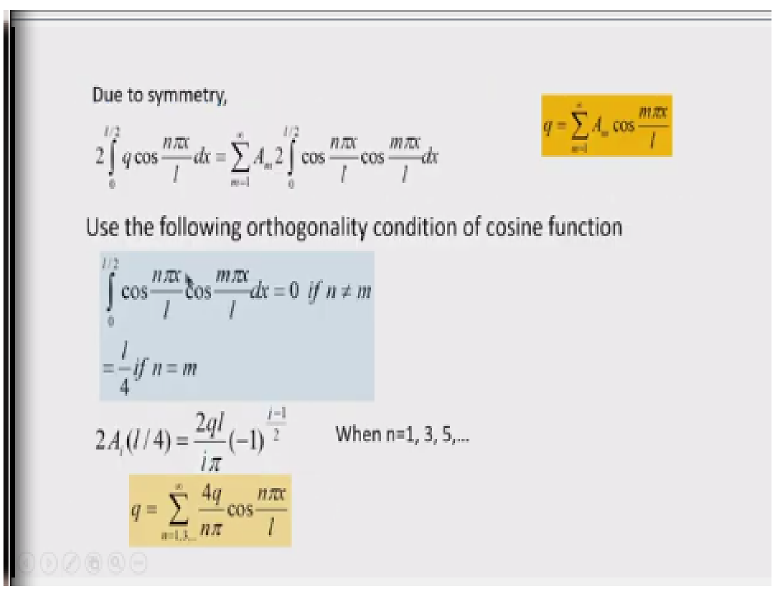

So, in the next step $2\int_0^{l/2} q\cos\frac{n\pi x}{l}dx = \sum_{m=1}^{\infty} A_m 2\int_0^{l/2}\cos\frac{n\pi x}{l}\cos\frac{m\pi x}{l}dx$. The left hand side now $q\cos\frac{n\pi x}{l}dx$ but since it is symmetric, so I multiplied it by 2 and apply the limit 0 to $\frac{l}{2}$. And right hand side you can see that A_m coefficient is there and it is inside the summation term and then the integral $\cos\frac{m\pi x}{l}$ cos this is multiplied by $\cos\frac{n\pi x}{l}$ then integrated and the limit a $\frac{l}{2}$ and because of symmetry we have used a factor 2, so this is integrated.

So, now we use the orthogonality condition of the cosine function, sine and cosine function follows a very important orthogonality relationship. And this relationship is due to orthogonal property of the function. And it can be verified that $\int_0^{l/2}\cos\frac{n\pi x}{l}\cos\frac{m\pi x}{l}$, when this product is integrated with respect to x between the limit 0 to $\frac{l}{2}$ or 0 to a, whatever may be you have seen in plate equations also where we use this property.

So, if I integrate it, if n $\neq$ m then the integral is 0, which means that suppose if I take $\varphi_c - \varphi$ and here $\cos\frac{3\pi x}{l}$ then integral is 0. If I take integration of $\cos\frac{2\pi x}{l}$ and $\cos\frac{\pi x}{l}$ dx is 0, that means n $\neq$ m. So, for that condition the integral is 0. But when n = m then in this case it becomes $\cos^2\frac{m\pi x}{l}$ because n and m is same, so the integration becomes $\frac{l}{4}$.

So, $\frac{l}{2}$ divided by 2 that is $\frac{l}{4}$ if n = m. So, using this orthogonality condition of the cosine function, we now write this equation again. So, this equation can be written as $2A_i(l/4) = \frac{2ql}{i\pi}(-1)^{\frac{i-1}{2}}$

Here you can see that where this i is varying from 1 to infinity, the index can be changed, i is also an integer. So, I have written here A_i, so $2A_i(l/4) = \frac{2ql}{i\pi}(-1)^{\frac{i-1}{2}}$

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This factor is taken to show that the value of integral exist only for odd number of integers. So, if n = 1, 3, 5, 7, 9, 11 like that, then only you will get the value of this integral otherwise this integral will be 0. So, therefore we now express this load q as because the coefficient now is known. Coefficient will be now we can get it coefficient will be $q = \sum_{n=1,3,..}^{\infty} \frac{4q}{n\pi} \cos\frac{n\pi x}{l}$ only for this odd integers, so n = 1, 3, 5. So, we have now got the coefficient of the Fourier loading.

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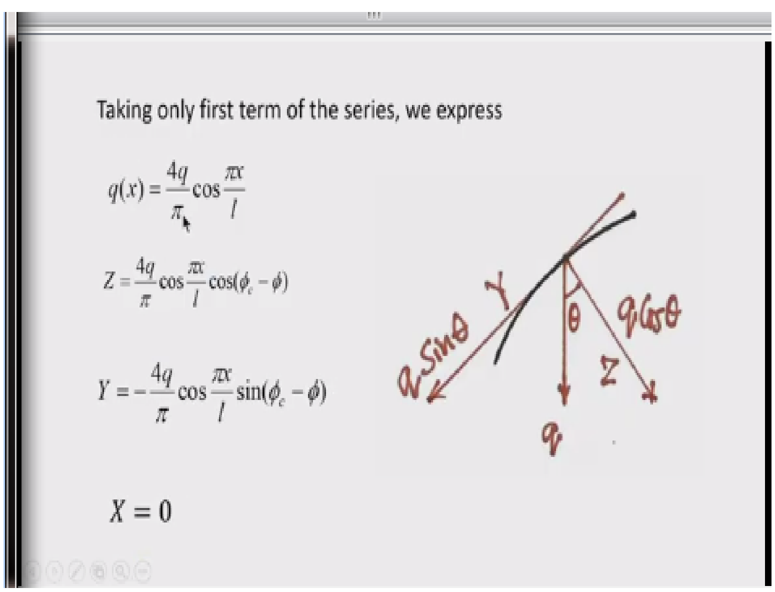

So, taking only first term because first term actually predominates and using the first time you will get almost accurate results in almost all cases especially for the first order terms. So, taking only first term of the series, we express q (x), say here it is q is a downward. This is the downward load that is q, it is varying along the x. Now in that case we are taking it as a constant loading, say for example self weight if the shell thickness is uniform, then self weight is also constant.

So, the q (x) here because the first term of the coefficient will be $q(x) = \frac{4q}{\pi}\cos\frac{\pi x}{l}$. So, this is the coefficient of the first term and $\cos\frac{\pi x}{l}$, so this is q. And if this angle is θ, then in the radial direction the component of q is $q\cos\theta$ in the tangential direction or meridional direction if you call it or circumferential direction this will be $q\sin\theta$. Now in the shell analysis using the Fourier loading, we have seen that in earlier slide in the first slide, I have demonstrated here.

That θ has to be replaced by $\varphi_c - \varphi$ because θ we are measuring from the crown, but we are expressing all the quantities of stress resultant in terms of φ. So, φ is measured from the edges that is a springing level. So, here the relation between θ and φ is $\varphi_c - \varphi$, if φ_c is the semi central angle. So, you can easily substitute now this here instead of theta we are now writing $\varphi_c - \varphi$.

So, the radial loading now Z is $Z = \frac{4q}{\pi}\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi)$. Then the tangential loading, that is the circumferential loading $Y = -\frac{4q}{\pi}\cos\frac{\pi x}{l}\sin(\varphi_c - \varphi)$. So, this is the circumferential loading and the loading in the longitudinal direction that is 0, so X = 0. So, knowing these 3 load components X, Y, Z, now we can proceed to find the membrane stresses.

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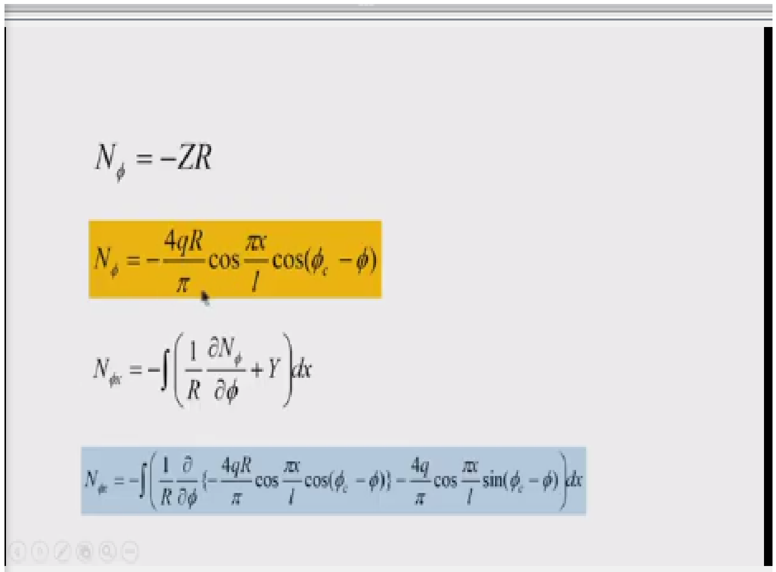

So, N_φ is directly obtained $N_\varphi = -ZR$. So, Z that is the radial load that we have obtained

$$N_\varphi = -\frac{4qR}{\pi}\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi)$$

So, you are getting this the expression for N_φ directly from this equation, there is no need to integrate anything. So, at any distance x or for a given $\frac{x}{l}$ ratio, where l is the span of the shell and knowing the angular distance π, where π is measured from the edge.

And φ_c is a given value for the shell that is the semi central angle, so φ varies from 0 to 2 φ_c, so range of φ will be 0 to 2 φ_c. So, under this condition because it is symmetrical, so we can only find up to the half part and then it will be similar thing for the other part because it is a symmetrical problem. N_φ is now $N_\varphi = -\frac{4qR}{\pi}\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi)$

So, at different values of φ for a certain $\frac{x}{l}$, you can now calculate the N_φ. That N_φ can be taken for the design purpose for checking the reinforcement in the tangential if it is reinforced concrete shell based on this N_φ we can now calculate this amount of steel needed in the shell. Then the membrane shear stress $N_{\varphi x}$, now it will be calculated from the equilibrium equation, if you see the equilibrium equation earlier that I have given let me here.

After finding N_θ, then we go to the 2nd equation. So, 2nd equation substitute $N_\theta = -ZR$ here. So, N_θ is completely determined, so we can now obtain $N_{x\theta}$ by integration. So, let us see how it is opting. So, $N_{x\varphi}$ or $N_{\varphi x}$ because this will be same equal to this integration of $\left(\frac{1}{R}\frac{\partial N_\varphi}{\partial \varphi} + Y\right) \times dx$. So, there the loading Y is coming and this N_φ is now obtained here, so we can substitute N_φ. So, N_φ is substituted you can see here $-\frac{4qR}{\pi}\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi)$, that is the N_φ.

And this quantity is the Y that is the component of the load in the tangential or circumferential direction. Now before integration let us differentiate. So, if you differentiate this quantity with respect to phi this term will be differentiated. So, it will be $\sin(\varphi_c - \varphi)$ and because the differentiation of cos with respect to cos phi with respect to phi will be accompanied with a negative sign, so negative sign will come. And again here if we differentiate this quantity with respect to phi the negative sign will be coming. So, this thing you have to note very carefully otherwise you will commit the mistake.

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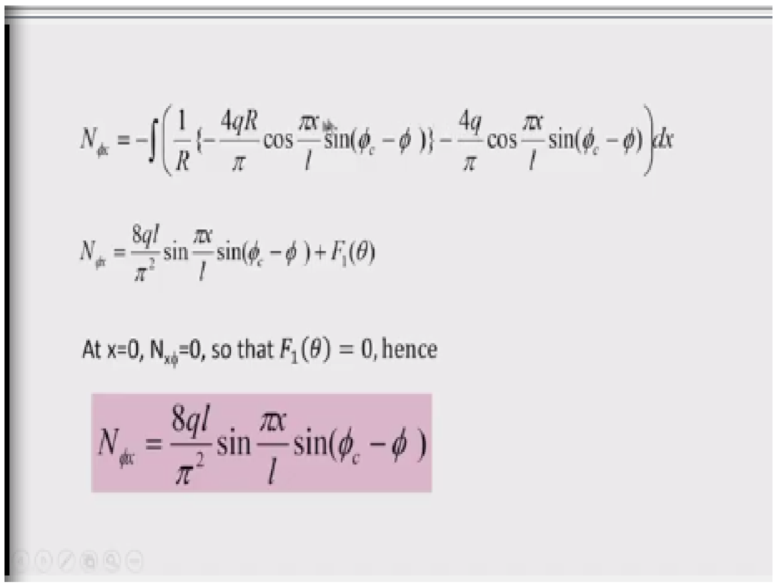

So, after this differentiation we get this

$$N_{\varphi x} = -\int\left(\frac{1}{R}\{-\frac{4qR}{\pi}\cos\frac{\pi x}{l}\sin(\varphi_c - \varphi)\} - \frac{4q}{\pi}\cos\frac{\pi x}{l}\sin(\varphi_c - \varphi)\right)dx$$

Now you can see this R, R will get cancelled, so ultimately when we integrate it with respect to dx and then we get this $N_{\varphi x} = \frac{8ql}{\pi^2}\sin\frac{\pi x}{l}\sin(\varphi_c - \varphi) + F_1(\theta)$

How π^2 is coming? Because if we add these 2 term after cancelling R, we will get this $-\frac{4qR}{\pi}\cos\frac{\pi x}{l}\sin(\varphi_c - \varphi)$ So, this quantity when it is integrated with respect to x that means we will integrate this parameter only, $\cos\frac{\pi x}{l}$. So, integration of $\cos\frac{\pi x}{l}$ will be $\sin\frac{\pi x}{l}$ and this $\frac{l}{\pi}$ factor will coming out, so therefore you are finding here $\frac{8ql}{\pi^2}$. So, because the $\frac{l}{\pi}$ is coming out we are getting here π^2. So, final expression for this is $N_{\varphi x} = \frac{8ql}{\pi^2}\sin\frac{\pi x}{l}\sin(\varphi_c - \varphi\ ) + F_1(\theta)$

or $N_{\varphi x} = \frac{8ql}{\pi^2}\sin\frac{\pi x}{l}\sin(\varphi_c - \varphi\ ) + F_1(\varphi)$. What is this $F_1(\varphi)$?

Because the constant of integration, now here constant of integration is taken as a function of theta because we are integrating with respect to x. So, therefore it is instead of taking a constant arbitrary constant we have taken it as a function of θ or φ. Now, we apply the boundary condition. So, we are considering a simply supported shell for our analysis and this condition we will assume here also.

Now, at x = 0 because x 0, origin we have taken at the centre of the shell which is a point of symmetry and here you can see that membrane shear stress is 0 because of symmetry. So, immediately we get $F_1(\theta)$ or $F_1(\varphi)$ = 0, so therefore, we can get completely

$$N_{\varphi x} = \frac{8ql}{\pi^2}\sin\frac{\pi x}{l}\sin(\varphi_c - \varphi\ )$$

So, that is one stress resultant that is completely found out.

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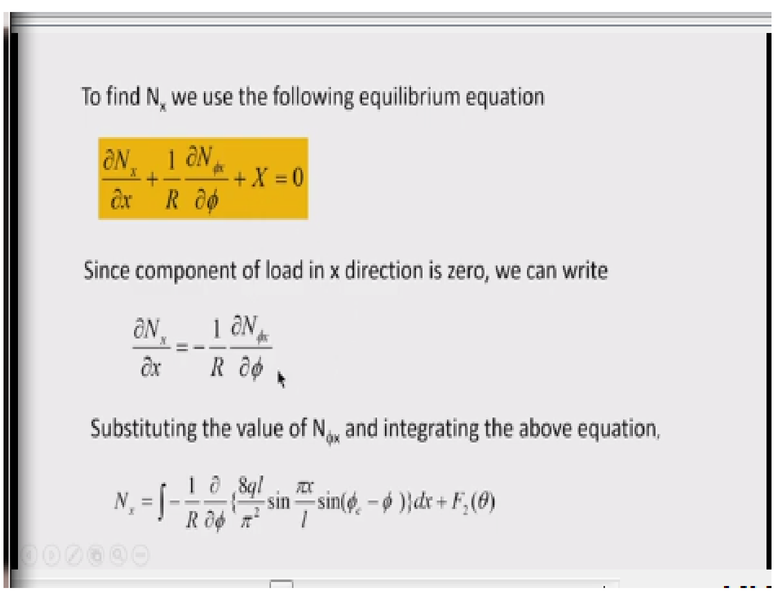

So, next we take the help of this equation.

$$\frac{\partial N_x}{\partial x}+\frac{1}{R}\frac{\partial N_{\varphi x}}{\partial \varphi}+X=0$$

Once we get $N_{\varphi x}$ then we are targeting to find this N_x the longitudinal stress. So, here we write it $\frac{\partial N_x}{\partial x}=-\frac{1}{R}\frac{\partial N_{\varphi x}}{\partial \varphi}$ and x is 0 because there is no component of the load in the longitudinal direction. Now substituting the $N_{\varphi x}$ whatever we got earlier, so this $N_{\varphi x}$ is here substituted and we write this equation. So, here 1 operation is remaining before integration, so this operation is that derivation. So, derivation of this quantity with respect to φ is carried out first and then integrated.

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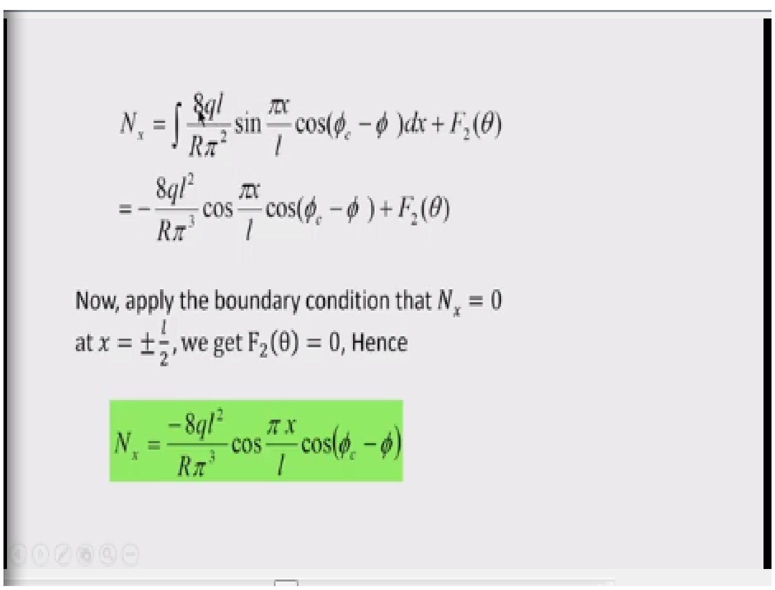

So, after taking the derivative of this quantity, we now get $\tau_{\varphi x}$ + a constant of integration, since we are we integrated with respect to x. So, we are taking a arbitrary constant as a function of φ

or θ. Now, this is integrated with respect to x and therefore this will be coming as $\cos\frac{\pi x}{l}$ and naturally a negative sign will come and the coefficient $\frac{1}{\pi}$ will be plugged here.

So, due to association of the coefficient that is coming from the integration of $\sin\frac{\pi x}{l}$ you are now getting $\frac{8ql^2}{R\pi^3}$. So, this is the coefficient and this is negative $\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi\ ) + F_2(\theta)$. So, this constant of integration now have to be found out. Now apply the boundary condition that $N_x = 0$. Now, this shell is supported at the ends that is the traverse, we call it reverse and reverse cannot receive any longitudinal load, so therefore N_x we take it as zero.

So, $x = \pm\frac{l}{2}$ the traverse is located, so putting $x = \frac{l}{2}$ here, if you put $x = \frac{l}{2}$ this becomes $\cos\frac{\pi}{2}$, so this term will get vanished, so $F_2(\theta)$ is again 0. So, now we finally arrive at the expression N_x. So, $N_x = \frac{-8ql^2}{R\pi^3}\cos\frac{\pi\, x}{l}\cos\left(\varphi_c - \varphi\right)$, so this is the expression for N_x.

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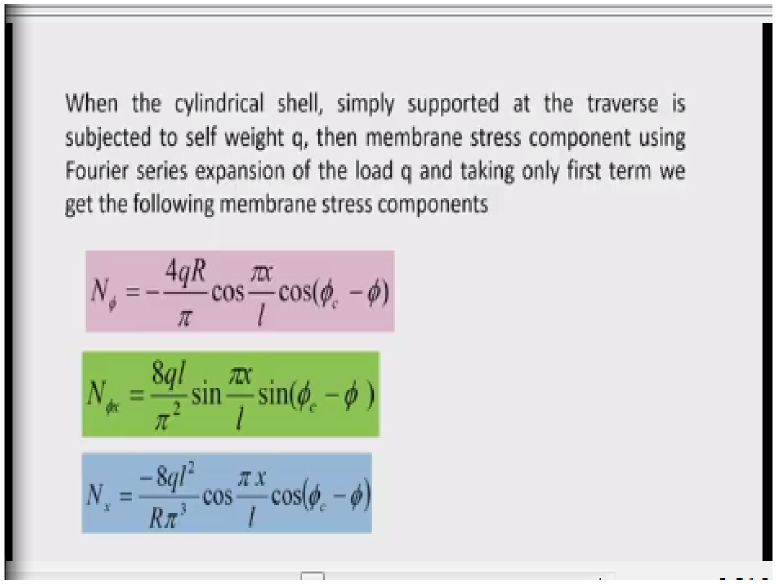

So, now we have obtained the 3 stress resultants N_φ, $N_{\varphi x}$, N_x, I am calling this as a stress resultant. But, in fact these are the membrane forces per unit length, you remember that the shell and plate, the all forces and moments we express in terms of appropriate unit per unit width or per unit length. So, N_φ is also the membrane force which is expressed suppose the force unit is Newton. So, N_φ will be expressed as Newton per metre or it may be say N expressed in Newton per millimetre as per the wish of the analyst.

So, $N_{\varphi x}$ is similarly expressed as Newton per millimeter, N_x is also expressed as Newton per millimetre. Now, if we want the stresses, suppose we want the shear stress, for example it is a concrete shell, so we want to check it for shear. What the permissible shear is exceeded or it is below the permissible shear of concrete of particular grade? So, in that case what we do actually, we will divide this $N_{\varphi x}$ divided by thickness of the shell.

Then $\frac{N_{\varphi x}}{h}$ will give you the $\tau_{\varphi x}$ that is the shear stress in the shell, that is the membrane shear stress. So, that shear stress can be checked with the design codes to comment on the safety of the shell against shear. So, like that other stresses can also be calculated and based on that the dimension of the shell reinforcement whatever quantities are required can be found out appropriately.

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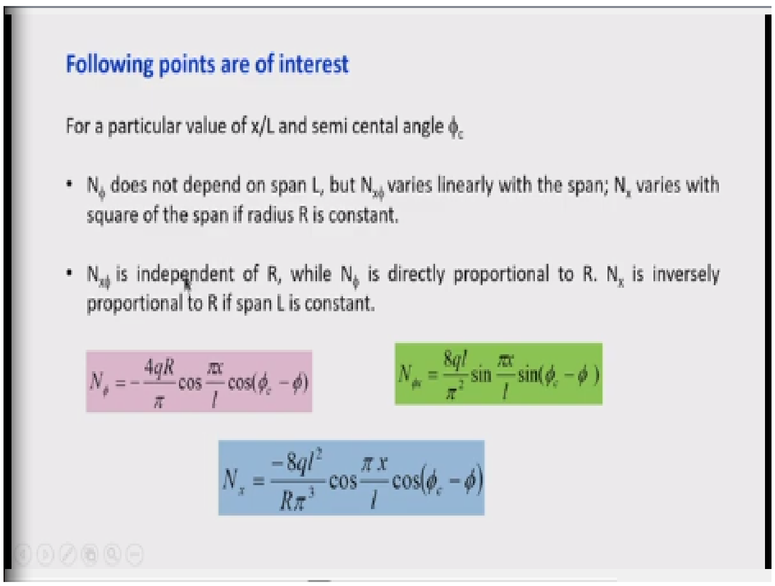

Now, following points are of interest. We have got these 3 stress resultants. Stress resultant is a general term actually this is the membrane forces. So, $N_\varphi = -\frac{4qR}{\pi}\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi)$, $N_{\varphi x} = \frac{8ql}{\pi^2}\sin\frac{\pi x}{l}\sin(\varphi_c - \varphi)$, $N_x = \frac{-8ql^2}{R\pi^3}\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi)$. So, 3 test results are there. Now you can see for a particular value of $\frac{x}{l}$ and for a given value of semi central angle φ_c.

Say for example, in a problem $\varphi_c = 60^0$, that means total angle subtended by the arc is 120^0 and at a particular ratio of x by l say $\frac{x}{l}$ = 0.25 at a quarter span. If we examine or any other value if we examine, then we can see that N_φ does not depend on span, it is very clear that for particular value of $\frac{x}{l}$ this quantity does not depend on the span length. But $N_{x\varphi}$ or $N_{\varphi x}$ varies linearly with the span and N_x varies with the square of the span if the radius is constant.

So, that characteristics we have noted from this 3 expression, then the second point is $N_{x\varphi}$ independent of R, you can see here there is no R that is radius of curvature of the shell appearing here while N_φ , N_φ is directly proportional to the radius of curvature. And you can also note that N_x is inversely proportional to R if l is constant that is span is constant. So, these are some points that you can note from the expression that we have derived.

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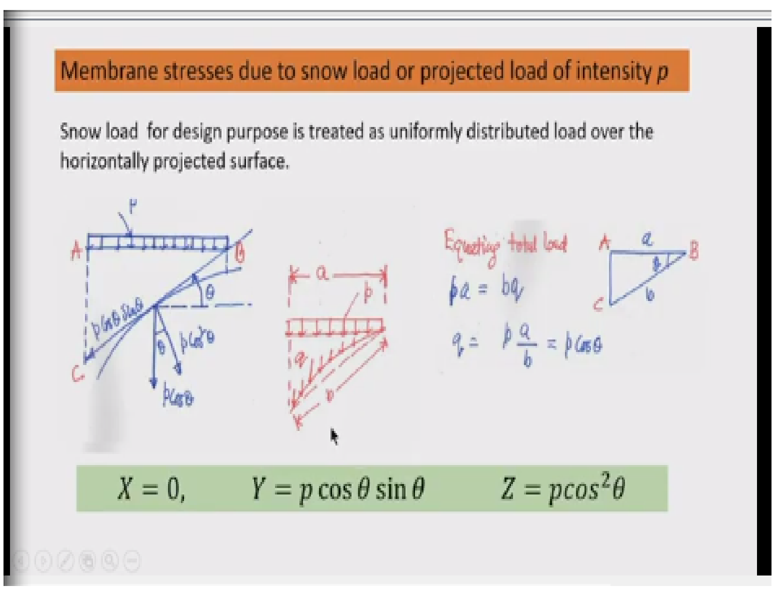

Now similar exercise can be carried out for finding the stresses due to snow load or projected load. Snow load we consider it as a projected load on the horizontal plane of intensity p. Suppose, this is the projected load p then the equivalent load along the length of the shell will be found as $p\cos\theta$, if p is the load and q is the equivalent load on the length of the shell, then it is found as $q = p\cos\theta$ and that is obvious from this simple relationship.

That is equating total load $pa = bq$ where the relation between a and b you can find from this triangle, so $\frac{a}{b} = \cos\theta$, so the q is nothing but $p\cos\theta$. So, this transformation is needed and other things are same. So, X = 0 because there is no load component along the longitudinal direction and then Y is $p\cos\theta\sin\theta$ and Z is $p\cos^2\theta$ that is obvious from this diagram.

So, total load earlier in case of gravity self weight or gravity load that we are considering self weight and live load. This was q and this component Z was $\mathrm{q}\cos\theta$, now this component is now $p\cos\theta$, so it becomes $p\cos^2\theta$. So, similarly the component of the load in the tangential direction is $p\cos\theta\sin\theta$, so 3 components we have got here.

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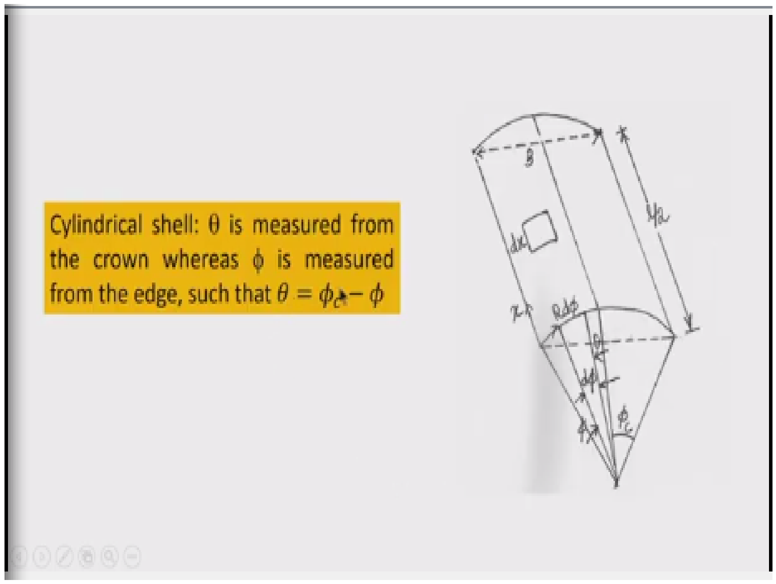

So, now in a similar way we can obtain this expression for the stress resultants.

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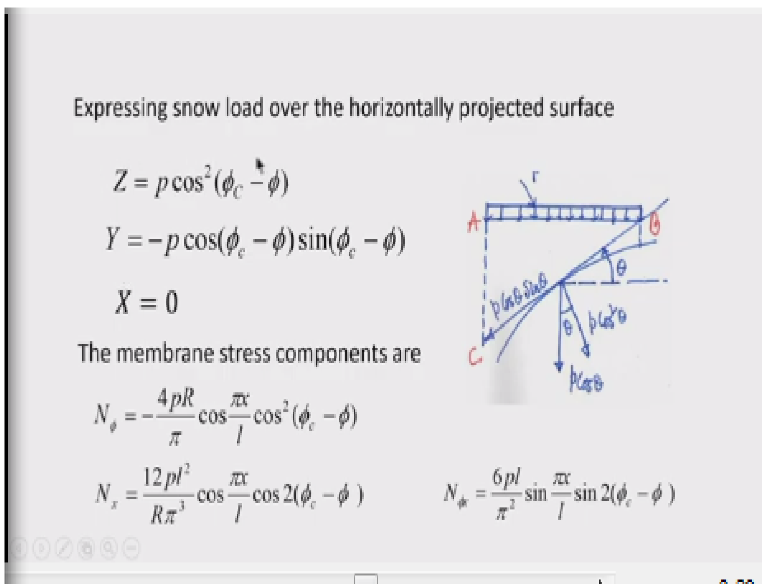

So, expression for stress resultants are calculated in a similar manner that we have carried out for the self weight, only the change is that the load quantity previously it was this $q\cos\theta$, now it is $p\cos^2\theta$, the radial load and this tangential load is $p\cos\theta\sin\theta$. So, it is very obvious that if the radial load is $p\cos^2(\varphi_c - \varphi)$ then N_φ is simply this -ZR, so that you know, so that have we obtain.

And we have already expressed the load into Fourier series. So, here the first term of the Fourier series is $\frac{4p}{\pi}$, p is the load $\frac{4p}{\pi}$ and it is component along this radial direction is $\frac{4p}{\pi}\cos^2(\varphi_c - \varphi)$. So, N_φ is this, N_x is this, N_x is found in this way, in the similar way that we have carried out, I am not repeating here, but you can easily come at this expression just by changing the load quantity. So, $N_{\varphi x} = \frac{6pl}{\pi^2}\sin\frac{\pi x}{l}\sin 2(\varphi_c - \varphi\ )$. So, these 3 expressions pertain to the projected loading that is the intensity is p.

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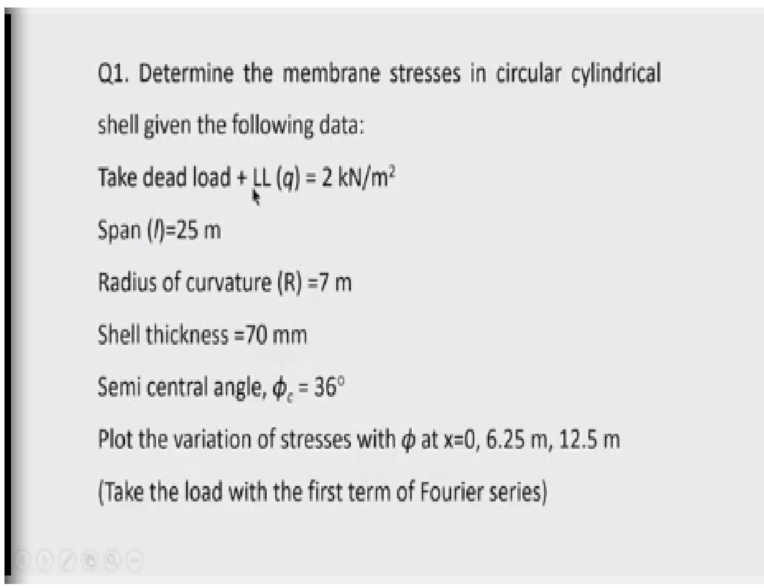

Now, let us see some problems. So, 1 problem is that determine the membrane stresses in circular cylindrical shell given the following data. Dead load plus live load is given as 2 kN/m^2 and it is assume to act along the shell surface, so it is not a projected load. Then span of the shell total span I have taken 25 metre, radius of curvature of shell because it is a circular directrix, so there is no variation of radius of curvature.

So, radius of curvature is taken as 7 metre, shell thickness I am taking here 70 millimetre, semi central angle that is φ_c is 36^0. Now we want to see the variation of stresses say first we take variation with φ, how it varies with the φ ? φ is measured from the edge if you go φ to say φ_c, then we will get complete picture of the stresses and then other part will also be similar because it is a symmetrical problem. So, at different longitudinal distance at x = 0 and x = 6.25 and x = 12.5 metre. So, that means $\frac{x}{l}$ is fixed. So, here $\frac{x}{l}$ is 0, here you can see $\frac{x}{l}$ is one fourth and here you can see $\frac{x}{l}$ is half.

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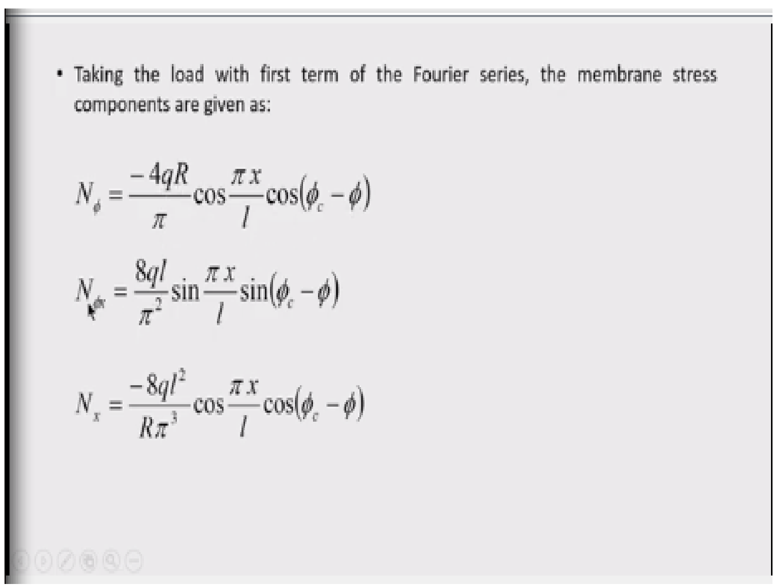

So, taking these data, we proceed to calculate the membrane forces N_φ taking this expression,

$$N_\varphi = \frac{-4qR}{\pi}\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi),$$

$$N_{\varphi x} = \frac{8ql}{\pi^2}\sin\frac{\pi x}{l}\sin(\varphi_c - \varphi)$$, then

$$N_x = \frac{-8ql^2}{R\pi^3}\cos\frac{\pi x}{l}\cos(\varphi_c - \varphi)$$

So, we will utilize this 3 quantities for 3 different values of $\frac{x}{l}$. First we will consider $\frac{x}{l}$ is 0, so in that case this term is 0. So, you are getting in the first case, that is at the crown where these x = 0.

So, you are getting at the centre, x = 0, you are getting $N_\varphi = \frac{-4qR}{\pi}\cos(\varphi_c - \varphi)$

And you can see here this quantity is completely 0 for any variation of φ because it is a point of symmetry. So, it is obvious that at x = 0, this shear stress does not exist because of symmetry.

And N_x, again it will be $N_x = \frac{-8ql^2}{R\pi^3}\cos(\varphi_c - \varphi)$. So, these 3 values are for $\frac{x}{l}$ = 0.

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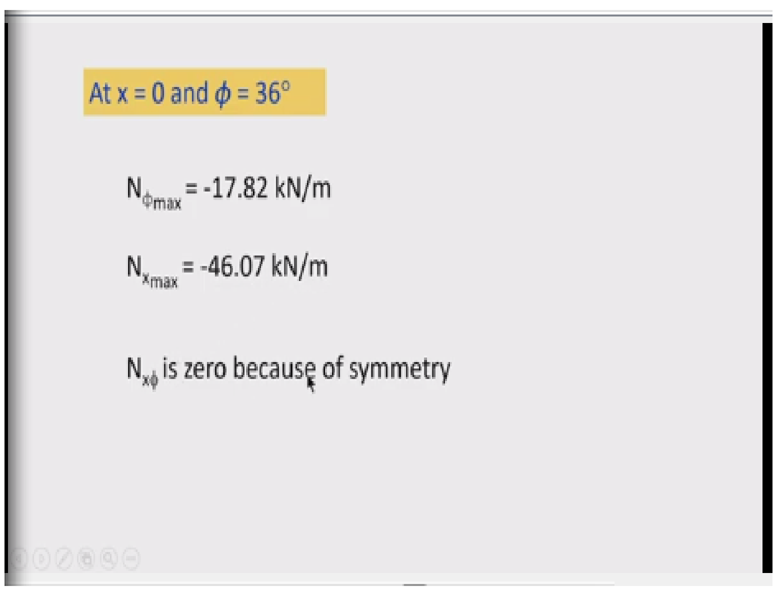

So, if we find for a semi central angle that is given here is 36^0, we find

$N_{\phi max}$ = -17.82 kN/m

N_{xmax} = -46.07 kN/m

and $N_{\varphi x}$ is 0 because of symmetry.

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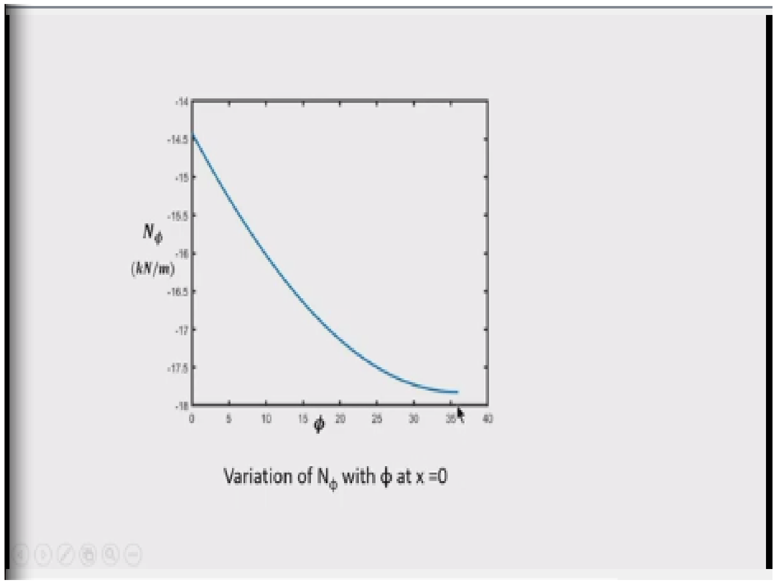

If we see the variation of stresses, then this stress variation is like that from 0 to φ_c . So, here it will be maximum value 14.5 and gradually decreasing and it is coming say value of 14.5 is the minimum value here. Because these are plotted on the negative scale and maximum value here you are finding that we have shown here 17.82. The stresses are plotted on the negative scale.

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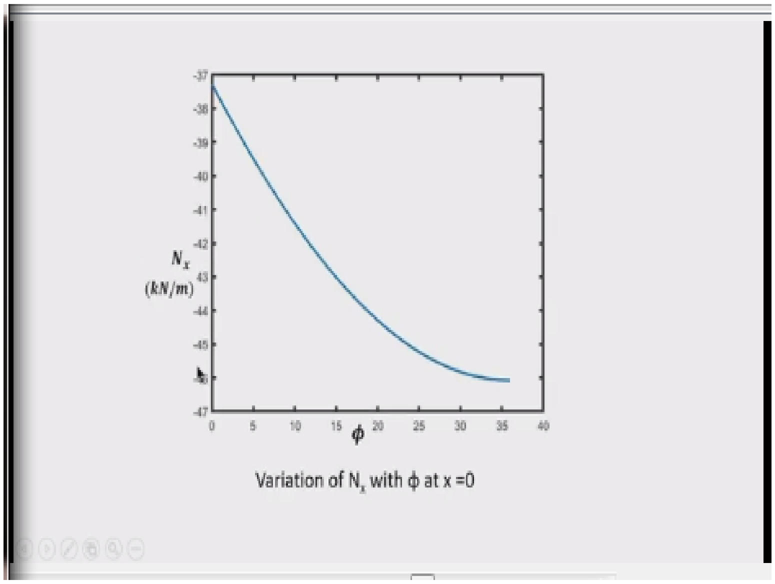

Then, let us see the variation of N_x, N_x is also the maximum value that you are getting here at φ_c are the crown and the value is tabulated here 46.07 and here also you are seeing these value is the maximum. So, N_x is this, the variation of N_x the longitudinal stress. In all cases, we are finding the magnitude of the maximum value is at the crown. So, this variation pertains to the x coordinate 0 that is at the centre of the shell.

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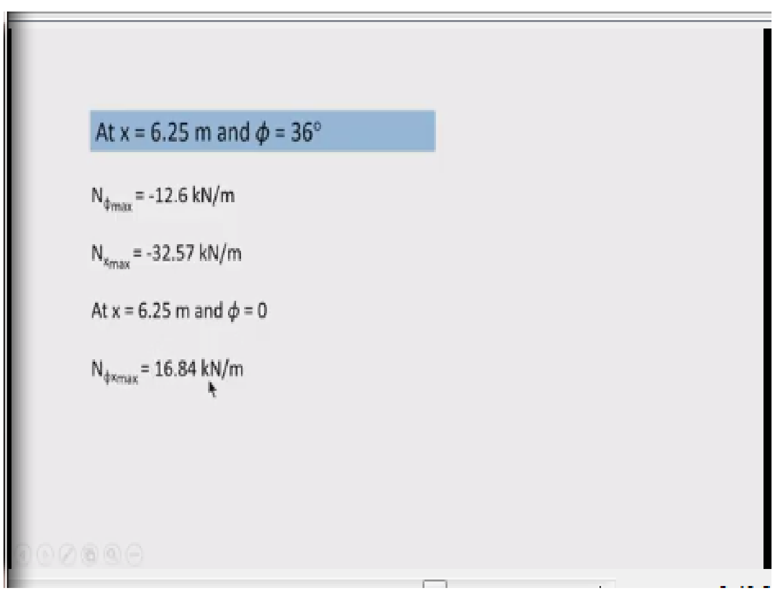

Now let us see the variation at x = 6.25, that is $\frac{x}{l}$ ratio is 0.25, 6.25 divided by 25 it will be 0.25.

So, $\frac{x}{l}$ = 0.25, we see the variation, $N_{\varphi \max}$ is -12.6, $N_{x\max}$ value is -32.57 and at φ = 0 we have obtained this $N_{\varphi \max}$ = 16.84.

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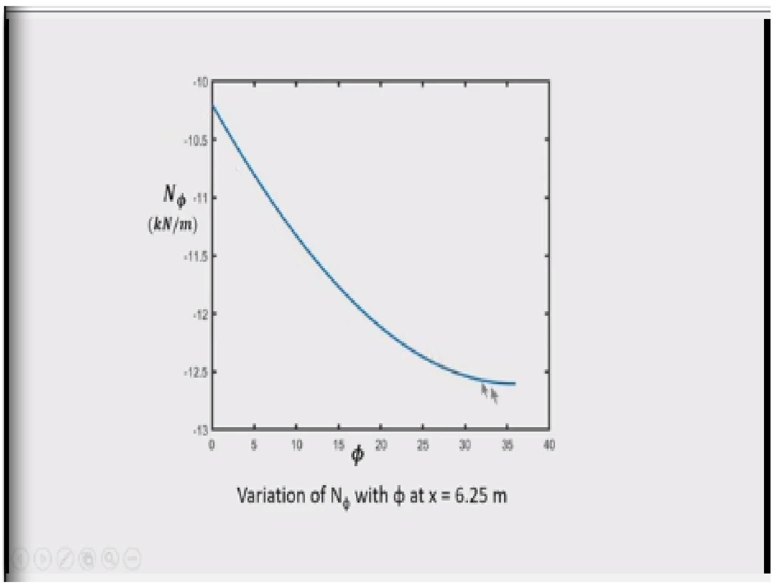

So, this variation if you see, you again it displays the similar kind of curve pattern. So, variation because the same expression we used only the change of parameter is changed, but the expression is same, so same nature of the curve we have seen here.

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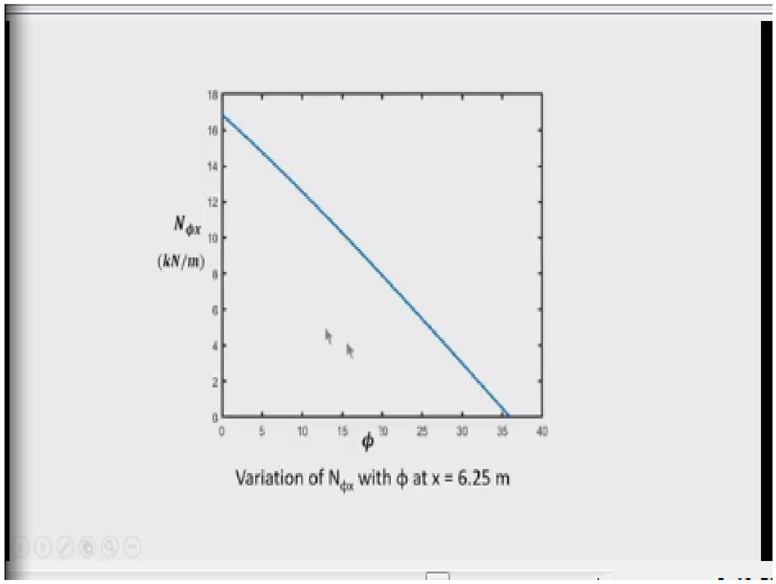

So, $N_{\varphi x}$ is also like that, maximum value is already noted.

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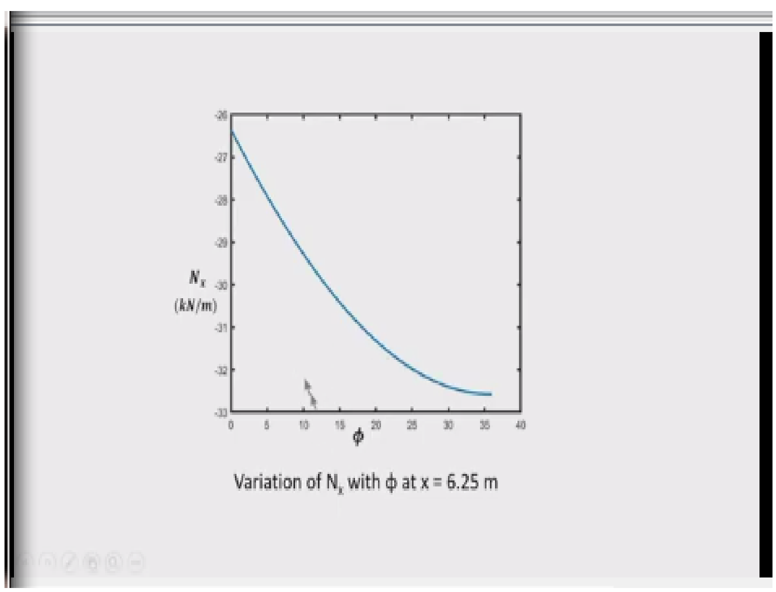

And this N_x, the variation of this longitudinal force in the shell is this. At all these variations are at x = 6.25 and the stress variations are shown with respect to φ up to semi central angle.

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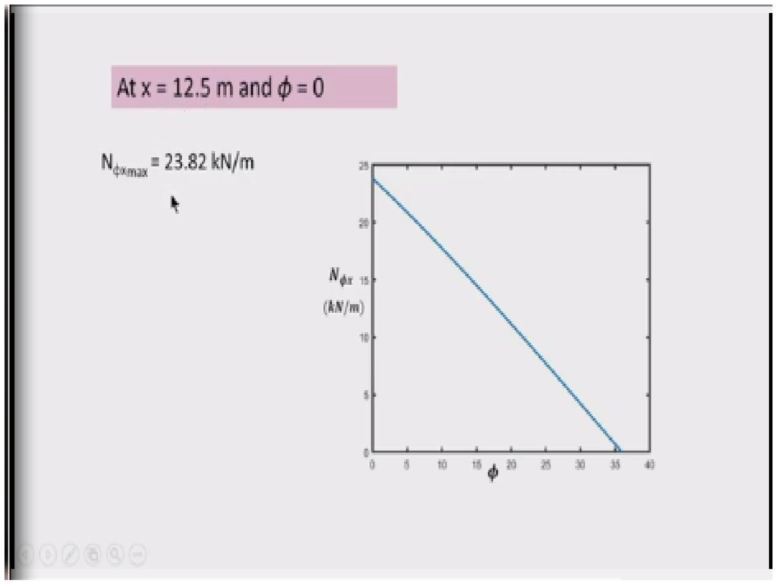

Next, at x = 12.5, that is $\frac{x}{l}$ ratio that you can tell if at l is 25 metre then it is 12.5/25, you can see 0.5. So, here this value you can see, how it varies? Maximum value is 23.82. So, this is obtained from direct application of the expression that we have obtained, but derivation is important and obtaining the Fourier coefficients is important. Numerical problem is just application of this equation that we have obtained and to plot this graph.

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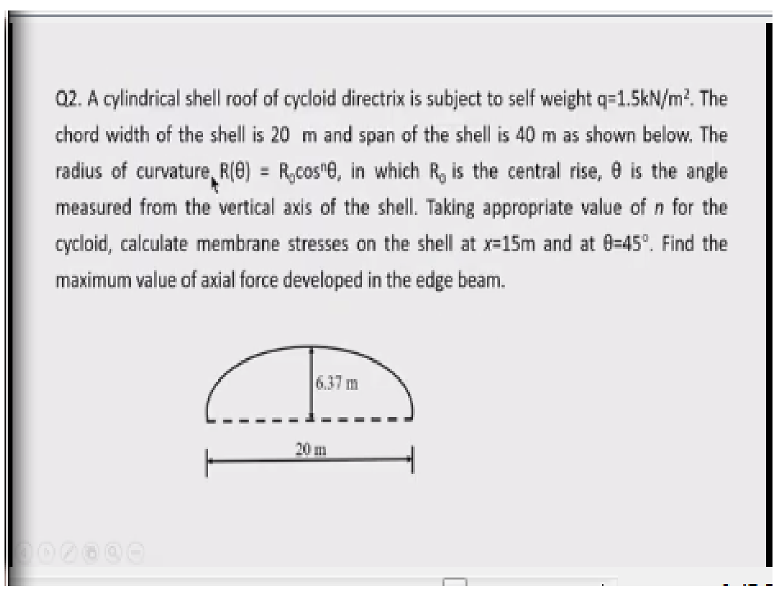

Next, let us see a problem of cylindrical shell roof. A cylindrical shell roof of cycloid directrix is subject to self weight q = 1.5 kN/m^2. The chord width of the shell is 20 metre, so this is the chord width and span of the shell is 40 metre. The span is of course here not shown, the radius of curvature here we take the intrinsic variation that is intrinsic equation that R is a function of θ radius of curvature and it is given by R_0 that is the radius of curvature at crown into cos raise to the power N_θ, where R_0 is the central rise.

And here of course it is a radius of curvature at the crown and θ is the angle measured from the vertical axis of the shell. Taking appropriate value of N for the cycloid, calculate membrane stresses on the shell at x = 15 metre and θ = 45 metre, find the maximum value of axial force develop in the edge beam. Suppose this shell is joined at the edges, of course the edges bending moment will be developed.

But the transfer of membrane stresses from shell to edge beam will cause some tension in the edge beam, so that we want to find out here. Now one thing you can note here, the cycloid is a curve that is formed when a point on a circle rolls along a straight line without slipping. So, therefore this distance if the radius of the circle is a, this distance will be $2\pi a$ and this the central rise here you can see, it will be 2a. So, accordingly if this distance is 20 metre, this chord width that is fixed, so this distance will be this is $2\pi a$, so 20 divided by π, so that will be 6.37.

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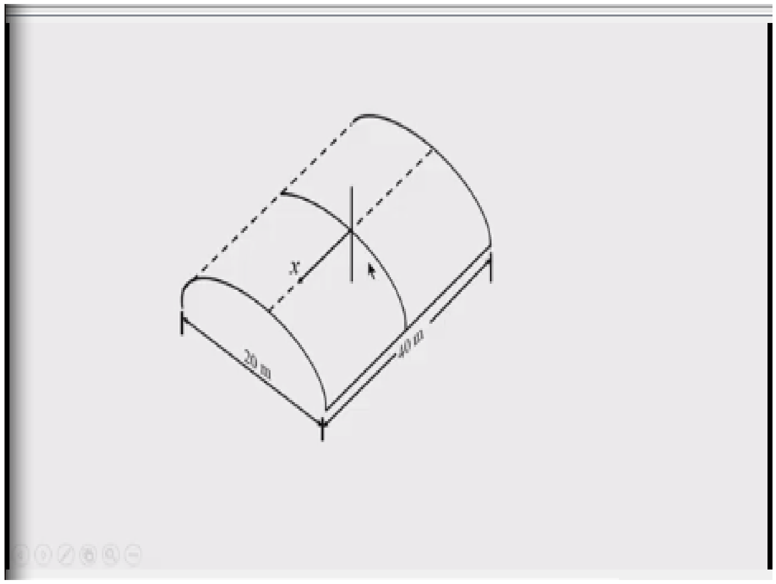

So, this is the shell isometric view of the shell 3D view, you can see at the span 40 metre.

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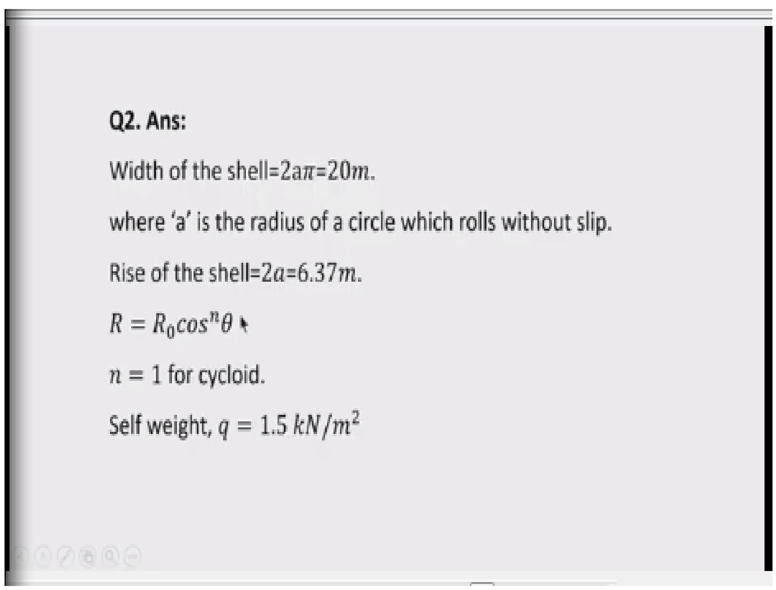

So, width of the shell is given this and this rise of the shell is this calculated, like that. So, if you refer my last lecture then we have obtained the n for different types of directrices like parabola, catenary, cycloid and also circle. For circle n = 0 because radius of curvature is constant at any angle θ. Now for cycloid type of directrix we have obtained n = 1. Now self weight q = 1.5 kN/m^2.

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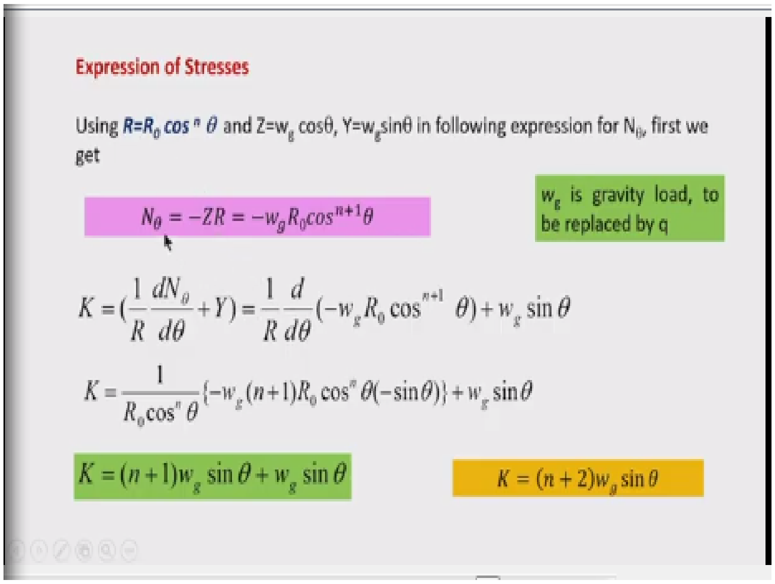

Application of these numerical data in the expression of stresses because the N_θ first we find out the tangential stress, circumstantial stress N_θ. Say N_θ = -ZR, so $-w_g R_0 \cos^{n+1}\theta$ $-w_g R0cos^{n+1}\theta$ because the Z, the component of the load in the radial direction is w g cos theta, so therefore this expression becomes this. w_g is the gravity load to be replaced by q, that in the earlier slide we have shown it.

And if you see the expression for N_θ in the earlier classes and also in the first 2 slides I have use this expression N_θ = -Kx. So, K value is this

$$K = (\frac{1}{R}\frac{dN_\theta}{d\theta} + Y) = \frac{1}{R}\frac{d}{d\theta}(-w_g R_0 \cos^{n+1}\theta) + w_g \sin\theta$$

N_θ we are substituting from here and this Y is $-w_g \sin\theta$. Here of course we are not using any Fourier expansion of the loading, just by normal self weight whatever is acting we are using. So, K is found out like that and K is

$$K = (n + 2)wg \sin sin\,\theta$$

So, after step by step derivation, you can arrive here $K = (n + 2)wg \sin sin\, \theta$. Now for cycloid type of directrix n = 1, so therefore k becomes $3w_g \sin\theta$, so that is the tricks. So, now you obtain $3w_g \sin\theta$ as K and when you required the derivative of K, it will be simple the derivative of $\frac{dk}{d\theta} = 3w_g \cos\theta$.

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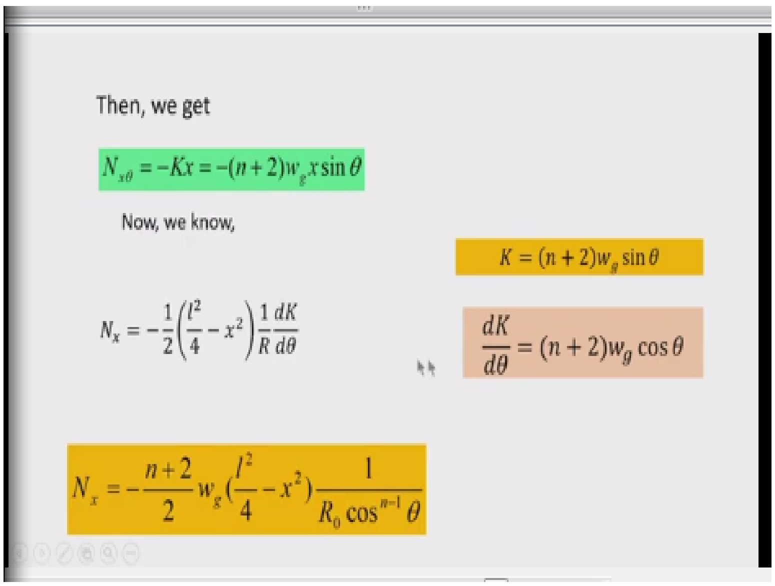

So, we have obtained this

$$\frac{dK}{d\theta} = (n + 2)w_g \cos cos\, \theta$$

I am putting n = 1, it will be $3w_g \cos\theta$. And the expression for $N_{x\theta}$ is

$$N_{x\theta} = -Kx = -(n+2)w_g x \sin\theta$$

.

Then expression for this N x

$$N_x = -\frac{1}{2}\left(\frac{l^2}{4} - x^2\right)\frac{1}{R}\frac{dK}{d\theta}$$

And you can see that this $\frac{dk}{d\theta} = 3w_g \cos\theta$, how 3 is coming?

Because n = 1 for a cycloid type of shell, cycloid type of directrix, so therefore the $\frac{dk}{d\theta} = 3w_g \cos\theta$. So, after substituting these value we now finally get the N_x here in this fashion.

$$N_x = -\frac{n+2}{2} w_g (\frac{l^2}{4} - x^2) \frac{1}{R_0 \cos^{n-1}\theta}$$

Here also we utilize the value of n, so if I substitute n = 1 here, then it becomes

$$N_x = -\frac{3}{2} w_g (\frac{l^2}{4} - x^2) \frac{1}{R_0}$$

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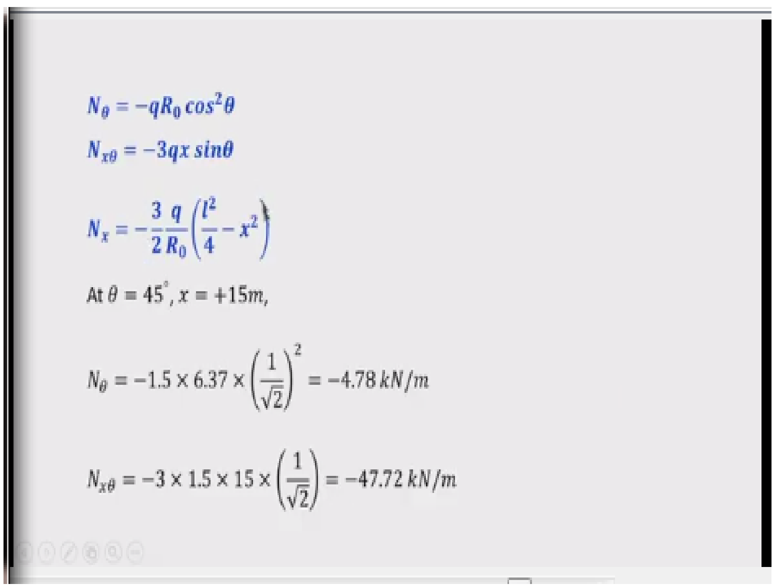

So, substitute the numerical values that is given in the problem. So, at $\theta = 45^0$, x = +15 metre, so we are getting N_θ = -4.78 kN/m. N_x theta we are getting -47.72 kN/m after a substitution of the given physical parameters of the shell given in the problem. $\frac{1}{\sqrt{2}}$ is coming because cos 45^0 or sin 45^0 whatever we take is $\frac{1}{\sqrt{2}}$.

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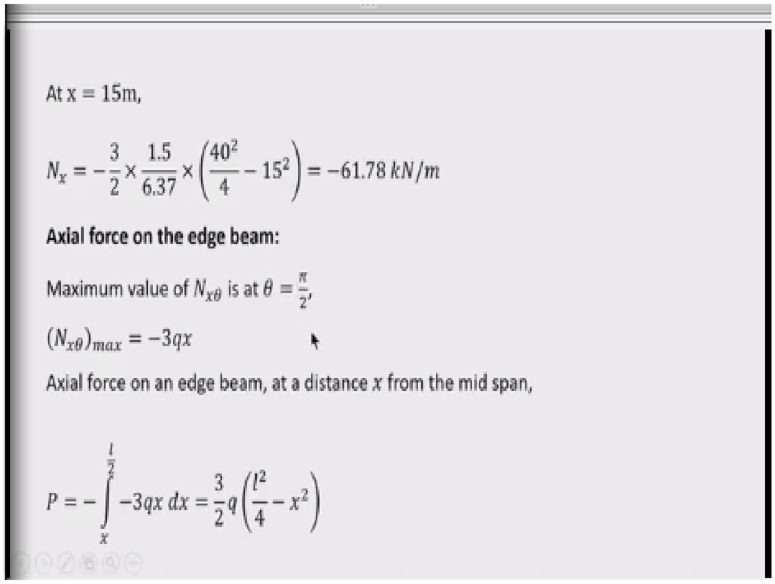

At x = 15 metres similarly we get this result

$$N_x = -\frac{3}{2} \times \frac{1.5}{6.37} \times \left(\frac{40^2}{4} - 15^2\right) = -\ 61.78\ kN/m$$

Now question comes how we determine the axial force on the edge beam? You can see the axial force is transferred on the edge beam due to membrane shear stress and $N_{x\theta}$ that is at the edge which is located at $\theta = \frac{\pi}{2}$ is simply given by this. Because we have found that $N_{x\theta}$ equal to if we put n = 1 then it will be $3w_g \sin\theta$.

So, at the edge $\theta = \frac{\pi}{2}$, so we are getting $3w_g x$. So, therefore w_g is replaced here as q. So, N $N_{x\theta}$ is $3qx \sin\theta$ and substituting $\theta = \frac{\pi}{2}$, we get -3qx as the edge shear force here. So, this is integrated along the length of the shell from any point say x to $\frac{l}{2}$ up to the end. So, after integration we are finding

$$P = -\int_{x}^{\frac{l}{2}} -\ 3qx\,dx = \frac{3}{2}q\left(\frac{l^2}{4} - x^2\right)$$

Now you can see here it is a tensile force and the maximum value you can see that is obtained at x = 0.

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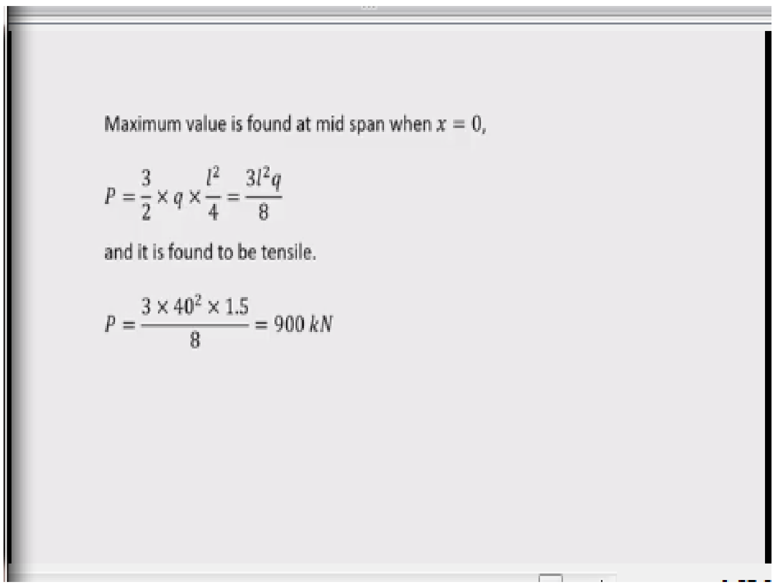

So, at x = 0 the maximum force at the mid span maximum force at x = 0 that is obtained on the edge beam

$$P = \frac{3}{2} \times q \times \frac{l^2}{4} = \frac{3l^2q}{8}$$

, 1.5 I have taken 3/ 2 and this also you can simplify it and you can write in decimal places. So, substituting the value of l and q, we now obtain this

$$P = \frac{3 \times 40^2 \times 1.5}{8},$$

so this is 900 kN. So, axial force that is the tensile nature is transferred on the edge beam or edge member whose magnitude is 900 kN

So, based on that magnitude we can calculate if it is RCC structure, we can calculate the reinforcement requirement on the beam due to transfer of membrane stresses at the edge member from this shell.

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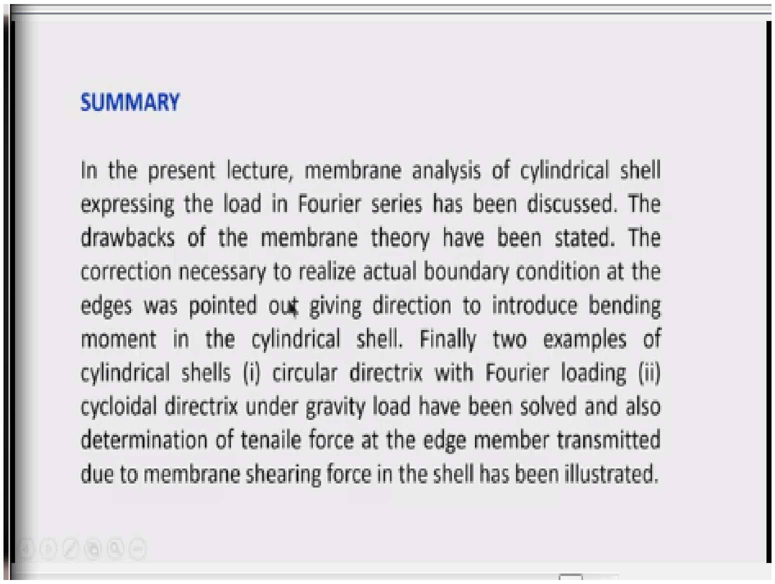

So, let us summarize this today's lecture. So, today's lecture we have discussed the membrane analysis of cylindrical shell using the load in Fourier series form. So, that is the new edition in this chapter membrane theory of cylindrical shell. The drawbacks of the membrane theory have been stated, the correction necessary to realize actual boundary condition at the edge was discussed giving a direction to introduce bending moment in the cylindrical shell. Because corrective load when you apply, this will produce a bending moment.

Finally, 2 examples of cylindrical shells, one is circular directrix with Fourier loading and second one is cycloidal directrix under self weight have been solved. And also determination of tensile force at the edge member transmitted due to membrane shearing force in the shell has been illustrated. So, you have learned the analysis of the membrane stresses in the cylindrical shell group and also the force that is transferred from the shell to the edge member. So if one uses the membrane theory for the design of shell group one can analyze like that for gravity load, self weight, live load and also for snow load. Thank you very much.