Plates and Shells
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Module-11
Lecture-31
Differential Equations of Equilibrium in Cylindrical Shell Using Membrane Hypothesis

Today I am starting module 11 of the subject plate and shell and it is the first lecture. So, far I have discussed the shell theory specially the membrane theory of shells in application of surface over evolution. So, you have seen that spherical dome and then a pressure vessels or a tank of any arbitrary meridian have been analyzed using the membrane theory of shell. Now, a very common type of shell exists in practical application and that is the cylindrical shell.

That is used for roof structure specially for workshop and warehouses and these types of structures, cylindrical shell is an example of single curved shell. That means, a generator which is a straight line is moved over a plain curve that is called directrix and as a result, the shell is form. So, single curve shell means it has Gaussian curvature 0.

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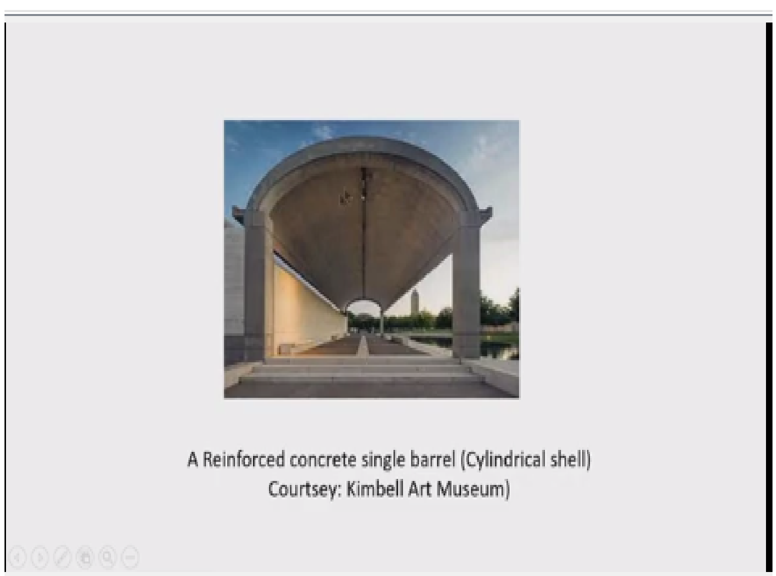

So, you can see an example of cylindrical shell. A large span has been covered with the help of very thin structural element which is a form of cylindrical shell; this is also called single barrel shell. Now, multiple barrel shells are also common in nature.

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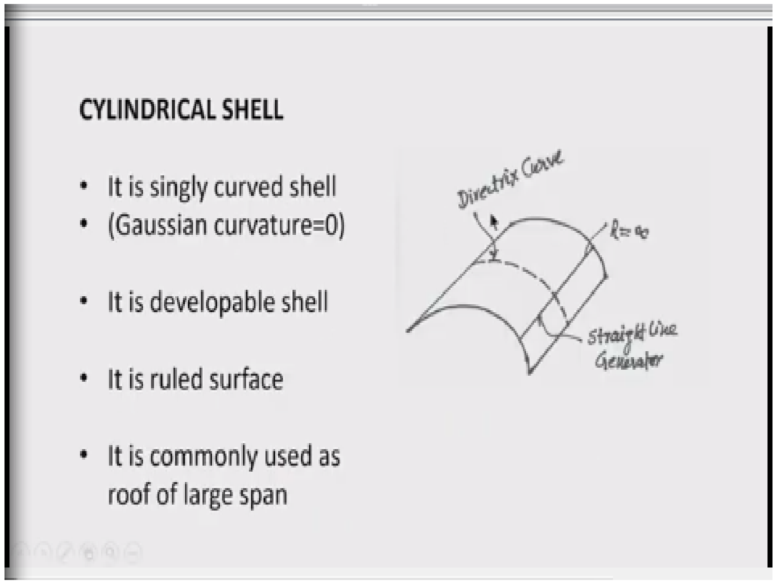

Cylindrical shells you can see, it is actually the singly curved shell whose Gaussian curvature is 0. So, Gaussian curvature 0 means the product of 2 principal curvatures that is $\frac{1}{r_1}$ and $\frac{1}{r_2}$ is 0 that means 1 radius of curvature is infinity. So, that is evident if I take a straight line generator which moves over the directrix that is a plane curve. So, the straight line generator has infinite radius of curvature, so therefore Gaussian curvature is 0.

Now this kind of shell also falls under the category of developable shell. That means, this shell can be flattened into a flat surface and it is also falling under the category of ruled surface. Because when the straight line is ruled over the curve the surface is formed, so therefore it is called ruled surface, it is commonly used as roof in a large span structure.

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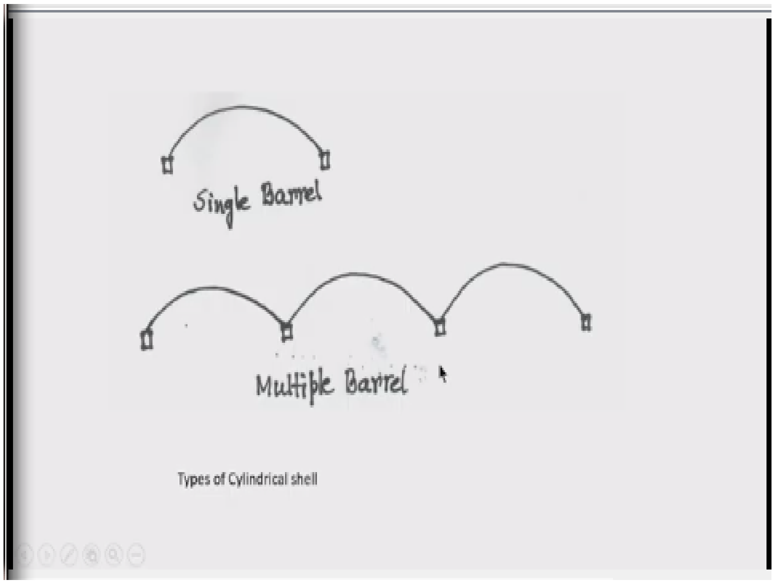

The cylindrical shell maybe of single barrel type that is here it is a single barrel cylindrical shell or it may be multiple barrel cylindrical shell. You can see that cylindrical shell has been provided with the edge been here, here the intermediate has edge beams also there, so it is having multiple support in different spans. So, these are the exterior support and these are the interior support. So, today our focus will be to derive the differential equations of equilibrium in cylindrical shell using membrane theory.

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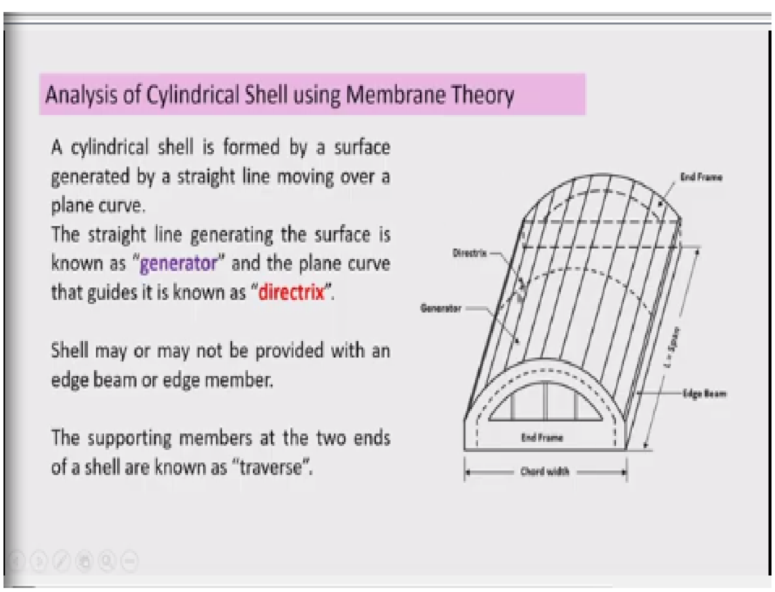

So, let me see what are the elements of the cylindrical shells? Here the cylindrical surface is shown and the dotted lines are the directrix. So, directrix may be of any form, any general curve can be taken or any known curve says circular arc or a parabola or elliptical curve can be taken. Then you can see that shell is supported at the ends by some kind of structure, that structure is known as traverse. So, at the end it is supported by traverse and it may be provided with edge beam or without edge beam also. The straight lines that are rolling over the curve are known as generator and the curve is known as directrix.

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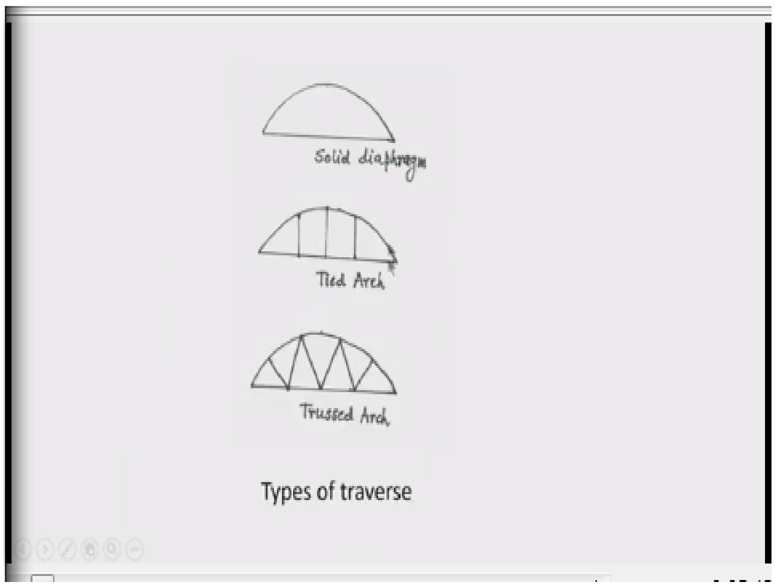

Now, if I see the different types of traverse, traverse maybe solid diaphragm a theme structural element which acts like a traverse, it is a solid, no opening is there. So, then the traverse maybe of tide arch that is in the form of arch but the vertical suspenders are there. Then it may be trussed arch, having a truss to join the rib of the arch with the horizontal member.

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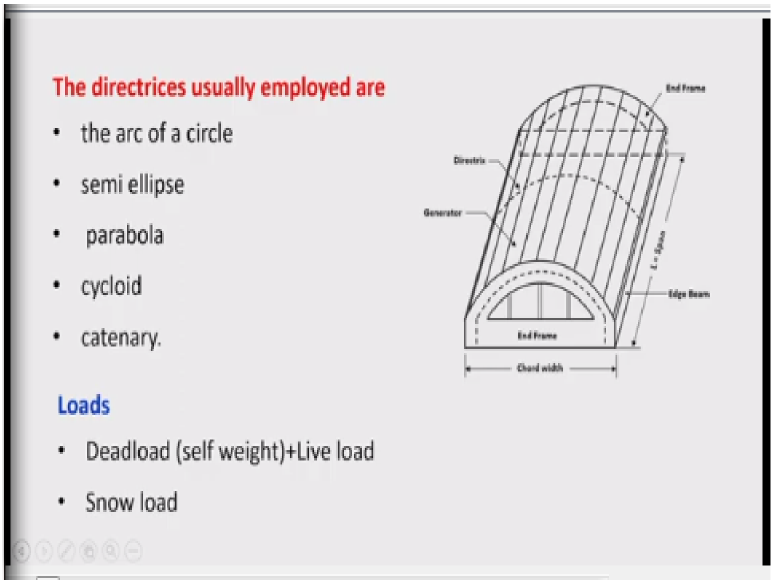

Now, directrices that are easily employed in the cylindrical shell maybe of an arc of circle. Arc of a circle that is a circular cylindrical shell is very common and analysis is also simple because of simple nature of the curve that is circle. But when we take the elliptical directrix, semi ellipse directrix or part of a parabola or a cycloid catenary, the analysis becomes involved because of complicacy in the geometry. You can see the circle has the constant radius of curvature, but other curves you will find the radius of curvature varies at any meridional angle.

So, that is the point where the difficulty arises in solving the cylindrical shell problem with the other types of directrix. But for architectural regions the directrix of other types are also adopted. So, we shall do the analysis of cylindrical shell with different types of directrix, arc of a circle, semi ellipse, parabola, cycloid, catenary, knowing the properties of the different types of curves. Load that I will consider for the analysis of cylindrical shell in this course are dead load, self weight plus live load. So, dead load and live load that are considered together is a gravity load acting on the shell. Then I will also take the snow load which is a load acting over the horizontal projected surface.

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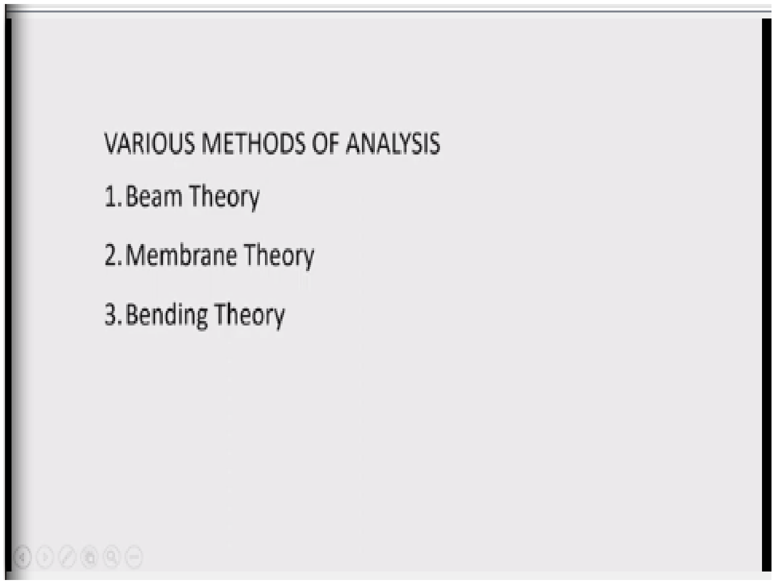

There are various methods of analysis of cylindrical shell. The one common method is beam theory, then second is membrane theory and bending theory. The beam theory is an approximate method; membrane theory is also simple but very attractive method. Because it gives the true picture of stresses except near this supports. Bending theory of course is complicated, that includes the bending moment near the edge of the shell. And that bending theory have been developed including the results of membrane theory. So, let us see what is beam theory?

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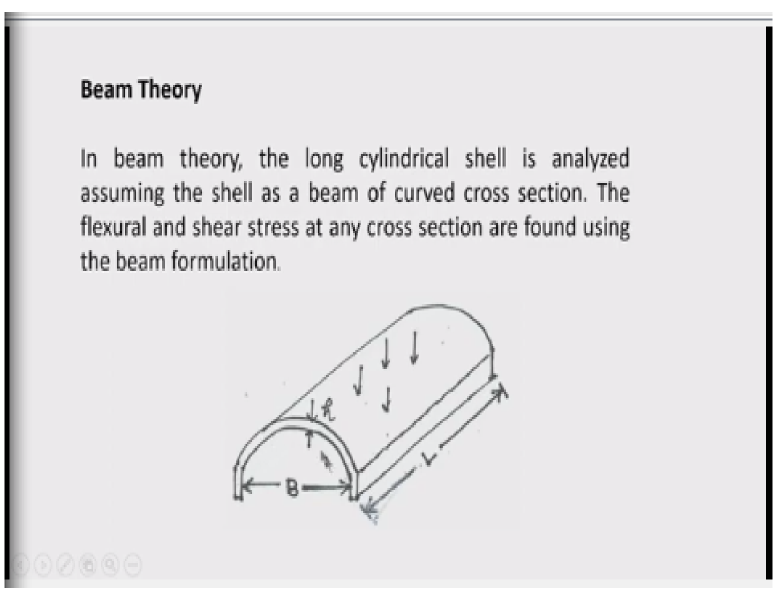

In beam theory the cylindrical shell can be idealized as a beam whose span is L and the cross section of the beam is curved, the curved cross section of the beam is seen here. So, the sectional property that to be used in the beam analysis will be slightly complicated because of the curved cross section. So, that have to be found that neutral axis of the cross section, moment of inertia about the neutral axis that have to be calculated for the curved cross section.

And other analysis will depend on the beam analysis that we are familiar in our UDL strength of material class. So, the maximum bending moment if it is simply supported shell then maximum bending moment will occur at the mid span. And if it is supported at the traverse, these are traverse is located here then maximum shear force will be generated here. So, we can find the maximum bending stress and maximum shear stress based on which the thickness of the shell and the reinforcement can be calculated, if it is used as a RCC roof shell.

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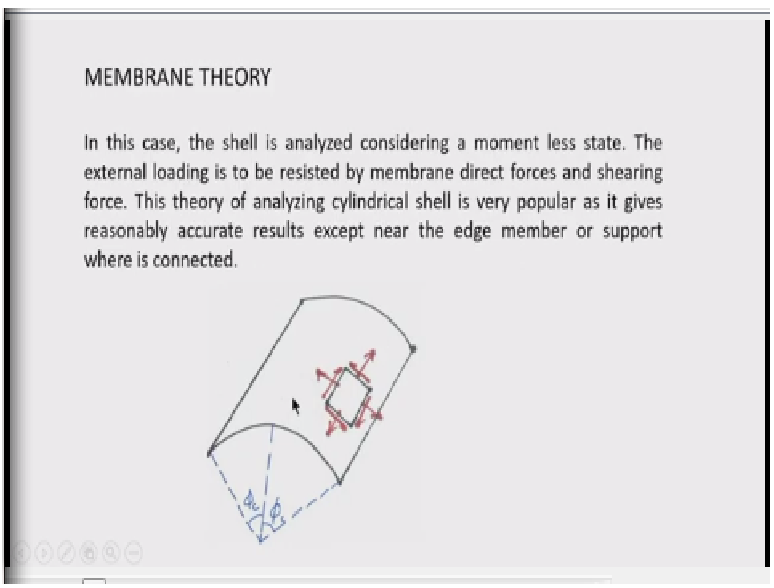

Then, the membrane theory which we will discuss in today's class and our whole analysis will depend on my membrane theory. In the last module I will introduce the bending of the shell. So, in this case, the shell is analyzed considering a moment less state. That means the bending rigidity of the shell is neglected. So, what does it mean that? The shell element is in a state of in plane forces. So, in plane forces are the only resisting forces in the shell to the external load.

The external loading is to be resisted by membrane forces that in a shell element I have shown by arrow, these are the membrane shearing forces and these are the direct in plane forces or membrane forces that you can see. One is in the direction of the generator that is in the longitudinal direction another is in the direction of the meridian that is in the direction of the directrix. That means tangential to the directrix curve and others are shearing force acting tangential at the edges. So, these are the forces in the membrane hypothesis and we will take these forces into consideration to derive the differential equations of equilibrium.

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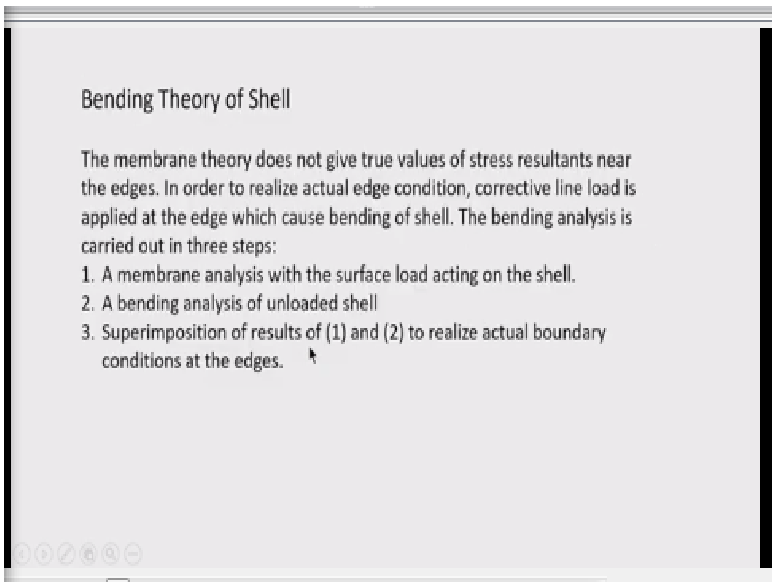

Now in the bending theory of shell, earlier also in some problem, I have demonstrated that the membrane theory fails to predict the actual nature of stress near the edges. Because the boundary condition of edges was not satisfied, if you recall this, the edge was free in our some of the examples that I took up in the earlier classes. And then due to free edge the shell at the edge should have 0 stress, but when we use the membrane theory and derive an expression, we found some stresses exist at the edges.

So, that is giving wrong information on the nature of stress that is produced or the stress that is actually produced at the edges. So, therefore, it is suggested that a corrective line load to be applied at the edge. So, when we apply some corrective line load at the edges, then bending will be developed in the shell near the edges, so that is the cause of bending. So, bending analysis is carried out in 3 steps.

Whatever membrane analysis we have done, this will be used as a particular solution on the bending of shell. So, membrane analysis with the surface load acting on the shell will be used as a particular solution then we will do the bending analysis of unloaded shell. Because when we will consider the bending moment in the shell, the order of differential equation increases.

So, therefore the characteristic equation will have a higher order that means it can go up to the 8th order equation. So, therefore if we use the unloaded shell analysis then we will find a equation of the wave and which will get damped out very quickly. So, bending analysis is necessary because of the edge disturbance. And edge disturbance is resulted near the edges and corrective load is applied. So, using these 2 results that means superimposing the 2 results 1 and 2, we can get the actual boundary condition at the edges and that will give you the correct analysis of the shell including the bending moment.

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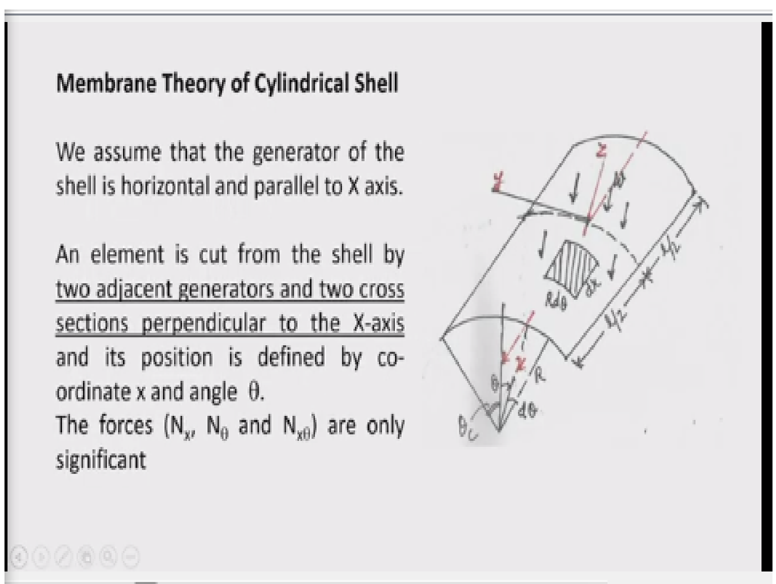

Let us now go to the membrane theory of cylindrical shell to develop the differential equations of equilibrium. We assume that generator of the shell is horizontal and parallel to X axis. So, this is the generator which rolls over the directrix which is a curved, this curve maybe circular arc, this may be your part of a parabola, this may be an elliptical curve or this may be a cycler, various types of possibilities are there for directrices.

So, now here our intention is to develop the differential equation of equilibrium. So, we take an element of the shell. The element is taken by cutting from the shell by 2 adjacent generators, 2 adjacent generators you can see and then 2 cross section perpendicular to X axis. So, the rectangular element is taken whose one side is dx and another side is $Rd\theta$, theta is measured from the crown, this is the crown of the shell that is the highest point.

And from this vertical axis, the angle θ is measured. So, at this edge the location is by θ and the adjacent element will be located at $d\theta$, where $d\theta$ is small angle. So, naturally the length of this curve is $Rd\theta$. So, now the area of the shell element is $Rd\theta \times dx$. So, the forces that will be considered on the element to resist the externally applied load are N_x, N_θ and $N_{x\theta}$. N_x is the membrane forces along the longitudinal direction, N_θ is the force in the circumferential direction and $N_{x\theta}$ is the membrane shearing force that I have shown in my earlier slide. That membrane shearing force, these are the membrane shearing force and this is N_x towards the X axis and this is N_θ.

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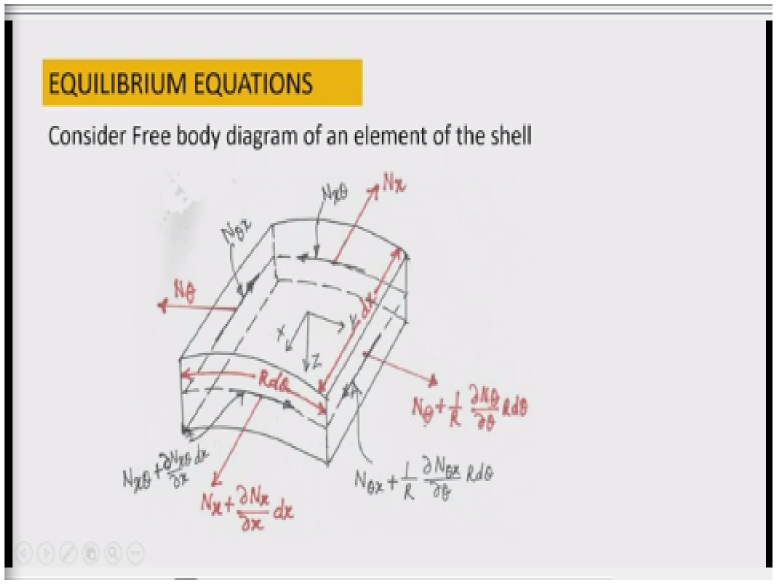

So, let us see the free body diagram of the shell element. So, if you see the element of the shell here, the length of the one side is $Rd\theta$ and another side is dx which is parallel to the generator. The X, Y, Z are the component of the load that is acting on the shell in the respective direction. X is the component of the load along the X direction, Y is the component of the load along the Y direction and Z is the component of the load along the radial direction.

So, here if we see a differential element, so in one edge this is the stress membrane force N_x which is acting at the medial surface. On the opposite edge the membrane stress is $N_x + \frac{\partial N_x}{\partial x} dx$, so these quantities the increment in the length of dx. Now, let me see the forces in the other edges. At the one edge the circumferential forces is N_θ, at the opposite edge the circumference force is $N_\theta + \frac{1}{R}\frac{dN_\theta}{d\theta} Rd\theta$. Now, how this quantity comes?

You can see the variation is within the length of $Rd\theta$. So, therefore $\frac{dN_\theta}{Rd\theta}$ is taken and the length over which it varies is $R\theta$, so it is multiplied by $Rd\theta$. Now, let me see the shearing forces in one edge that is the edge which is parallel to the circumferential direction. They are the membrane stresses are $N_{x\theta}$ and on the opposite edge the increment will take place in a length of dx. So, therefore $N_x + \frac{\partial N_x}{\partial x}dx$ is the membrane force at the opposite edges. Let us see the other shearing forces, in one edge where the N_θ is acting the membrane shearing forces $N_{\theta x}$

On the opposite edge it is $N_\theta + \frac{1}{R}\frac{dN_{\theta x}}{d\theta}Rd\theta$. This increment is taking place within a length of $Rd\theta$, so therefore $Rd\theta$ term is coming here and it is also multiplied by $Rd\theta$. So, knowing the forces acting on the element and component of the loads in the respective direction axis, one is along the x axis and one is along the circumferential axis and one is along the radial direction. So, now we can establish the equilibrium of the forces.

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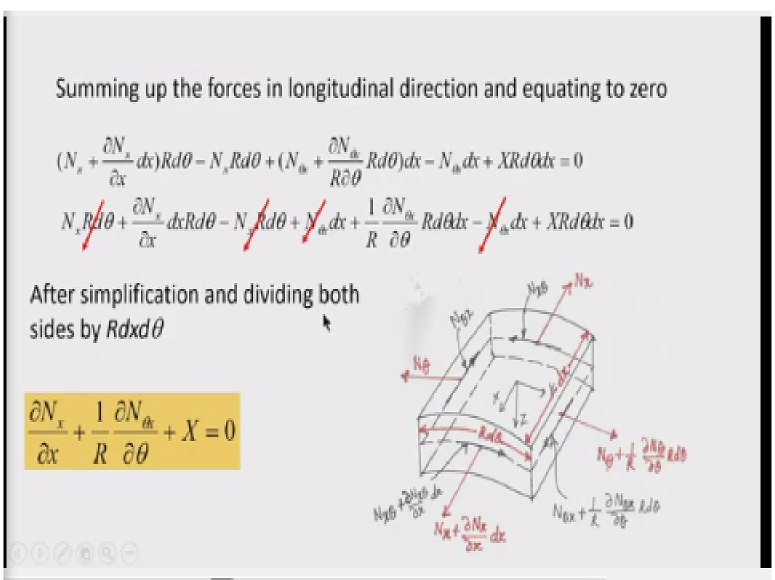

So, summing up the forces in longitudinal direction first and then equate to 0. Now let us see what are the forces in the longer general direction? One is N_x, so if I take this direction as the positive direction of the x axis, then $N_x + \frac{\partial N_x}{\partial x} dx R d\theta$ because the length is $Rd\theta$ over which it is acting, so it is multiplied by $Rd\theta$. On the opposite edges, the force is N_x, so it is $N_x \; Rd\theta$ but the direction is opposite, so therefore negative sign is given.

Then also find out other forces in the x direction. Other forces in the x directions are you can see here $N_{\theta x}$ and it is increment on the other edges. So, if I take this force first, so that will be $N_{\theta x} + \frac{\partial N_{\theta x}}{R d\theta} R d\theta \times dx$ because it is acting over the length dx. And it is multiplied with the force per unit length because these N_x, $N_{\theta x}$ and N_θ are all expressed in terms of force per unit length.

So, therefore to find out the total force on the edge of the element, we have to multiply by the corresponding length of the element. Then on the opposite edges, you are seeing that $N_{\theta x}$. Component of the load along the x direction is X and the area of the middle surface is $Rd\theta dx$. So, total load on the element is $XRd\theta dx$ and it is equated to 0 to establish the equilibrium.

Now multiplying the terms that means, $Rd\theta$ multiplying this term inside the bracket with the $Rd\theta$ will $N_x R d\theta + \frac{\partial N_x}{\partial x} dx R d\theta$. Then this term is existing as it is - $N_x R d\theta$ then again we break this term, so $N_{\theta x}dx+\frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta}Rd\theta dx$, then we get this term $-N_{\theta x}dx+XRd\theta dx$ equal to 0. So, you can see there are some common terms which can be cancelled.

So, $N_x Rd\theta$, $N_x Rd\theta$ is cancel, then $N_x\theta$, $N_x\theta$ are cancel. So, ultimately we are getting an equation of this $\frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta}Rd\theta dx$ + $XRd\theta dx$ and already this term is there $\frac{\partial N_x}{\partial x}dxRd\theta$.

Now you can see here with the all terms the $dxRd\theta$ theta is common. So, $dxRd\theta$ theta indicates the area of the element which is non zero. So, therefore divide throughout by $dxRd\theta$, so, after dividing throughout by $dxRd\theta$ and after simplification we get

$$\frac{\partial N_x}{\partial x}+\frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta}+X=0$$

So, this is the first differential equation that we have obtained using the equilibrium condition in the longitudinal direction that is the direction parallel to the generators. Now, in this equation you can see that 2 unknowns are involved, one is N_x that is the membrane force along the x direction and another is membrane shearing force $N_{\theta x}$ because X will be known and R will be known from the given geometry of the shell, so we have to find this N_x and $N_{\theta x}$. But we need other equation because a single equation with more than one variable cannot be solved; we require the help of other equations.

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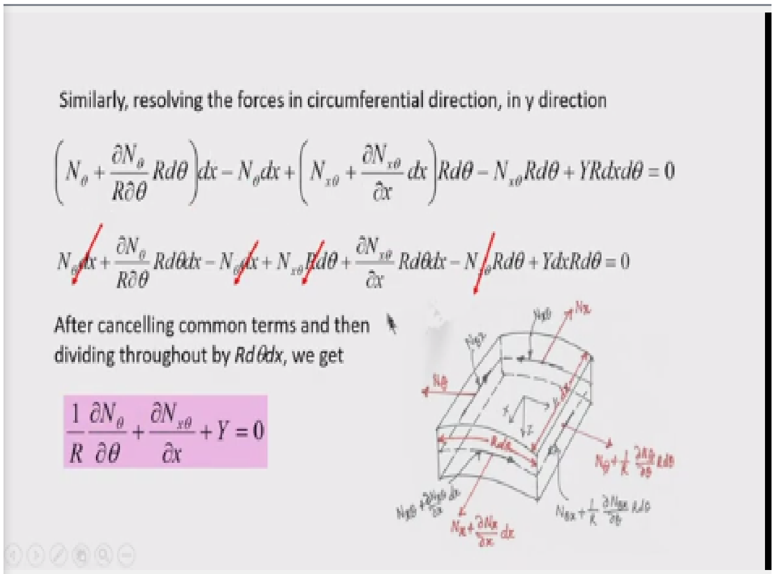

So, now let us establish the equilibrium in the y direction, y direction I mean here the direction that is tangential to the directrix. So, you can see what are the forces in the y direction? One is N_θ that is acting here and on the opposite direction $N_\theta + \frac{1}{R}\frac{\partial N_\theta}{\partial\theta}Rd\theta$. Then other forces in the y direction is $N_{\theta x}$ that you are seeing and here you are seeing $N_{x\theta} + \frac{\partial N_x}{\partial x}dx$.

Now establish the equilibrium. So, equilibrium will be established here one is $\left(N_\theta + \frac{\partial N_\theta}{R\partial\theta}Rd\theta\right)dx$ that is the total force along this edge that we have taken. Then $-N_\theta dx$ on this edge because dx is the length, similarly $\left(N_{x\theta} + \frac{\partial N_{x\theta}}{\partial x}dx\right)Rd\theta$ is the force in this direction along the circumferential direction. And on the opposite edges you will get $N_{x\theta}Rd\theta$ and the total load in the circumferential direction because y is the component per unit area. So, when we multiply by the area of the element it will be $YRdxd\theta$, so we will get this equation.

$$\left(N_\theta + \frac{\partial N_\theta}{R\partial\theta} Rd\theta\right)dx - N_\theta dx + \left(N_{x\theta} + \frac{\partial N_{x\theta}}{\partial x} dx\right)Rd\theta - N_{x\theta} Rd\theta + YRdxd\theta = 0$$

Now again term by term multiplication you carry out and you will find that it is decomposed into

$$N_\theta dx + \frac{\partial N_\theta}{R\partial\theta} Rd\theta dx - N_\theta dx + N_{x\theta} Rd\theta + \frac{\partial N_{x\theta}}{\partial x} Rd\theta dx - N_{x\theta} Rd\theta + YdxRd\theta = 0 .$$

Cancelling some common terms, common terms are there, so you can cancel it and divide throughout by $Rd\theta dx$, ultimately you will arrive at this equation. The second equation of equilibrium which relates N_θ and $N_{x\theta}$. In the previous case we related N_θ and $N_{\theta x}$ here we related N_θ and $N_{x\theta}$. So, this equation is

$$\frac{1}{R}\frac{\partial N_\theta}{\partial\theta} + \frac{\partial N_{x\theta}}{\partial x} + Y = 0$$

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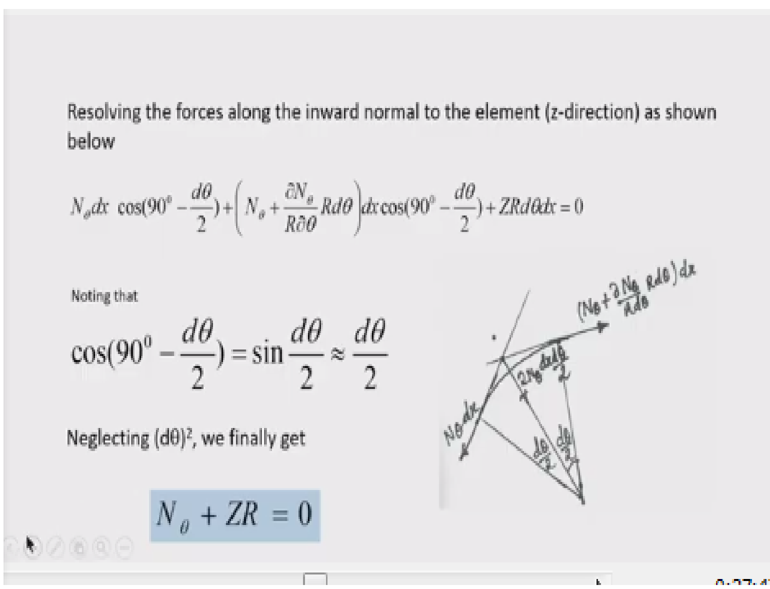

Now, let us go to resolve the forces along the radial direction and then sum it and equate to 0 to establish the equilibrium in the radial direction. You can see here that, if this is the element along the circumferential direction whose length is $Rd\theta$, but $N\theta dx$ acts along this direction, that is parallel to the direction of generator. So, the total force here is $N\theta dx$, on the opposite edge it will be $\left(N_\theta + \frac{\partial N_\theta}{Rd\theta} Rd\theta\right)dx$.

So, this term is taken $\left(N_\theta + \frac{\partial N_\theta}{Rd\theta} Rd\theta\right)$ dx. Now resolve the forces along the normal direction. So, if the total angle that is subtended by the element at the centre is d theta, then semi angle is $\frac{d\theta}{2}$. So, when we resolve the forces along the radial direction, suppose this angle is $\frac{d\theta}{2}$ and this is your 90 degree, so this angle will be $(90^0 - \frac{d\theta}{2})$.

So, resolving these forces along the radial direction **N** N theta dx cos because the component of this along the radial direction if this angle is $(90^0 - \frac{d\theta}{2})$ will be $N_\theta dx \ \cos(90^0 - \frac{d\theta}{2})$. On the other side $\left(N_\theta + \frac{\partial N_\theta}{R\partial\theta} Rd\theta\right)dx \cos(90^0 - \frac{d\theta}{2})$. Then the component of the load along the radial direction is $ZRd\theta dx$. You can see here the components of the forces on the 2 opposite edges arising due to circumferential membrane stress N_θ is additive. Because component are acting in the same direction, so these are added and it is combined with the component of the external load along the Z direction that is $ZRd\theta dx$ equated to 0. So, note it that $\cos(90^0 - \frac{d\theta}{2})$ is nothing but $\sin\frac{d\theta}{2}$. And because $d\theta$ is a small angle, so we can write $\sin\frac{d\theta}{2}$ is approximately equal to

$\frac{d\theta}{2}$. So, based on that this quantity will be $N_x dx \frac{d\theta}{2}$ and these component will be $\left(N_\theta + \frac{\partial N_\theta}{R\partial\theta} Rd\theta\right)\frac{d\theta}{2} + ZRd\theta dx$. [Time 31:02]

Now, in this middle term you will find that $\frac{1}{R}\frac{\partial N_{\theta x}}{\partial\theta}$ will be multiplied by $\frac{d\theta}{2}$, so that means d $d\theta^2$ will come. So, since $d\theta$ is a small quantity it is square is neglected, so this term can be neglected. So, ultimately neglecting the square of this small term and combining the other term because it will be $N_\theta dx$ $\frac{d\theta}{2}$ and here also it will be $N_\theta dx \frac{d\theta}{2}$.

So, that means, the component will be $2N_\theta dx \frac{d\theta}{2}$, so this component has to be balanced by the component of the load along the radial direction. So, we get actually these $2N_\theta dx \frac{d\theta}{2} + ZRd\theta dx = 0$. So, dividing both sides by $dxd\theta$ ultimately you will arrive at this equation $N_\theta + ZR = 0$. So, this is the 3rd equilibrium equation. Now, we have got 3 equations of equilibrium in the membrane theory for this cylindrical shell.

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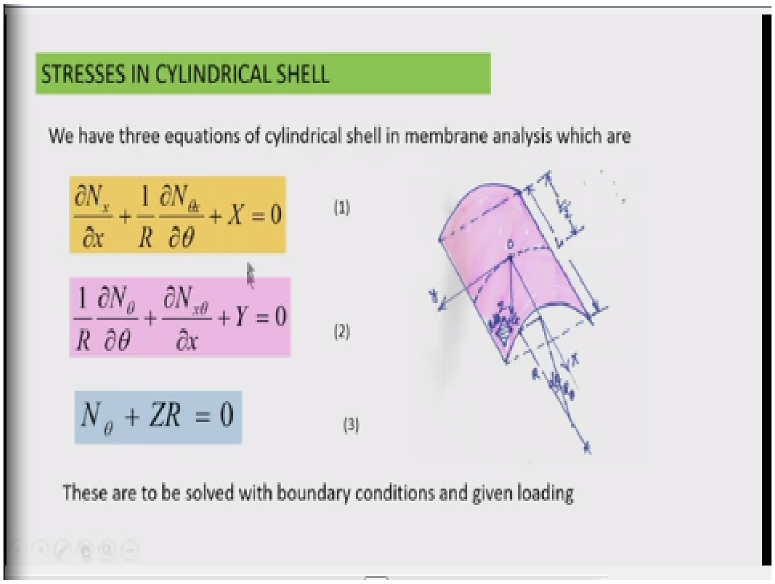

So, 3 equations of equilibrium are these $\frac{\partial N_x}{\partial x}+\frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta}+X=0$, that is first equation.

Second equation is $\frac{1}{R}\frac{\partial N_\theta}{\partial \theta}+\frac{\partial N_{x\theta}}{\partial x}+Y=0$ and third equation is $N_\theta + ZR = 0$.

You may note here that we are dealing with 3 unknown quantities, one is N_x, N_θ and membrane shearing force $N_{\theta x}$. Because the component of the load will be known from the given loading condition on the shell.

Now, it is obvious that 3 equations and we have only 3 unknown quantities though this problem of cylindrical shell in membrane theory is a statically determinate problem. So, now the $N_\theta + ZR = 0$ is an algebraic equations, so knowing the component of load along the radial direction and knowing the variation of radius of curvature R, we can find the N_θ at any angle that is measured from the crown. One thing can be noted that the radius of curvature R is not constant except in the circular cylindrical shell which has having constant radius of curvature. But other case of directrices the radius of curvature to be first obtained by the theory of calculus.

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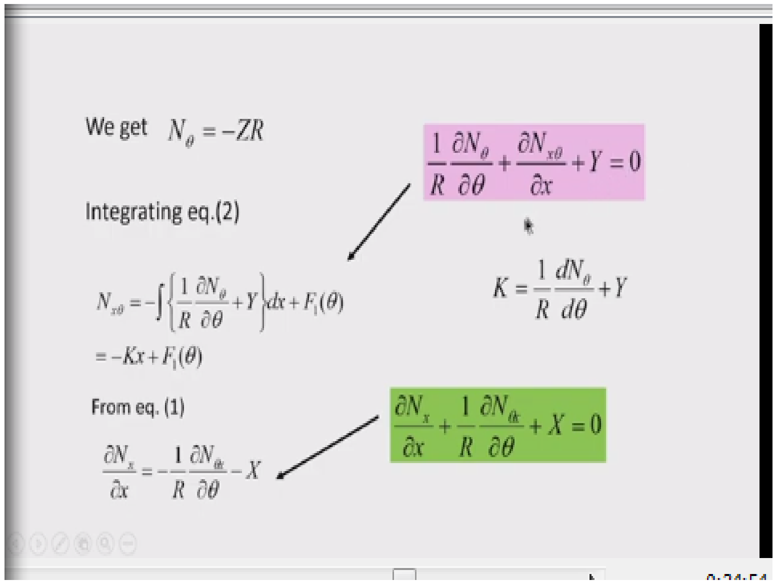

So, from the 3rd equation we get $N_\theta = -ZR$. Now, if I substitute this $N_\theta = -ZR$ in this equation, then we are targeting to obtain this $N_{x\theta}$. So, we get ultimately if I take $\frac{\partial N_{x\theta}}{\partial x}$ in the left hand side and transfer $\frac{1}{R}\frac{\partial N_\theta}{\partial \theta}$ in the right hand side with a negative sign and capital Y is also in the right hand side with negative sign. Then, after integrating we can get because these N_θ and Y these will be a function of θ.

So, I group these together because R is also a function of θ and it is integrated with respect to x because this differentiation is with respect to x. So, integrated with respect to x and the constant of integration. Now since this expression is depending on the θ, so therefore we are taking a constant as a function of θ. So, since this is not depending on the x, so I take it as a constant.

But it is not purely constant, it is depending on the variable θ but not with x. So, therefore, when I integrate it, it becomes $-Kx + F_1(\theta)$ actually K is your this quantity $\frac{1}{R}\frac{dN_\theta}{d\theta} + Y$. From equation 1, so once we got this $N_{x\theta}$ then we substitute this $N_{x\theta}$ in the 1, $\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta} + X = 0$ this equation is 1 is written here again. So, you can substitute this $\frac{\partial N_x}{\partial x}$ is kept in the left hand side. In the right hand side the quantities $-\frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta} - X$. So, now this expression has to be integrated.

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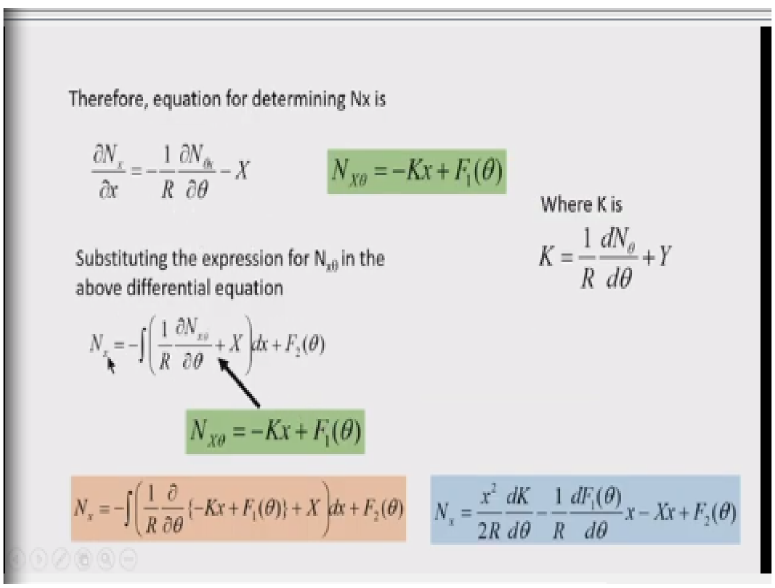

Integrated this expression one can obtain this $N_x = -\int\left(\frac{1}{R}\frac{\partial N_{x\varphi}}{\partial \theta} + X\right)dx + F_2(\theta)$, and $F_2(\theta)$ is another constant of integration. Now this quantity that is here N x theta we earlier obtain $N_{x\theta}$ is obtained as $-Kx + F_1(\theta)$. So, we substitute $N_{x\theta}$ here, when you substitute $N_{x\theta}$ here and carryout the differentiation then we obtain

$$N_x = -\int\left(\frac{1}{R}\frac{\partial}{\partial\theta}\{-Kx + F_1(\theta)\} + X\right)dx + F_2(\theta)$$

$$N_x = \frac{x^2}{2R}\frac{dK}{d\theta} - \frac{1}{R}\frac{dF_1(\theta)}{d\theta}x - Xx + F_2(\theta)$$

X is the component of the load along the x direction, along the longitudinal direction that is that direction parallel to the generator. Now, integrating this expression we get now, you see here K is a function of θ.

So, therefore $\frac{dK}{d\theta}$ is kept here and x is integrated with respect to dx, X is integrated here. So, after integrating this quantity X it becomes $\frac{x^2}{2}$ and R is there. So, 1st term integration is carried out, 2nd term integration you can see this $\frac{dF_1(\theta)}{d\theta}$ term is appearing here and integration of this 1 with respect to x is x and $\frac{1}{R}$ term is appearing here. Then the 3rd term is X which is the component of the load in the longitudinal direction that is also a constant quantity.

So, it is integration will be $-Xx$ plus the constant of integration is there. So, we have obtained the 3 expressions of the stresses individually. One is your $N_\theta = -ZR$ that is the first stress this circumferential stress is easily obtained without solving any differential equation because the 3rd equation of equilibrium is an algebraic equation. Then secondly we got $N_{x\theta} = -Kx + F_1(\theta)$ and lastly we obtain the longitudinal stress N x which is given by

$$N_x = \frac{x^2}{2R}\frac{dK}{d\theta} - \frac{1}{R}\frac{dF_1(\theta)}{d\theta}x - Xx + F_2(\theta)$$

So, here the task remains to find out the differential coefficient $\frac{dK}{d\theta}$ and also to evaluate the unknown constant $F_1(\theta)$ and $F_2(\theta)$ by substituting the boundary conditions.

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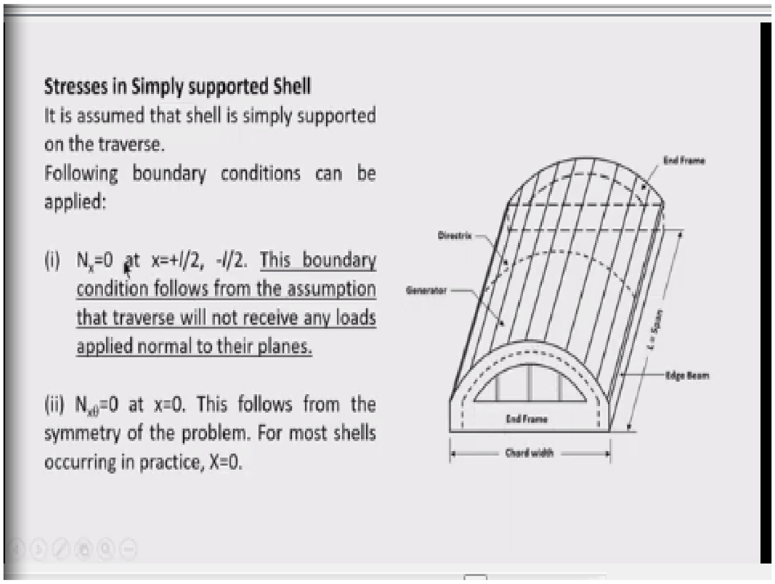

So, we shall illustrate one type of boundary condition that is possible here to implement. Other type of boundary condition requires rigorous solution and that is not possible with this membrane hypothesis. So, here we obtain these stresses and we found the unknown constant $F_1(\theta)$ and $F_2(\theta)$ make their appearance in the expression of the stresses. So, let us find out the unknown constant $F_1(\theta)$ and $F_2(\theta)$. Now, at the traverse when this shell is supported we take N_x =0 . That means boundary condition is from the assumption that traverse will not receive any loads applied normal to their planes.

So, based on that condition we take $N_x = 0$ provided we take the origin at the centre of the shell.

So, if the span of the shell is L then at $x = +\frac{l}{2}$ and $x = -\frac{l}{2}$ traverse are there and there the $N_x = 0$. So, substituting this boundary condition that $N_x = 0$ at $x = +\frac{l}{2}, -\frac{l}{2}$, we can get 1 constant. The 2nd boundary condition is $N_{x\theta} = 0$ at x = 0 because x = 0 is the point of symmetry and there the shear force cannot exist. So, most of the shell occurring in practice the load along the x direction i.e longitudinal direction is also 0.

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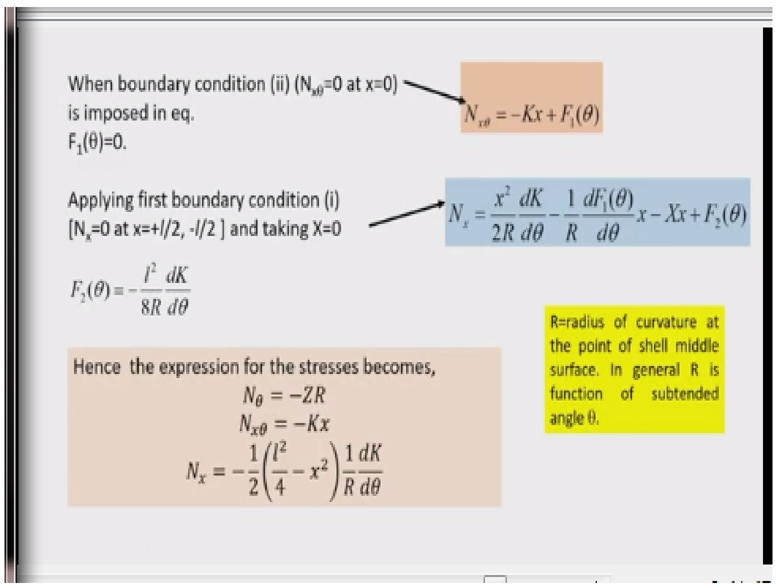

So, therefore we obtain the 2 constant of integration, one is $N_{x\theta} = 0$ at x = 0, that means $F_1(\theta) = 0$. Applying the 1st boundary condition that is the 2nd boundary condition on $N_{x\theta}$ we apply. First boundary condition was on the N_x and the N_x was the longitudinal stress which is 0 at $x = +\frac{l}{2}, -\frac{l}{2}$. And taking the component of the load along the longitudinal direction X = 0.

We now put this $x = \frac{l}{2}$ here and then let us evaluate $F_2(\theta)$.

So, $F_2(\theta)$ can be evaluated by putting $x = \frac{l}{2}$ here and we get $F_2(\theta)$ is nothing but $F_2(\theta) = -\frac{l^2}{8R}\frac{dK}{d\theta}$

Hence the expression for these stresses becomes $N_\theta = -ZR$ $N_\theta = - ZR$ and $N_{x\theta} = -Kx$ .because $F_1(\theta)$ is 0 and $F_2(\theta)$ we have evaluated as $F_2(\theta) = -\frac{l^2}{8R}\frac{dK}{d\theta}$ and this is when substituted in the expression for N_x we can simplify this expression as

$$N_x = -\frac{1}{2}\left(\frac{l^2}{4} - x^2\right) \times \frac{1}{R}\frac{dK}{d\theta}$$

R is the radius of curvature at the point of shell meet surface. And in general R is a function of subtended angle θ.

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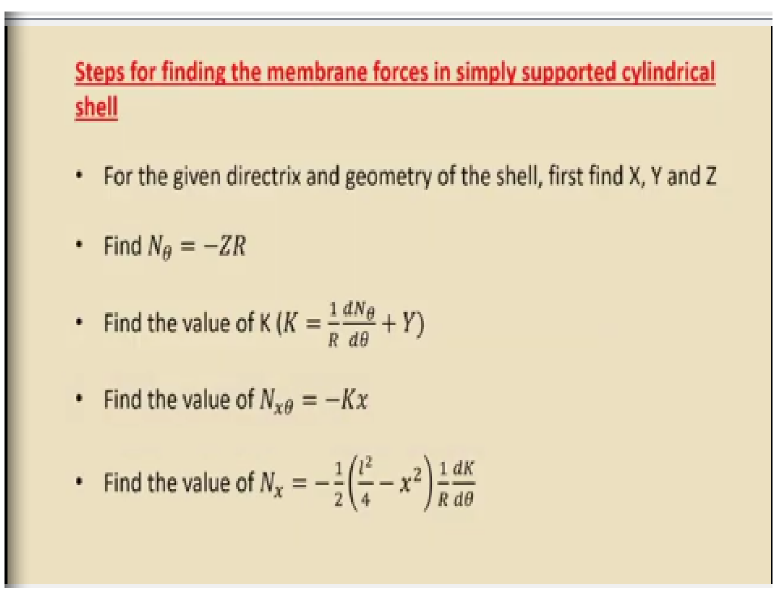

Now in calculation of membrane stresses some steps have to be followed. So, systematic steps are given here for the given directrix and geometry of the shell. First find this X, Y and Z and also the expression for radius of curvature that is the most important. So, that task have to be done completed first, given the load in any direction we have to find the component of the load along the longitudinal direction along the directrix and along the radial direction.

So, 3 components of loads are found and also the radius of curvature of the directrix curve is to be found out. So, once these quantities are found out then we go for obtaining N_θ. N_θ is the circumferential stress that is developed on the shell. So, $N_\theta = -ZR$, so this is simply found from the algebraic 3rd equilibrium equation which is a algebraic equation. Then after finding N_θ, find the value of K.

Because you are seen that K is appearing in many places, so we have to know what is K. So, after finding this N_θ K is to be obtained and K is nothing but $K = \frac{1}{R}\frac{dN_\theta}{d\theta} + Y$

Once K is obtained then go to this step, find $N_{x\theta}$. $N_{x\theta} = -Kx$, then after finding $N_{x\theta}$ find the value of N_x because N_x depends on the x, so x can be fixed at any location and then the K is already evaluated here, so K is in terms of θ. So, therefore $\frac{1}{R}\frac{dK}{d\theta}$ can be calculated and N_x is completely known.

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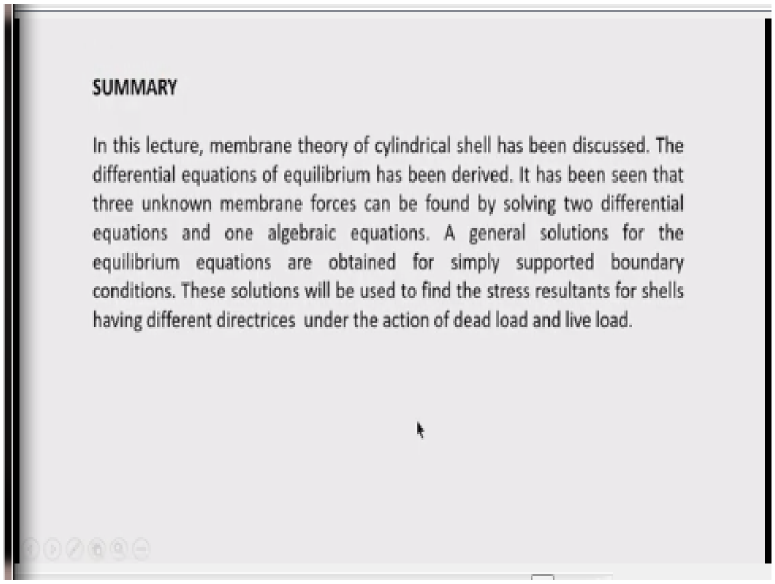

So, today we have completed the derivation of the differential equations of equilibrium of a cylindrical shell and is in the membrane hypothesis. And it has been seen that 3 unknown membrane forces are existing in the differential equations and there are 3 differential equations also. So, this problem is statically determinate problem and we have attempted a general solution for the equilibrium equations for simply supported boundary condition. What I mean that?

It is a general equation, general solution whatever be the radius of curvature and loading. That means depending on the directrix, whether it is arc of a circle whether it is a semi ellipse, whether it is a cycloid, whether it is a parabola whatever maybe we can give input to these general solution required quantities, required quantities are radius of curvature and the value of X, Y and Z.

So, after giving this input we can derive the equation or expression for membrane forces in the cylindrical shell for any type of directrix. Any type of directrix, there is no restriction that it should be a circular cylindrical shell or it may be a paraboloid shell any type of directrix you take and this expression can be used. So, this solution will be used to find the stresses in our next class for shells of different directrices and that the action of dead load and live load, thank you very much.