Plates and Shells
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Lecture - 30
Membrane Theory of Hyperboloid of Revolution: Application of Cooling Tower

Hello everyone, today it is my third lecture of the module 10. So, far I was discussing about the application of membrane theory in pressure vessels and tank in this module. Today I will give a further application on a membrane theory in the surface of revolution in the form of hyperboloid of evolution. So, that type of shell is very common in cooling tower in thermal power station and we will see how this type of shell can be analysed with the help of membrane theory. So, topic of discussion is hyperboloid of revolution and its application to cooling tower.

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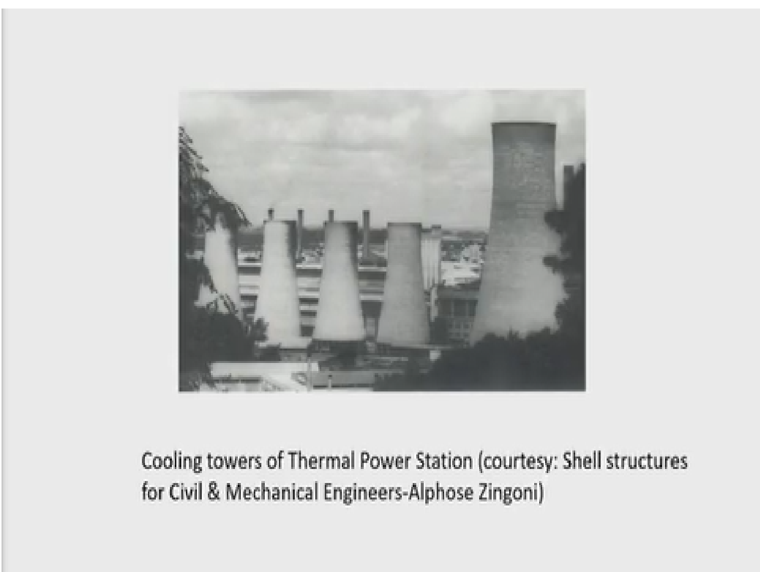

So, cooling tower that are actually a very essential part of the thermal power station and it is having a hyperbolic paraboloid type of shell. So, that is constructed in concrete made of reinforced concrete and steel and has to resist the self-weight and other imposed load. So, therefore, the design of this type of cooling tower is very important and significant for the safety of the industrial operation and the design is generally done using the membrane theory of shell.

So, we have discussed the membrane theory of shell and we have seen that in case of membrane theory the membrane forces that is in-plane forces N_{ϕ} N_{θ} and membrane shearing forces and $N_{\theta\phi}$ are of significant. But in case of shell, axisymmetric loading we do not have to consider this $N_{\theta\phi}$ or $N_{\phi\theta}$. Therefore, it becomes a problem of finding only two unknown variable N_{θ} and N_{ϕ} which are the circumferential stress and meridionals stress respectively.

Once you find these N_{θ} and N_{ϕ}, N_{θ} and N_{ϕ} it is a membrane force in expressed as force per unit length. But when we convert it into stress unit that means, the N_{θ} and N_{ϕ} is divided by the thickness of the shell we get this the stress, corresponding stress. So, N_{θ} divided by thickness of the shell say h will give you the circumferential stress which is also commonly known as Hoops stress.

And when N_{ϕ} divided by h will give this meridional stress or it is commonly known as the longitudinal stress.

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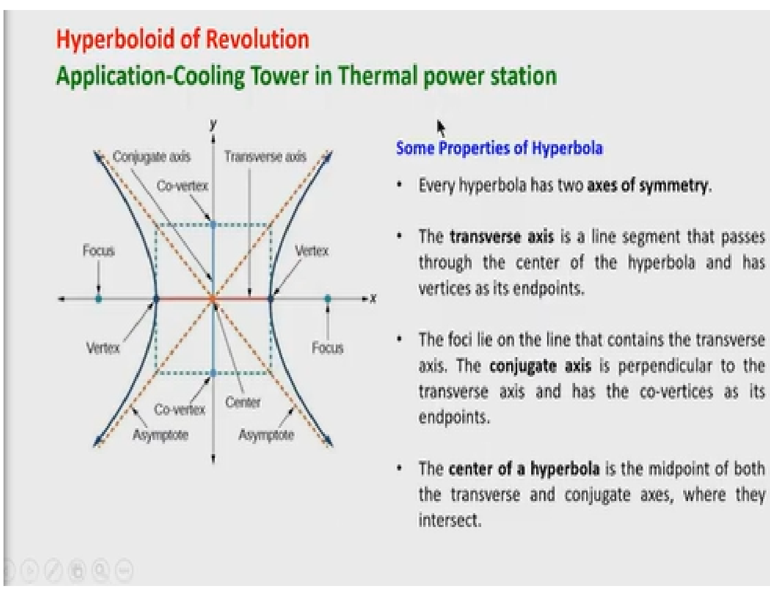

So, hyperboloid of revolution of one shape is formed when a hyperbola is revolved around a vertical axis of rotation. So, this is the vertical axis of rotation and this is the hyperbola. So, when it is rotated around this vertical axis of rotation the hyper shell of surface of revolution is formed and it is nothing but hyperboloid of revolution. Now, we have to know the properties of the hyperbola that is actually the meridional curve in this case.

And it is rotated around the vertical axis of rotation. So, let us see what are the properties of the parabola? That will be essential for the analysis of the stresses in the hyperboloid revolution. So, every hyperbola has two axes of symmetry that you are seeing, the axes of symmetry is your this x-axis and this y-axis. So, this hyperbola that it is x; symmetrical shell ups revolution. So, whatever load that is applied here is symmetrical about the axis of rotation.

The transverse axis that exists is a line segment that passes through the centre of the hyperbola. So, centre of the hyperbola is this and it passes through the centre of the hyperbola and has vertex for each hyperbola that is for meridian. Meridian is rotated about the vertical axis of rotation. So, vertex of the parabola is located at the transverse axis. So, this is one vertex point and another vertex point is there.

So, the distance of the vertex of hyperbola from the axis of rotation is effector known as a. Now, let us see the parabola hyperbola has a focus. So, one focus is here for this hyperbola and when it is rotated and it is on the other side there will be another focus. So, the focus that is lying on axis it is lying on transverse axis and conjugate axis is the another axis which is perpendicular to the transverse axis. So, y-axis here is a conjugate axis.

Conjugate axis also has co-vertex. So, co-vertex is located by knowing the side of the rectangle. The centre of the hyperbola is the midpoint of both the transverse and conjugate axes.

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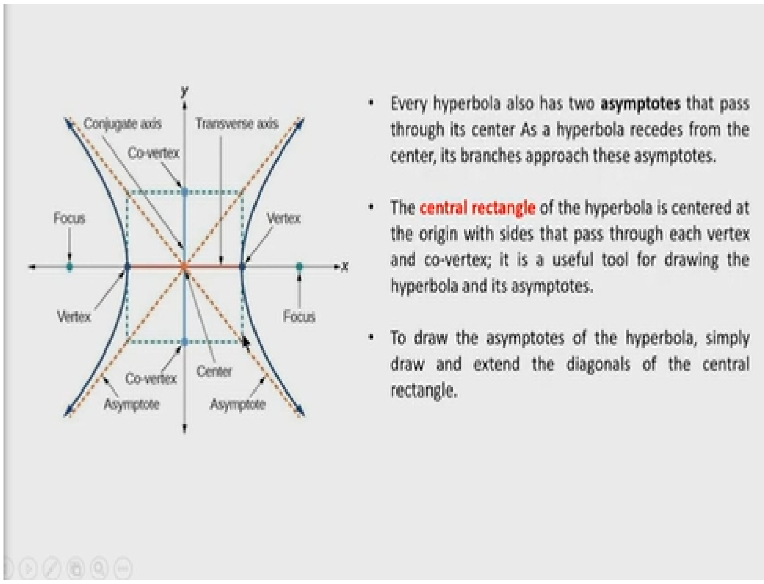

Now every hyperbola has two asymptotes. So, two asymptotes you are seeing here that is hyperbola receives from the centre gradually and its branches become asymptotic that is the approach this asymptote. So, you can see here in the upper portion as well as lower portion it is the hyperbolic curve is asymptotic to the straight line. So, asymptotes are straight line passing through the centre of the hyperbola.

The central rectangle that is formed here is centred at the origin which sides that passes through each vertex. So, one side is this distance between the two vertex that is a is the distance of one vertex from the axis of rotation then the side of this rectangle is twice a. Now co-vertexes are at a distance b from the transverse axis. So, if it is a distance of b from the transverse axis the length of the central rectangle is $2b$.

So, $2b$ is the length and $2a$ is the width. So, we get the size of the rectangle that is the centre rectangle. And the asymptote is diagonal to this central rectangle that it has to pass through the centre of the rectangle as well as through the corner of the rectangle. So, two asymptotes are there and the branch of the hyperbola becomes asymptotic as it goes away and away from the centre of the hyperbola.

To draw the asymptotes of the hyperbola simply draw and extend the diagonals of the central rectangle. So, if I draw the diagonals of the central rectangle and extend it, we will get these asymptotes. So, asymptotes slope is important and it can be easily found by knowing the length and breadth of the central rectangle.

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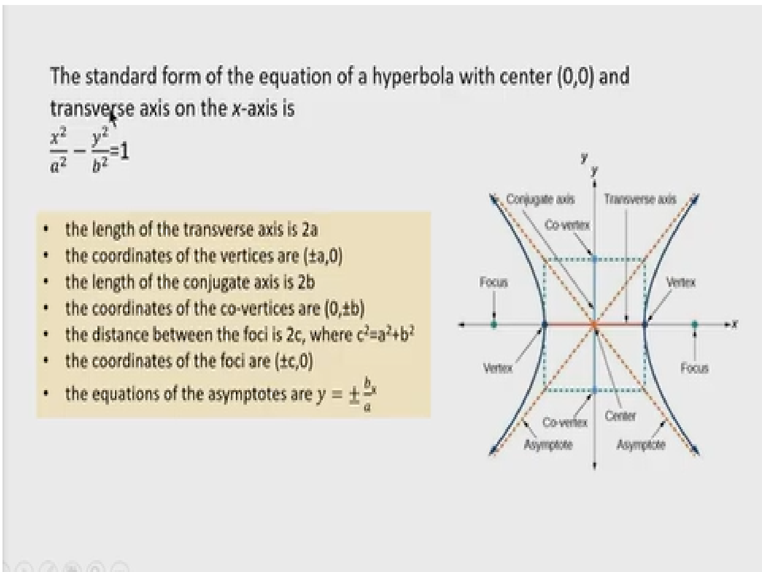

The standard form of the equation of the hyperbola with centre as 0, 0 centre is the origin which is taken as the coordinate is 0, 0 and transverse axis lying on the x-axis. So, this is the x-axis conjugate x-axis is your y-axis. So, then equation the parabola, hyperbola can be written as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. So, that is the equation of the hyperbola. The length of the transverse axis is $2a$.

And co-ordinate of the vertices are $\pm a,\ 0$ that is on the right-hand side towards the positive axis. The focus is located at a distance of c from the centre of the parabola and on the left-hand side towards the negative side of the x-axis the focus is located at a distance of c from the conjugate axis. So, the vertex are located at a distance a from these conjugate axis whereas the focus foci, 2 foci are there 2 foci are located at a distance of c from the conjugate axis.

But this distance between 2 foci and this length of the transverse axis and length of the conjugate axes are related by this relation. So, $c^2 = a^2 + b^2$ so, this relation is valid for this hyperbola.

Now, you can see here that in the equation of the parabola, a is simply known as this distance that is the distance of the vertex of the parabola from the vertical axis of rotation that is the conjugate exists.

Whereas the; b has to be found if we know the distance between 2 foci. So, if a is known, then b can be found if c is obtained from the distance between 2 foci. The coordinate of each foci is $\pm c$, on towards the positive axis, it will be $(c,\ 0)$ and 2 as the negative axis it will be $(-\ c,\ 0)$. The equation of the asymptotes you are seeing that it is passing through the centre of the parabola. So, it is a straight line. So, its slope will $\frac{b}{a}$, because this side is b and this is a.

So, slope is simply $\frac{b}{a}$, equation of the straight line is in the form standard form $y = mx$ where m is the slope of this straight line. So, slope of the straight line is b/a in the positive direction you will get $+\ b/a$, in the negative direction you will get $-\ b/a$. So, these properties are essential to solve the equation for meridional stresses. Now, let us consider the hyperbola a revolution of one shell and meridian is a hyperbola.

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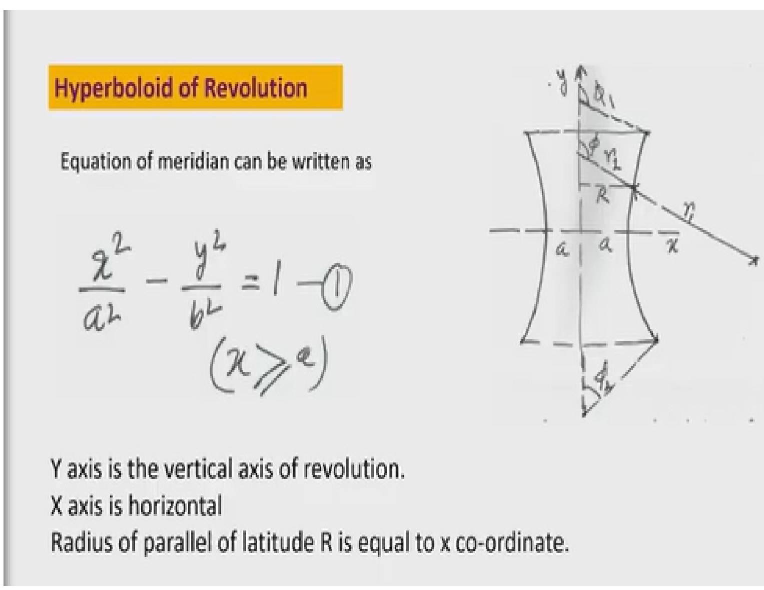

So, this is the meridian which is a hyperbola and equation of the meridian can be written as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a is the distance between the vertex of the parabola from the axis of rotation. Whereas the b has to be found from the distance of the foci that is also lying on the transverse axis as the distance between two foci are related by the equation $c^2 = a^2 + b^2$.

So, $2a$ is known as the length of the transverse axis whereas $2b$ has to be known from the length of the conjugate axis. That means a central rectangle with the ends in central tangle the width is $2a$ and the length is $2b$. So, this is the co-vertex is located at a distance b from the transverse axis in the upwards direction and another co vertex is in the downward direction it is located at a coordinate $(0, \; -b)$.

So, we know all the geometrical parameter of the hyperbola now, we can write the equation where x has to be greater than or equal to a. So, radius of the parallel circle R is equal to the x coordinate. So, at any level suppose we want to find this parallel circle that is the parallel of latitude you can call it. So, here this x coordinate will give the radius of the parallel circle.

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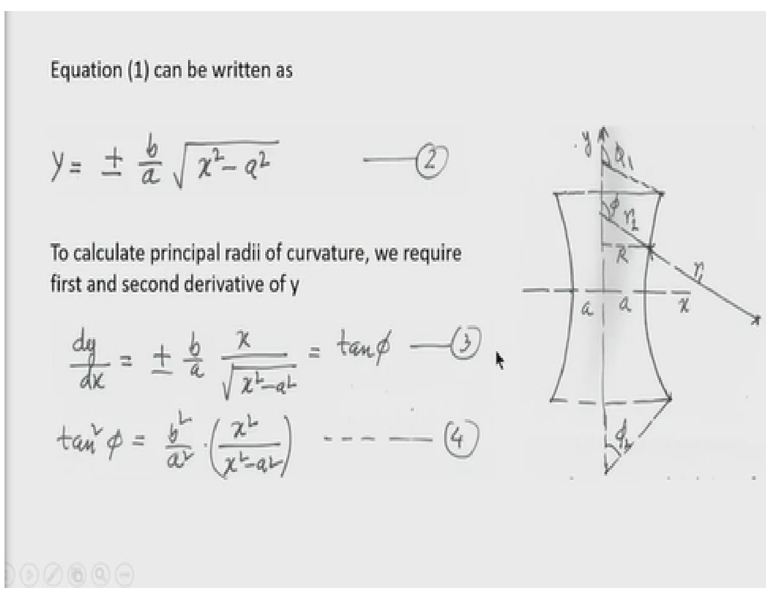

Now this number 1 equation can be written because we want y to find out the slope and curvature. So, y can we returned from the equation one has $y = \pm \frac{b}{a}\sqrt{x^2 - a^2}$. To calculate principal radii of curvature, we require first and second derivative of y. Now, you can see this type of shell that is hyperboloid double revolution is essentially a double click shell but it is anti-elastic.

So, due to anticlastic nature, you can see that one radius of curvature that is centre of one radius of curvature is in the opposite direction as compared to other radius of curvature. So, two area of curvature r_1 and r_2 are shown here where r_1 the radius of curvature for r_1 will meet the centre in the opposite direction whereas, r_2 will meet the vertical axis of rotation at this point. So, they are on the opposite side of the middle surface of the shell.

So, differentiating this equation number 2 with respect to x we can find the slope. So, $\frac{dy}{dx}$ can be obtained as $\frac{dy}{dx} = \pm\frac{b}{2a} \times \frac{2x}{\sqrt{x^2-a^2}}$. So, ultimately cancelling two from both numerator and denominator we obtain that $\frac{dy}{dx} = \pm\frac{b}{a} \times \frac{x}{\sqrt{x^2-a^2}} = \tan tan\, \phi$. So, tan square phi can be obtained from this equation, tan square phi is b square by a square into x square by x square - a square.

So, this type of shell is an example of anticlastic shell which we have not earlier analysed. We are earlier analysed a shell which are doubly curved of course. But they were simplistic nature and in one case we have found that the Gaussian curvature is 0. So, such type of shell is formed by rotating a straight line say example a cylindrical shell or conical shell that we have found earlier.

Now, here the Gaussian curvature if you want to calculate, Gaussian curvature is nothing but product of two principal curvature. So, if the radius of curvature is r 1 and r 2 so, naturally the Gaussian curvature will be $\frac{1}{r_1} \times \frac{1}{r_2}$ and out of that $1/r_1$ will be negative. So, therefore, you will get Gaussian curvature is always negative for that type of shell and it indicates that it is a anticlastic shell. Now, from equation four we can obtain the x.

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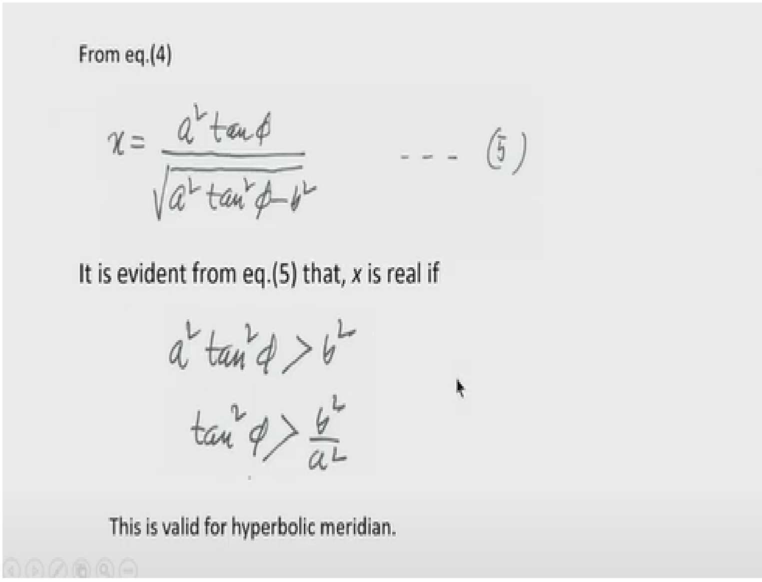

So, just by simple algebraic manipulation will obtain axes, $\frac{a^2 \tan tan\ \phi}{\sqrt{a^2 \phi - b^2}}$. Now, this equation can be examined very carefully to see the validity of or the range of x in which x is valid. So, you can note here that if $a^2 \phi\ - b^2$ is negative quantity then x is imaginary which is not possible. So, therefore, from this equation 5, we can impose a condition for this hyperbola that $a^2 \phi$ should be greater than b^2.

So, which means ϕ should be greater than $\frac{b^2}{a^2}$. So, this is the characteristic of hyperbolic meridian.

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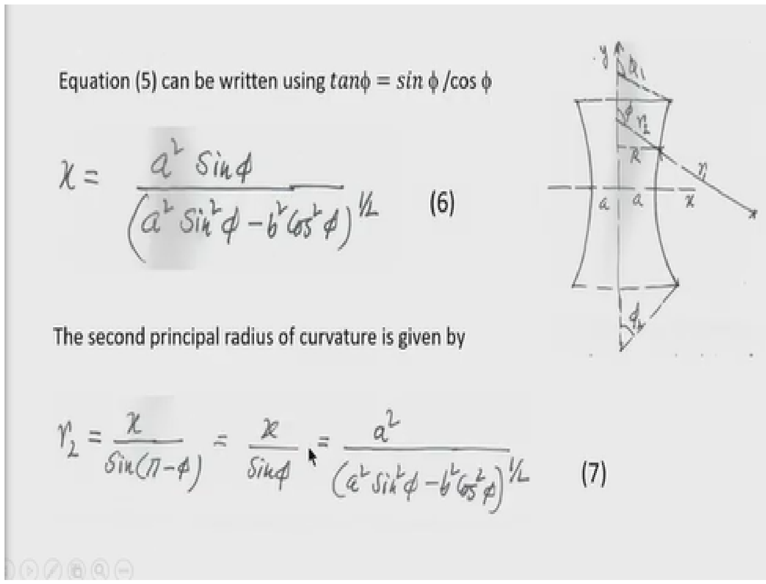

Now, from equation 5, equation 5 is this. So, here tan $tan\ \phi$ is again substituted as $\frac{\sin sin\ \phi}{\cos cos\ \phi}$. So, substituting tan $tan\ \phi = \frac{\sin sin\ \phi}{\cos cos\ \phi}$. The equation 5 can now be written as $x = \frac{a^2 \sin sin\ \phi}{\left(a^2\phi - b^2\phi\right)^{\frac{1}{2}}}$. So, that means this term is under the square root. The second principal radius of curvature r_2 can be easily expressed as r_2 equal to if this coordinate is x or r.

So, this $\frac{x}{\sin sin\ (\pi - \phi)}$. So, $\frac{x}{\sin sin\ \phi}$ and sin $sin\ \phi$ can be obtained because tan $tan\ \phi$ is known. Once the tan $tan\ \phi$ is known, then you can open the sin $sin\ \phi$ also. So, substituting the value of sin $sin\ \phi$ then we get this r_2 as $r_2 = \frac{a^2}{\left(a^2\phi - b^2\phi\right)^{\frac{1}{2}}}$ or directly you put x is here, x is this quantity from equation number six.

If you put x from equation six you will also arrive at this quantity. So, the second principal curvature is known $\frac{a^2}{\left(a^2\phi - b^2\phi\right)^{\frac{1}{2}}}$. And first principal radius of curvature we now have to obtained.

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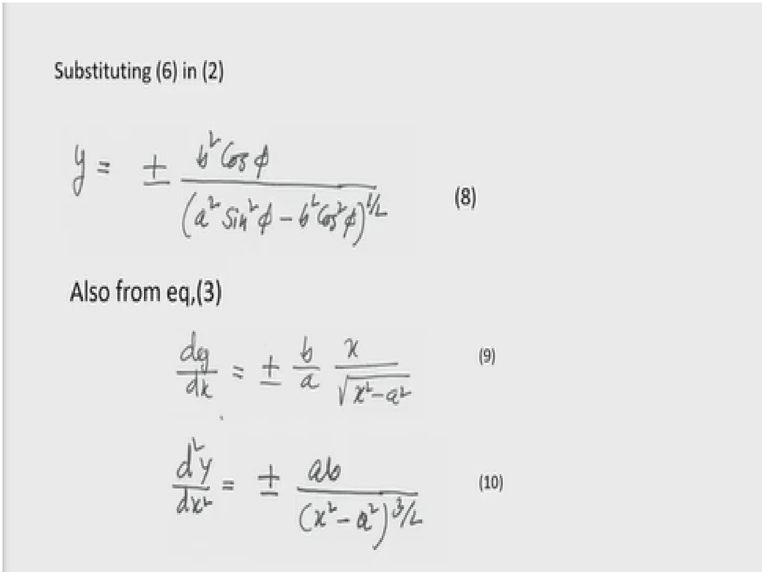

From the equation of this calculus that we know to determine the curvature we need first derivative and secondary derivative that we have already calculated.

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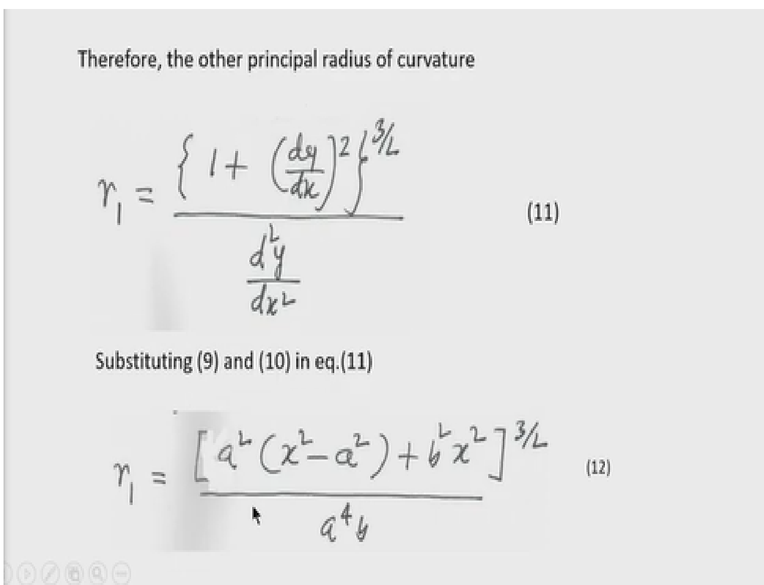

So, r 1 the other principal curvature that is the curvature of the meridional curve which is a hyperbola and for that meridional curve at this point we walked in $\frac{dy}{dx}$ as well as the $\frac{d^2y}{dx^2}$. So, with this quantity $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ that will give second derivative of the meridional curve. The function which defines the meridian curve we can obtain now r_1.

So, $r_1 = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$. So, this is the r_1 and substitute now the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ that we obtained earlier for the hyperbolic meridian. So, substituting these two quantities $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ that is equation number 9 and 10 that is substitute here. We will get the r_1 in terms of x and other parameters of the hyperbola.

So, $r_1 = \frac{\left[a^2(x^2-a^2)+b^2x^2\right]^{\frac{3}{2}}}{a^4b}$. So, we have obtained r_1 the first principal curvature and r_2 is the second principal curvature. So, product of these $\frac{1}{r_1}$ and $\frac{1}{r_2}$ will give you the Gaussian curvature.

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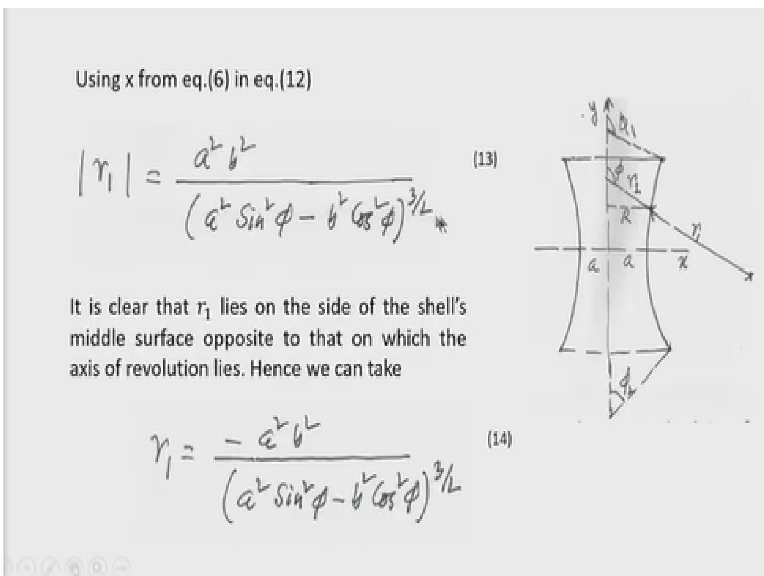

Since r_1 is to be given a negative sign because r_1 lies on the side of the shells middle surface opposite to that on who is the axis of revolution lies. So, axis of revolution lies on that side centre of curvature of the hyperbolic curve at this point will lie on the opposite side. So, therefore, r_1 is negative quantity and when actual calculation will put r_1 $is\ negative$. But this absolute value of r_1 is given as $\frac{-a^2b^2}{(a^2\phi - b^2\phi)^{\frac{3}{2}}}$

So, these two quantities are very important to principal area of curvature that is obtained from the first principle of calculus.

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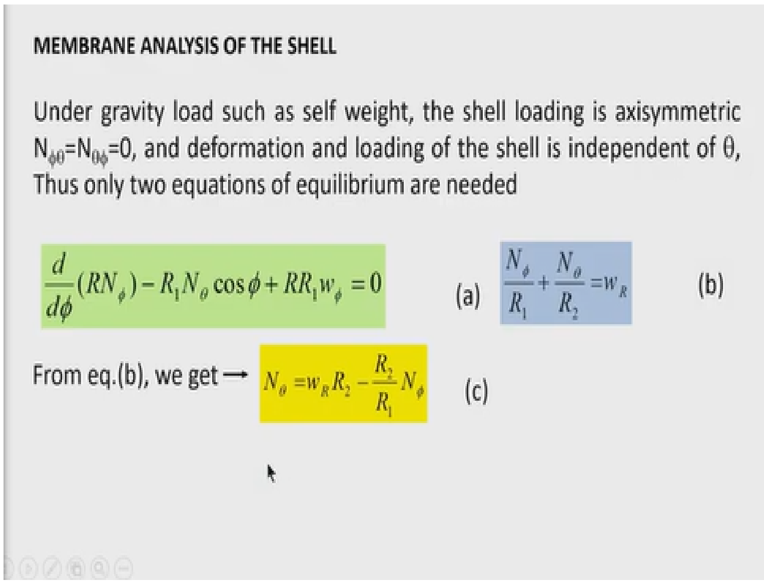

So, next, we go for membrane analysis of the shell. The shell here is under the axisymmetric loading although it is a anticlastic shell but it is of doubly curved nature and it is non-developable surface. So, in this type of shell under the self-weight acting along the surface of the shell, the loading condition is symmetrical with respect to the axis of revolution. So, therefore, the membrane sharing force $N_{\theta\phi}$ $N_{\phi\theta}$ is taken or to be taken to 0.

And deformation that displacements, stresses or loading of the shell is independent of the angle theta. So, those only two equations of equilibrium are essential to find the unknown quantities that is N_ϕ and N_θ. So, first equation is $\frac{d}{d\phi}(RN_\phi) - R_1 N_\theta \cos cos\,\phi + RR_1 w_\phi = 0$ is the first principal radius of curvature here in this example we have taken the symbol R_1. So, capital R_1 has to be replaced by symbol R_1.

This is the general equation we have derived earlier and we are referring this to obtain the member stresses in N_ϕ and N_θ. So, R is the radius of the parabolic circle at certain level defined by the angle ϕ. And you can see here this N_θ and N_ϕ both are appearing in the first equation. But second equation of equilibrium is an algebraic equation and it is given by $\frac{N_\phi}{R_1} + \frac{N_\theta}{R_2} = w_R$, w_R is the component of the load in the radial direction.

So, from this equation b, there is the second equilibrium equation in that case. Equilibrium equation here pertains to the balance of forces along the radial direction. So, from this equation N_θ is obtained as $N_\theta = w_R R_2 - \frac{R_2}{R_1} N_\varphi$.

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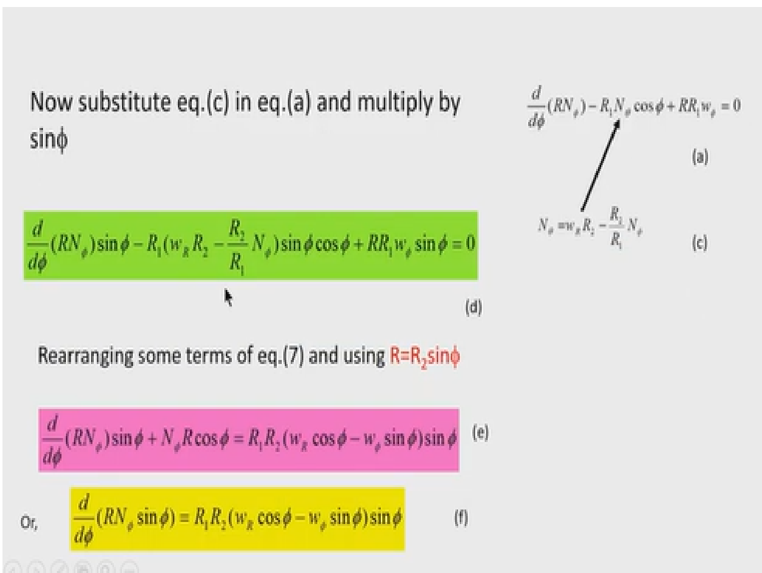

So, substitute this N_θ here and we will get is now this equation substituting this N_θ value here we now get this differential equation in this form and after rearranging. So, this form is $\frac{d}{d\phi}\left(RN_\phi\right) \sin sin\, \phi\ - R_1(\ldots$ and this is actually it was N_θ so, this value is substituted here. Then other quantities are there already and then rearranging some terms of the equation 7 and using $R = R_2 \sin sin\, \phi$.

We now obtain the differential equation in this form $\frac{d}{d\phi}(RN_\phi) \sin sin\, \phi\ + N_\phi R \cos cos\, \phi\ = R_1 R_2 (w_R \cos cos\, \phi\ - w_\phi \sin sin\, \phi\ ) \sin sin\, \phi$.

This equation now has to be integrated. So, again these two terms can be written in this form. So, $\frac{d}{d\phi}(RN_{\phi} \sin sin\, \phi) = R_1 R_2 (w_R \cos cos\, \phi - w_{\phi} \sin sin\, \phi) \sin sin\, \phi$. So, we have grouped these two terms together and you can see here there are is nothing but your $R_2 \sin sin\, \phi$.

So, in some of the equation we have substituted R as $R_2 \sin sin\, \phi$. So, therefore, the $\sin sin\, \phi$ term is appearing in some of the quantities. So, this is due to change of a variable, say $R = R_2 \sin sin\, \phi$.

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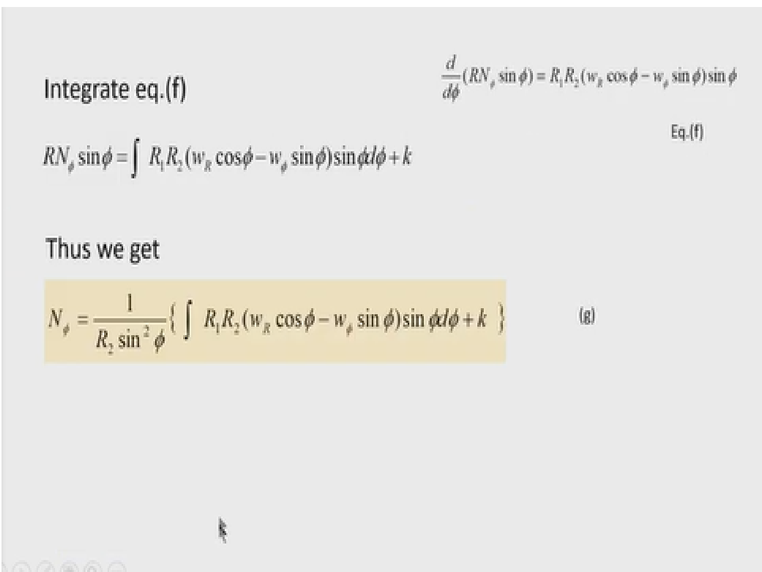

So, this equation can be easily integrated and after integration we get $RN_{\varphi} \sin sin\, \varphi = \int R_1 R_2 (w_R \cos cos\, \varphi - w_{\varphi} \sin sin\, \varphi) \sin sin\, \varphi\, d\varphi + k$, k is a constant of integration. Now, here we get that if we substitute $R = R_2 \sin sin\, \phi$ then we can write $N_{\varphi} = \frac{1}{R_2 \varphi}\left\{\int R_1 R_2 (w_R \cos cos\, \varphi - w_{\varphi} \sin sin\, \varphi) \sin sin\, \varphi\, d\varphi + k\right\}$. So, this is the equation to find the one of the membrane force N_{ϕ} that is in the meridional direction that exists due to self-weight.

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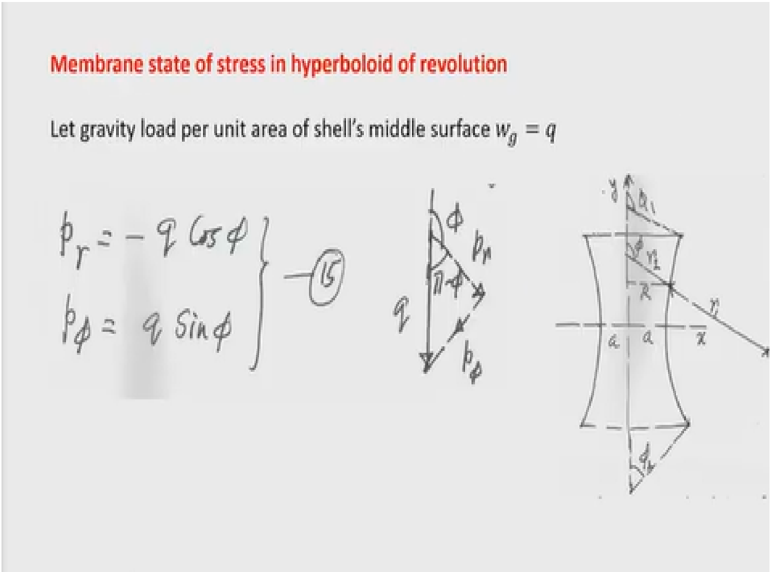

So, membrane state of stress in the hyperboloid revolution. So, this is the general formulation. Now, we have to come to the our problem that is the cooling tower which is a hyperbolic of revolution. So, here the cooling tower is subjected to self-weight that is the gravity load acting vertically and if this angle is ϕ then this angle is $\pi - \phi$. So, component of q along the radial direction along the radial direction is p_r and along the meridional direction is p_ϕ. So, p_r is nothing but $-\ q \cos cos\ \phi$ and p_ϕ from this angle you can see it will be $q \sin sin\ \phi$.

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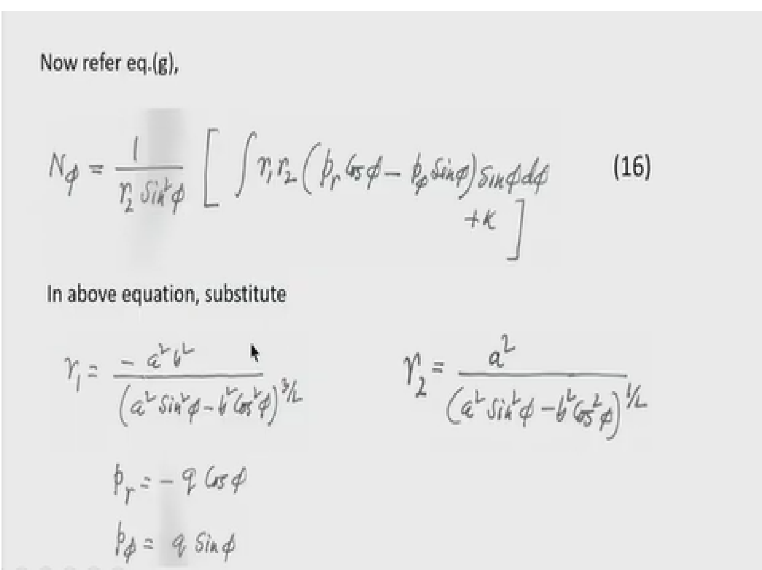

So, these two terms will substitute here in this integration. So, $p_r \cos cos\, \phi$, $p_\phi \sin sin\, \phi$ and we have obtained this in terms of this gravity load q. So, p_r is substituted as $q \cos cos\, \phi$ and p_ϕ is substituted $q \sin sin\, \phi$, r_1 is this that we have obtained earlier and r_2 we have obtained earlier with as this quantity $\frac{a^2}{\left(a^2\phi - b^2\phi\right)^{\frac{1}{2}}}$.

So, these two quantities r_1 and r_2 is obtained from the first principle of calculus and very important parameter is here.

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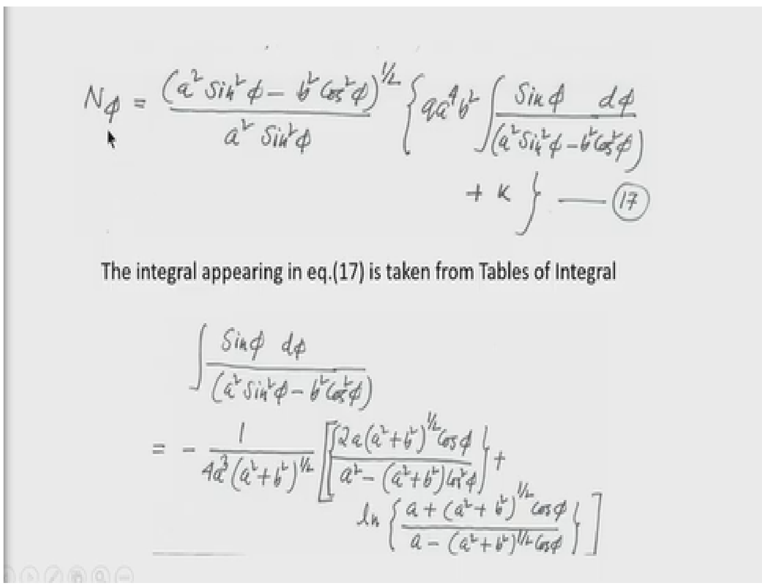

So, after integration, we obtain $N_\phi = \frac{\left(a^2\phi - b^2\phi\right)^{\frac{1}{2}}}{a^2\phi}\left\{qa^4b^2 \int \frac{\sin sin\, \phi}{(a^2\phi - b^2\phi)} d\phi + k\right\}$. So, that integration has to be done. So, that integration can be done and tables of integral provide such type of integral.

Various integrals are available in standard integrals are available in tables of integral or it can be done using the principle of integral calculus. So, integral appearing in equation 17 is taken from tables of integral and then

$$\int \frac{\sin\sin \phi}{(a^2\phi - b^2\phi)} d\phi = -\frac{1}{4a^3(a^2+b^2)^{\frac{1}{2}}}\left[\left\{\frac{2a(a^2+b^2)^{\frac{1}{2}}\cos\cos \phi}{a^2-(a^2+b^2)\phi}\right\} + \ln ln\left\{\frac{a+(a^2+b^2)^{\frac{1}{2}}\cos\cos \phi}{a^2-(a^2+b^2)\phi}\right\}\right].$$

Ln means logarithmic log with the natural base. So, log ln means natural logarithm. So, this integral though it is lengthy but it can be found after integrating this expression. So, these have to be substituted now in this equation 17.

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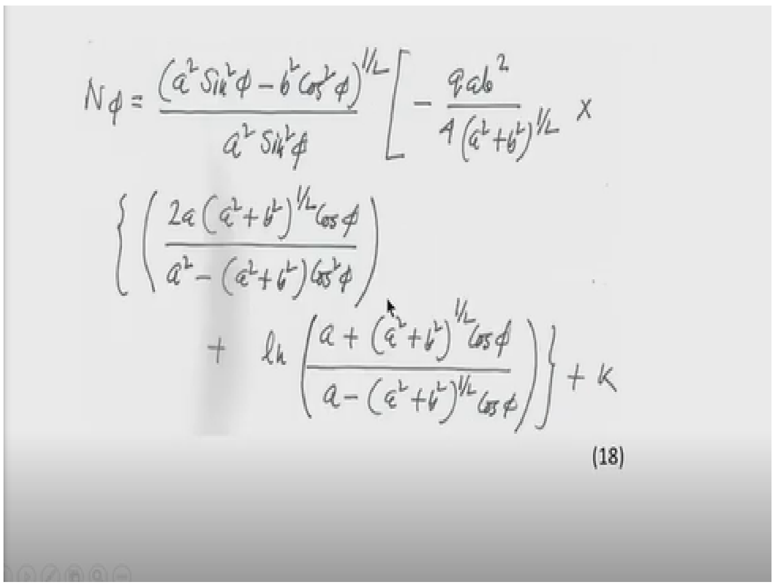

After substituting we get this

$$N_\phi = \frac{(a^2\phi - b^2\phi)^{\frac{1}{2}}}{a^2\phi} \times \left[-\frac{qab^2}{4(a^2+b^2)^{\frac{1}{2}}} \times \left\{\left(\frac{2a(a^2+b^2)^{\frac{1}{2}}\cos\cos \phi}{a^2-(a^2+b^2)\phi}\right) + \ln ln\left(\frac{a+(a^2+b^2)^{\frac{1}{2}}\cos\cos \phi}{a^2-(a^2+b^2)\phi}\right)\right\} + k\right].$$

So, all this quantity again have to be multiplied by this factor and then the constant of integration appears we could not find it till now. So, we will see how the constant of integration can be found out or eliminated.

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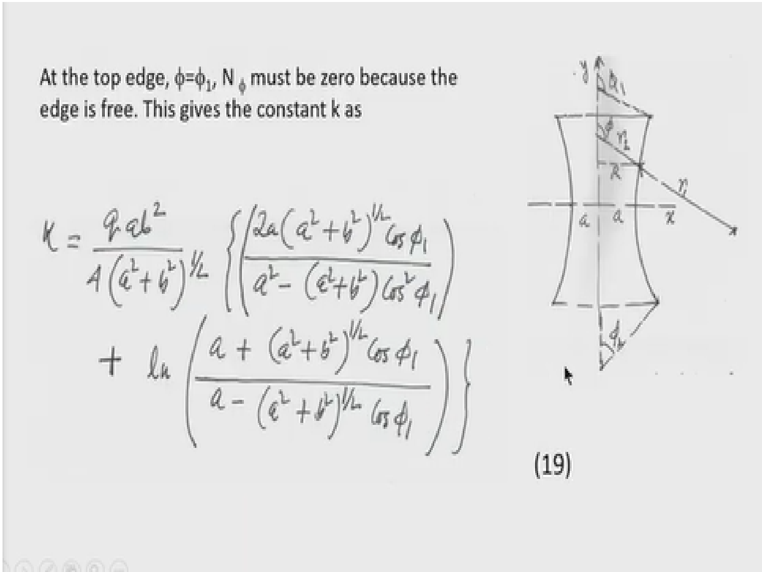

Now here you can see, the shell at the upper surface is free so, at $\phi = \phi_1$ that you are seeing the reference plane is this vertical axis of revolution and at this point, the meridional angle is ϕ_1. So, at this point, this shell is free that means there should not be any stress so, $N_\phi = 0$ at this point. So, therefore putting $N_\phi = 0$ here at $\phi = \phi_1$.

We can find $k = \frac{qab^2}{4(a^2+b^2)^{\frac{1}{2}}} \times \left\{ \left(\frac{2a(a^2+b^2)^{\frac{1}{2}} \cos\cos\,\phi_1}{a^2-(a^2+b^2)\phi_1} \right) + \ln ln \left(\frac{a+(a^2+b^2)^{\frac{1}{2}} \cos\cos\,\phi_1}{a^2-(a^2+b^2)\phi_1} \right) \right\}$. So, constant of integration is found out.

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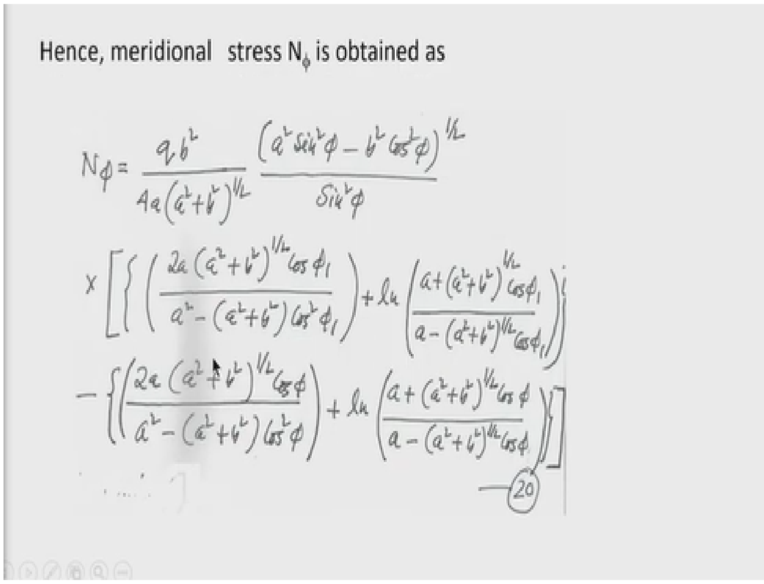

So, total expression that is complete expression for the meridional stress N_ϕ is now calculated as

$$N_\phi = \frac{qb^2}{4a\left(a^2+b^2\right)^{\frac{1}{2}}} \frac{\left(a^2\phi - b^2\phi\right)^{\frac{1}{2}}}{\phi} \times \left[\left\{\left(\frac{2a\left(a^2+b^2\right)^{\frac{1}{2}} \cos cos\, \phi_1}{a^2-\left(a^2+b^2\right)\phi_1}\right) + \ln ln \left(\frac{a+\left(a^2+b^2\right)^{\frac{1}{2}} \cos cos\, \phi_1}{a^2-\left(a^2+b^2\right)\phi_1}\right)\right\} - \left\{\left(\frac{2a\left(a^2+b^2\right)^{\frac{1}{2}} \cos cos\, \phi}{a^2-\left(a^2+b^2\right)\phi}\right)\right.\right.$$

, where $\cos cos\, \phi_1$ is known quantity. Because the boundaries of the shell is $\cos cos\, \phi$ so, we can find the ϕ_1. The same term is repeated here you can see, but here angle is ϕ that means it any meridional angle ϕ you can calculate the stress.

So, first quantity is coming due to the constant k that is evaluated by imposing the boundary condition at $\phi = \phi_1$ and second term you see that it contains the variable ϕ and by putting the value of ϕ at any meridional angle you can calculate the stresses. So, that is the expression for the meridional stress N_ϕ though it is a long-expression but very useful expression for design of cooling tower.

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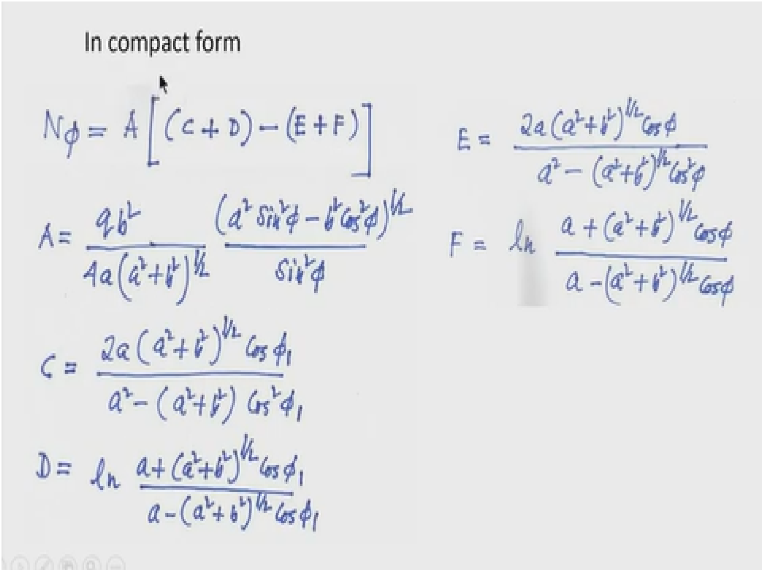

In compact from let us write like that we write in compact form $N_\phi = [(C + D) - (E + F)]$.

So, $A = \frac{qb^2}{4a\left(a^2+b^2\right)^{\frac{1}{2}}} \frac{\left(a^2\phi - b^2\phi\right)^{\frac{1}{2}}}{\phi}$.

$$C = \left(\frac{2a\left(a^2+b^2\right)^{\frac{1}{2}} coscos\ \phi_1}{a^2-\left(a^2+b^2\right)\phi_1}\right).$$

$$D = \ln ln \left(\frac{a+\left(a^2+b^2\right)^{\frac{1}{2}} coscos\ \phi_1}{a^2-\left(a^2+b^2\right)\phi_1}\right).$$

$$E = \left(\frac{2a\left(a^2+b^2\right)^{\frac{1}{2}} coscos\ \phi}{a^2-\left(a^2+b^2\right)\phi}\right).$$

$F = \ln ln \left(\frac{a+\left(a^2+b^2\right)^{\frac{1}{2}} coscos\ \phi}{a^2-\left(a^2+b^2\right)\phi}\right)$. So, that at any phi you can put this but this is a constant term. Because we impose the edge condition at $\phi = \phi_1$. edge condition at $\phi = \phi_1$ was that the $N_\phi = 0$ at the free edge.

So, imposing this condition we obtain this constant K and as a result of this, we get all these terms.. So, the N_ϕ is known completely if you know the q, suppose a in a RCC shell or a concrete shell thickness is known. Then thickness multiplied by the density of the concrete will give you the load per unit area as a self-weight of the shell.

So, q is known in that way and other parameter B and A has to be found out from the geometry of the hyperbola.

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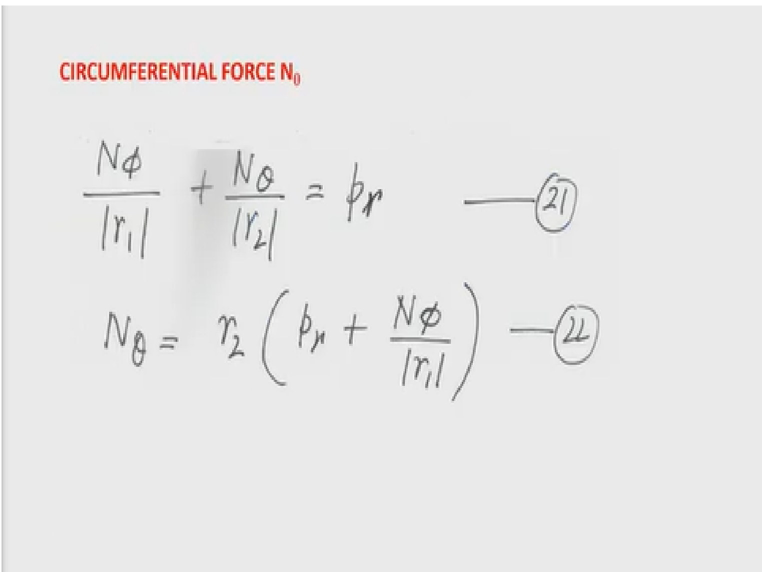

So, once you obtain this N_ϕ then N_θ is obtained by this equation $\frac{N_\phi}{|r_1|} + \frac{N_\theta}{|r_2|} = p_r$, p_r is the radial load and this load we have seen that we have this resolved the component of q in the radial direction p_r and $p_r =- q \cos cos\, \phi$. So, in the equation here $p_r =- q \cos cos\, \phi$ and other quantity and N_ϕ that we have obtained just now. This N_ϕ we have to substitute it r_1 is obtained earlier or r_2 is obtained earlier. So, N_θ is obtained as r 2 p r + and N phi by r 1.

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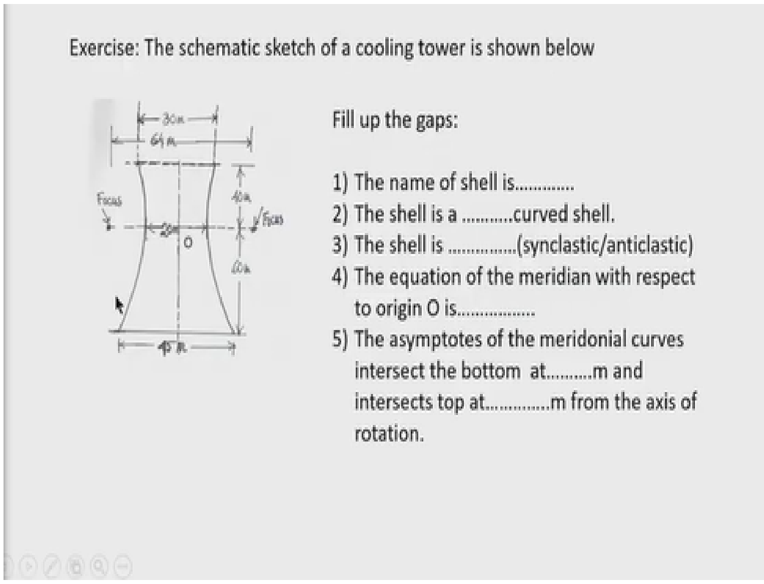

So, N_θ and N_ϕ both are obtained now, and we can design the shell or we can analyse the shell. So, let us see some exercise on this hyperboloid of evolution that is a cooling tower problem. So, a schematic sketch of the cooling tower is before you and you can see this cooling tower height here is 100 meter and the excess of the cooling tower is at a distance of 60 meter from the base and the top is at a distance of 40 meter.

So, let us see what are the some salient features of this cooling tower the name of the shell is so, first question the name of the shell is this name is hyperboloid revolution and this shell is a curved shell. What is the type of curved shell? It may be singly curved or doubly curved shell but this type of shell is the doubly curved shell. Number three question is it synclastic or anticlastic. But here because of the curvature into opposite direction, we call this shell as synclastic shell.

The equation of the meridian can be defined as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Now asymptotes of intersect the bottom and at which point m asymptotes will intersect the top of the shell from the axis of rotation. So, that we want to find out.

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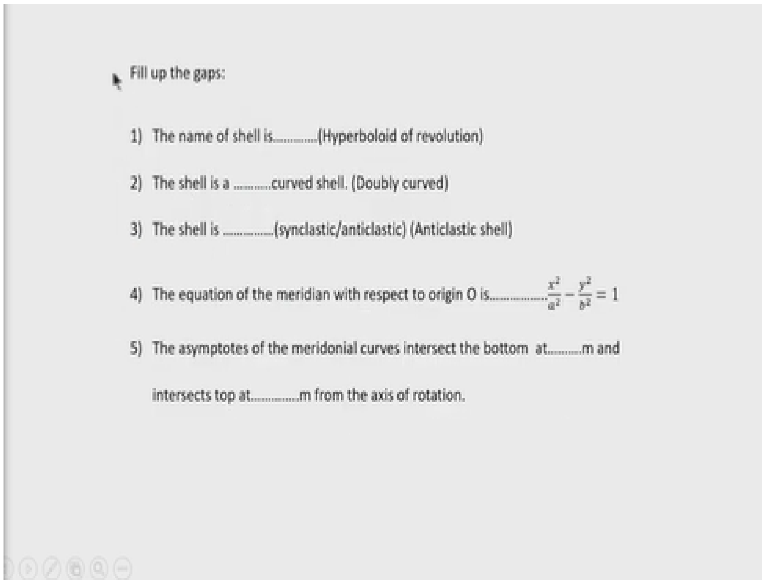

So, the question on the fill up the gaps is answered. And now the last question the asymptotes of the meridional curves intersect the bottom at this meter or intersects top at this meter from the axis of rotation let us find it.

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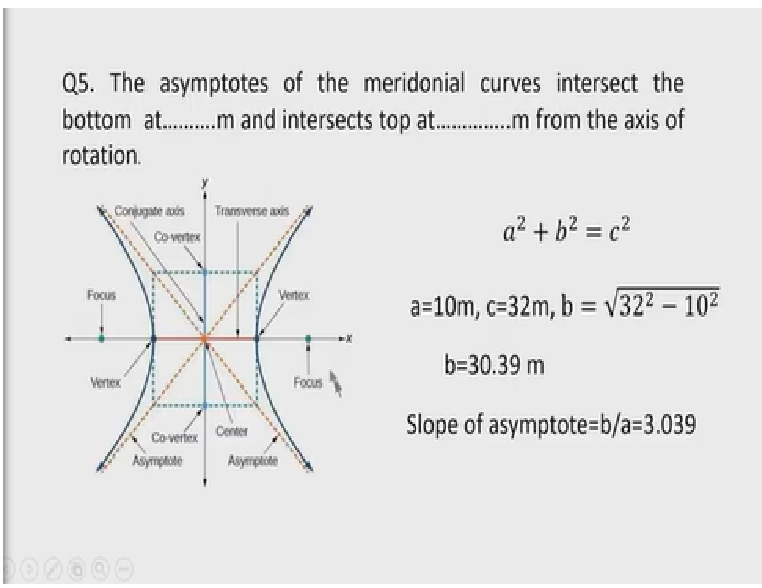

So, this is the question now, we know that $a^2 + b^2 = c^2$, where c is the distance of one of the foci from the vertical axis of rotation. So, here you can see from the given dimension of the shell this c is located for classes located at a distance of 32 meter from the axis of rotation. So, c is 32 meter and you can see this the length of the transverse axis is 20 meter that is a is 10 meter. So, a is 10 meter.

So, therefore, $b = \sqrt{32^2 - 10^2}$ so b is 30.39 meter. Now, knowing the b and a we can now find the slope of the asymptotes. Slope of the asymptote can be found as $\frac{b}{a}$. So, $\frac{b}{a} = 3.039$ that is the slope and if we see that if this is the slope so in angle and this is the you are this considered a triangle now with slope here there is tan $tan\,\phi = 3.039$. So, at the bottom, there are some total intersects this base at a distance of $\frac{60}{3.039}$ that is 19.74 meter from the axis of rotation.

And at the top, it will intersect the top and then distance of $\frac{40}{3.039}$ from the vertical axis of rotation.

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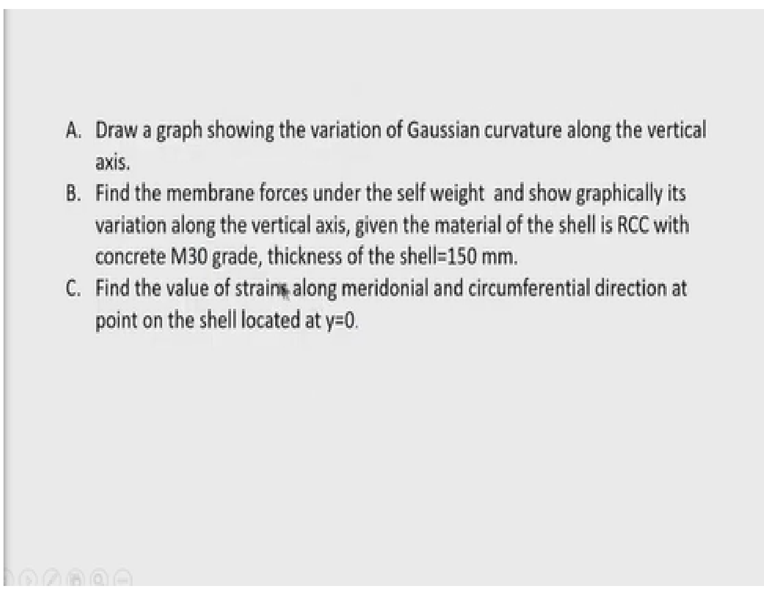

Now let us see the stress variation or the main force variation. So, first, let us see how the Gaussian curvature varies along the vertical axis. Now, Gaussian curvature is known as the product of two principal curvatures. So, radius of curvature r_1 and r_2 are obtained. So, we can now find the gaussian curvature as $\frac{1}{r_1} \times \frac{1}{r_2}$. So, that variation we have to show along the vertical axis.

Then second question is find the membrane forces under the self-weight and show graphically its variation along the vertical axis. Given the material of the shell is RCC with concrete M30 grade. So, M30 concrete will be the permissible stress in bending compression will be 8 N/mm^2, thickness of the shell is 150 millimetre and the steel is not specified here. But we can find the steel area also.

For example, steel can be taken as Fe 415 whose permissible stress is 230 N/mm^2. Then find the value of strain along the meridional and circumferential direction at point on the shell located at $y = 0$.

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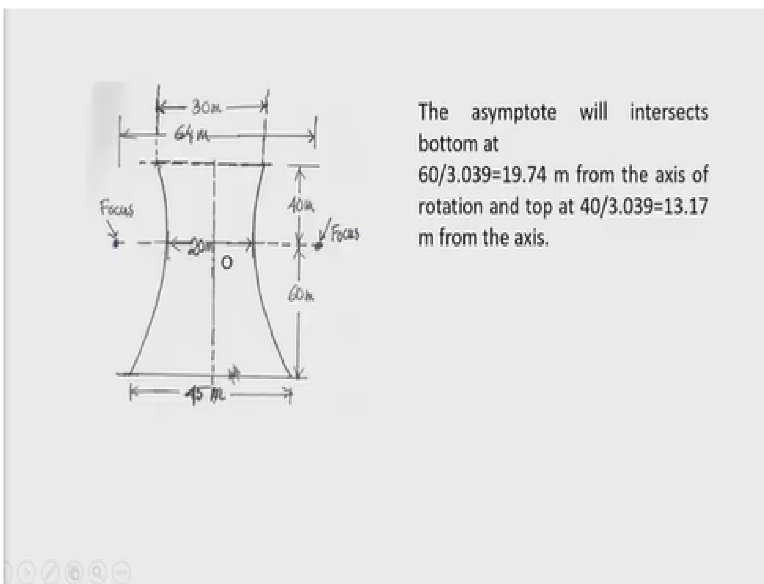

So, that means $y = 0$ if this is the origin so, $y = 0$ that means, you have to find that $y = 0$ and at this distance you can find at 10 $meters$.

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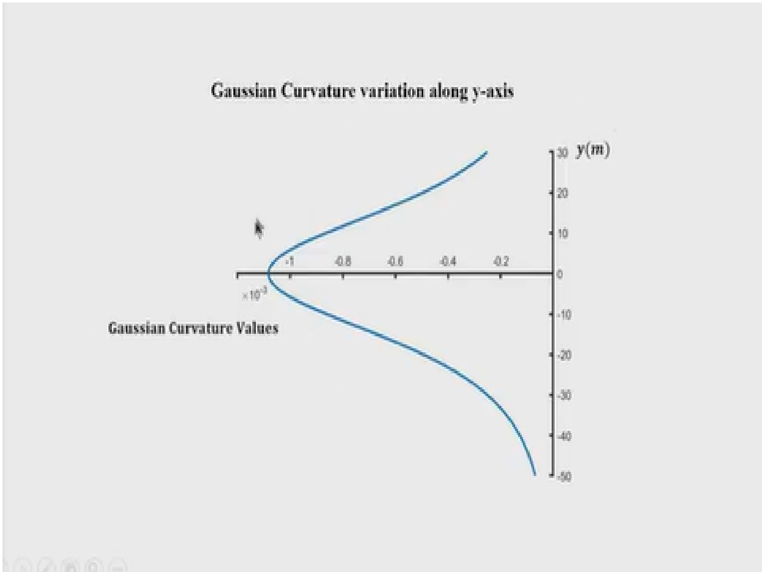

Let us see the variation of Gaussian curvature. So, Gaussian curvature varies like that and you can see that the Gaussian curvature is negative.

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Table 1. Variation of Gaussian curvature along the vertical axis

y (m)	Gaussian curvature (m^{-2})
30	-2.5×10^{-4}
20	-4.94×10^{-4}
10	-8.63×10^{-4}
0	-0.0011
-20	-4.94×10^{-4}
-30	-2.5×10^{-4}
-50	-6.77×10^{-5}

And the value of Gaussian curvature is tabulated here, if it is 0 at 10 meter intervals about the top and 20 meter interval about the towards the bottom, we have tabulated this and then this Gaussian curvature is written here. So, you can see that at 0 the Gaussian curvature is 0.0011 that means, at the centre of the hyperbola.

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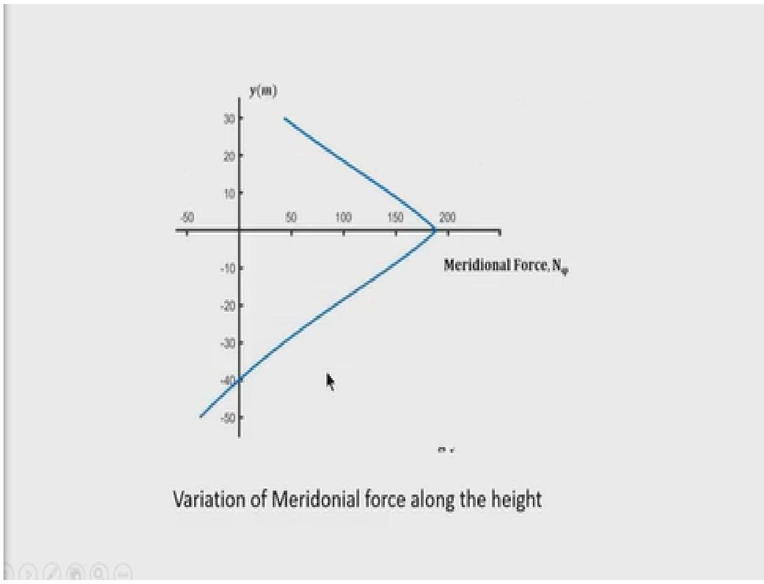

Variation of Meridonial force along the height

So, now, let us see the meridional force along the height of this shell. Meridional forces N_{ϕ} and along the height if I plot it, you can see here that up to say this region all forces are comprehensive in nature. And maximum forces you can see here say around say 180 and this is also tabulated here, tabulated in the second table. So, meridional stress at here say 190 for example.

So, 190 if it is a stress force kN/m here, then in N/mm it will be $190\,N/mm$ and if the thickness of the shell is 150. So, $\frac{190}{150}$ so that ratio is far less than the permissible compressive stress in bending compression in concrete of grade M30. Permissible stress in bending compression of concrete of weight M30 is taken as a N/mm^2.

And we are getting the induced stress compressive stress maximum as say $190/150$, $190/150$ is again very low quantity is less than 2. So, the structure is safe in regard to the permissible compressive stress of concrete. Now, if you want to see the N_{θ} variation you can see this how the N_{θ} varies and you can find the N_{θ} becomes tensile that is negative in some of the regions. So, here we will get this tensile stress.

And one thing is that the if this is the maximum stress then based on that you can also find the quantity of reinforcement.

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Table 2. Variation of Meridional Force and Circumferential Force along the vertical axis

y (m)	Meridional Force, N_φ (kN/m)	Circumferential Force, N_θ (kN/m)
30	43.08	-9.93
20	92.24	-1.37
10	144.23	9.87
0	189.28	20.48
-20	92.24	-1.37
-30	43.08	-9.93
-50	-37.48	-21.30

Quality of reinforcement can be found suppose if you know the force per unit length then maximum force per unit length that is obtained by nature divided by permissible stress in tension of steel. So, that is taken as $230 N/mm^2$. So, that quantity will give you the area of steel per meter length. So, area of steel in millimetre square per meter length. Now, choosing a diameter of the steel bar generally in thin structure here the thickness is 150 mm.

So, maximum you can provide the bar diameter from 8 mm to 12 mm within this. So, you can see the spacing of the reinforcement will also not be very large, requirement of reinforcement will be less. So, that means here suppose A_{st} is the required area of the steel and a_{st} is the area of one bar that you choose the diameter of the bar either 8 mm or 10 mm or 12 mm. Then the spacing of the steel can be found as $(a_{st} \times 1000)/A_{st}$.

So, that is the spacing of the bar in millimetres centre to centre. So, that is found based on the area of steel that we have calculated depending on the maximum tensile stress that is found in this table. So, maximum tensile stress is found in the table is very low that you are seeing here.

And therefore, your the requirement of reinforcement will also be low. So, the requirement of reinforcement and checking the thickness of the; shell adequacy of the thickness of the shell.

So, that the; induced compressive stress remains within the permissible limit. That type of problem I have earlier demonstrated in case of spherical domes. So, same principle can be used here to check the safety of the shell in regard to its thickness and also to specify the reinforcement requirement in the shell. So, this is the analysis of the hyperboloid revolution which is a very important structure in thermal power station.

And obviously, it is falling under a doubly curve anticlastic shell and that is a shell of surface of revolution.

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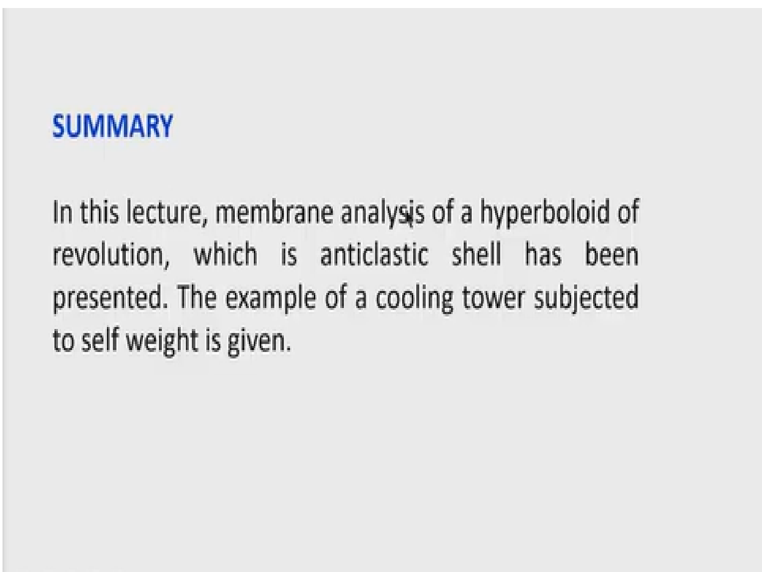

So, let us see the summary of today's lecture. So, in this lecture membrane analysis of hyperboloid revolution, which is anticlastic shell, has been presented. The example of a cooling tower subjected to self-weight is given. Thank you very much.