Plates and Shells
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Module-1
Lecture-03
Plate Equations and Boundary Conditions with Examples

Hi everybody, today I am continuing my lecture and let me first tell what I have covered in the last class and then the outlines of the today's lecture will be given to you.

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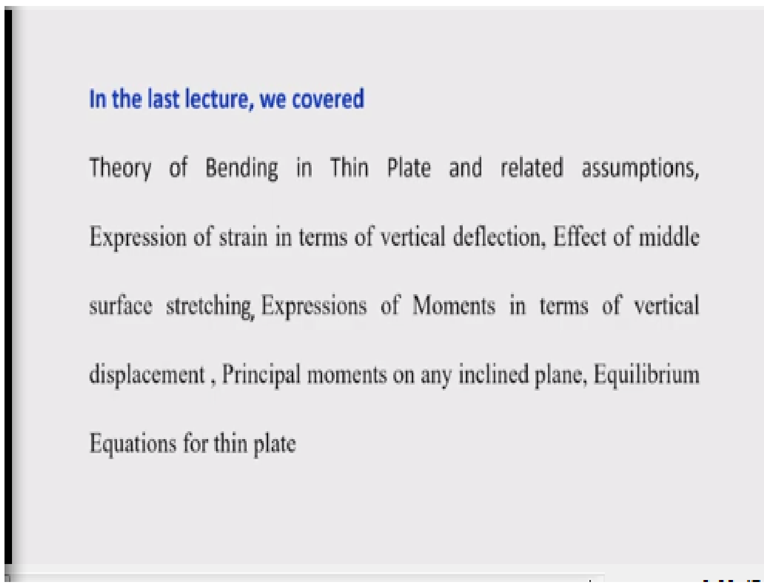

So, in the last lecture you have learned the theory of bending in thin plates and what are the assumptions related to bending. Then we have obtained expression of strain in terms of vertical displacement. Then effect of multiple middle surface stretching, that we have obtained. Then expression of moments in terms of vertical displacement, principal moments on any inclined plane, equilibrium equations for thin plate.

So, that was our last item covered in the last class and you remember that I have derived fourth order partial differential equation with a biharmonic operator.

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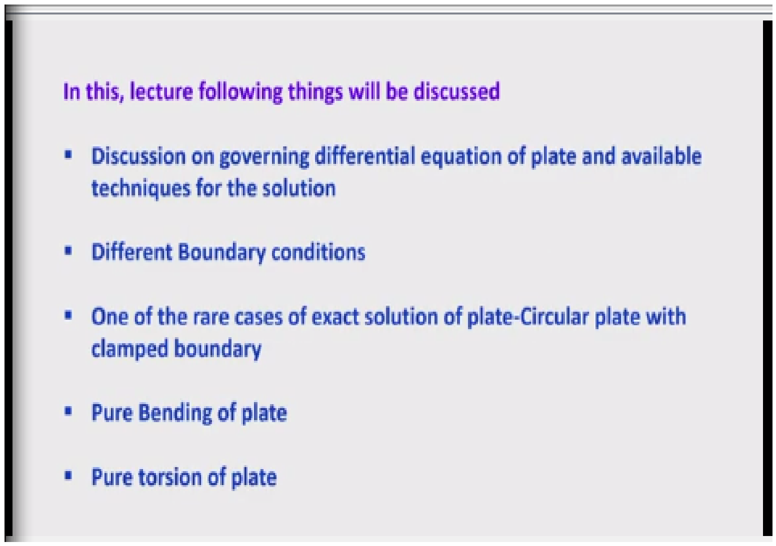

The equation has some specialty in mathematical physics and you know that this type of equation also represents the airy stress function equation. So, we shall now proceed how to solve these type of equation for different boundary conditions of the plate and what are the general method available and whether the method yield the exact solution or we required to adopt any approximate method. All these will be discussed in our class.

So, discussion on governing differential equation, that will be covered first. Then. what are the available methods or techniques for the solution. Especially today I will discuss about the exact solution, approximate solution will be covered later in separate classes. Then boundary conditions in the plate because plate has straight edge or it may have curve boundary also. So, today I will specially discuss the boundary condition for straight edges later on I will go to the boundary condition for the curved edge.

Then there are some cases where the closed form (02:51) solution is readily obtained, very simple function is chosen and that function is seen to satisfy the boundary condition as well as your differential equation. So, nobody can deny that this function is not a correct solution, it will yield the exact solution for them. But there are some specific boundary condition and specific loading for which only this is possible.

So, one of the rare cases of exact solution of plate and that there are some examples from which today I will cover the circular plate with clamped boundary. For that you can readily obtain the exact solution say choosing a simple function. Then I will discuss the pure bending of plate when the plate is subjected to pure bending moment along the edges, how it deforms and pure torsion of the plate.

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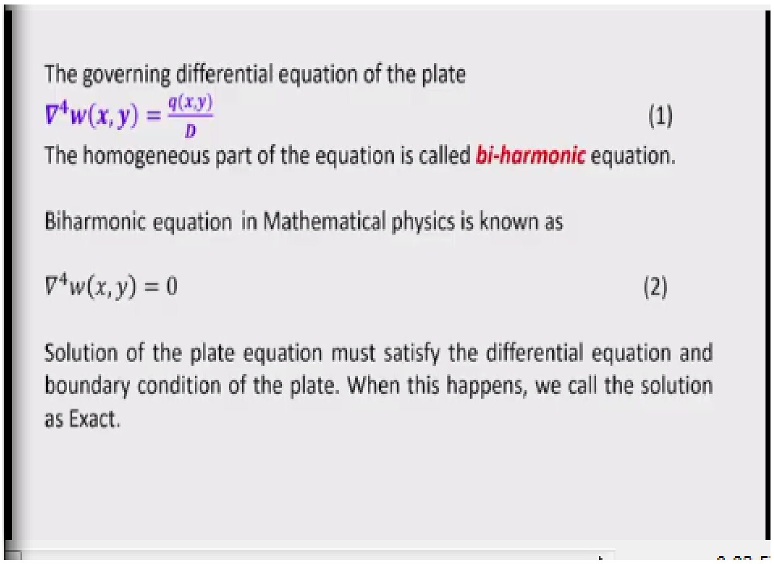

Now if you see the nature of the differential equation, Δ^4 is the biharmonic operator and if the right hand side of this equation is 0, then in mathematical physics it is known as biharmonic equation. So, the homogeneous solution for that type of biharmonic equation is already in existence in different theories of mathematical physics. And here if we go for the plate subjected to a transverse loading or it may be some edge loading also, where the homogeneous part has to be solved, imposing the boundary condition at the edges, so that condition may also arise.

So, differential equation of the plate must be satisfied by the solution and the solution must satisfy the boundary condition. If two conditions are satisfied simultaneously, then we call the solution as exact solution. Now we will focus on the exact solution in this class specially, later on we will cover some approximate method where exact solutions are not possible.

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In general there are four types of mathematically "exact" solutions available for the plate problem:

- Closed form solution
- Solution of Biharmonic Equation upon which particular solution is superimposed.
- Double trigonometrical series solution
- Single series solution

The rigorous solution of the plate problem is essentially a boundary value problem.

Closed form solutions of plate problems are limited. The general solution is usually obtained in case of linear analysis as

$$w(x,y) = w_h(x,y) + w_p(x,y) \qquad (3)$$

where w_h and w_p represent the homogeneous and particular solution respectively

So, let us see the general techniques that are used for the exact solution or solution of the plate problem are closed from solution that satisfy the boundary condition as well as differential equation. Then solution of biharmonic equation that is you get an homogeneous solution and over which the particular solution due to forcing term has to be super imposed, then only you will get the total solution or complete solution for a linear case.

So, for a linear case homogeneous solution plus particular integral due to forcing function will give you the complete solution, so that case may also arise. Now in certain cases the trigonometry series provide very useful tool or very useful way of finding the exact solution of the plate problem for the specific boundary conditions or for specific loading, specially for some specific boundary condition, single trigonometric series or double trigonometric series can be conveniently use to find their closed form solution.

The rigorous solution of the plate is essentially a boundary value problem, because boundary condition has to be imposed. When you find the homogeneous solution of the biharmonic equation then you will get because it is a fourth order equation. So, imposing the boundary condition at the 2 edges you will get the 4 constants or 4 equations that has to be solved simultaneously and you will get the problem of finding the constant of integration.

In many cases to simplify this we adopt the symmetric loading, anti symmetric loading, so many techniques are used to simplify the calculation of the constants of integration because it sometimes involves a lengthy calculation. So, closed form solution of the plate problems are limited that statement I am making here and it is true in many cases you will not get. Then you have to adopt a numerical technique, either finite difference or finite element techniques.

Or any other numerical techniques solving the differential equation, integrating the equations you can use any numerical methods. So, here w_h is the homogeneous solution of the homogeneous solution of the plate equation that means a biharmonic equation. And w_p is the solution for the forcing function that I call a particular integral or particular solution.

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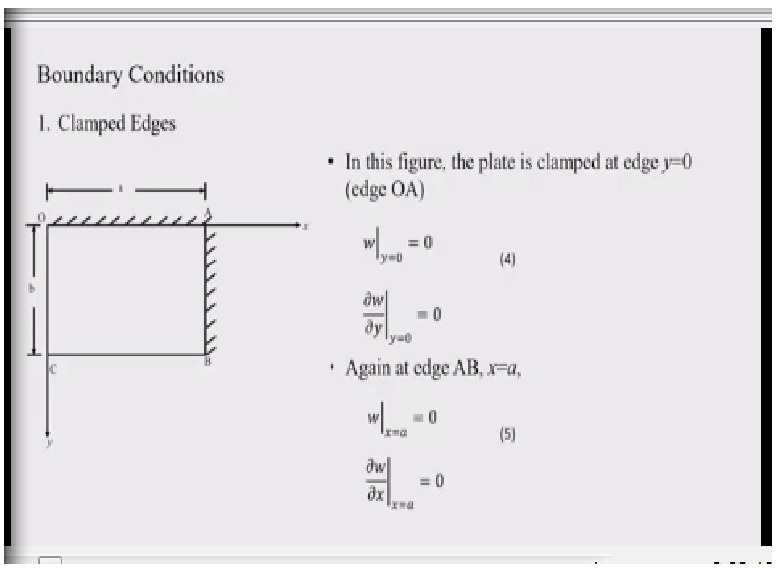

Now let us see the boundary condition. Because I told you that boundary conditions are the important components of the plate problem. When the boundary conditions are satisfied then you will get the solution which is close to the exact value. So, in boundary condition, there are two types of boundary condition, one is your geometric boundary condition and another I can term it as a force boundary condition.

In the geometric boundary condition the deflection and slopes are the parameters that have to be known at the edges. And for fourth boundary condition the shearing force and twisting moment or bending moment have to be known at the edges. Now here I take an example of a rectangular plate which is having straight edges. So, the rectangular plate is say OA, BC that you are seeing here i.e the rectangular plate.

The length of the plate is 'a' which is along the x.-axis and width of the plate is b along the y-axis. And you can see here that edge OA is fixed, it is clamped and edge AB is also fixed. For example, we have a steel plate at one edge along the edge throughout continuously it is welded to another plate or another element of steel. For example, base plate is welded or flange is welded to the web, so this type of condition may occur.

So, that means it has a welded edge OA which represents the fixed boundary condition. Similarly, AB is also an edge which is welded, so it represents a fixed boundary condition or clamped boundary condition. Now the boundary condition OA, I will specify that, you see along this y is 0, so along OA, the y value is 0. If I refer O as the origin then along OA y is 0, so that means deflection is 0 along OA because it is fixed.

So, w at y equal to 0 is one condition that have to be imposed here. Then slope along the y direction. So, since this is fixed, so the slope along the y direction the deflection curve will be such that the tangent here will be almost horizontal if the plate is horizontal. So, the tangent will have 0 angle making with the horizontal plane.

Therefore, the slope is 0 at the fixed edges, hence.

$$\left.\frac{\partial w}{\partial y}\right|_{y=0} = 0$$

So, $\frac{\partial w}{\partial y}\frac{\partial w}{\partial y}$ that is partial derivative of $\partial y\,\partial y$, deflection is taken, because deflection is a function of two variables x and y. Again, you see at the edge AB, so you have to identify what is the edge. At the edge AB, x is A and this AB is parallel to y axis. So, any y coordinate on AB will vary from 0 to b. But x is fixed; x is 'a' along the AB. So, again at x = a, you are getting the deflection at x = a is equal to 0. So, whatever deflection solution you obtain involving the 4 constants of integration then you can impose this condition at x = a to relate the constant with their 0 value.

Now since this AB is fixed along the y direction, so the slope of the deflected curve along the x axis, i.e along x direction will be 0 because tangent will be making 0 angle with the x axis at the fixed edges. So, therefore we have taken

$$\left.\frac{\partial w}{\partial x}\right|_{x=a} = 0$$

Substituted the value of x = a. So, this is one important condition that we generally encounter in our practical situation of particular life that is the clamped edges.

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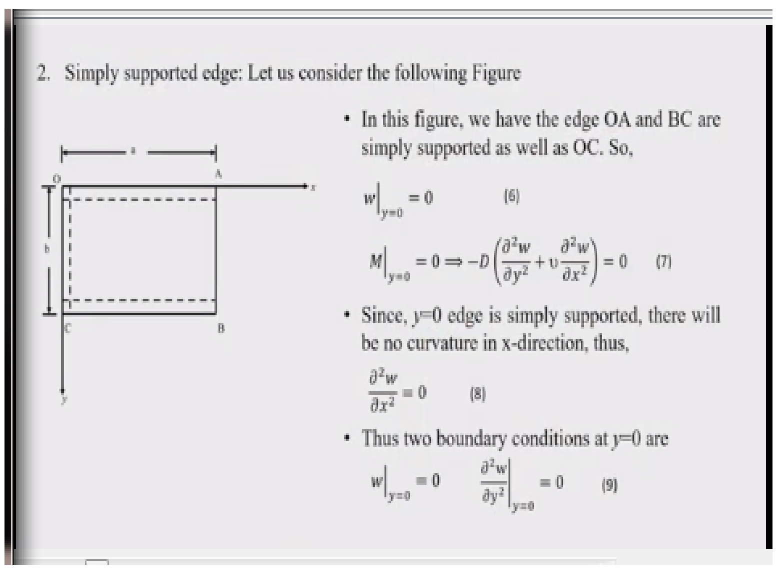

Next let us go to simply supported edges. That is very common condition and it is mostly useful in simplifying the work. Because simply supported condition although in the previous case you have seen that deflection and single derivative of the deflection is taken. But simply supported, although it is a very simple condition but the higher derivative is required. So, I will now discuss what is simply supported condition.

So, simply supported conditions are here in the drawing it is seen that OA is also simply supported, OC is also simply supported and BC is also simply supported. That means at x = 0 it is simply supported, (x = 0 is here OC). So, OC edge is simply supported, then y = 0 that means, the edge OA is also simply supported. And then y = b that is the side CB or BC is also simply supported.

So, let us see what are the conditions that to be written for simply supported edges. In a simply supported edges we know that deflection and bending moment has to be 0. So, rotation is allowed in this simple support that we know. So, w is 0 that is the deflection at y = 0 = 0 for the edge OA, I am writing the condition for edge OA first. So, if I write the condition for OA first, y is 0, so therefore w at y = 0 is equal to 0 that is the one condition.

Then second condition bending moment along y direction since this is simply supported along the y axis, so slope at y = a along y axis cannot vanish. So, therefore the bending moment curvature exist and therefore bending moment is 0. So, rotation is allowed, so moment is 0. So, bending moment expression we know that $-D\left(\frac{\partial^2 w}{\partial y^2} + \upsilon \frac{\partial^2 w}{\partial x^2}\right) = 0$ $-D\left(\frac{\partial^2 w}{\partial y^2} + \upsilon \frac{\partial^2 w}{\partial x^2}\right) = 0$ (where $\frac{\partial^2 w}{\partial y^2} \frac{\partial^2 w}{\partial y^2}$ is the curvature in y direction and $\frac{\partial^2 w}{\partial x^2}$ $\frac{\partial^2 w}{\partial x^2}$ curvature in x direction).

Now you can see this edge OA is simply supported, so there cannot be any curvature along the x direction, it is simply supported along the x axis. So, there cannot be any curvature or slope along the x axis or along the edges. So, naturally we take this $\frac{\partial^2 w}{\partial x^2} = 0 \frac{\partial^2 w}{\partial x^2} = 0$, hence the second condition is simplified as $\frac{\partial^2 w}{\partial x^2} = 0 \frac{\partial^2 w}{\partial x^2} = 0$

So, second derivative of w with respect to x square = 0, so this is the condition at y = 0, so 2 condition we got at y = 0. Now if I want to write the condition for the edge OC then how we will write. So, edge OC is the x = 0, so boundary condition at OC will be w = 0, that is one condition and second condition will be, the bending moment along the x direction is 0. So, bending moment along the x direction is $-D\left(\frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2}\right) = 0$ $-D\left(\frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2}\right) = 0$

But since it is supported along the y axis, i.e along the edge OC which is parallel to Y axis, so there cannot be curvature along the y direction. So, that means $\frac{\partial^2 w}{\partial y^2} = 0 \frac{\partial^2 w}{\partial y^2} = 0$, so for that edges the condition will be w at x = 0 is 0 and $\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=0} = 0 \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=0} = 0$. So, it can be written in the similar way at this edge OC alright. Now, let us go to other condition.

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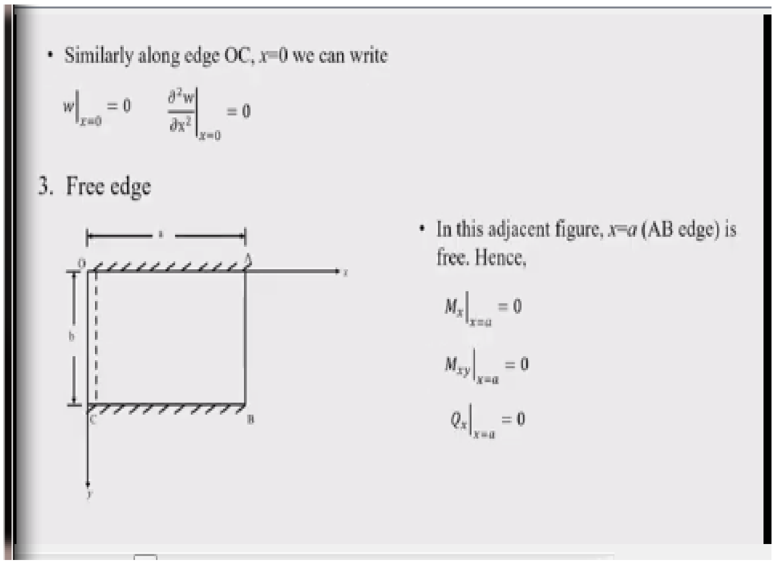

That condition I have written along OC, that I have told you now it is written and you will get in my note also. So, free edge, let us see what the free edge indicates. Free edge, though there is not any support but condition imposed is very complex at the free edge. Because at the free edge we know that forces or stresses vanishes. So, therefore the force boundary condition has to be imposed on the free edges, not geometric boundary condition.

Because at the free as both slope and deflection cannot be 0. In the simply supported edges, we imposed one geometric condition that is w = 0, but one fourth boundary condition that bending moment = 0. In clamped edges, we imposed two geometric conditions only that w = 0 and slope = 0. Simply supported edges one geometric condition and one force condition. But in the free edges geometric condition cannot vanish because w has to be non-zero and slope also has to be non-zero.

So, in that case we have to impose the force boundary condition that is condition for shearing force and bending moment. Now in the plate you know that due to generation of shear stress, there will be twisting moment. So, at the free edge we have three quantities to be vanished, one is bending moment 0, twisting moment 0 and shear force 0. Now since this edge AB is located by x coordinate as x = a, x = a denotes the edge AB completely.

This along AB, the y coordinate varies from 0 to b but x coordinate is fixed x = a along AB**.** Now here three conditions are there, these three conditions have been given by poisson.

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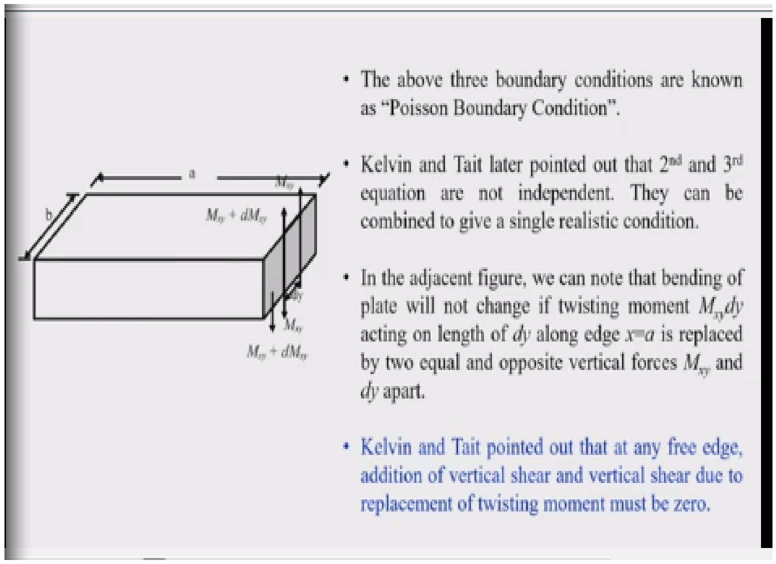

And later on, this other authors, Kelvin and Tait found that three conditions are not necessary. Because these last 2 conditions can be combined that is the shear force and twisting moment can be combined to give a single condition because these are not independent quantities. So, Kelvin and Tait pointed out that second and third equation that I have shown you that for bending moment and the twisting moment and shear force (0) can be combined because they are not independent.

And if you combine it you will get a realistic boundary condition. Now let us see how we can combine it. In the adjacent figure we see that the twisting moment M_{xy} acting on a element dy. Because all quantities in the plate (i.e whatever stress resultant etc.) are per unit width that you remember. So, M xy is also per unit width.

So, if it acts on a length dy, so total twisting moment on this length will be M_{xy} dy. Now you can see this, this twisting moment M_x dy can be represented by 2 equivalent opposite forces M_{xy} and M_{xy}, so that it give rise to a couple M $_{xy}$ dy. So, this M $_{xy}$ dy is equivalent to a total M_{xy} and d xy equivalent opposite forces is equivalent to a couple M_{xy} dy. So, based on that it is found that in adjacent element there will be some increment $M_{xy} + dM_{xy}$. And dM_{xy} is the increment which in the full form is $\frac{\partial M_{xy}}{\partial y} dy \frac{\partial M_{xy}}{\partial y} dy$

That is coming from Taylor series expansion that I have given you in the equilibrium of equation lecture. So, how the incremental quantity is written on the opposite faces or at a distance given by at a distance dx or dy apart. So, this on the adjacent element you will get that $M_x + dM_{xy}$ this is the twisting moment. So, Kelvin and Tait pointed out that any free edge addition of vertical shear, vertical shear is already there.

So, in this edge the vertical shear Q_x is already there. If Q_x is added to the shear force generated due to twisting moment and then it is equated to the edge shear (if there is some edge shear), then it will represent the true boundary condition. So, at the free edge there is no shear force, so edge shear is 0. So, that means the edge shear that is to be balanced by the component of shear force acting on this edge Q_x plus the increment of shear force due to twisting moment. So, let us say how this can be found out?

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- Now, vertical force at any point due to replacement of twisting moment M_{xy}

$$(M_{xy} + \frac{\partial M_{xy}}{\partial y} dy) - M_{xy} = \frac{\partial M_{xy}}{\partial y} dy$$

- Hence, total vertical shear at the edge,

$$V_x dy = Q_x dy + \frac{\partial M_{xy}}{\partial y} dy$$

$$\Rightarrow V_x = Q_x + \frac{\partial M_{xy}}{\partial y}$$

$$\Rightarrow V_x = -D \frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - D \frac{\partial}{\partial y}\left((1-\upsilon)\frac{\partial^2 w}{\partial x \partial y}\right)$$

$$\Rightarrow V_x = -D\left(\frac{\partial^3 w}{\partial x^3} + (2-\upsilon)\frac{\partial^3 w}{\partial x \partial y^2}\right)$$

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y}$$

$$Q_x = -D \frac{\partial}{\partial x}\left\{\frac{\partial^2 w}{\partial x^2} + \vartheta \frac{\partial^2 w}{\partial y^2}\right\} - D(1-\vartheta)\frac{\partial}{\partial y}\left\{\frac{\partial^2 w}{\partial x \partial y}\right\}$$

$$Q_x = -D \frac{\partial}{\partial x}\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right\}$$

Similarly, $Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} = -D \frac{\partial}{\partial y} \nabla^2 w$

So, Kristov has given an expression for the edge shear therefore it is popularly known as Kristov edge shear. So, in this slide you can see that net vertical force in the element of dy you will find

$\frac{\partial M_{xy}}{\partial y} dy \frac{\partial M_{xy}}{\partial y} dy$. So, you can see this is equivalent to the unit of force. Because the M_{xy} is the force per unit length and this is your this y, so this will be again the unit of force per unit length.

Because the shear force is expressed as the unit of force per unit length. Hence total vertical shear at the edge should be V_x, which is total vertical shear. So, V_x multiplied by the length of the edge which is dy or length of the element dy should be equal to the shear force that is generated due to vertical force Q_x, Q_x into dy for this element plus this is the component of shear force produced by twisting moments.

So, if these two forces are added and equated to the edge shear then it will reflect the true boundary condition at the free edge. So, Kristov has given this expression and dividing both sides by dy you will get $V_x = Q_x + dM_{xy}$ by dy. So, this is the edge shear force. Now, at the free edge since there is no shear force acting, so edge shear is 0, so V_x will be 0 at the fixed end.

Now let us find out what is the expression for V_x in terms of deflection, because we will solve the equation of deflection, plate equation is expressed in terms of deflection, the generic equation. So, we will find the solution in terms of deflection which is a function of x and y. So, therefore let us express the edge shear in terms of deflection. Now here you can see that Q_x from the equilibrium equation if you remember in my last class.

We have obtained the equilibrium equation that $Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y}$ $Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y}$. So, what is M_x. M_x is the bending moment in the x direction and bending moment in the x direction is given as $-D \frac{\partial}{\partial x} \left\{ \frac{\partial^2 w}{\partial x^2} + \vartheta \frac{\partial^2 w}{\partial y^2} \right\}$

$-D \frac{\partial}{\partial x} \left\{ \frac{\partial^2 w}{\partial x^2} + \vartheta \frac{\partial^2 w}{\partial y^2} \right\}$, this is the bending moment expression along the x direction. So, the first derivative of bending moment is taken and then the first derivative of twisting moment is taken.

So, with the twisting moment expression that is the twisting curvature $\frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y}$ with the twisting moment you know that a term $D(1-\vartheta)\, D(1-\vartheta)$ is associated. So, this term is there again and it is minus, so $-D(1-\vartheta) -D(1-\vartheta)$ is there and the derivative of the twisting curvature is taken with respect to y. Now if you take this differentiation, then you will find that cubic the third derivative will be resulting.

And after simplification it is a very interesting thing that after simplification in the edge shear expression in the shear force expression you will not get any poisson ratio term. So, there is no poisson ratio term in the shear force expression. Shear force due to vertical force or transverse force. So, shear force due to transverse force transverse load in the plate is simply - $-D \frac{\partial}{\partial x}$ $-D \frac{\partial}{\partial x}$ and this you know this is the by him say Laplacian equation.

So, we can now say that Q_x is nothing but $-D \frac{\partial}{\partial x} -D \frac{\partial}{\partial x}$ into $\partial^2 \; \partial^2$ that is the Laplacian operator multiplied by w. And here for Q_y the derivative has to be taken with respect to y. So, once we know the expression for Q_x, now substitute the expression for Q_x here, from here to here. I brought these two here and dM_{xy} you know that the expression for dM_{xy} is this term and multiplied by this term.

So, I have brought this expression for dM_{xy} and I differentiated this with respect to y. So, after differentiation and rearranging, you will find that edge shear is giving a very interesting expression. That is edge shear equal $-D\left(\frac{\partial^3 w}{\partial x^3}+(2-\upsilon)\frac{\partial^3 w}{\partial x \partial y^2}\right)$

$-D\left(\frac{\partial^3 w}{\partial x^3}+(2-\upsilon)\frac{\partial^3 w}{\partial x \partial y^2}\right)$. Now you can see here this is the edge shear force at x = A, that is at the x edge means x coordinate is constant along the edges**,** x coordinate is constant but y is varying.

For example, we want to find out say, in the previous case let me show the figure. Say this is the free edge that I have taken AB where the x coordinate is constant x = A. For example, if OA is free instead of clamp as suppose OA is free, then we have to write at y = 0, edge shear V_y is 0. So, expressions for V_y can be written with an analogy of that expression, V_y will be

$$V_x = -D\left(\frac{\partial^3 w}{\partial y^3}+(2-\upsilon)\frac{\partial^3 w}{\partial y \partial x^2}\right)$$

$$V_x = -D\left(\frac{\partial^3 w}{\partial y^3}+(2-\upsilon)\frac{\partial^3 w}{\partial y \partial x^2}\right)$$

So, for depending on the edge, where the coordinates are specified, you can express the edge shear and the edge shear has to be equated to 0 at the free edge that is the first condition. Second condition as the free edge there will not be any bending moment.

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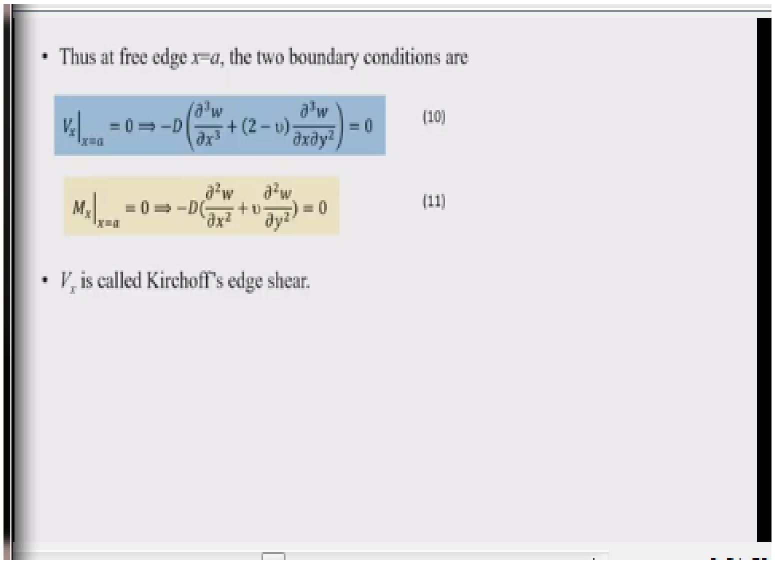

So, bending moment because this is the x edge, so x = a bending moment is 0, so bending moment expression is 0. But here in the simply supported edge compare to the two expression here for free edge we are equating bending moment 0. In the simply supported edge we also equated bending moment equal to 0. But you can see the distinction or difference between these two expressions. In the simply supported case where the bending moment at x = a that edge was simply supported, we take it to 0 and then we neglected the curvature along the y direction.

So, only $\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2}$ was there. But here interestingly you can see because it is the free edge, so curvature cannot vanish. So, it is going to be deflected and free edge deflection will be measurable, significant deflection will be there at the free edge, slope will be there. So, therefore you have to take two terms in the moment expression and V_x is known as the Kristov edge shear.

So, that is very important condition for the free edge because originally poisson has given 3 condition the bending moment 0, twisting moment 0 and edge shear 0. But twisting moment and vertical shear are combined to give a Kristov edge shear. So, ultimately the final condition reduced from 3 to 2. So, it becomes easier after combining these two-expression twisting moment and these vertical shear force.

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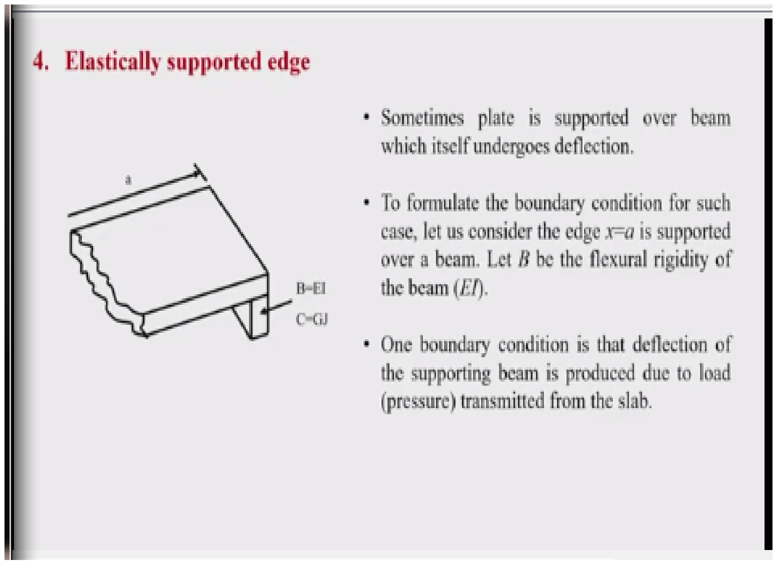

Now let us go to the other condition that is sometimes we get this type of situation a plate or slab Say for example; a slab is supported by a beam where beam is also a elastic element**.** Because when the slab load is transferred to the beam, beam has to react to this load; beam is not a rigid element. So, when the load is applied to a elastic element or a flexible element it has to undergo some displacement whatever small maybe.

So, similarly here due to load transfer from the slab to the beam there is some deflection in the beam, nobody can deny. And you can also see that this edge where it is supported by the elastic beam the moment when bending moment acts along the lengthwise direction here, will also try to rotate the beam. So, this bending moment has to be resisted by the beam torsional capacity.

So, you are understanding here the two conditions are necessary, one is that the beam deflection here will be same as the plate deflection. But beam deflection will only can be found when we know the distributed load over the beam. Distributed load over the beam is nothing but the edge shear along this edge**,** whatever edge shear you are finding along this edge this will be the load acting on the beam.

Similarly, the bending moment whatever you get here for the plate this will be your distributed twisting moment over the beam which has to be resisted by the torsional moment or torque in the beam. So, let us find the mathematical expression for these conditions. So, two conditions I were expressed and I have written here. We should know two parameters for the beam one is B, that is the flexural rigidity of the beam, E is the modulus of elasticity of the beam and I is the moment of inertia of the beam.

It is not necessary that plate and beam should have same modulus of elasticity, beam maybe of different materials. Suppose if you construct a slab say of concrete of say 20 MPa then beam maybe of concrete of 30 MPa. So, there will be difference of modulus of elasticity again two different types of metal can also be used when the metallic plate is resting on another metallic support.

So, therefore this modulus of elasticity of the beam not necessarily the same as the modulus of elasticity of the plate, so that you should remember. Then the torsional rigidity of the beam, torsional rigidity is defined as the product of shear modulus into torsional constant J. Torsional constant J is slightly difficult to find out specially for rectangular section because we will find that a rigorous solution is obtained for the torsional constant and a chart was given in the book of theory of elasticity by Timoshenko.

For rectangular beam of different width depth ratio, so that you will find. So, coefficients are given and you can find from that coefficient. For a circular section that torsional constant is very easy, for circular section J is nothing but polar moment of inertia. But for rectangular section or other thin wall section the torsional constant is not readily determined as the sum of the moment of inertia, it is not equal to the polar moment of inertia.

Polar moment of inertia is nothing but $I_x + I_y$, $I_x + I_y$ gives a moment of inertia perpendicular to the x and y axis. But it is not the case of other type of section, only it is true for circular section where you can easily get the polar moment of inertia as is $I_x + I_y$. However, for rectangular beam or rectangular support that rectangular sectional support that we are showing here the torsional constant has to be found out by proper method. So, these are two parameters that should be known for beam then only we can write the boundary condition.

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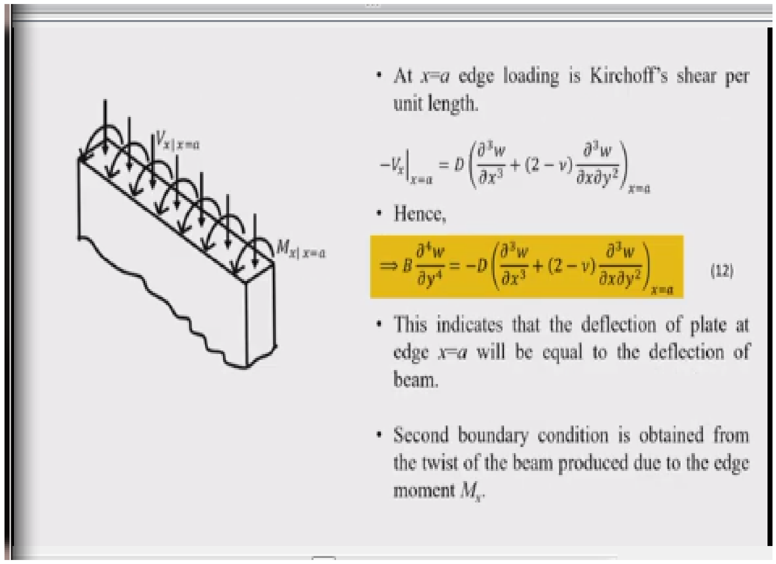

So, for the beam if we see the deflection of the beam, deflection of the beam is to be written as a fourth order equation $EI\frac{\partial^4 w}{\partial y^4} EI\frac{\partial^4 w}{\partial y^4}$, that is the differential equation or the very well-known equation for beam deflection subjected to uniformly distributed load of constant cross section 'B'. Then you can see the right-hand side is nothing but the load transmitted on the beam and load transmitted on the beam is nothing but the edge shear.

You can see this quantity is edge shear, there is the edge shear per unit length will be distributed load on the beam. So, knowing this quantity at x = a, we can find out or we can impose the boundary condition to evaluate the unknown constants of integration. So, this indicate that the deflection of the plate at edge x = a will be equal to the deflection of the beam. Then second boundary condition is obtained from the twist of the beam produced due to edge moment M_x.

So, at the edge x there will be the bending moment M_x along the x direction and you can see the resistive moment is offered by the beam due to it is torsional rigidity. So, this moment has to be found, bending moment expression we already know it.

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- From equilibrium condition, twisting moment in the beam should be balanced by distributed moment at $x=a$.

$$M_t + \frac{\partial M_t}{\partial y}dy - M_t = M_x dy \qquad \text{or,} \quad \frac{\partial M_t}{\partial y} = M_x\Big|_{x=a}$$

- Now, let C be the torsional rigidity of the beam i.e., C=GJ where G is the shear modulus and J is the torsional constant.
- Thus, $M_t = -C\left(\frac{\partial^2 w}{\partial x \partial y}\right)$

$$\frac{\partial}{\partial y}\left(-C\frac{\partial^2 w}{\partial x \partial y}\right)\Big|_{x=a} = -D\left\{\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right\}\Big|_{x=a}$$

- M_t is found from the rate of change of angle of rotation at any cross section multiplied by the torsional rigidity.

$$\Rightarrow C\frac{\partial^3 w}{\partial x \partial y^2}\Big|_{x=a} = D\left\{\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right\}\Big|_{x=a} \qquad (13)$$

So, bending moment expression is known here and we have equated these two torsional moment or torque of the beam and how it is evaluated. For example, at one end of the beam or at one element of the beam the couple is empty, torsional couple is empty at a distance dy the torsional couple will be $M_t + \frac{\partial M_t}{\partial y}dy$ $M_t + \frac{\partial M_t}{\partial y}dy$ at a distance dy. So, that means if I take the equilibrium of the element for torsional moment because at the other side the sign will be different.

So, this minus this will be your bending moment M_x dy. So, you can see that this quantity, the first line equation is nothing but $\frac{\partial M_t}{\partial y} = M_x|_{x=a}$ $\frac{\partial M_t}{\partial y} = M_x|_{x=a}$. That means here the torsional moment that is you have to find out. And torsional moment is nothing but, Torsional moment of a beam if we know the twist, rate of twist multiplied by the torsional rigidity is your torsional moment.

So, rate of twist is nothing but $\left(\frac{\partial^2 w}{\partial x \partial y}\right)$ and it is multiplied by the torsional rigidity C which is nothing but equal to G j, G is the shear modulus of elasticity and it is derivative is taken with respect to y and it is equated to the bending moment at x = a. So, both the quantities are evaluated at x = a. So, this gives the second boundary condition for this case, when the plate is supported along the elastic beam, continuous support, there is no discontinuity of the support that you should remember.

So, M_t a is found from the rate of change of angle of rotation at any cross section multiplied by the torsional rigidity. So, after simplifying you will get this, this is the equation i.e second equation for the boundary condition of this case when the beam is plate is resting on the elastic beam and the first boundary condition is this. So, 2 boundary conditions we have written.

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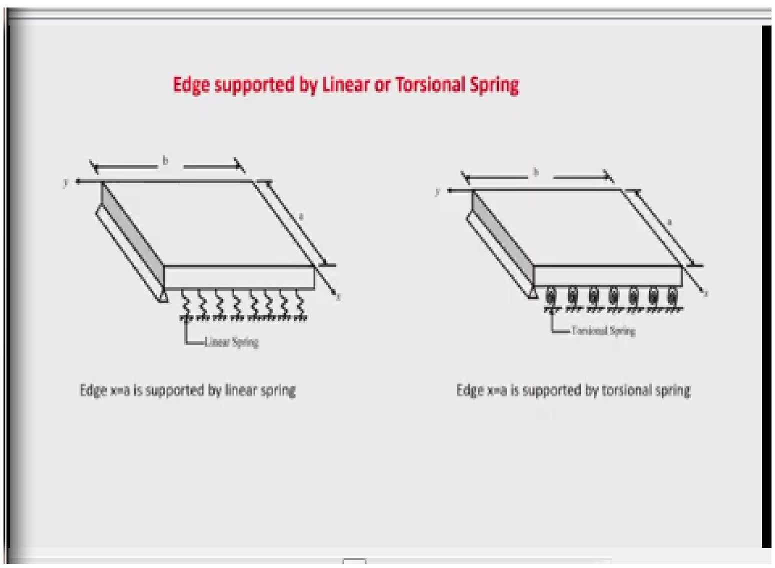

Now there may be case when one of the edges is supported by the linear spring continuously distributed, there is no break. Although in the figure you can see that a break is there, but it is with a very close spacing. So, it is a distributed spring, it is acting on this plate edge if this is the origin of the coordinate system then x = a, the spring is distributed or the spring is attached at x = a, so this is linear spring.

Another case maybe there when they torsional spring or rotational spring is attached and this edge x = a. So, these are the 2 non-classical boundary condition. Previous one also non-classical boundary condition. Classical boundary conditions are clamped, pinned or simply supported and your free clamp, simply supported, free, these three are classical boundary condition.

Non-classical boundary condition I have shown that the beam resting on the elastic support**.** Then plate resting on the elastic beam then plate resting on the linear spring and plate resting on the rotational spring. So, let us see what may be the equation for boundary conditions in these two cases separately.

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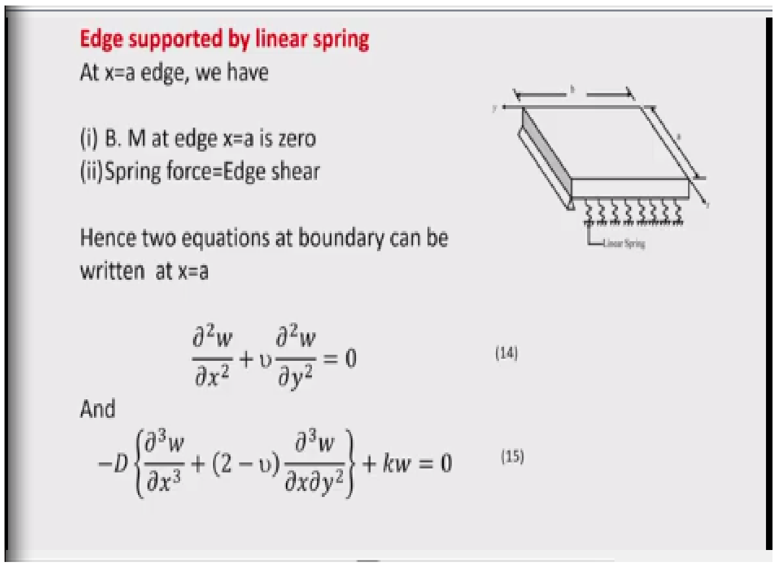

So, first let us see edge supported by linear spring, so linear spring is supported. So, 2 condition we have to impose, bending moment at edge x = a is 0, here bending moment is 0 and spring force, here due to deflection at the edges spring will offer resistance. So, spring resistance at these edges k stiffness of the spring multiplied by the deflection this should be equal to your edge shear. So, spring force should be equal to edge shear, so that is one condition. But first only condition is bending moment equal to 0.

$$\frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2} = 0$$

So, bending moment equal to 0 gives the implication

$$\frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2} = 0$$

This is the bending moment expression at x = a, you have to substitute in the expression x = a, then only you can write the equation. And the second condition spring force = edge shear. So, this is the edge shear expression at x = a that we have derived earlier + *kw* that is the spring force equal to 0.

That means you can write the D into this whatever quantity is there within this second bracket equal to *kw*. So, it indicates that spring force has to be resistant or has to be balanced by the edge shear. So, these are the condition when the one edge is supported by linear spring**.**

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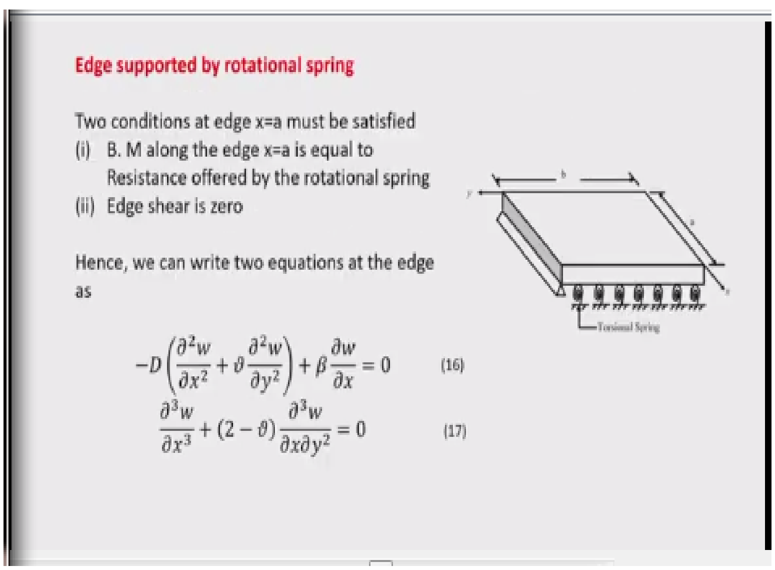

Then edge supported by the rotational spring, 2 condition at the edge x, a must be satisfied. So, let us see what are the condition. Since, it is rotated by the rotational spring or torsional spring, it will offer resistance to the bending moment along the edge. So that means along the edge, the bending moment transferred from the plate should be balanced by the resisting moment offered by the rotational speed.

So that is one thing, the bending moment along the edge x = a as equal to the resistance or resisting torque offered by the rotational speed and second condition is edge shear is 0. So hence, we can write two equations at the edges. One is this, this is your bending moment plus this is the rotational or torsional resistance offered by the spring, 'β' is the torsional constant or rotational spring constant that we take.

Beta is the rotational spring constant multiplied by the angle of rotation $\frac{\partial w}{\partial x}$ $\frac{\partial w}{\partial x}$. So, this should be equated at x = a for such cases to get first condition of boundary. Second condition of boundary is obtained by equating edge shear = 0. So, we have discussed boundary condition that we generally encounter in practice, classical type and non-classical type. In classical types are common but non-classical types also we have to impose in certain cases alright.

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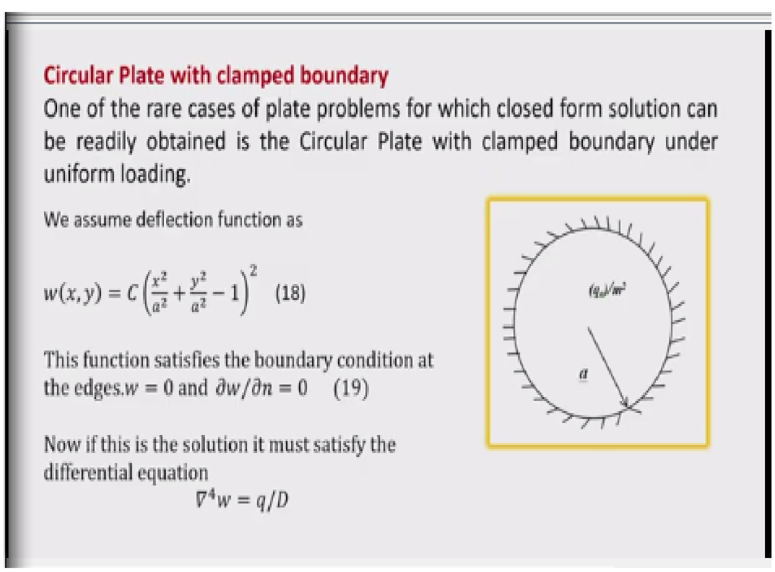

Now I want to discuss that in certain cases of plate problem, we can obtain very interesting solution without a very rigorous calculation. And that are some rare cases, one of the cases is such that when a circular plate is clamped along the edges, so that is one case, a rare case.

Circular plate clamped along the edges and subjected to UDL throughout the plate, so Q_0 per meter square Q_0 per unit square a square is the load intensity here.

And a is the radius of the circle. Now you can see how this problem can be solved. This is proceeded like that. If I assume a deflection function, say this, C is a unknown constant and

$$w(x, y) = C\left(\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1\right)^2$$

$$w(x, y) = C\left(\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1\right)^2$$

$$C\left(\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1\right)^2$$

If I assume this as the deflection function, then it shows that it readily satisfies the boundary condition. So, what are the boundary condition?

Boundary condition at the fixed these edges, fixed edges will be your slope, it is deflection is 0. And the slope, because this is a curved edge, so I take slope with respect to normal direction, ∂n . And this can be decomposed into a Cartesian coordinate slope also that we have seen if the angle of inclination with the original x axis is known, that derivation I have done in my earlier classes.

So, these two conditions have to be satisfied at the edges, and taking this function, you can see that at the boundary $x^2 + y^2 = a$. So, this quantity becomes 1, so 1 - 1 is 0, so deflection is satisfied. Again, if you take this derivative of that quantity with respect to x, or with respect to y, this expression within the bracket will come first, and then we will take the derivative with respect to x or y whatever maybe.

So, again it will be 0 because this term is again appearing in the expression of slope. So, therefore the same condition is also satisfied. So, you have to take a quadratic term with this expression, $\left(\frac{x^2}{a^2}+\frac{y^2}{a^2}-1\right)^2$ $\left(\frac{x^2}{a^2}+\frac{y^2}{a^2}-1\right)^2\left(\frac{x^2}{a^2}+\frac{y^2}{a^2}-1\right)$.

So, the expression shows that boundary conditions are satisfied at once without any difficulty for clamped edges, nobody can deny.

Now if this condition is satisfied and if I think that it is the solution of the differential equation, that it must satisfy the differential equation. So, if I take certain function as a solution, I have to substitute it in the differential equation to see whether both sides are equal. So, that means, this function or w_{xy}, now have to be substituted in this differential equation.

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In expanded form

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial x^4}=\frac{q_0}{D} \qquad (20)$$

Let us evaluate the derivatives of the w(x,y) to be substituted in above equation (20)

$$\frac{\partial w}{\partial x}=4C\left(\frac{x^2}{a^2}+\frac{y^2}{a^2}-1\right)\left(\frac{x}{a^2}\right)$$

$$\frac{\partial^2 w}{\partial x^2}=\frac{4C}{a^2}\left(\frac{x^2}{a^2}+\frac{y^2}{a^2}-1\right)\left(\frac{4}{a^2}\right)+\frac{8C}{a^4}x^2$$

$$\frac{\partial^3 w}{\partial x^3}=\frac{8Cx}{a^4}+\frac{16Cx}{a^4} \qquad \text{Now,} \quad \frac{\partial^4 w}{\partial x^4}=\frac{24C}{a^4}$$

Let us see, if I substitute this in the differential equation $\nabla^4 w = q/D$ $\nabla^4 w = q/D$ which is expanded in this form

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial x^4} = \frac{q_0}{D}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial x^4} = \frac{q_0}{D}$$

Q_0 is the uniformly distributed load. Now you require here to evaluate or know the derivatives up to fourth order because the expression is given for w, as $\left(\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1\right)^2$. So, you can successively obtain the derivative**.**

So, if you obtained the derivative $\frac{\partial w}{\partial x}$ is obtain, $\frac{\partial^2 w}{\partial x^2}$ $\frac{\partial^2 w}{\partial x^2}$ is obtain then $\frac{\partial^3 w}{\partial x^3}$ is obtain.

So, lastly, we go to the 4th derivative, so 4th derivative is $\frac{24C}{a^4}$ $\frac{24C}{a^4}$, it can be easily verified.

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Similarly, we have

$\frac{\partial^4 w}{\partial y^4} = \frac{24C}{a^4}$; $\qquad \frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2}\right) = \frac{8C}{a^4}$

Hence, substituting the derivatives in plate equation, we get

$C\left(\frac{24C}{a^4} + 2 \times \frac{8C}{a^4} + \frac{24C}{a^4}\right) = \frac{q_0}{D}$

Hence

$C = \frac{q_0 a^4}{64D}$ (21)

Hence deflection surface is given by

$w(x,y) = \frac{q_0 a^4}{64D}\left(\frac{x^2}{a^2} + \frac{y^2}{a^2} - 1\right)^2$ (22)

It can be seen maximum deflection is found at the centre and is equal to

$w_{max} = \frac{q_0 a^4}{64D}$ (23)

And similarly, we can obtain the derivative with respect to y and the cross derivative that is required for the second term, that is $2\frac{\partial^4 w}{\partial x^2 \partial y^2}$ $2\frac{\partial^4 w}{\partial x^2 \partial y^2}$, so that quantities also evaluated. Now substituting all these derivatives in the differential equation, in the equation number 20, we will get this constant C, C is easily evaluated as $\frac{q_0 a^4}{64D}$ $\frac{q_0 a^4}{64D}$

.

So, hence the deflection surface is defined by this, C is known, so exactly deflection surface is obtained. So, from the deflection surface now you can go for other quantities like bending moment, shear force, twisting moment etc. whatever you may be but for x is symmetrical case is there will be no twisting moment, so bending moment you can obtain. And it can be seen that maximum deflection will occur at the center.

So, substituting x = 0, y = 0 in this expression, expression 22 you get the maximum deflection of the plate as $\frac{q_0 a^4}{64D}$ $\frac{q_0 a^4}{64D}$. So, that is very rare cases where the solution is obtained just by assuming as suitable function which is satisfying first the boundary condition and after imposing this into the differential equation, we get the unknown constant.

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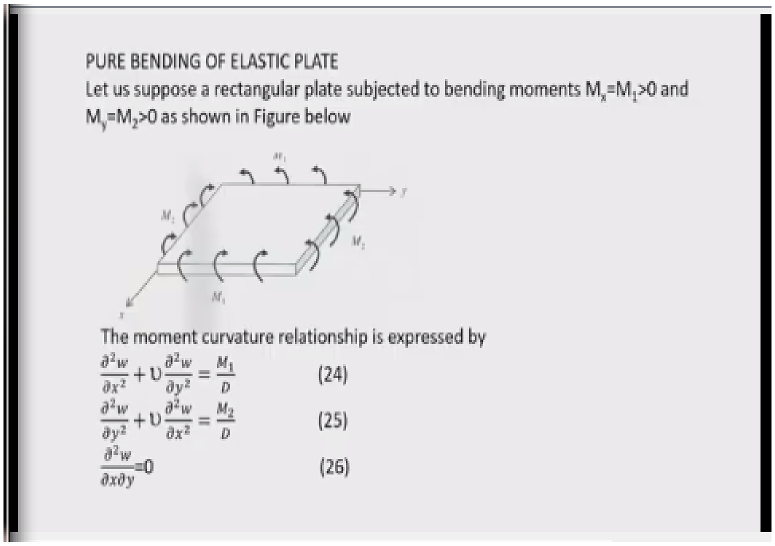

Now let us see the pure bending case of a plate, that is very interesting, a interesting surface is obtained when we find out this deflection of the plate. Now, the plate is subjected to pure bending along the edges M_1, and it is a symmetrical case. This is symmetrical case it is shown here, but there may be at anti symmetrical case also. The moment curvature relationship is expressed you know that moment curvature relationship is this.

And since there is no twisting moment, so $\frac{\partial^2 w}{\partial x \partial y} = 0$

Now, equation number 24 and 25, you can see here this is the curvature in the x direction and this is the curvature in y direction. Here this is curvature in y direction, this is curvature in x direction and these are the moments that is applied moment M_1 and M_2 on the plate.

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Solving for the curvatures from first two of the above equations, one get

$$\frac{\partial^2 w}{\partial x^2} = \frac{M_1 - \vartheta M_2}{D(1-\vartheta^2)} \qquad (27)$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{M_2 - \vartheta M_1}{D(1-\vartheta^2)} \qquad (28)$$

$$\frac{\partial^2 w}{\partial x \partial y} = 0 \qquad (29)$$

Integration of first eq.

$$\frac{\partial w}{\partial x} = \frac{M_1 - \upsilon M_2}{D(1-\upsilon^2)} x + f_1(y) \qquad (30)$$

So, solving 24 and 25 simultaneously, one can obtain this expression of curvature in x direction, and expression of curvature in y direction. And twisting moment as usual, we have written $\frac{\partial^2 w}{\partial x \partial y} = 0$ Now let us integrate these, because we are targeting these w, w has to be found out. So, integrating the first equation, with respect to x, we get $\frac{\partial w}{\partial x}$ $\frac{\partial w}{\partial x}$ and it is x and constant of integration. Now constant of integration here I take as a function of y because I am doing the integration with respect to x.

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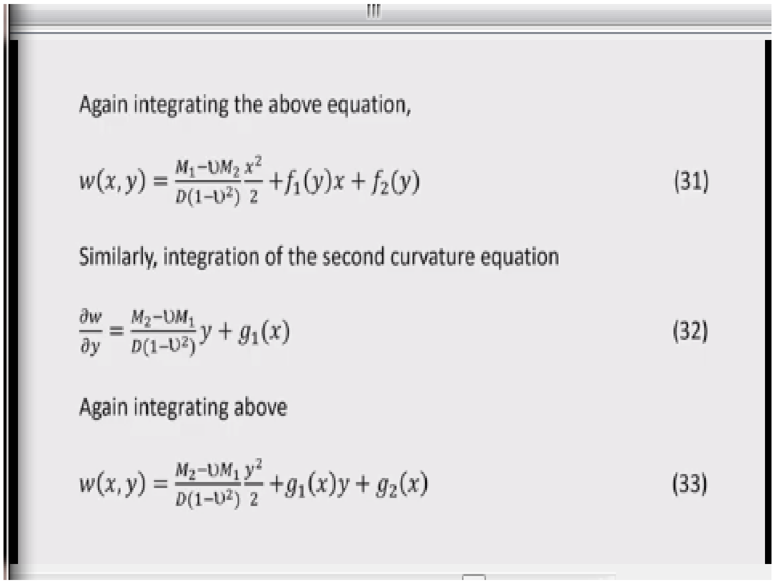

Again integrating the above equation,

$$w(x,y) = \frac{M_1 - \upsilon M_2}{D(1-\upsilon^2)}\frac{x^2}{2} + f_1(y)x + f_2(y) \qquad (31)$$

Similarly, integration of the second curvature equation

$$\frac{\partial w}{\partial y} = \frac{M_2 - \upsilon M_1}{D(1-\upsilon^2)}y + g_1(x) \qquad (32)$$

Again integrating above

$$w(x,y) = \frac{M_2 - \upsilon M_1}{D(1-\upsilon^2)}\frac{y^2}{2} + g_1(x)y + g_2(x) \qquad (33)$$

Then again integrating the first expression, this expression I get this expression will be $\frac{x^2}{2}\frac{x^2}{2}$. So, $\frac{x^2}{2}\frac{x^2}{2}$ is coming, and then $f_1(y)\,x$ and then another constant of integration which also I will take as a function of y because I am integrating with respect to x. Similarly, integration of the second curvature equation, second curvature equation is this equation number 28. So, integrating equation number 28, first we get this, so here the function of x will appear. And after second integration we will get another function of x.

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Now twisting curvature can be expressed as

$$\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right)=0 \qquad (34)$$

Integration of these equations gives

$$\frac{\partial w}{\partial x}=G(x) \text{ and } \frac{\partial w}{\partial y}=F(y) \qquad (35)$$

which means

$$f_1(y)=A \text{ and } g_1(x)=B \qquad (36)$$

Equating both the expressions for w(x,y)

$$\frac{M_1-\upsilon M_2}{D(1-\upsilon^2)}\frac{x^2}{2}+Ax-g_2(x)=\frac{M_2-\upsilon M_1}{D(1-\upsilon^2)}\frac{y^2}{2}+By-f_2(y)$$

Now integrating the twist curvature equation, twist curvature now I am writing in this way, an integration of this equation simply results. Say if I integrate this equation first equation, this equation equal to 0. Then I will get $\frac{\partial w}{\partial x}$ $\frac{\partial w}{\partial x}$ and function of G(x) function of x, because I am integrating with respect to y. And in the second case, I am integrating with respect to x. So, I am getting a function of y as a constant, which means that earlier cases, if you compare this you will get the $f_1(y)$ = constant in $g_1(x)$ = constant.

So, these constants I termed as A and B. Equating both the expressions for w_{xy}, we have got after twice integration of curvature in x direction and after twice integration of curvature in x direction. So, equating these we get this**.** So, there are some interesting conclusion will arise, you can see here in the left-hand side, the expressions are function of x. In the right-hand side, the expressions are function of y, this clearly indicates because M_1, M_2, poisson ratio and capital D all are constant, B is a constant, A is a constant.

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Now twisting curvature can be expressed as

$$\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right) = 0 \qquad (34)$$

Integration of these equations gives

$$\frac{\partial w}{\partial x} = G(x) \text{ and } \frac{\partial w}{\partial y} = F(y) \qquad (35)$$

which means

$$f_1(y) = A \text{ and } g_1(x) = B \qquad (36)$$

Equating both the expressions for w(x,y)

$$\frac{M_1 - \upsilon M_2}{D(1-\upsilon^2)}\frac{x^2}{2} + Ax - g_2(x) = \frac{M_2 - \upsilon M_1}{D(1-\upsilon^2)}\frac{y^2}{2} + By - f_2(y)$$

But how a function of x can be equal to a function of y. This is not possible unless both are constant**.** So, that is very important conclusion.

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$$\frac{M_1 - \upsilon M_2}{2D(1-\upsilon^2)}\frac{x^2}{2} + Ax - g_2(x) = \frac{M_2 - \upsilon M_1}{2D(1-\upsilon^2)}\frac{y^2}{2} + By - f_2(y)$$

The left hand side of the equation is function of x where as right hand side is a function of y. This holds good only when both the sides are constant say –C

Hence we can write the deflection surface as,

$$w(x,y) = \frac{M_1 - \upsilon M_2}{D(1-\upsilon^2)}\frac{x^2}{2} + \frac{M_2 - \upsilon M_1}{D(1-\upsilon^2)}\frac{y^2}{2} + Ax + By + C \qquad (37)$$

The three arbitrary constants represent rigid body motion. To obtain these constants, let us fix the origin at the centre such that deflection w and slope $\partial w/\partial x$, $\partial w/\partial y$ are zero at x=y=0. In that we can A=B=C=0, so all deformations will be measured relative to this reference.

And based on that we take these holds good only when both the sides are constant and that constant I am taking at -C. So, that means in the first equation, see this is the first equation of deflection obtained from integration of curvature in x direction, and this is the second equation of deflection obtained from curvature of y direction, so both are constant. So, if I add this to a constant a constant in as added to a first equation and we get this.

Now these three arbitrary constant A, B, C, now have to be evaluated based on the prescribed condition at the boundary. Because here boundary conditions are not specified, it is just a plate on which the moments are applied. That is in the boundary condition we know the moments. But these constants can be eliminated if we take the center of the plate as the reference point from which the deflections are measured, deflection or slopes are measured.

So, if I take the origin as the reference point, then I get that at x = 0, y = 0, the slope and deflection will be vanished because it is a symmetrical bending. So, therefore in that case we will get A = 0, B = 0, C = 0.

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Hence,

$$w(x,y) = \frac{M_2}{2D(1-\upsilon^2)}[\{(M_1/M_2) - \upsilon\}x^2 + \{1 - \upsilon(M_1/M_2)\}y^2] \quad (38)$$

Once, the deflection w(x,y), other quantities in the plate can be determined. In the special case of equal bending moments, with $M_1=M_2=M_0$, one has

$$w(x,y) = \frac{M_0}{2D(1+\upsilon)}(x^2 + y^2) \quad (39)$$

One can find in this case, radius of curvature is

$$r_x = r_y = \frac{D(1+\upsilon)}{M_0} \quad (40)$$

So, in that case the deflection surface is simply written just like as a function of x^2 and y^2. And the coefficient of x^2 you know and coefficient of y^2 you know here. That can be arranged after arranging the multiplier in a suitable way. So once the deflection w_{xy} is known, other quantities in the plate can be determined, that you know the usual practice.

But if I take $M_1 = M_2$ is equal to some constant value. That means plate is bended by equal couple on all sides, then you will get interesting thing like that. The w_{xy} is becoming this, so this indicates the surface deflects in the form of a part of a sphere, so one can find in this case the radius of curvature as $r_x = r_y = \frac{D(1+\vartheta)}{M_0}$ $r_x = r_y = \frac{D(1+\vartheta)}{M_0}$.

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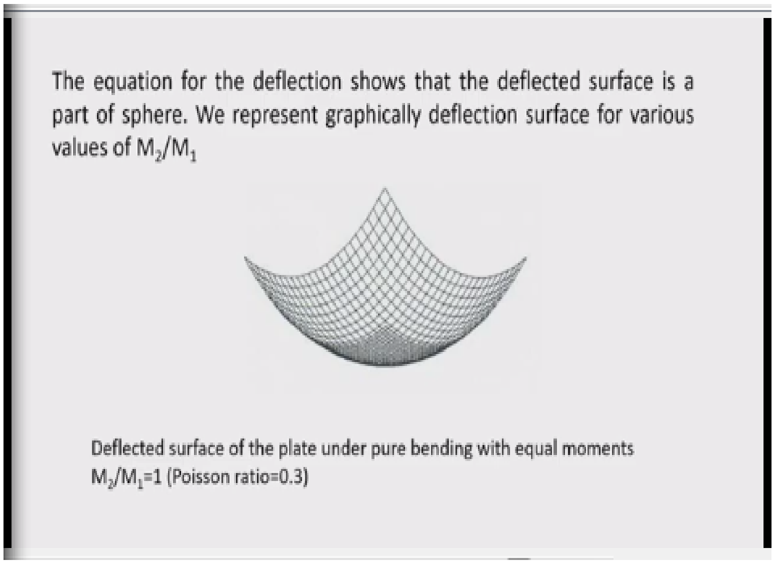

So, the surface will look like that when M_2 by M_1 = 1 and poisson ratio is 0.3, for that I have plotted.

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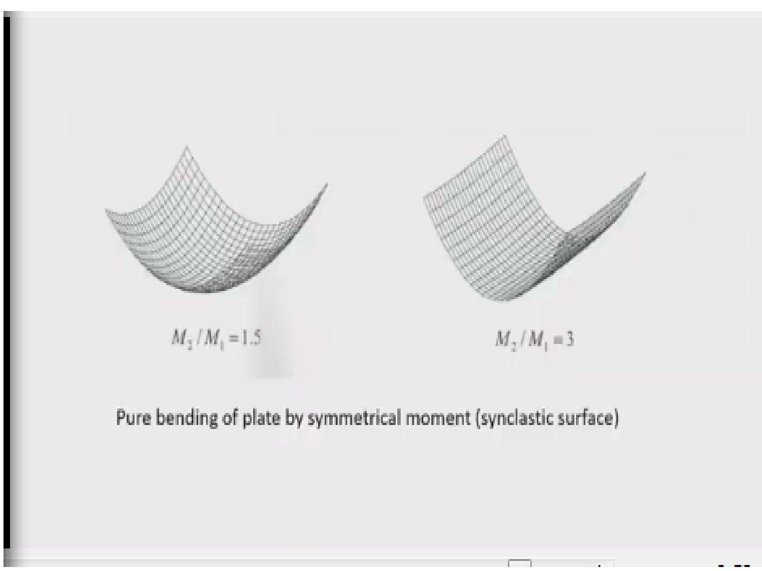

And for different values of M_2 by M_1 ratio, 1.53, but all are symmetrical cases, so symmetrical bending you are finding that curvature in one direction**,** so it is a simplistic surface.

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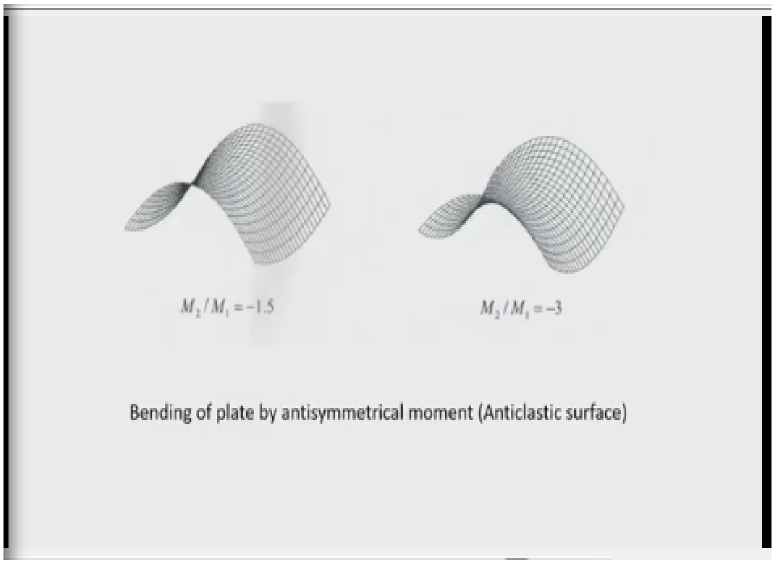

Now when there is a anti symmetric moment, then you will find the reverse curvature. So, reverse curvature is seen when the moments are not equal, or not symmetrical. I should not tell it equal magnitude is maybe whatever maybe but this is not symmetrical**.** So, in that case the M_2 by M_1 is -1.5 that the senses are different and here M_2 by M_1 is -3. So, here you will get a reverse curvature and therefore this type of surface is known as anticlastic surface.

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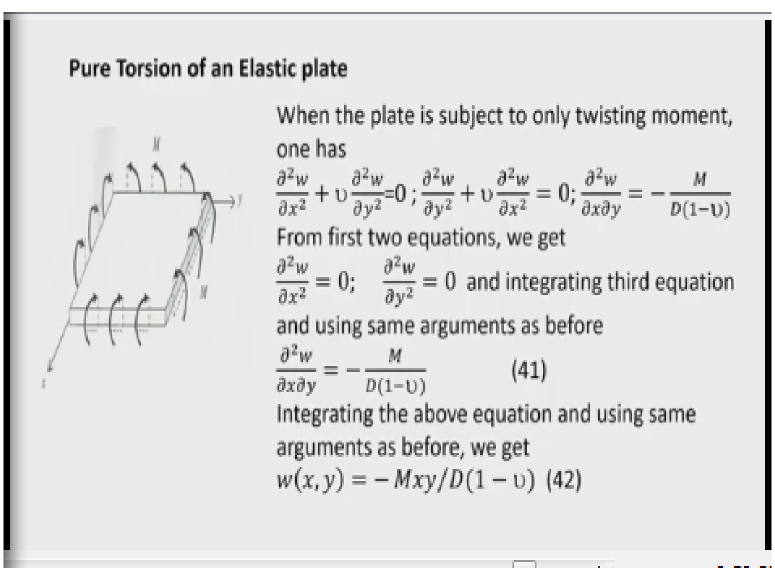

If I see the pure torsion of the plate, so in that case bending moments are 0, only the torsional moment is applied. And torsional moment equation we know, this is the torsional moment equation this and we can write this torsional moment equation as this. So, integrating above equation and using same argument as before, we can arrive

$$w(x,y) = -Mxy/D(1-\vartheta)$$

$$w(x,y) = -Mxy/D(1-\vartheta).$$

So, this is another equation for deflection of a flake subjected to pure torque, so pure moment is there.

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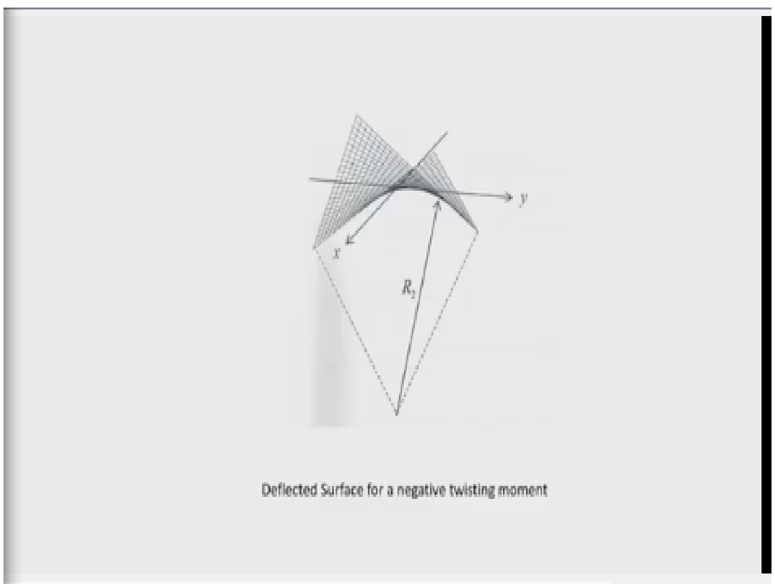

Deflected Surface for a negative twisting moment

So, in that case the deflection surface will look like that.

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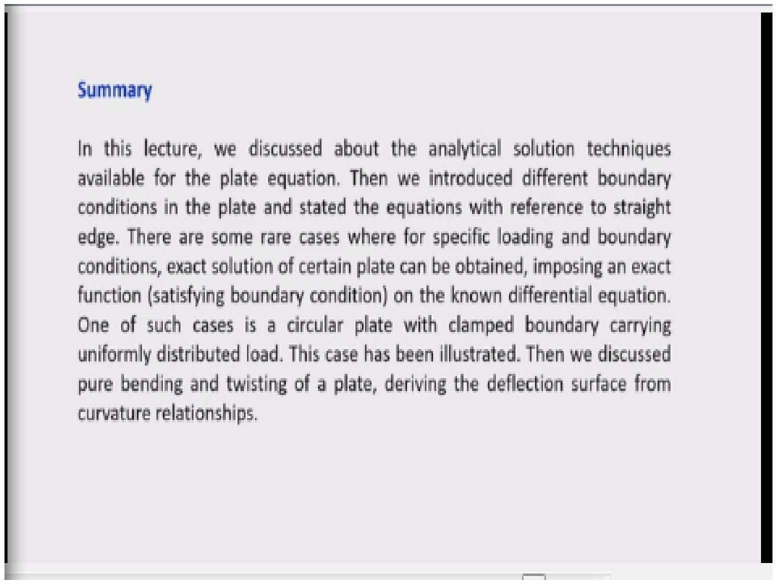

So, let us see whatever we have covered in this lecture, let me summarize this. So, in this lecture we discussed about the analytical solution technique available for the plate equation. Then we have introduced different boundary condition in the plate, classical boundary condition, non-classical boundary condition, plate edges, simply supported, clamp, free. These are 3 classical boundary condition, non-classical boundary condition as resting on the elastic beam.

Then your edge resting on the elastic beam, then edge resting on the elastic spring, then edge resting on the rotational spring. In the first case the edge resting on the linear elastic spring and in the second case the edge resting on the rotational spring. So, this type of boundary condition we have seen. Then I have focused that there are some cases where the closed form solution can be readily obtained.

So, these are the rare cases, it happens for some particular boundary condition and particular type of loading, not for all cases, very limited cases it can be obtained. So, one of such cases is the circular plate clamped along the edges and subjected to uniformly distributed load. So, that case I have demonstrated, how to find the deflection, taking a suitable deflected surface to find the unknown constant associated with the deflection function.

And from that we have seen that the maximum deflection occurs, as we expected at the center and magnitude is given in terms of the radius square and this load intensity and your radius to the power 4, load intensity and this flexural stiffness of the plate. As usual in case of beam also we have seen that deflection decreases if we increase the depth of the beam. Here also in the plate equation, we have seen that deflection decreases when the thickness of the plate is increased.

Because in the flexural rigidity of the plate the h cube term is appearing. So, there is similarity with the beam and plate also. So, then we have seen that pure bending of a plate subjected to a moment at the edge moment same sense but of different magnitude. And then when the magnitude of the moment and sense of the moments are equal, a purely symmetrical case of equal magnitude.

Then we have seen that deflected surface is a part of a sphere, and radius of the sphere can be easily computed from this deflected surface by taking the second derivative, the curvature and reciprocal of this will be radius of curvature. Then we have seen that for a anti symmetric moment, there will be reversal of curvature. So, the two terms I have introduced one is simplistic surface but the curvature is in the same direction.

And anticlastic curvature the bending takes in a reverse way, so reversible curvature occurs due to inter symmetrical loading. Then we have seen the pure torsion of a plate, and how it is solved from the twist equation in a very simple way. And we have demonstrated the form of the surface that is formed due to pure twisting. So, thank you, again we will meet in the next class to proceed further, thank you.