Plates and Shells

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Module-11
Lecture-32
Membrane Analysis of Cylindrical Shell Roof Subjected to Self Weight and Snow load

Hello everybody, today it is my 2nd lecture of the module 11. In the module 11 I was discussing the membrane theory of cylindrical shell. In the last class I derived the general equations for the equilibrium of cylindrical shell. And today I will go for the analysis of cylindrical shell subjected to vertical loading, that is as a self weight along the surface of the shell and also this snow load along the projected area. So, today our discussion will be on the derivation of the expression of these membrane stresses for cylindrical shell roof subjected to self weight and snow load.

And we will be incorporating the different types of directrices in the expression. So, a general expression is targeted to find out the stresses membrane stresses in this cylindrical shell roof. And with the help of the general expression, we can apply this formulation to any type of directrices. Because cylindrical shell, this is actually formed when the straight line generator moves over a directrix, that is the plain curve. So, the different types of directrices are possible and we will see the some common type of directrices for the analysis of cylindrical shells.

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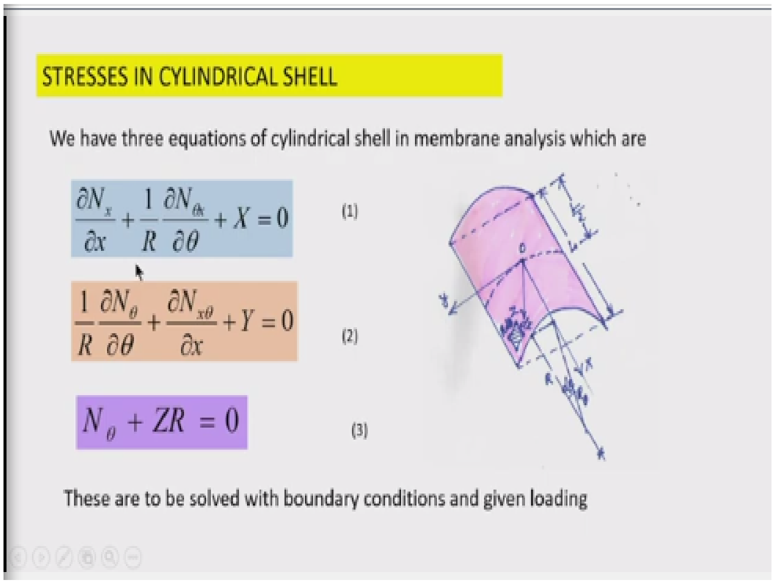

Now the equations of equilibrium in 3 directions have been obtained in the last class and these equations are in this form. The first 2 are the differential equations relating the membrane forces in the longitudinal direction that is N_x and $N_{\theta x}$ with the component of the load along the longitudinal direction. The 2nd equation is the equation of equilibrium in the y direction, y direction is the direction along the tangent to the directrix. So, this 2nd equation connects the membrane force N_θ and $N_{x\theta}$ and the load in the y direction.

Third equation you can see, it is an algebraic equation and it is directly giving you the value of $N_\theta = -ZR$, what is Z? Z is the component of the load along the radial direction. Z is the radial direction that is the direction directed towards the centre of curvature. So, these equations need to be solved with the help of boundary conditions and given loading, then we will be able to know the value of the membrane stresses which can be used for design purpose.

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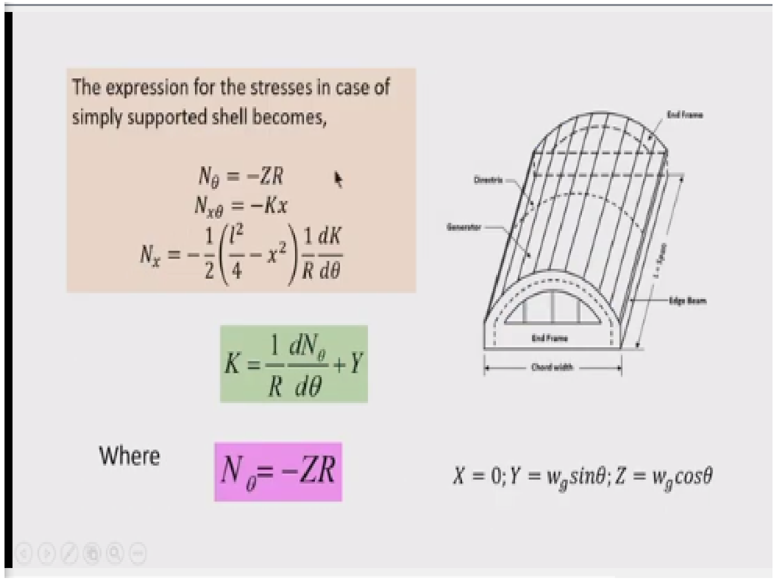

The equation of stresses in case of simply supported shell have been obtained in my last class also. And we have seen that for simply supported edge, means here the simply supported condition exist at the traverse, traverse at the support at the 2 ends of the shell. So, the simply supported condition at the traverse the 2 conditions have been imposed, one is the N_x is 0 and another is $N_{x\theta}$ is 0.

So, based on that 2 conditions, we have obtained the constants of integration and the final expression for N_θ, $N_{x\theta}$ and N_x are given here for simply supported cylindrical shell. So, $N_\theta = -ZR$ where R is the radius of curvature at any point that is point is located by the distance x and the angle measured from the vertical axis passing through the centre. And K is a constant that K is given by $w_g \sin\theta$, Y is the component of the load along the tangential direction to the meridian.

So, with the help of this equation, we can obtain the membrane stresses for cylindrical shell, if the ends are simply supported. Ends are supported on traverse and traverse may be of different form this may be of solid diaphragm type or it may be a tide arch or it may be a trust arch. So, here you are seeing it is a tide arch form. So, the condition here is that N_x is 0 $N_{x\theta}$ is 0, origin is taken at the centre of the shell.

So, if origin is here then boundary condition is specified at x = $+\frac{l}{2}$ and l is the length of the shell or span of the shell. Component of the load along 3 principle directions are given as x = 0, X is the component of load, Y is the component of the load along the tangential direction to the directrix that is $w_g \sin\theta$ and Z is the component of the self weight that is $w_g \cos\theta$, if w_g is the self weight acting on this shell. However, this expression will slightly alter when this snow load is considered because snow load is assumed to act over the horizontal projected area.

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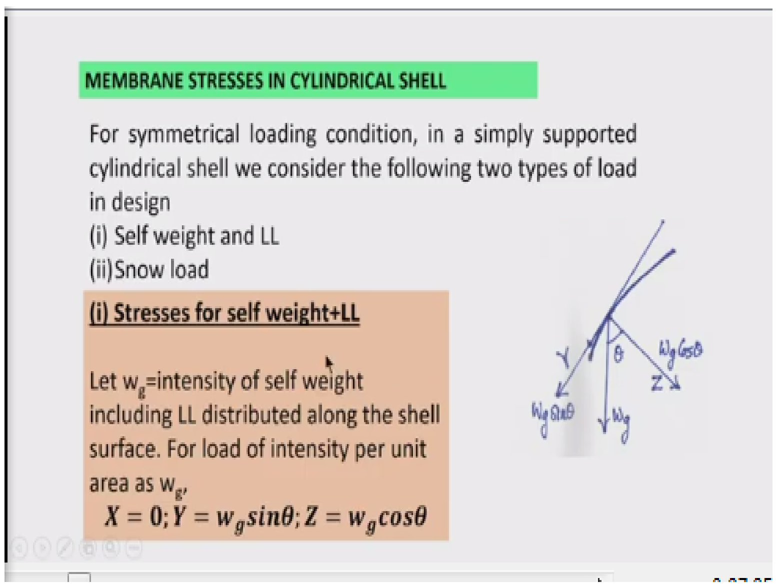

Let us now calculate the membrane stresses individually for self weight and this self weight + live load and snow load. First we will cover the self weight plus live load and then we will go for snow load. Now here you can see that a arc of a portion of this shell is represented here and the w_g is the self weight acting at the point vertically downward and it is component along the radial direction is $w_g \cos\theta$ and in the meridional direction it is this $w_g \sin\theta$.

Because this angle is Θ, so after resolving this in the radial direction it becomes $w_g \cos\theta$ and resolving in the tangential direction at this point it becomes $w_g \sin\theta$. So, the component of the loads are specified and X is 0 the longitudinal force along the shell surface is assumed to be 0.

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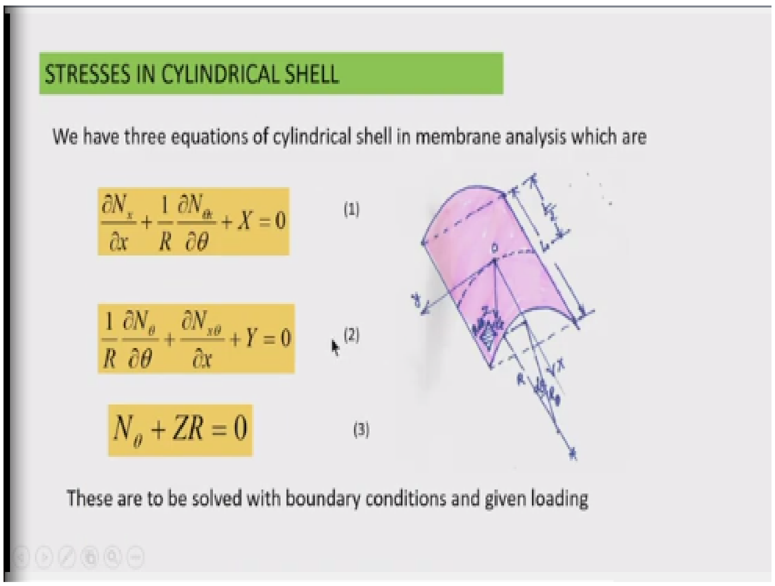

So, these are the equations of equilibrium, from that we get step by step the different stresses. First I shall go for obtaining N_θ, so N_θ = -ZR, so N_θ is directly given by $-w_g \cos\theta$ theta because this is the component of the load along the radial direction that is what is Z, you can see here Z. So, substituting Z here in R, R is a function of angle θ, so therefore R is written in terms of θ. But when we consider a circular cylindrical shell R is a constant, so it does not vary with theta.

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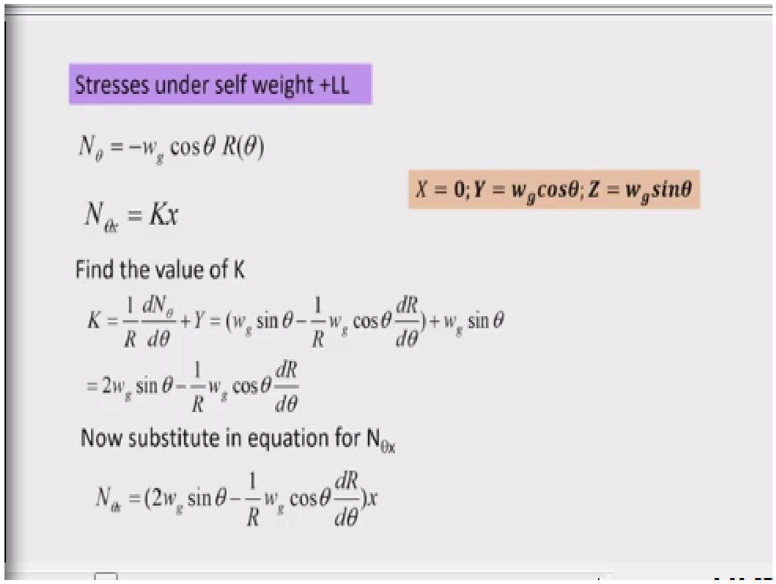

Then we shall go for calculating $N_{\theta x}$, the membrane shear force. Now, you can see in my earlier slide that $N_{x\theta}$ = -Kx where $K = \frac{1}{R}\frac{dN_\theta}{d\theta} + Y$, Y is the component of the load along the tangent to the directrix. So, the $N_{\theta x}$ is given by Kx. Now, K is $K = \frac{1}{R}\frac{dN_\theta}{d\theta} + Y$, so this is the value of K. Now substitute this value of N_θ , N_θ we already obtained - $w_g \cos\theta R\theta$.

So, after differentiating with respect to theta and then substituting the value of Y as $w_g \cos\theta$, we get this expression. And after simplification the final expression is $2w_g \sin\theta - \frac{1}{R} w_g \cos\theta \frac{dR}{d\theta}$ 2w. Because here you can see when we differentiate this expression K, this N_θ which is given by this, the 2 functions are involved here one is cos theta and another is $R\theta$.

So, you have to differentiate suppose if you differentiate first cos theta then $R\theta$ will remain as it is. Then secondly you differentiate $R\theta$ and $\cos\theta$ will remain associated as coefficient, so you are getting this term. Therefore, $\frac{dR}{d\theta}$ term is coming because R is a function of θ, when this R is constant for in case of circular cylindrical shell then this term vanishes.

So, after obtaining K, we now write the expression for $N_{\theta x}$. So, $N_{\theta x}$ is given by $(2w_g \sin\theta - \frac{1}{R} w_g \cos\theta \frac{dR}{d\theta})x$. So, you can see these quantities are dependent on theta and this w_g is constant quantity. So, at any position, at any meridional angle the $N_{\theta x}$ is linearly varying with x. So, that is noted from this expression.

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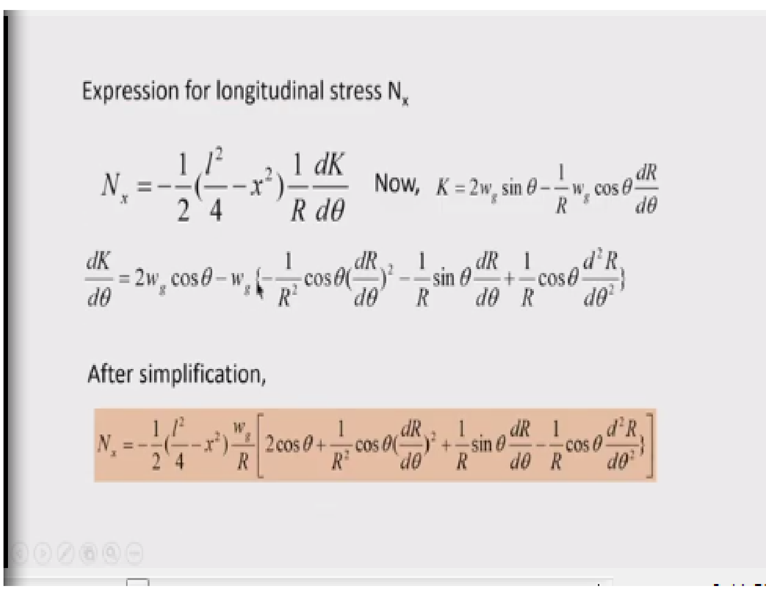

Expression for longitudinal stress N_x. So, longitudinal stress N_x for simply supported shell we have obtained as this $N_x = -\frac{1}{2}(\frac{l^2}{4} - x^2)\frac{1}{R}\frac{dK}{d\theta}$. So, this expression now we will take to find out the expression for longitudinal stress. So, N_x is written as $N_x = -\frac{1}{2}(\frac{l^2}{4} - x^2)\frac{1}{R}\frac{dK}{d\theta}$. Now, this K is obtain as this $K = 2w_g \sin\theta - \frac{1}{R} w_g \cos\theta \frac{dR}{d\theta}$.

So, $\frac{dK}{d\theta}$ can be obtained by differentiating this expression with respect to theta. So, if I differentiate this expression with respect to theta then I will get

$$\frac{dK}{d\theta} = 2w_g \cos\theta - w_g \{-\frac{1}{R^2}\cos\theta(\frac{dR}{d\theta})^2 - \frac{1}{R}\sin\theta\frac{dR}{d\theta} + \frac{1}{R}\cos\theta\frac{d^2R}{d\theta^2}\}$$

w_g I have taken as a factor because it is a constant term then I have to differentiate 3 terms $\frac{1}{R}\cos\theta\frac{dR}{d\theta}$. So, these 3 terms are in the product form, so I have to differentiate using the differentiation rule of product of functions.

So, here it is differentiate first I differentiated $\frac{1}{R}$. So, $\frac{1}{R}$ when I differentiate it, it becomes $\frac{1}{R^2}$ then $\frac{dR}{d\theta}$ will be there. So, $\frac{dR}{d\theta}$ again combined with this it will give $\left(\frac{dR}{d\theta}\right)^2$ whole square and $\cos\theta$ will be here as it is. Then I will go for differentiating the other quantity. So, $\cos\theta$ I differentiated, so it becomes sine theta and then these $\frac{dR}{d\theta}$ is there and $\frac{1}{R}$ is there.

After that the differentiation is done on this term $\frac{dR}{d\theta}$ and it becomes now d^2R, that is second derivative $\frac{d^2R}{d\theta^2}$ and other terms remain as it is $\frac{1}{R}\cos\theta$. So, we have got the term $\frac{dK}{d\theta}$, now it is possible to write the expression of N_x. So, the expression for N_x becomes

$$N_x = -\frac{1}{2}(\frac{l^2}{4} - x^2)\frac{w_g}{R}\left[2\cos\theta + \frac{1}{R^2}\cos\theta(\frac{dR}{d\theta})^2 + \frac{1}{R}\sin\theta\frac{dR}{d\theta} - \frac{1}{R}\cos\theta\frac{d^2R}{d\theta^2}\}\right]$$

So, the value of N_x $N_{\theta x}$ and N_θ are known in general form for this self weight. Now depending on the directrices, the value of R and this R will change and this will be substituted in the proper places to evaluate the value of membrane stresses.

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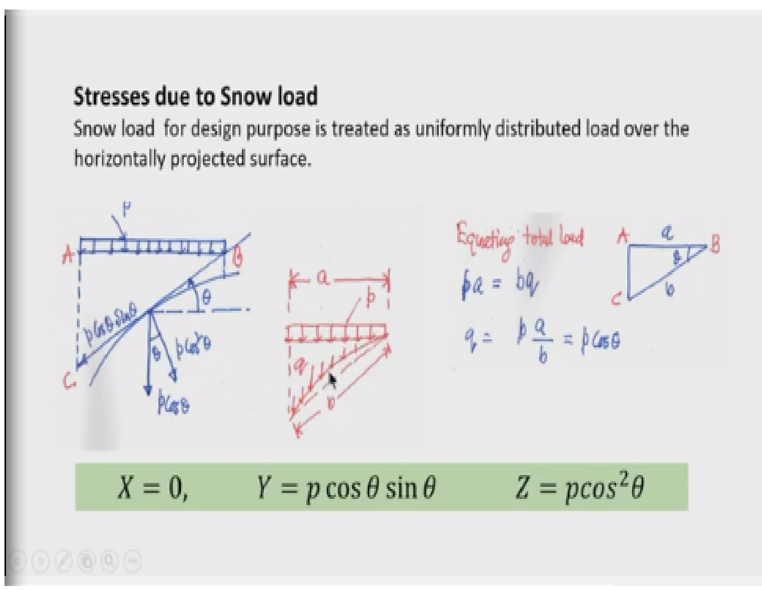

Let us see the case of snow load. Snow load for design purpose is treated as uniformly distributed load over the horizontally projected surface. So, the snow load is acting here over the horizontally projected surface. And it is the shell surface and we have to convert this in terms of the load that is acting along the surface of the shell. But that is done in this way suppose on a element, the total load will remain constant.

So, if I use this say if a is the horizontal length of this, so $p \times a$ equal to say this is the curved lenght arc length for example, so $R = \dfrac{\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$. So, q becomes $p\dfrac{a}{b}$. Now from this triangle it can be seen that $a/b = \cos\theta$. So, q that is the vertical component of the load is nothing but , so therefore this load is written as $p\cos\theta$. Now once this load is written because p is the intensity of the load over the horizontally projected surface.

So, now it is converted to the load along the surface of the shell, so it becomes $p\cos\theta$. So, now the component of $p\cos\theta$ along the radial direction is $p\cos^2\theta$ and along the tangential direction it will be $p\cos\theta\sin\theta$. So, 3 components of the loads are known now, X is 0 along the longitudinal axis, Y is $p\sin\theta\cos\theta$ and Z is $p\cos^2\theta$.

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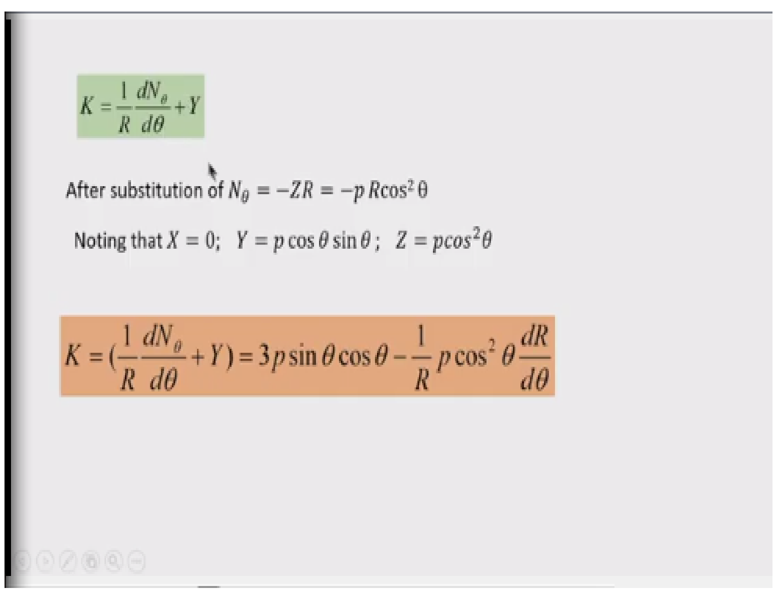

Then we can find out this value of K. So, N_θ is directly -ZR = Z was $p\cos^2\theta$, so we can write $-pR\cos^2\theta$. Once you obtain N_θ, then you can evaluate K and K is obtained as you see this term N theta is $-pR\cos^2\theta$. So, after differentiating this N_θ with respect to theta, we get now $k = 3p\sin\theta\cos\theta - \frac{1}{R}p\cos^2\theta\frac{dR}{d\theta}$, so this term is obtained after adding Y.

Then the simplification is possible when you know the value of R and if you know the R as a function of θ again you can differentiate this quantity. For a directrix which is having constant curvature, that is radius of curvature is constant along the length of the curve then the last quantity this $\frac{dR}{d\theta}$ will be 0. So, this will not be existing in case of shell with constant radius of curvature like a circular cylindrical shell.

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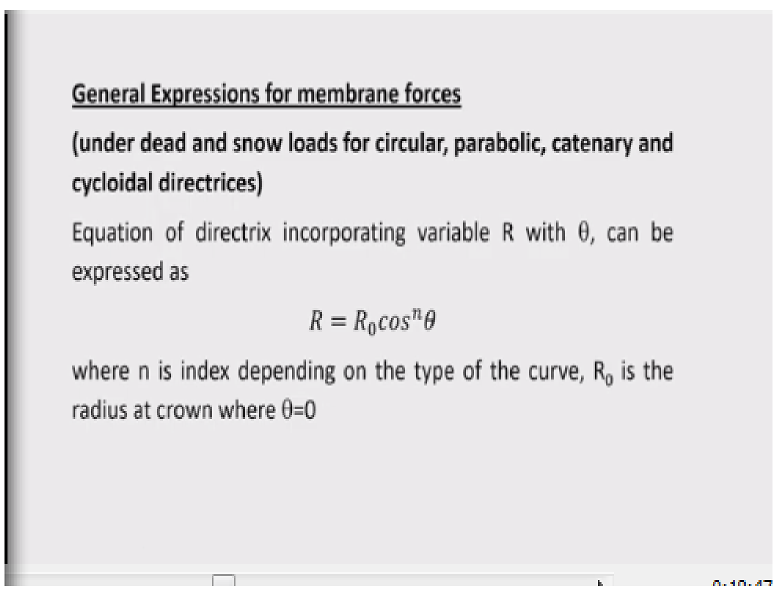

Now we have obtained a general expression for the membrane forces for the gravity load as well as snow load. Gravity load here I mean that self weight plus live load and snow load is also gravity load but it is acting over the horizontally projected surface. So, we shall consider now here 4 kinds of directrices, one is circular, then parabolic, catenary and cycloidal directrices.

The other type of directrices that are common is semi ellipse but here I consider only 4, semi ellipse can be treated in a different way. The 4 directrices are considered here because it is possible to express the equation of the directrix in the intrinsic form relating the radius of curvature and the angle θ. So, it can be written in this form the equation of the directrix the general form of equation of the directrix is

$$R = R_0 \cos^n \theta$$

So, here R_0 represents the radius of curvature at the crown that is the highest point on the shell. And n is an number which may be negative, positive depending of the nature of the curve. So, R is varying with $R = R_0 \cos^n \theta$, n is a index which depends on the type of the curve and R_0 is the radius at the crown where θ is the 0. Because theta is measured with respect to the vertical axis passing through the meridian.

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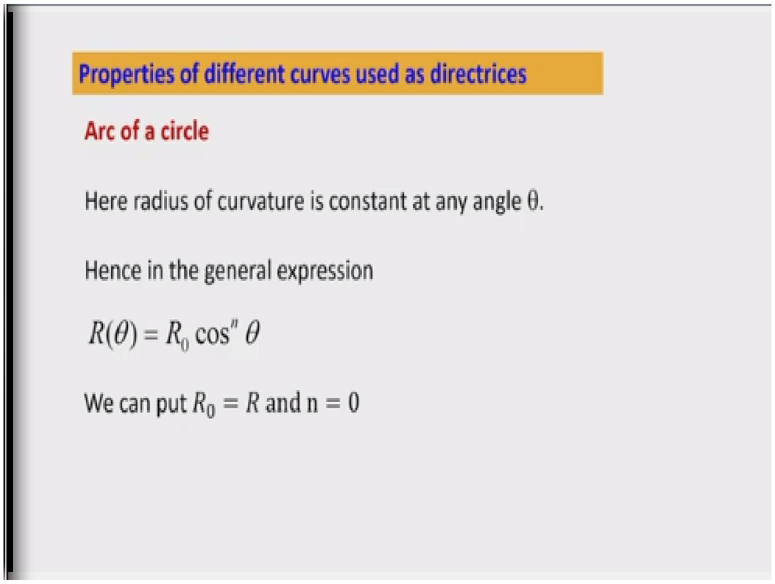

So, properties of different directrices are to be found out before finding the expression for that type of cylindrical shell. To solve the numerical problems on the cylindrical shell, we should know the properties of different directrices. And properties of databases were expressed in the intrinsic form $R = R_0 \cos^n \theta$. So, therefore the different curve are treated here to find out their equation in the form of $R_0 \cos^n \theta$.

First let us see which is a very common type of directrix is a arc of a circle. So, circular cylinder is very common in application and here you can see the radius of the curvature is constant at any angle θ. So, therefore your this $R = R_0 = R$. So, R is constant everywhere along the circular directrix in n is 0 because R should be equal to R_0 or R a constant quantity. So, this term should be 1, so it is possible only when n is 0.

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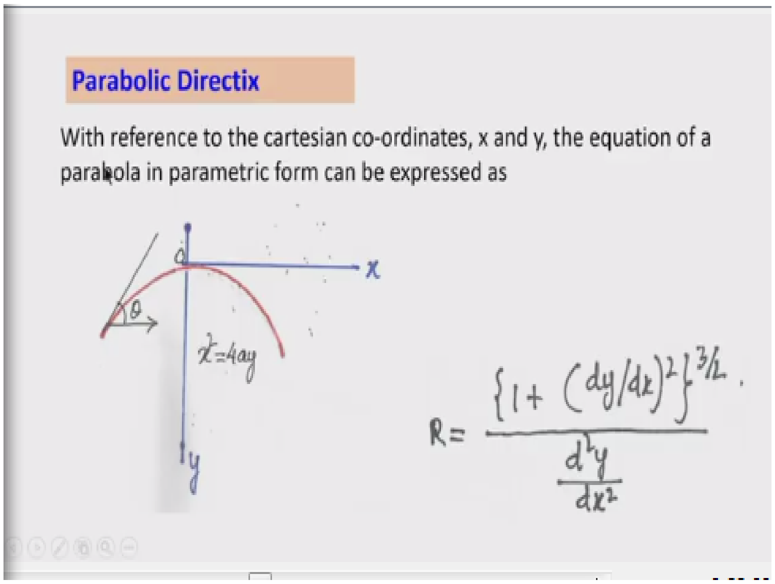

Then we go for a parabolic directrix. Parabolic dielectrix are also seen in construction because there is no harden fast rule, that for cylindrical shell the directrix should be only this circle, it can be of any form. So, we are now considering the parabolic directrix. In parabolic dielectrix the equation of the parabola is to be written in the intrinsic form. So, intrinsic form is $R = R_0 \cos^n \theta$.

So, see this with reference to x and y axis, the equation of the parabola is written $x^2 = 4ay$. And at any point along the tangent the tangent makes an angle θ with the horizontal and $\tan\theta$ is the slope of the curve. So, if the equation of the curve is given by y, then we can find the radius of curvature at any point by the expression that is known from the calculus

$$R = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

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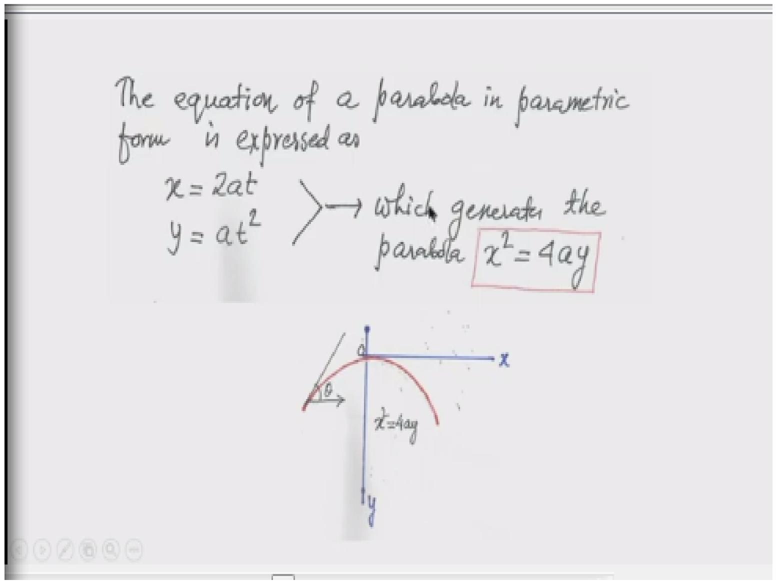

So, equation of the parabola in parametric form is expressed now. Because to convert the equation of the curve or to find out the properties of the curve with a equation in the form of $R = 2a\left(1+t^2\right)^{\frac{3}{2}} = 2a\left(1+\tan^2\theta\right)^{\frac{3}{2}}$, we should express the equation of the curve in the parameteric form first then we can find the R_0 and n. So, let us see this example, see the parametric form of the equation of the parabola is written as x = 2at and y = at^2 which generates the parabola that can be verified from this expression.

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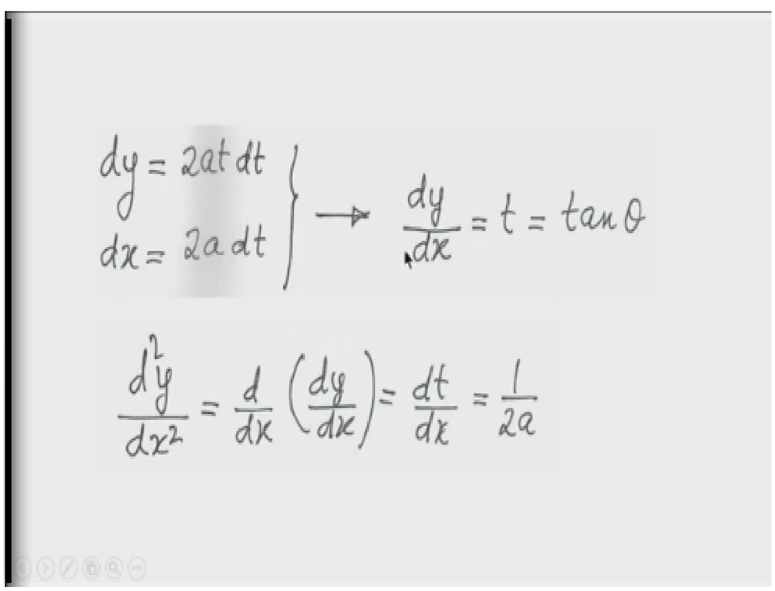

Now this is the parametric form of the equation $\frac{dy}{dx} = t = \tan\theta$. Because first we differentiated y and it becomes 2t because if I differentiate t^2 it becomes 2 t dt and a is there. And when we differentiate dx, that is dx then it will be 2a because differentiation of t is 1, so 2a dt. So, now dividing this by this we get the parameter t which is nothing but $\frac{dy}{dx} = \tan\theta$. Now let us find the 2nd derivative. Because 2nd derivative is needed for finding the radius of curvature. So, if I differentiate this quantity $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ and then we can write $\frac{dt}{dx} = \frac{1}{2a}$.

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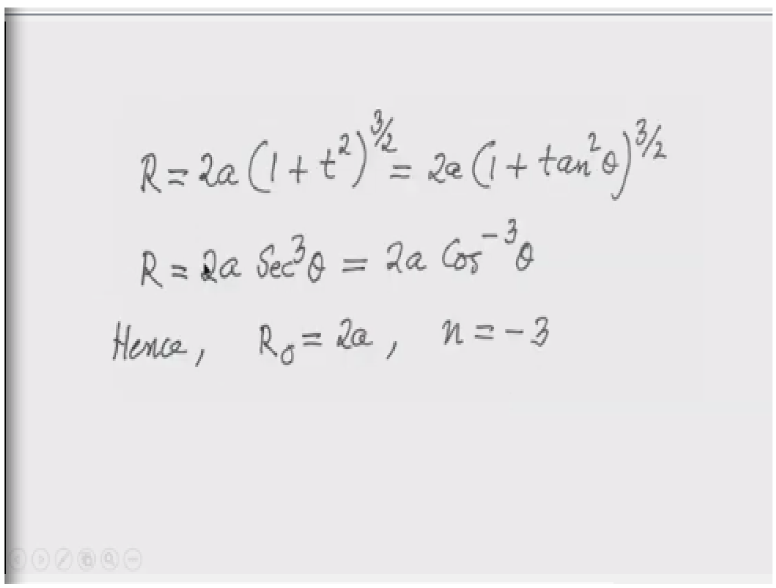

So, R equal to, now R is found using this formula $R = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$. So, once we know $\left(\frac{dy}{dx}\right)$ and $\frac{d^2y}{dx^2}$, now we can find the R. So, R is computed as $R = 2a\left(1+t^2\right)^{\frac{3}{2}}$ equal to $2a\left(1+\tan^2\theta\right)^{\frac{3}{2}}$, so $\left(1+\tan^2\theta\right)^{\frac{3}{2}}$. So, now this term $\left(1+\tan^2\theta\right)$ is nothing but $\sec^2\theta$.

So, once it is raise to the power 3/2 then it becomes 2a $\sec^3\theta$. So, that is nothing but because sec theta is reciprocal of $\cos\theta$. So, we can now write $2a\cos^{-3}\theta$. So, the equation of the parabola in this form is found out.

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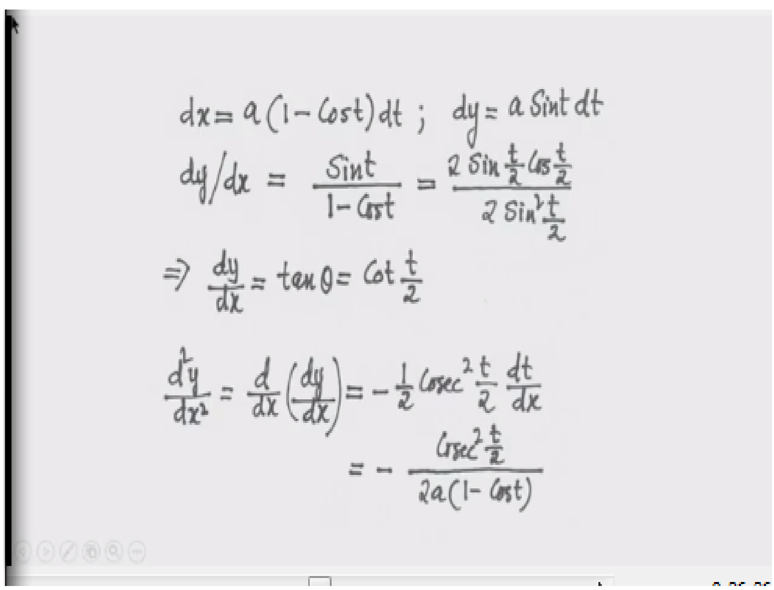

The form is this $R_0\cos^n\theta$. So R_0 for parabolic curve is 2a and n = -3.

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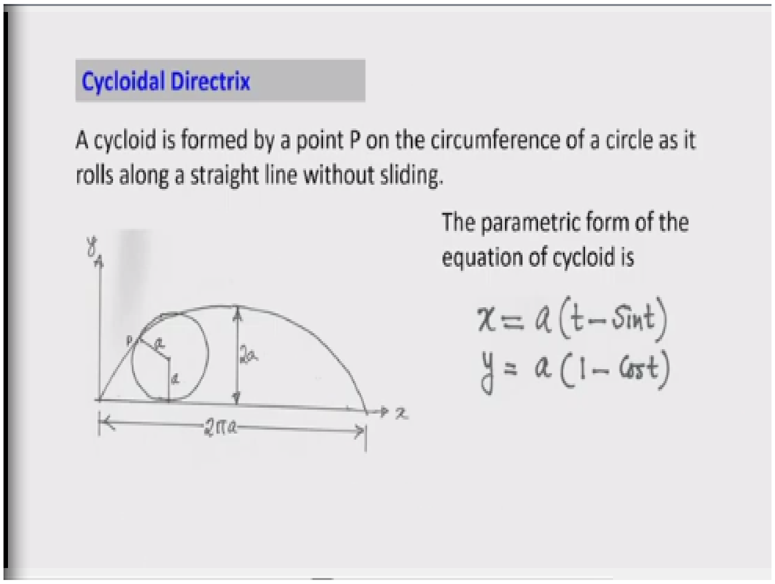

Now let us go to the cycloidal directrix. A cycloid is formed by a point on the circumference of a circle as it rolls along the straight line without sliding, so this is the definition of cycloid. The parametric form of the equation of the cycloid is $x = a(t - \sin t)$ and $y = a(t - \cos t)$. So, these 2 equations are seen. So, parametric form of this cycloid is $x = a(t - \sin t)$ and $y = a(t - \cos t)$.

So, dx we can now find, dx will be $a(1 - \cos t)dt$. So, $a(1 - \cos t)dt$, dy can be found, dy will be $a \sin t dt$, so $a \sin t dt$. So, dy, dx is now calculated $\frac{dy}{dx}$ will be $\frac{\sin t}{1 - \cos t} =$, sin t can be written as $2\sin\frac{t}{2}\cos\frac{t}{2}$ divided by $2\sin^2\frac{t}{2}$, because $\frac{1 - \cos t}{2}$ is $2\sin^2\frac{t}{2}$. And after simplification of this quantity, it can be written as $\cot\frac{t}{2}$.

Now let us differentiate again to find out the curvature So, $\frac{d^2y}{dx^2}$ is now found out, so it is $\frac{d}{dx}\left(\frac{dy}{dx}\right)$. So, after differentiating this quantity one can find it that it is equal to $-\frac{1}{2}\cos ec^2\frac{t}{2}\frac{dt}{dx}$. Because first I have differentiated this expression with respect to t and then t is differentiated with respect to x. So, after simplification you can write $-\frac{\cos ec^2\frac{t}{2}}{2a(1-\cos t)}$.

Further simplification is necessary to bring this equation in the form $R = R_0\cos^n\theta$.

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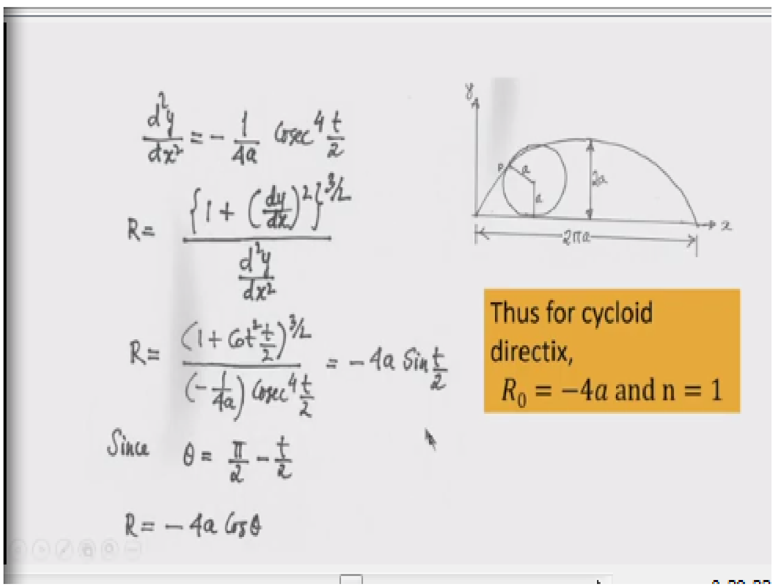

So, $\frac{d^2y}{dx^2} = -\frac{1}{4a}\cos ec^4\left(\frac{t}{2}\right)$. So, now put these value of these dy, dx that you have found $\cot\frac{t}{2}$ and $\frac{d^2y}{dx^2} = -\frac{1}{4a}\cos ec^4\left(\frac{t}{2}\right)$ in this expression for radius of curvature. So, after simplification we get it is equal to radius of curvature is equal to $R = -4a\sin\frac{t}{2}$. Now you can note that the

$$y = a\cosh\left(\frac{x}{a}\right)$$
$$\frac{dy}{dx} = \tan\theta = \sinh\frac{x}{a}$$
$$p\cos^2\theta$$
$$-pR_0\cos^{n+2}\theta$$

.

So, this $\frac{t}{2}$ if I substitute here as $\frac{\pi}{2} - \theta$, then it becomes $R = -4a\cos\theta$. So, that means, here now we have been able to write the expression in the intrinsic form that is $R = R_0\cos^n\theta$. So, now find out what is R_0? R_0 here is -4a and n is here 1, because it is $\cos\theta$, so n is 1. So, for cycloidal directrix the equation of the cycloid to be used in our expression for finding the membrane stress is given by $-4a\cos\theta$, where R_0 is -4a and n is 1.

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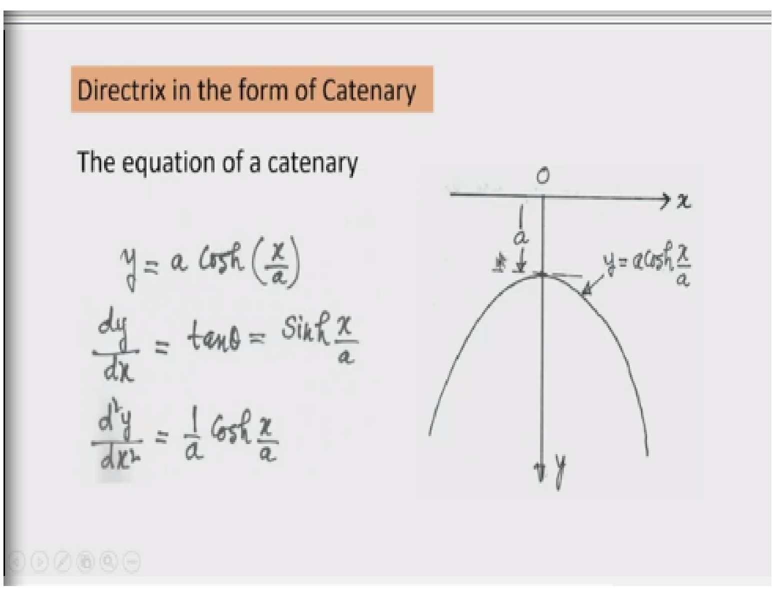

Next directrix is the catenary which is also common in some application. Now equation of the catenary can be written as $y = a\cosh\left(\frac{x}{a}\right)$ So, dy, dx is calculated $\frac{dy}{dx} = \tan\theta = \sinh\frac{x}{a}$ $\frac{d^2y}{dx^2} = \frac{1}{a}\cosh\frac{x}{a}$ that is the first derivative is calculated which is nothing but $\tan\theta$ and it is equal to . Because $\frac{1}{a}$ will come out and it will be cancelled with the a, so the term remains only the $\sinh\frac{x}{a}$. 2nd derivative is calculated and it is equal to $\frac{d^2y}{dx^2} = \frac{1}{a}\cosh\frac{x}{a}$. So, 2 derivatives are found, now we can go for calculating the radius of curvature.

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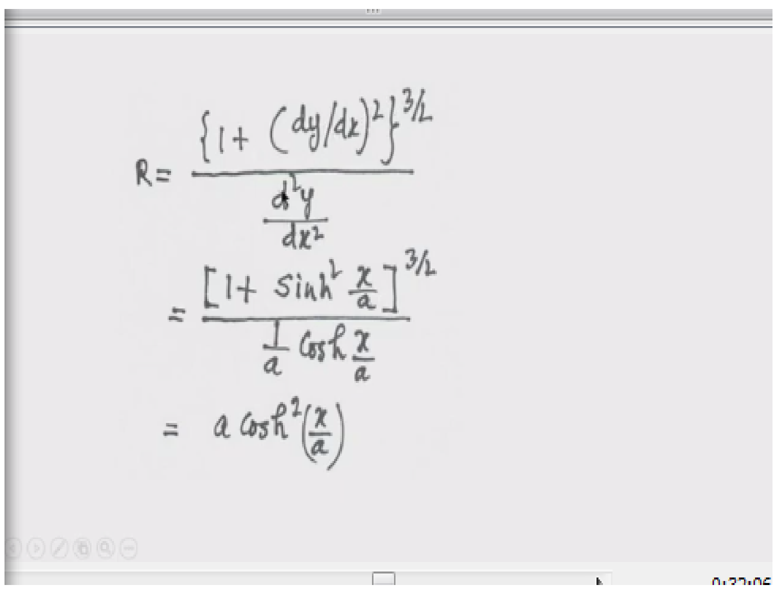

So, radius of curvature is given by

$$R = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

Substituting the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, now we can write this we have already got $\frac{dy}{dx}$ as $\sinh\frac{x}{a}$ and $\frac{d^2y}{dx^2} = \frac{1}{a}\cosh\frac{x}{a}$. So, substituting these values we now get R in the form of a $\cosh\frac{x}{a}$.

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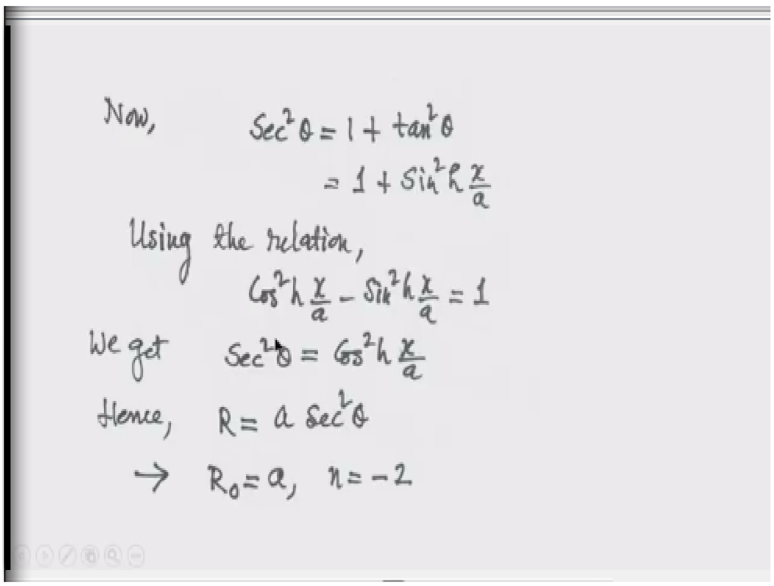

So, now it has to be agai $R = R_0 \cos^n \theta$, so trignometrical function cosine has to be brought here.

So, to do this we take this $\cosh^2\frac{x}{a} - \sin^2\frac{x}{a} = 1$ and this $\tan\theta$ we already obtain, $\tan\theta$ is your $\sinh\frac{x}{a}$. So, now substituting this $\tan\theta$ as $\sinh\frac{x}{a}$, we can write $\sec^2\theta = 1 + \sinh^2\frac{x}{a}$.

Now we know this trigonometrical identity that is $\cosh^2\frac{x}{a} - \sin^2\frac{x}{a} = 1$. So, that means $1 + \sinh^2\frac{x}{a}$ is nothing but you can find it, it will become $\cosh^2\frac{x}{a}$. So, this is written here in this form and this is nothing but $\sec^2\theta$. So, now we can relate because this is related to $\cos\theta$ because Sec is related to cos, so $\sec^2\theta = \cosh^2\frac{x}{a}$.

So, therefore the equation of the curve can be written as $R = a\sec^2\theta$ and therefore R_0 = a, n = -2. So, in this way we can find the equation of the directrix in this very interesting form that is $R = R_0\cos^n\theta$, so that the value of R_0 and n can be substituted in the general expression that we have derived for any kind of directrices.

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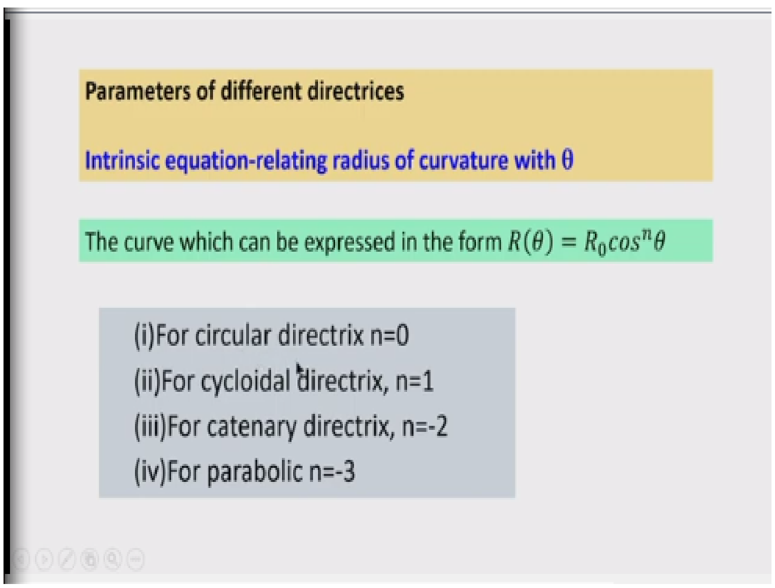

So, now we summarise the parameter of the different directrices. For circular directrix takes n = 0, for cycloidal directrix n = 1, for catenary directrix n = -2, for parabolic it is -3. The equation of the curve is $R(\theta) = R_0\cos^n\theta$. One directrix I have not included here because these cannot be expressed in this form, that is the semi ellipse and that have to be found from the general procedure that finding the radius of curvature after successive differentiation of the equation of the curve.

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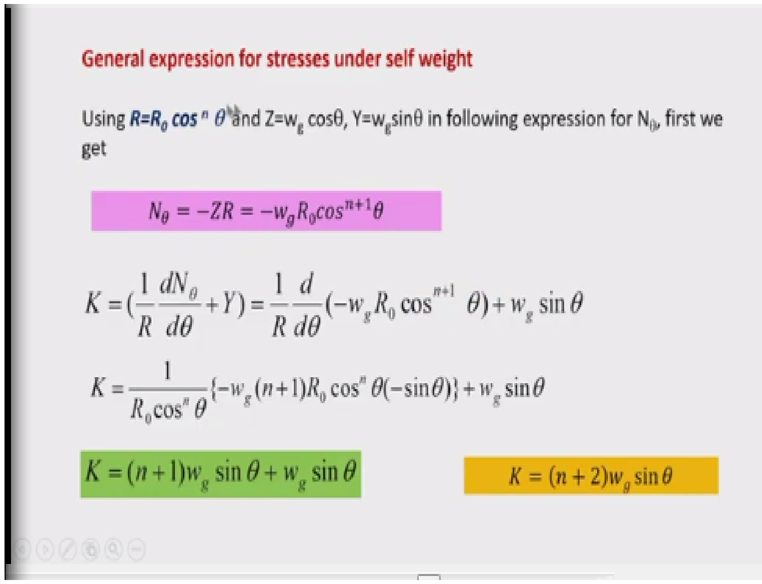

So, let us now find the general expression for stresses under self weight. Using this relation $R = R_0 \cos^n \theta$, the radial component of the vertical load is $w_g \cos\theta$ and the component along the tangential direction is $w_g \sin\theta$. So, substituting this we can now find the expression for my membrane stresses one by one. First let us find the N_θ, $N_\theta = -ZR$, so $w_g \mathrm{R}_o \cos^{n+1}\theta$. So, this is the expression for this N_θ and suppose it is a circular cylindrical roof then n is 0 and R is your constant quantity, so we can find this value now.

For say other type of directrices we have found R_0 as well as n, so we can put here. So, K can be found now, $K = (\frac{1}{R}\frac{dN_\theta}{d\theta} + Y)$, that means N_θ is differentiated with respect to θ. And since this is the expression for N_θ when I differentiate this it becomes $-w_g (n+1) R_0 \cos^n \theta(-\sin\theta)$. And here R is substituted as $R_0 \cos^n \theta$ and this is the Y, this is the component of the load Y. So, after simplification K becomes $K=(n+1)w_g \sin\theta + w_g \sin\theta$ which is nothing but $\mathrm{k}=(n+2)w_g \sin\theta$. So, once you find K, now you can find $N_{\theta x}$.

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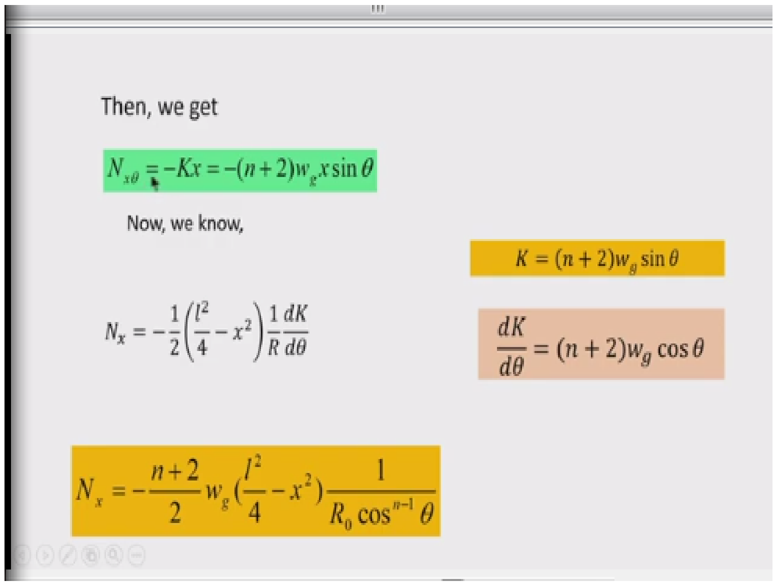

So, $N_{x\theta} = -Kx = -(n+2)w_g x\sin\theta$. Then we come to the N_x the longitudinal membrane stress that has to be found. The expression for longitudinal membrane stress for simply supported shell was derived as this

$$N_x = -\frac{1}{2}\left(\frac{l^2}{4} - x^2\right)\frac{1}{R}\frac{dK}{d\theta}$$

So, if K is known as this

$$K = (n + 2)wg \sin sin\, \theta$$

differentiation of this quantity with respect to theta will give you this

$$\frac{dK}{d\theta} = (n + 2)w_g \cos cos\, \theta$$

So, after substituting this in this expression, now we get

$$N_x = -\frac{n+2}{2}w_g(\frac{l^2}{4}-x^2)\frac{1}{R_0\cos^{n-1}\theta}$$

So, 3 membrane stresses we have evaluated.

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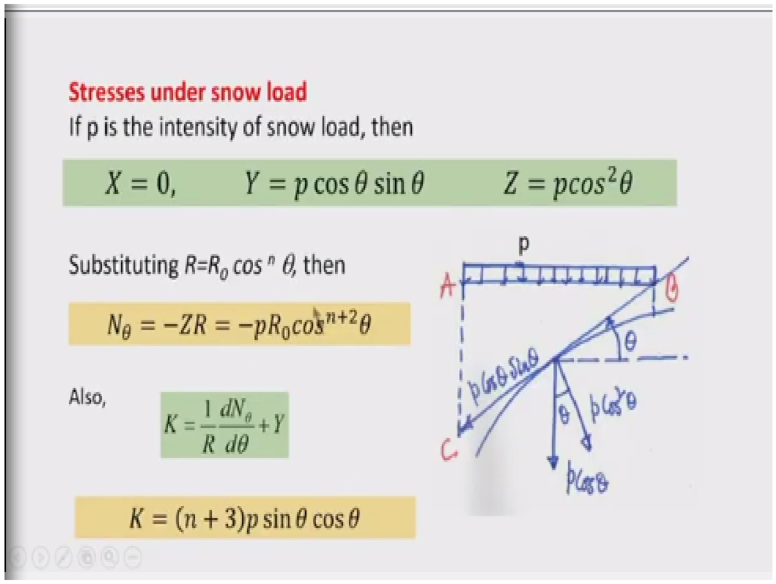

Now we go for calculating the stresses under the snow load. So, component of this snow load along this radial direction and the circumferential direction is given as, in the radial direction it is $p\cos^2\theta$ as you can see from this figure. And in the circumferential direction it is $p\cos\theta\sin\theta$, this is evident from figure and also discussed earlier. So, substituting this $R = R_0\cos^n\theta$.

And this expression for the load component we now first obtain $N_\theta = -ZR$. So, Z is $p\cos^2\theta$ and this expression R is substituted here. So, final expression will be $-pR_0\cos^{n+2}\theta$, so this is the expression for N_θ. Now, find out these K value, $K = \frac{1}{R}\frac{dN_\theta}{d\theta} + Y$, Y is known as $p\cos\theta\sin\theta$. So, we differentiated this quantity and then finally it is found

$$K = (n + 3)p \sin sin\,\theta \cos cos\,\theta$$

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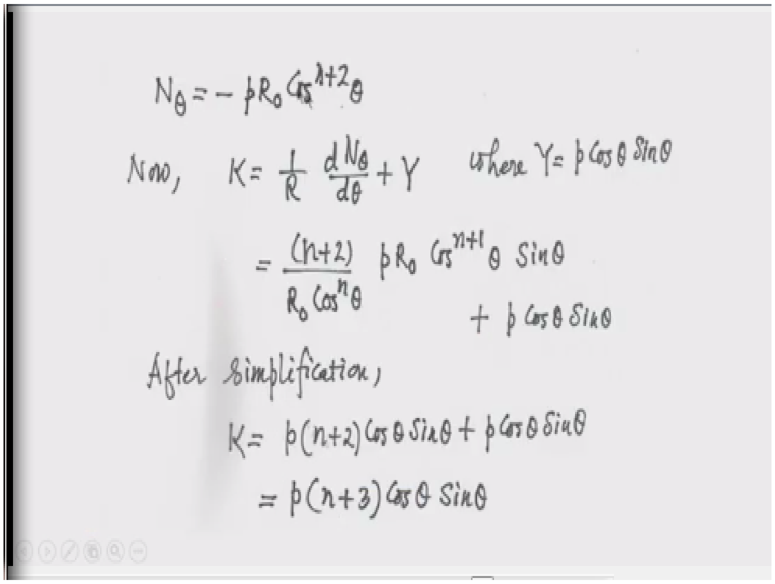

This is of course is illustrated by step by step procedure. So, $N_\theta = -pR_0 \cos^{n+2}\theta$ and K is $K = \frac{1}{R}\frac{dN_\theta}{d\theta} + Y$, where Y is $p\cos\theta\sin\theta$. So, it is written like that, substituting this value of N theta means N_θ is differentiated with respect to θ. We can see that this expression is obtain, this

$$\frac{n+2}{R_0 \cos^n\theta} pR_0 \cos^{n+1}\theta \sin\theta + p\cos\theta\sin\theta$$

So, then taking common terms or adding this we finally get $k = p(n+3)\cos\theta\sin\theta$, so that is written here. So, p is the intensity of the snow load.

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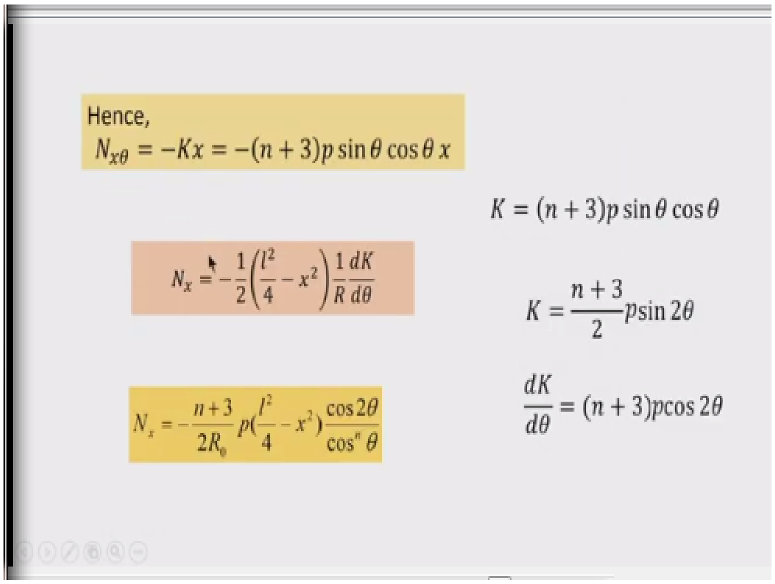

So, once you get the $N_{x\theta}$ and $\frac{dk}{d\theta}$ then other quantities can be easily found out. So, N_x is nothing but

$$N_x = -\frac{1}{2}\left(\frac{l^2}{4} - x^2\right)\frac{1}{R}\frac{dK}{d\theta}$$

We have obtained K as

$$K = \frac{n+3}{2}\sin sin\, 2\theta$$

Because $\cos\theta\sin\theta$ is nothing but $\frac{\sin 2\theta}{2}$, so it is written in this form. So, $\frac{dk}{d\theta}$ is nothing but

$$p = \int_x^{l/2} \left(N_{x\theta}\right)_{\theta=\theta_c} dx$$

$$w_g\left(\frac{l^2}{4} - x^2\right)\sin\theta_c$$

$$p_{\max} = \frac{w_g l^2 \sin\theta_c}{4}$$

, because 2 will get cancelled with this 2 in the denominator.

So, therefore the N_x the expression for longitudianl stress can be written as

$$N_x = -\frac{n+3}{2R_0} p(\frac{l^2}{4} - x^2) \frac{\cos 2\theta}{\cos^n \theta}$$

So, in this way we have obtained the membrane stresses 3 component of stresses that is N_x, N_θ and $N_{x\theta}$ for cylindrical shell having any general type of directrices here include the following type. One is circular, parabola, cycloid and this catenary. So, these type of directrices are included in this expression, for other types of directrices the general procedure should be adopted. So, after obtaining this 3 stress component for snow load and dead load.

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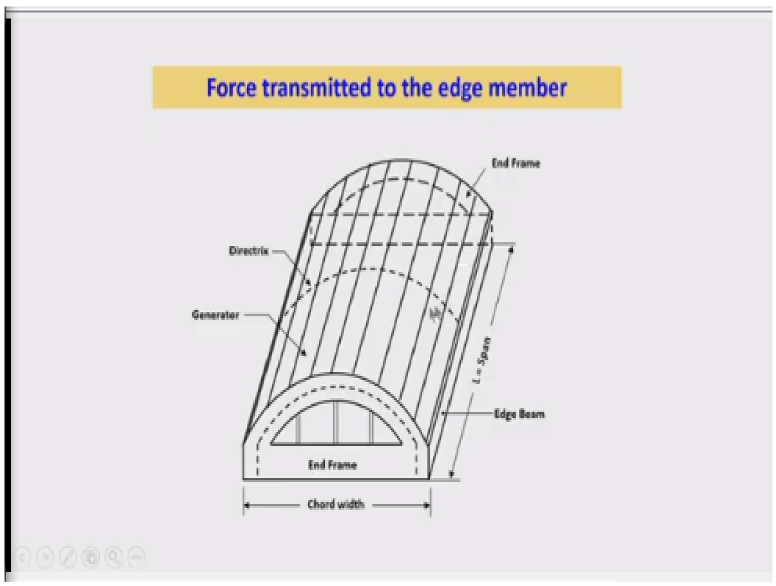

We can now find the force transmitted to the edge member. Now you can see that shell is supported on the edge member, so what is the force transmitted on the edge member? Because edge member will be subjected to force that will be up to certain region it will be compressive and up to certain region it will be tensile, so let us find out this.

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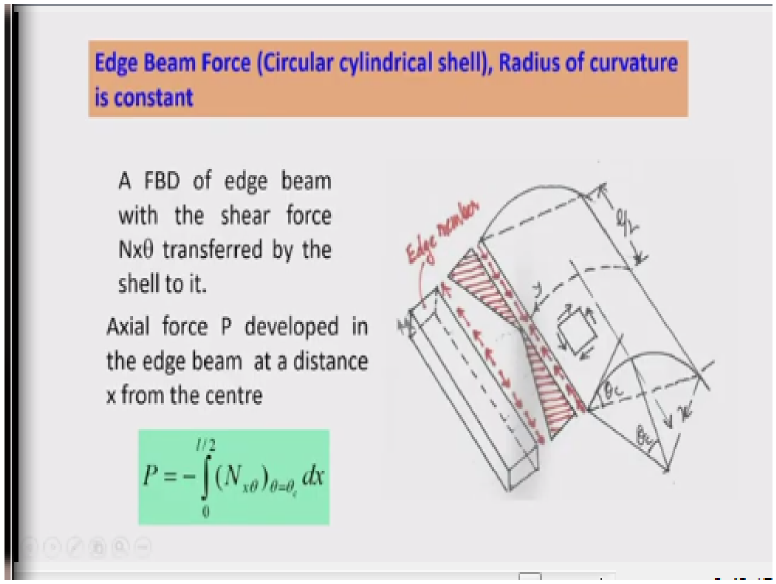

You can see here the $N_{x\theta}$, if we see the expression for membrane shear force it is $N_{x\theta}$ = -K x, that is x = 0, membrane shear stress is 0 and it is maximum at the ends. So, one is in the negative direction and other in the positive direction. So, let us find the resultant force. So, axial force P developed in the edge being at a distance x from the centre can be given as integration of 0 to $\frac{l}{2}$ $N_{x\theta}$, $N_{x\theta}$ has to be found at this edge that is $\theta = \theta_c$. So, at this point the $N_{x\theta}$ has to be found out. So, it is integrated from 0 to $\frac{l}{2}$ for the one part.

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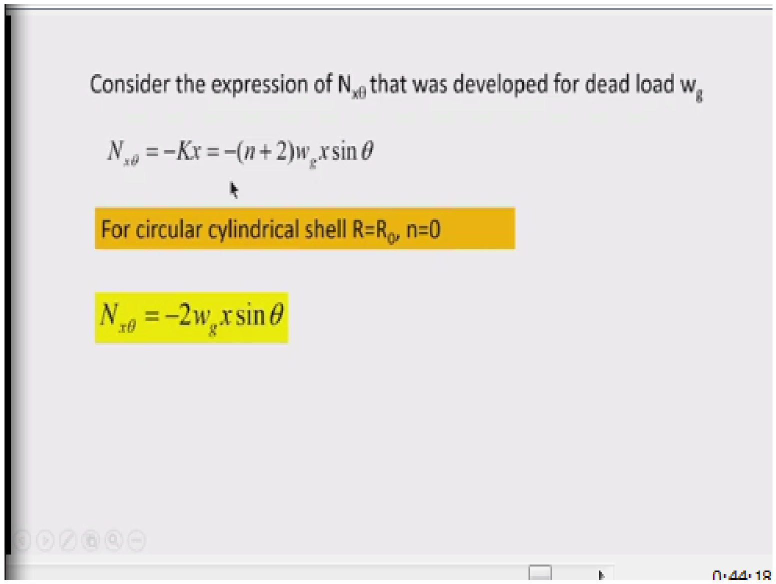

And after integration this N x theta is given as

$$N_{x\theta} = -Kx = -(n+2)w_g x\sin\theta$$

And let us do it for a circular cylindrical shell first, so R = R_0 and n = 0 for circular cylindrical shell. So, therefore $N_{x\theta}$ can be written as $2w_g \sin\theta$.

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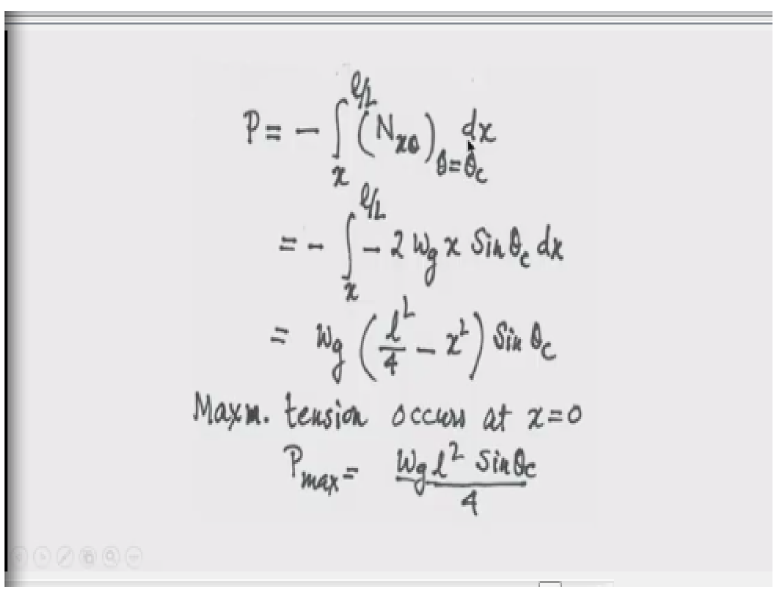

So, P is now calculated as at a distance x, so therefore we put the lower limit x. But if it is calculated from 0, that is centered then we put the lower limit 0. So,

$$p = \int_{x}^{l/2} \left(N_{x\theta} \right)_{\theta=\theta_c} dx$$

So, substituting the value of $N_{x\theta}$ as $-2w_g \sin\theta_c$ dx. Now after integration we find the expression for the edge member force as $w_g \left(\frac{l^2}{4} - x^2 \right) \sin\theta_c$. Now it is seen that maximum tension occurs because it is now becoming positive, so tensile force is treated as positive. So, maximum tension occurs at x = 0, so by putting x = 0 we get $p_{\max} = \frac{w_g l^2 \sin\theta_c}{4}$.

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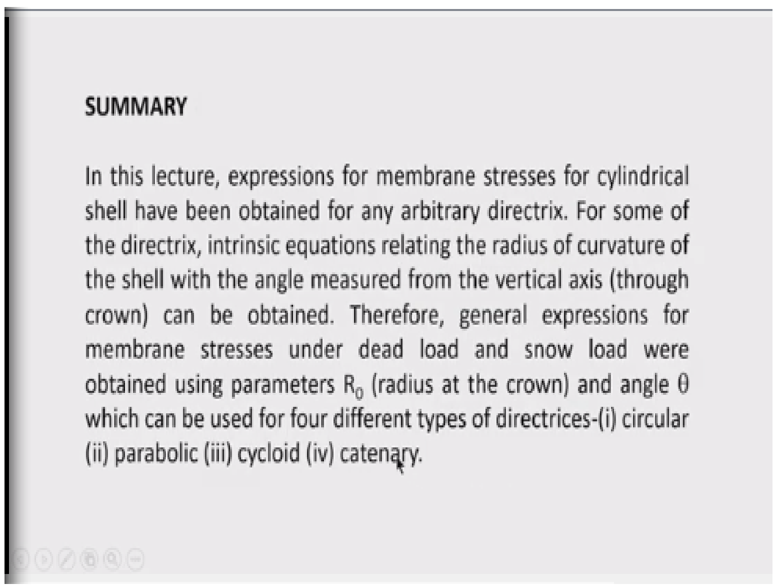

So, let me summarize what we have done today. In this lecture, expression for membrane stresses for cylindrical shell have been obtained for any arbitrary directrix. For some of the directrices, intrinsic equations relating the radius of curvature of the shell with the angle measured from the vertical axis can be obtained, that is R and θ is related. Therefore the general expressions for membrane stresses under dead load and snow load were obtained using parameters R_0 and angle

θ. This expression can be used for 4 different types of directrices, circular, parabolic, cycloid and catenary. We will see some numerical problem in the next class, thanking you.