Plates and Shells
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Lecture - 28
Membrane Theory in Pressure Vessels

Hello everybody, today I am starting module 10, and in this module, I want to discuss the membrane analysis of pressure vessels and tank. Now, so far, I have discussed the theory of membrane analysis that how a moment less state in a thin shell is developed, and this is useful for analysing the stresses under the external load, and the results produced by the membrane analysis are reasonably accurate except at the boundaries.

So, there are things I have shown with several examples of spherical domes, conical shells and then domes with opening and therefore, today, I want to extend this membrane analysis to a kind of structure which is also a thin shell of evolution with the help of membrane theory, I want to find out the state of stresses in the pressure vessels. So, our topic will be my membrane theory of pressure vessels.

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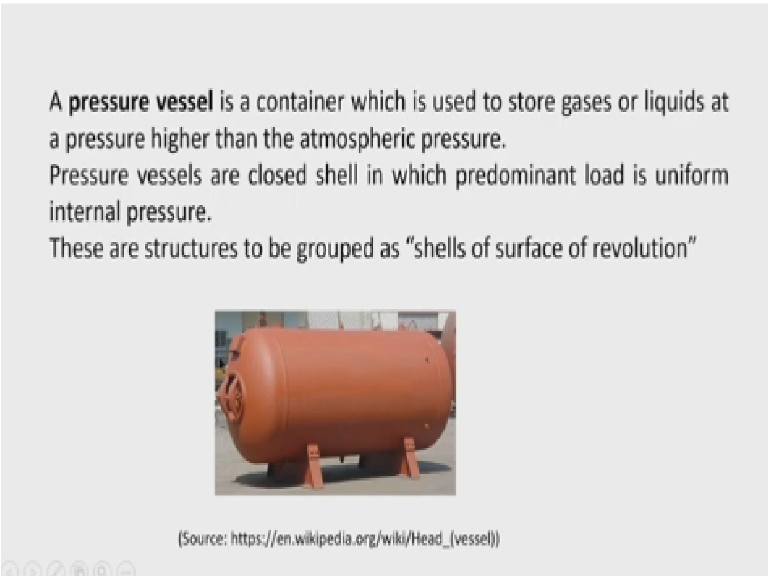

Now, first, let us discuss what is pressure vessel? Pressure vessel is a container which is used to store gases or liquids at high pressure. Generally, the pressure in the vessel is more than the atmospheric pressure. So, the container wall the shell has to be designed properly so that it can resist the higher internal pressure. Now, you can see the pressure vessels are mainly made of metal structure and especially the mild steel is used to fabricate the pressure vessel.

The pressure vessel are generally of cylindrical type and it has head at both end which may be is part of a sphere that is spherical part or it may be ellipsoidal part or it may be any other form. So mainly the ellipsoidal and spherical parts are common at the closer end enclosure. So, pressure vessels are close shell actually and predominant load is internal pressure. Shell force, it is negligible compared to the internal pressure. These are structures to be grouped as shell of surface of revolution.

So, this type of pressure vessel is formed when curve is rotated about an axis of revolution. So, here access of revolution is horizontal and here a straight line is rotated to form a cylindrical portion of the pressure vessel. Similarly, here this say here it is a spherical part the head that you are seeing at the end the spherical part so arc of a circle is rotated about the axis of rotation to form the closure of the shell enclosure.

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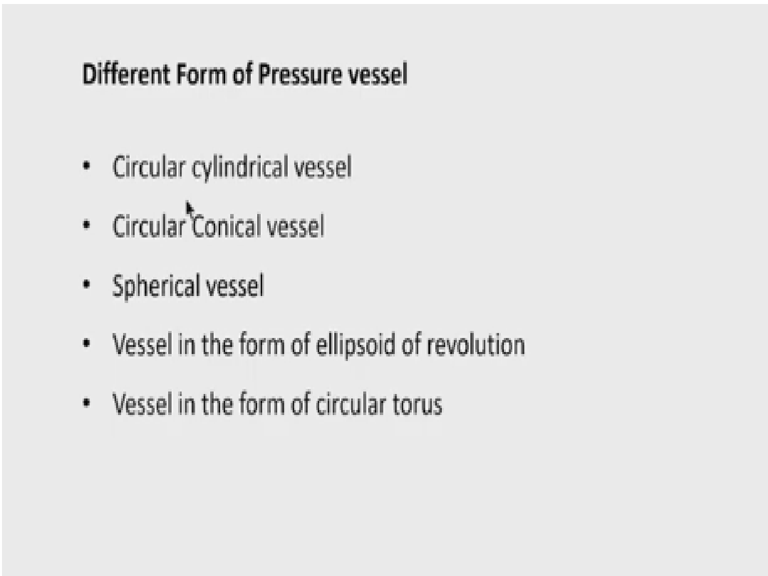

Pressure vessel is demand in the industry for carrying these LPG or you can say the oxygen cylinder or petroleum liquid or gaseous substances are stored in the pressure vessel and carry it to the site for different use. In a different form of pressure vessels are found, one is circular cylindrical vessel, then circular conical vessel, spherical vessel, vessel in the form of ellipsoid of revolution, vessel in the form of circular torus. So, you can find the different types of geometrical shapes are given in the pressure vessel.

So, one by one we will discuss and we will see how the theory is developed. Theory is our memory analysis for axisymmetric loading condition and our target is to find the meridional stress N_ϕ and circumferential stress N_θ for the design purpose.

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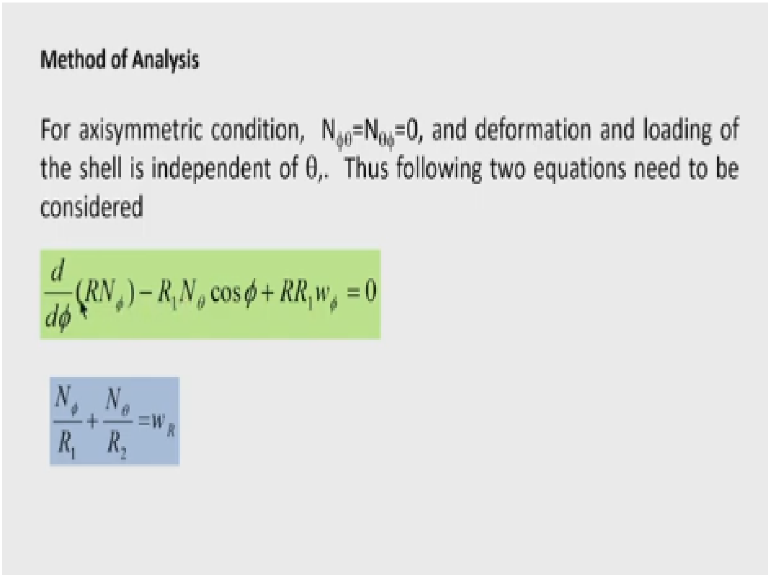

Now, since in the axisymmetric condition the analysis is carried out. So, we assume that the main sharing force $N_{\theta\phi} = N_{\phi\theta} = 0$ and deformation and loading whatever is produced in the shell or acted on the shell due to internal pressure are all independent of θ. So, thus following two equations are to be considered only for the analysis. So, one is a differential equation that you are seeing $\frac{d}{d\phi}(RN_\phi) - R_1N_\theta \cos cos\,\phi + RR_1w_\phi = 0$.

Now, here this the N_θ is the circumferential stress and N_ϕ is the meridional stress R_1 is the principal radius of curvature of the meridional curve and R is the radius of the parallel of latitude. So, these are the symbols and it is a differential equation of first order, but you can see in the first equation the variable N_ϕ and N_θ the unknown variable N_ϕ and N_θ are involved and in this second equation you can note that it is an algebraic equation given by $\frac{N_\phi}{R_1} + \frac{N_\theta}{R_2} = w_R$.

Where w_ϕ and w_R are the component of the load in the meridional direction and radial direction respectively, w phi is the component of the load in the meridional direction and w_R is the component of the load in the radial direction. Radial direction I mean that is a direction along the normal to the tangent at the point of meridian. So, solution of these two equations will give you the N_ϕ and N_θ.

Now, from the second equation here we can express N_θ in terms of N_ϕ and then we can substitute back in the first equation so that the first differential equation is converted into a differential equation of single unknown variable N_ϕ.

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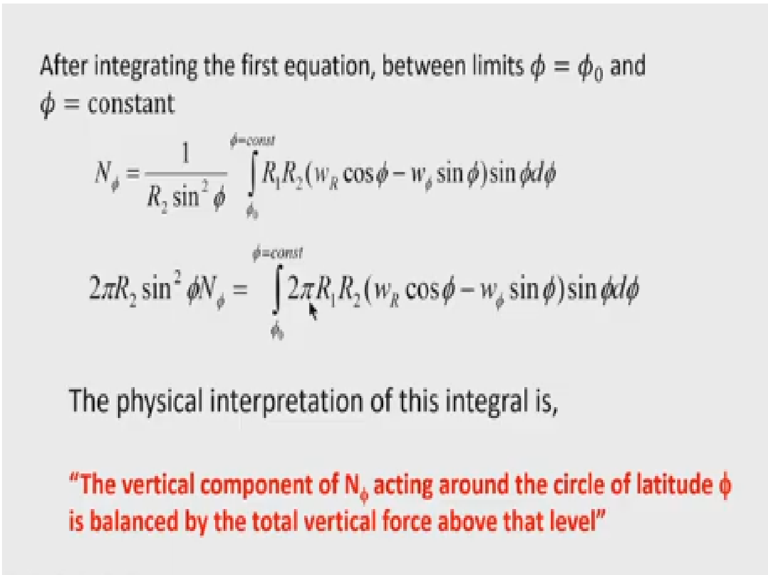

So, then N_ϕ can be found out by integration of the differential equation. So, N_ϕ is given by

$N_\varphi = \frac{1}{R_2\varphi} \int_{\varphi_0}^{\varphi=const} R_1 R_2 (w_R \cos cos\, \varphi - w_\varphi \sin sin\, \varphi) \sin sin\, \varphi\, d\varphi$, the ϕ is measured from the axis of rotation R_1, R_2 are the principal radii of curvature and w_R is the radial component of the load $\left(w_R \cos cos\, \phi - w_\phi \sin sin\, \phi\right) \sin sin\, \phi\, d\phi$.

So, right-hand side and left-hand side, left-hand side is N_ϕ and right-hand side contains this integral expression. Now, because we are carrying out a definite-integrals so constant of integration is not written here. Now, if I multiply this equation by 2π, 2π both the sides and being these R_2 here then we get this term. The left-hand term is $2\pi R_2 \phi\, N_{\phi}$ and right-hand term becomes the integral expression whatever you are getting only the multiplication is done with 2π. So, therefore, 2π term is coming here.

So, this expression can be interpreted physically, the physical interpretation is that if we see the left-hand side N_{ϕ} sin $sin\, \phi$ will be the say here.

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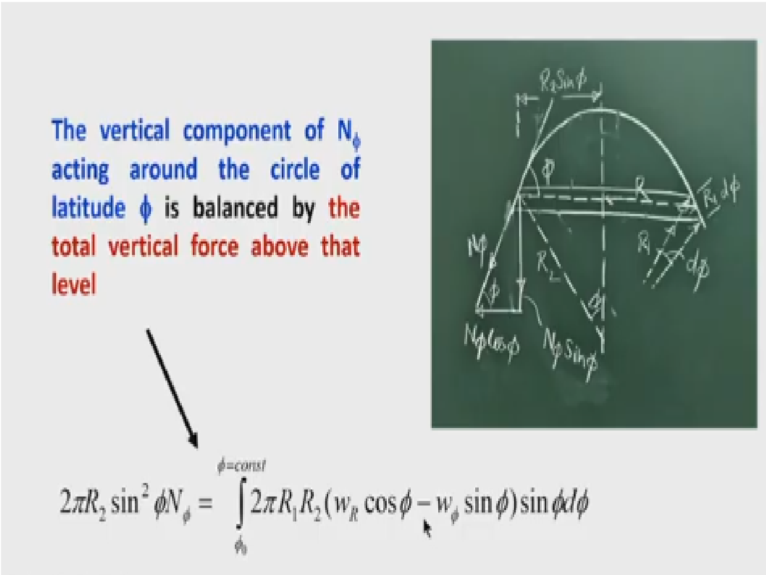

It is a part of the shell the surface of revolution in N_{ϕ} is the meridional stress that is acting along the tangent and then meridian. So, N_{ϕ} sin $sin\, \phi$ is the vertical component of the meridional force at an angle ϕ. Now, when N_{ϕ} sin $sin\, \phi$, this is the force per unit length and when it is multiplied by the circumference and this parallel of latitude then it becomes the total vertical component of N_{ϕ}.

So, left hand side of this equation we present the total vertical component of N_{ϕ}. So, $2\pi R_2 \phi\, N_{\phi}$ is the total vertical component of N_{ϕ}. Now, if I see this term this w_R cos $cos\, \phi$ is the component of the load along the radial direction and w_{ϕ} sin $sin\, \phi$ is the component of the load in the meridional direction. So, these some represent the total vertical load.

So total vertical load multiplied by the surface area what is surface area if I take a strip here $R_1\, d\phi$ is the width of the strip if the differential angle is $d\phi$ and the circumference is $2\pi R_2\; pi\; R$ is the circumference. Now R can be related with R_2, that is R we can write R equal to $R_2 \sin sin\, \phi$. So, therefore, you are seeing that here $R_2 \sin sin\, \phi$ term is there R 1 d phi term is there and 2 pi term is there.

So, the surface area is inside this integral term elemental surface area and the term inside the bracket represents the net vertical load, because w_R is their radial component of the load and w_ϕ is the component of the load along the meridional direction. So, when it is resolved along the vertical direction and then added it represents the net vertical load. So, right-hand side represents the net vertical load acting above the level ϕ.

So, this equation represents a force balance along the vertical direction. So, with the help of this meaning that physical meaning that total component vertical component of N_ϕ at any level phi should be balanced by the total vertical load above that level. We can find out the unknown stress component N_ϕ. Ones N_ϕ is found then N_θ can be calculated from this algebraic equation. Now, let us go one by one different types of pressure visuals.

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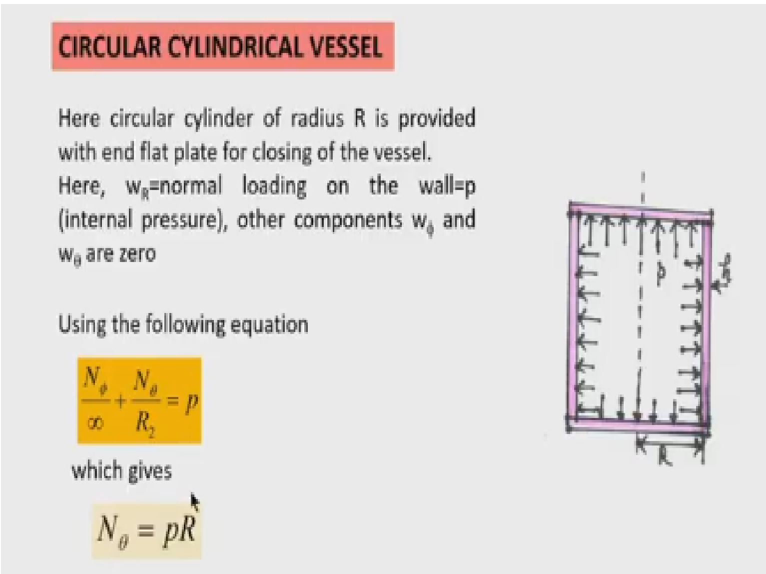

First, let us take the circular cylindrical mission. Now here circular cylinder of radius R is considered and it is acted upon by internal pressure p. The cylinder is provided with an enclosure, an enclosure is in the form of a flat plate. But in particular purpose, the enclosure may not be a flat plate it may be also a shell a spherical shell or an ellipsoid or any other form. So, here for the simplicity, we have taken the enclosure as the flat plate because our intention is to find out the membrane stresses in this cylinder.

So, we will use the two equations, one is this equation that is algebraic equation $\frac{N_\phi}{R_1} + \frac{N_\theta}{R_2} = p$. Now here the w_R the radial load is p and it is positive because the positive direction of w_R is this away from the centre up area from the axis of evolution. So, here the positive convention is taken and therefore this is the w_R component w_R simply is the radial pressure p. That is actually internal pressure in the pressure vessel.

Now, these types of shells of revolution have one curve as a straight line because when a straight line is revolved around the axis of rotation, then a circular cylinder is formed. So, the radius of curvature of the straight line is ∞. So, therefore, this first equation the N_ϕ by ∞, so, this goes to 0. So, only this term remains in N_θ/R_2 equal to p. Now, here for the cylinder $R_1 = R_2 = R$, so, radius of the circular cylinder.

So, therefore, $N_\theta = pR$. So, the value of circumferential stress N_θ which is called the hoop stress in this case hoop stress is acting along the circumference is pR . Now, let us find out this other component of this stress that is N_ϕ we are calling N_ϕ that is along the direction of the meridian, here meridian is a straight line. So, sometimes in case of analysis of cylindrical shell, the notation N_ϕ can be replaced by the N_x.

Sometime in some book or some author prefers to write the circumference a meridional stress in the cylindrical one as N_x, instead of N_ϕ because N_ϕ is related to sphere. But anyway, will keep this notation consistent. Here N_ϕ is understood as the remembrance stress which is acting along the meridian. Even meridian is a straight line this will be denoted by N_{ϕ} but somebody can replace it by the another term N_x. So, many other prefers to replace this N_{ϕ} in case of cylindrical shell with the notation N_x.

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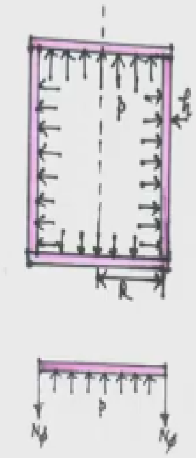
For finding, the value of N_{ϕ}, consider the free body diagram at the end of the shell to write the vertical equilibrium,

$$2\pi R N_{\phi} = p\pi R^2$$

$$N_{\varphi} = \frac{pR}{2}$$

Therefore, hoop and longitudinal stresses are

$$\sigma_{\theta} = \frac{pR}{h} \qquad \sigma_{\phi} = \frac{pR}{2h}$$



So, for finding the value of n phi, consider the free body diagram at the end of the shell. So, if I consider the free body diagram at the end of the shells say, for example, this top plate if I consider the top plate is acted upon by the pressure internal pressure p this is the only load in the self-weight is neglected. So, it has to be balanced by the vertical component of N_{ϕ}. Now, N_{ϕ} is acting along the circumference of the plate.

Now plate is a circular so the total N_{ϕ} acting at this level is $2\pi R N_{\varphi}$ where R is the radius of the circular cylinder and it should be balanced by the external is the internal pressure that is acting on the plate circular plate which is of radius R total load due to internal pressure upward load here you can see the acting on this plate is $p\pi R^2$. So, therefore, N_{ϕ} is calculated as $\frac{pR}{2}$. So, this is generally referred as the hoop and longitudinal stresses respectively.

So, hoop stress we can find out this N_{ϕ} hoop force is the N_{θ} there is the circumferential force. If I divided by h if I divide N_{θ}/h I will get the hoop stress. So, hoop stress is $\frac{pR}{h}$ and the longitudinal stress that is the meridional stress. I call it in general sense will be $\sigma_{\varphi} = \frac{pR}{2h}$.

So, this is the analysis of circular cylindrical pressure vessel.

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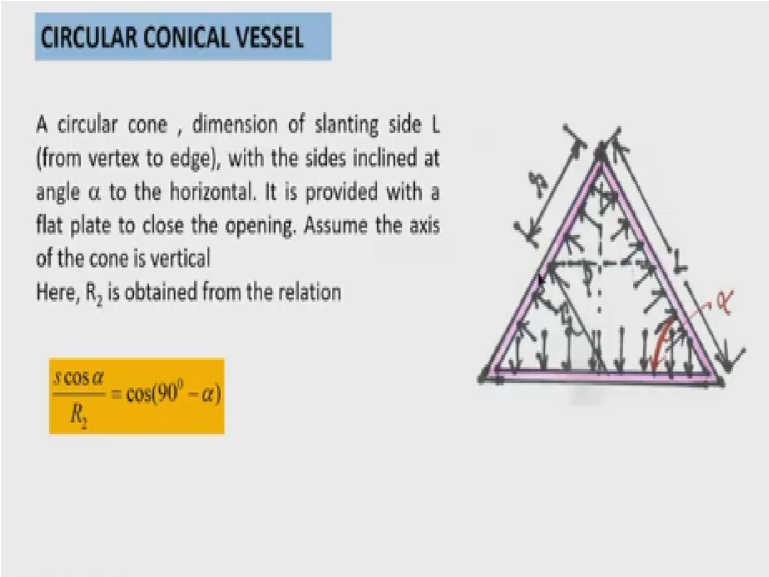

Let us take a circular conical vessel. Circular cone of the dimension of slanting side L, so you can see this is the circular cone, and it is closed at the end. It is formed by generating a slanting straight line incline straight line about the axis of rotation and then we form a cone however the closure is provided with a flat plate. So, the α is the angle that the slanting side makes with the horizontal, L is the length of the slanting side.

And then other parameters are say we will find it out from the geometry take a level say at this level you want to calculate. So, this plane is at a distance of s from the apex of the cone here. So, in this position we can relate this the radius of this circle parallel circle with the inclined length and also, we can relate this second principal curvature r_2 to with this incline length, one thing can be noted that this type of shell has 0 gaussian curvature because the one curve straight line.

So, for straight line the radius of curvature is infinity. So, therefore, the curvature is 0. So, product of two principal curvatures is known as Gaussian curvature and hence the Gaussian curvature for this conical shell as well as cylindrical shell is 0. Now, if I make use of this triangle to write this relation, $s \cos cos\, \alpha$ is your this site, $\frac{scoscos\,\alpha}{R_2}$ will be $\cos cos\,(90^0 - \alpha)$.

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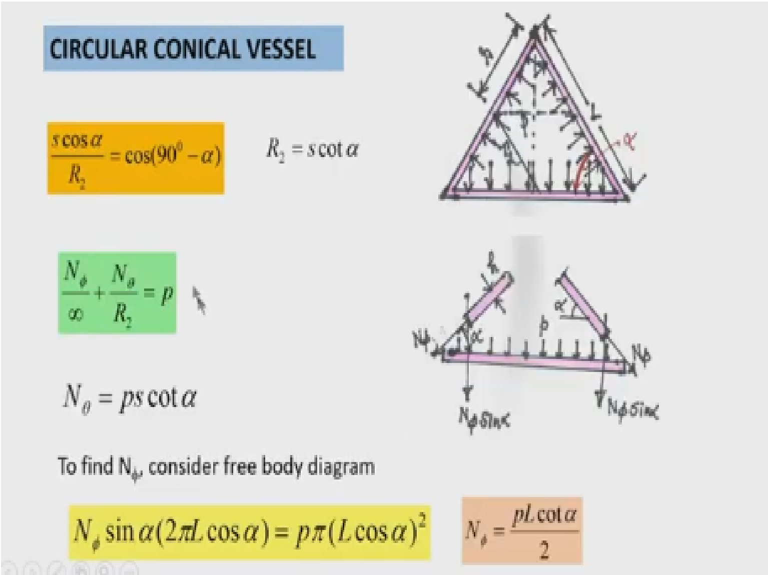

So that relation is used to find r_2 the second principal radius of curvature in terms of the inclined distance and the semi apex angle. So, it becomes $s \cot cot\, \alpha$, here of course this α is the angle that we have denoted as the angle made by the inclined side with the horizontal axis. So, semi apex angle will be $\alpha/2$. So, the how about this relation is coming out in terms of full angle α. So, r_2 the second principal radius of curvature is equal to $s \cot cot\, \alpha$.

Now, we make use of this relation $\frac{N_\phi}{R_1} + \frac{N_\theta}{R_2} = p$. But you can note here you have already noted that one of the principal curvature is infinity that is R_1 is ∞ the meridional curve here straight line. So, therefore, $\frac{N_\phi}{\infty} + \frac{N_\theta}{R_2} = p$, p is the internal pressure and this term goes to 0. So, therefore, this is the circumferential stress or you can call it hoop stress is equal to $N_\theta = pR_2$ and $R_2 = s \cot cot\, \alpha$. So, we can get the circumferential stresses $N_\theta = ps \cot cot\, \alpha$.

Now, we want to find the meridional stress? Meridional stress can be found by integral expression or using the physical meaning of the integral expression, we can find the value of the meridional stress and find. Now, here we will use the physical meaning of the integral expression instead of carrying out the full integration. So, to find N_{ϕ} consider the free body diagram at the endplate.

So, here you can see the endplate is subjected to this loading which is produced due to internal pressure in form internal pressure and this plate is again a circular plate. So, therefore the total load here will be the informed pressure intensity p multiplied by the area of the circle. Now, here you can see for the endplate which is of circular shape the radius will be $L \cos cos\, \alpha$.

So, therefore the total load on this endplate will be p is the intensity of the load area of the circle is $\pi(L \cos cos\, \alpha)^2$. So, this is the total, load this total load has to be balanced by the total vertical component of N_{ϕ} at this level. So, if I see the N_{ϕ} which is acting along the incline slide side that is along the meridian and you can see the vertical component of N_{ϕ} here is $N_{\phi} \sin sin\, \alpha$ and the circumference here is again the circumference of this endplate.

So, we write here $N_{\phi} \sin sin\, \alpha$ is the vertical component of this meridional stress N_{ϕ} multiplied by the circumference of the circular plate to $(2\pi L \cos cos\, \alpha)$ equal to total load at this level. So, total load at this level is produced due to uniform internal pressure acting on the plate. So, it is $p\pi(L \cos cos\, \alpha)^2$. Now, you can see here this is balanced. Now, the total internal force that is acting due to the pressure uniform pressure is downward.

So, to balance this load the N_{ϕ} should be acting upward. So, below of N_{ϕ} is now after simplification is coming as $\frac{pL \cot cot\, \alpha}{2}$. So, in this way in a circular conical vessel we can find the N_{θ} and N_{ϕ}.

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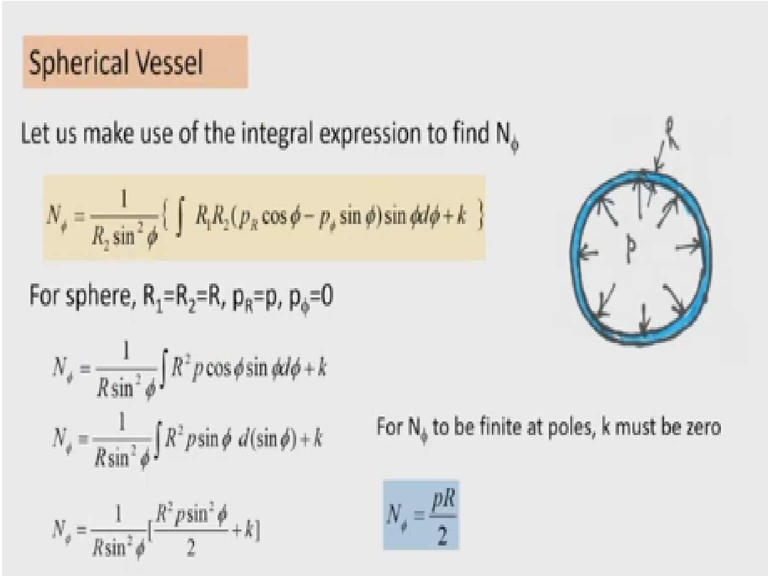

Let us come to this spherical version which is very common. And due to spherical symmetry, the designer prefer this because the analysis is very simple and even in this some pressure vessel the enclosure is made in the form of a spherical is provided with a spherical part spherical component. So, therefore, this analysis of spherical vessel is also important. So, let us make use of the integral expression to find the value of N_{ϕ}.

Now, here instead of these force balance that is physically making use of the physical interpretation of this integral. We can find it but instead of this we now doing the integration. So, let us see how we can arrive at the result $N_{\varphi} = \frac{1}{R_2\varphi}\left\{\int R_1R_2(p_R \cos cos\,\varphi\, - p_{\varphi} \sin sin\,\varphi\,) \sin sin\,\varphi\, d\varphi + k\right\}$. k is a constant of integration because this is a indefinite integral.

Now, p_R is the component of the load in the radial direction and now here load is your uniform internal pressure p and p_{ϕ} is the load that is acting along the meridional direction. Now, for this spherical vessel the principal radius of curvature $R_1 = R_2 = R$, the radius of the sphere R. The radius of this sphere here in this problem is taken as R.

p_R that is the radial pressure acting on the wall internal pressure is your p that is we have taken p as the internal pressure and p_{ϕ} that is the pressure along the material reaction is 0. So, that quantity we know. So, this integral now becomes $N_{\varphi} = \frac{1}{R\varphi}\int R^{2}p\cos cos\,\varphi\ \sin sin\,\varphi\, d\varphi + k$. Now, for N_{ϕ} to be finite at poles k must be 0 because at poles when you integrate it, you will find that only it is the final bill is possible only if you put $k = 0$.

So, therefore, the N_{ϕ} is found out omitting this constant and N_{ϕ} after integration becomes $N_{\varphi} = \frac{1}{R\varphi}[\frac{R^{2}p\varphi}{2} + k]$. So, k is taken as 0. So, therefore, the final result for $N_{\phi} = \frac{pR}{2}$ is pR by 2. To find the value of N_{θ} we make use of the equation $\frac{N_{\varphi}}{R_{1}} + \frac{N_{\theta}}{R_{2}} = p$. Now, since $R_{1} = R_{2} = R$ we get N_{θ} equal to $pR - pR/2$. So, $pR - \frac{pR}{2}$ and again N_{θ} coming as $\frac{pR}{2}$.

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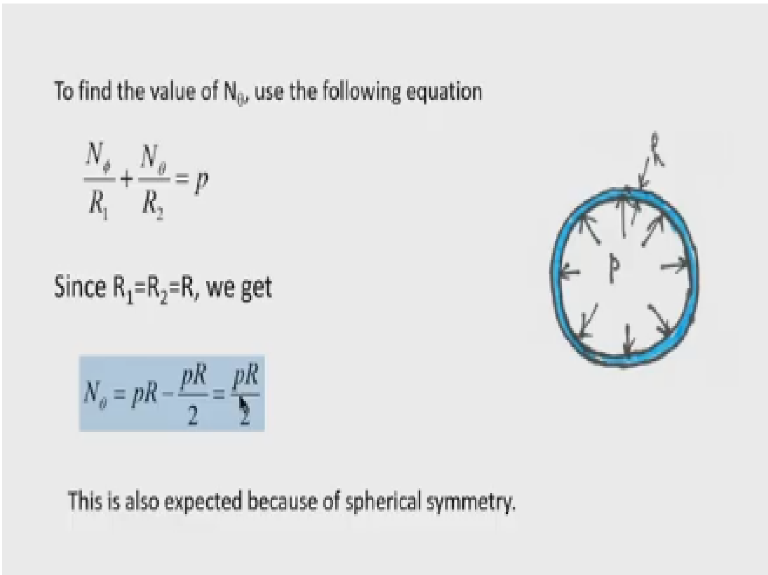

Now in this problem for a spherical pressure vessel both the stresses N_{ϕ} and N_{θ} are equal $\frac{pR}{2}$ and this is expected because of spherical symmetry.

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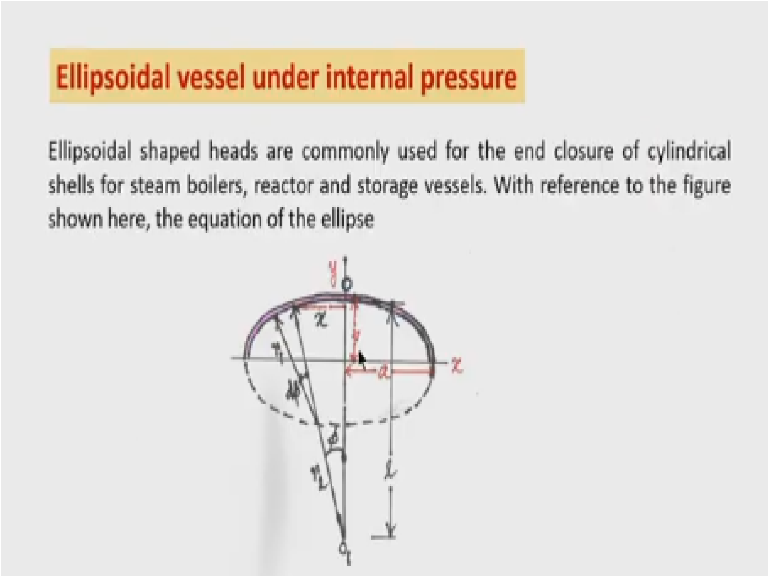

Now, let us come to the ellipsoidal vessel under internal pressure. Ellipsoidal shaped heads are commonly used for the enclosure of cylinder shells, in many cases for steam boilers, reactor and other storage vessels. So, analysis of ellipsoidal vessel is slightly complicated because of the difficulty in finding the expression for the principal radii of curvature. You have to do a lengthy differentiation to find the principal radii of curvature.

Now let us see how it can be done, we have taken a elliptical vessel acted upon by internal pressure p inform internal pressure p with reference to the figure shown here the equation of the ellipse can be written as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where this origin is here and your x-axis positive x-axis is along this direction positive y-axis is along the upward direction and a is the semi-major axis and b is semi-minor axis.

So, a/b ratio is the very important factor here you will find it later which influences the nature of the stress. Now, here in this figure, we see that at any point the meridional angle is ϕ and principal radius of curvature for the meridional curve which is ellipse is R_1 and R_2 is second principal curvature at this point. Now, the other parameters are at this point the coordinate is x the coordinate of this point is (x, y) whereas, x has to be measured negative at this point.

And this is taken as positive this is the positive direction of the x-axis but however, since this is symmetrical about this y-axis the analysis of the elliptical part taking a point here or towards the first quadrant will be same. Now l is the length denoted here from the point O_1 to the crown of the shell. Now, this type of shell that is the ellipsoid of revolution has application in steam boilers in reactor or other storage vessel mainly in industry and there are some technical name for different regions.

This part this Apex point is known as crown and this is known as this level is known as equator and in between these this is known as knuckle region. So, there are name given in the three region, this is crown, this is equator and knuckle region.

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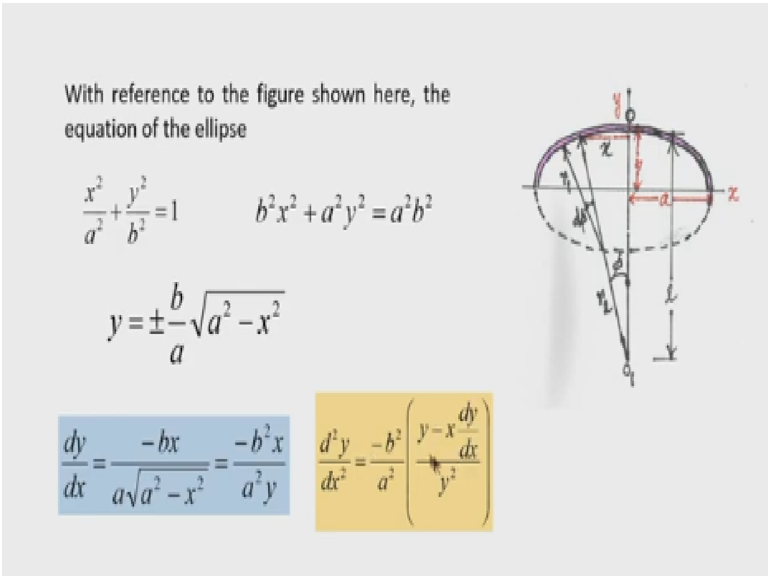

Now, let us go to the mathematics part with reference to the figure shown here the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Now, this equation can be written as $b^2x^2 + a^2y^2 = a^2b^2$, then y is found as $y = \pm\frac{b}{a}\sqrt{a^2 - x^2}$. We want to find the principal radius of curvature of the elliptical curve. So, therefore, we need the derivative first-order derivative and second derivative.

Differentiating it once we get this expression first, we differentiate this so, it will be $-\frac{1}{2\sqrt{a^2-x^2}}$. So, and then differentiating inside this $-2x$ this term will come so 2, 2 will get cancelled so ultimately, we will get $\frac{dy}{dx} = \frac{-bx}{a\sqrt{a^2 - x^2}}$. And you can see here again $y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$.

So, we can write this in terms of y. So, $\frac{-b^2x}{a^2y}$. Now taking second derivative this expression, we can now find that second derivative is $\frac{-b^2}{a^2}$ and this $\frac{x}{y}$ derivative we have to take because both are functions of x. So, we differentiate we take the derivative with the rule that we know the differentiation of the expression say expression which is a ratio of two functions of x.

So, first, we differentiate this numerator and then it is multiplied by the denominator. So, differentiation of numerator is 1 multiplied by denominator is y so, first term is y then minus the numerator into differentiation of the denominator so differentiation of the denominator $\frac{dy}{dx}$ so it is over. So, numerator of the derivative is complete denominator will be this denominator square so, y^2. So, we have got this quantity.

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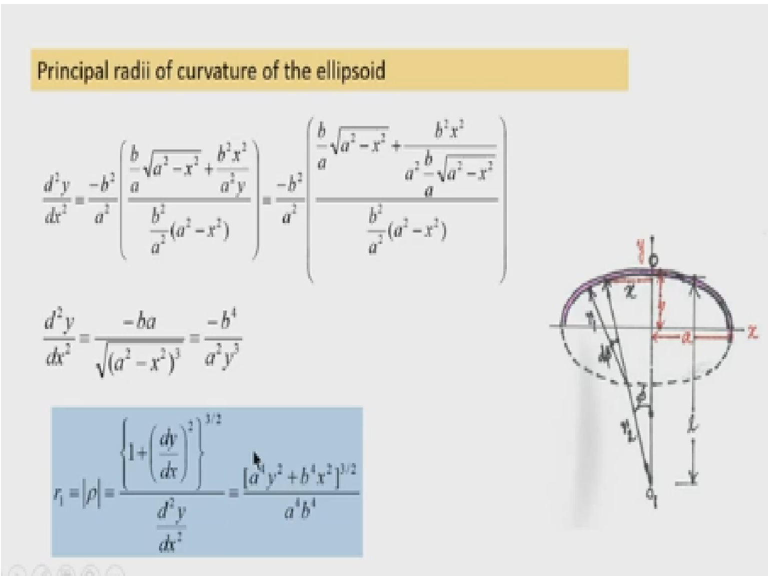

Now, after several simplification that is bringing this term this why that we got it here and $\frac{dy}{dx}$ also we got earlier and y^2 we can now square it. So, all these terms are now substituted in this expression and then after simplification, we will finally get $\frac{d^2y}{dx^2} = \frac{-ba}{\sqrt{(a^2-x^2)^3}}$ that means, we can write also this like that $\frac{-ba}{(a^2-x^2)^{\frac{3}{2}}}$.

So, this is again using the first expression that $y = \pm\frac{b}{a}\sqrt{a^2 - x^2}$ we can write it in a compact form as $\frac{-b^4}{a^2y^3}$. So, this term second derivative and first derivative is needed to compute the radius of curvature principal radius of curvature. So, principal radius of curvature r_1 I am taking the absolute below is given by $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$.

So, after substituting the values that we have obtained here for this second derivative and first derivative in the earlier expression and after simplification with care you have to take care so, that no term is missed and the power is appropriately given. So, we will find $r_1 = \frac{[a^4y^2+b^4x^2]^{3/2}}{a^4b^4}$. So, this is first principle of principle radius of curvature that is the radius of curvature of the ellipse actually you can call at any point x y.

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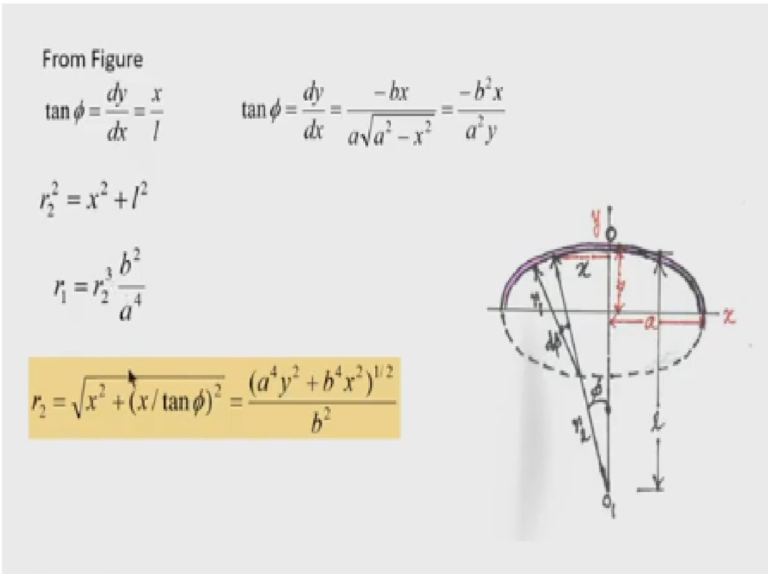

Now from figure here we can see that the $\tan tan\,\phi$ is nothing but $\frac{dy}{dx} = \frac{x}{l}$. So, $\tan tan\,\phi = \frac{dy}{dx}$ and $\frac{dy}{dx}$ we earlier obtain so, we write it $\frac{-bx}{a\sqrt{a^2-x^2}}$ and this is after simplification we can write it in terms of y. Now, the second principal curvature that is r_2 that line you are seeing here from this triangle using the Pythagoras theorem, we can find $r_2^2 = x^2 + l^2$.

So, we can find the $r_2 = \sqrt{x^2 + (x/\tan tan\,\phi\,)^2}$. So, substituting the value of $\tan tan\,\phi$ here and after simplification, we obtain the value of $r_2 = \frac{(a^4y^2+b^4x^2)^{1/2}}{b^2}$. So, we obtained 2 principal radii of curvature. So, r_1 is one principal radius of curvature and r_2 is also one principal radius of curvature. r_1 is expressed in terms of r_2.

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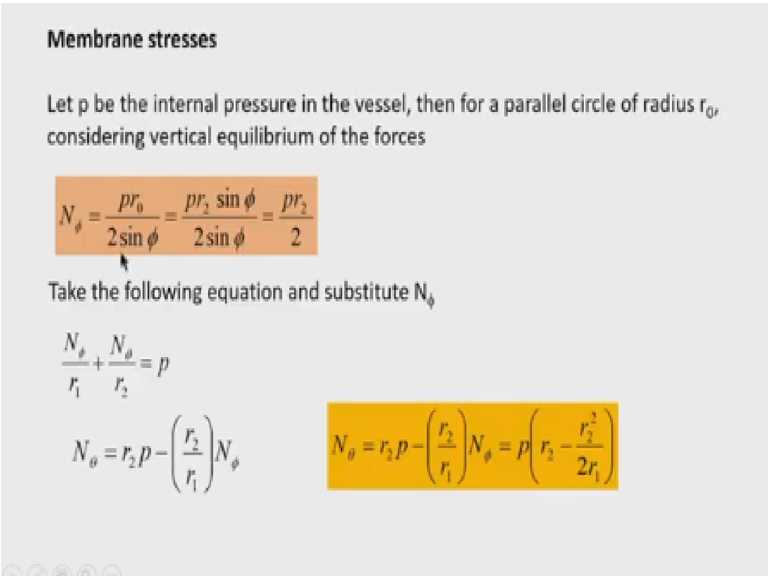
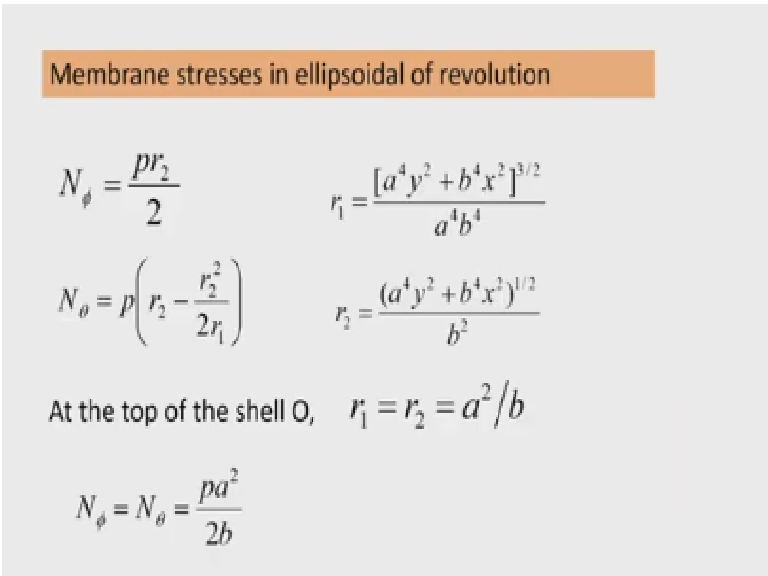


So, let us now see the membrane stresses let p be the internal pressure in the vessel, then for any parallel circular radius or not, say here r not is x here. So, at any parallel circle if this is the direction of N_ϕ then the resolving the forces the N_ϕ along the vertical direction and equating this to the total load to total force produce due to internal pressure above that level, we can find out the value up N_ϕ.

So, that principle used here and for a parallel circle of radius r_0. r_0 here as we can find that $N_\phi \sin sin\,\phi$ that is the vertical component multiplied by say $2\pi r_0$ that is the your $2\pi r_0$ will be your circumference of the parallel circle at that level so $N_\phi \sin sin\,\phi \times 2\pi r_0$ will be the total vertical component of N_{ϕ} and it is equated to the total force produced by the internal pressure at this level. So, it is $p\pi r_0^2$.

So, after simplification we get $N_{\phi} = \frac{pr_0}{2\sin sin\,\phi}$ and substituting $r_0 = r_2 \sin sin\,\phi$ because r_0 you can see in this figure this is r_0, $x = r_0$. So, r_0 and r_2 can be related so r_0 is nothing but $r_2 \sin sin\,\phi$, so substituting this we now get $\frac{pr_2 \sin sin\,\phi}{2 sin\,\phi} = \frac{pr_2}{2}$. In the figure r_0 is written as x but here it is denoted at this level the radius of this circle is r_0 which is equal to x.

Now take the following equation and substitute N_{ϕ} here so after substituting we can get $N_{\theta} = r_2 p - \left(\frac{r_2}{r_1}\right) N_{\phi}$ is now found in the simplified form $p\left(r_2 - \frac{r_2^2}{2r_1}\right)$, so 2 values of membrane stresses are obtained one is N_{ϕ} another is N_{θ} let us say how this forces are varying along the meridional angle ϕ. **(Refer Slide Time: 42:34)**



So, we have got these two expressions and we also know the 2 radii of curvature r 1 and r 2. Now at the top of the shell that is the crown r 1 equal to r 2 and therefore you can see that r 1 and r 2 at the top of the shell that is the x 0 y is equal to b, you can get here r 1 r 2 equal to a square by b. So, N phi equal to n theta at the top of the shell and it is equal to pa square by 2b, top of the shell is the pole of the shell and here the 2 membrane stresses are equal and magnitude is pa square by 2b, where a is the length of this semi major axis.

At the equator you can see here equator is this. So, at this point, your this x is a and y is 0. So, y is 0 here and x coordinate is a.

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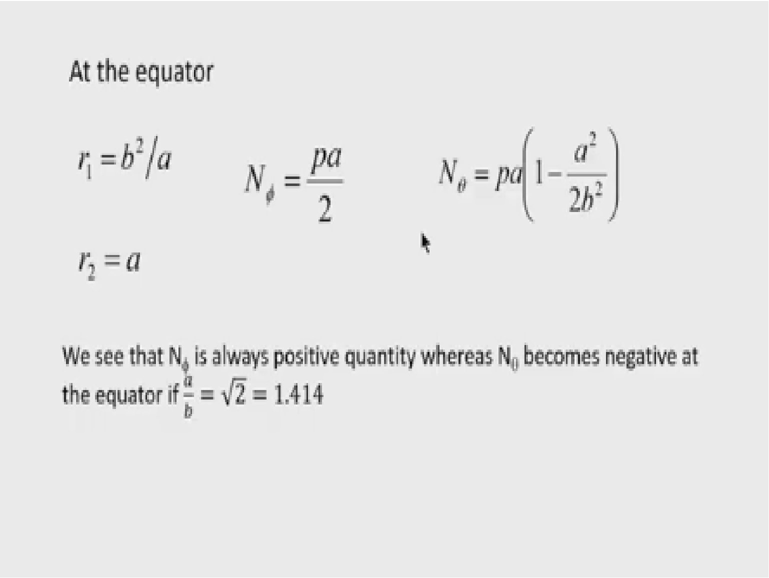

So, substituting this in the expression for r_1 and r_2 we can find the at the equator $r_1 = \frac{b^2}{a}$ and $r_2 = a$. So, therefore $N_\phi = \frac{pa}{2}$ and $N_\theta = pa\left(1 - \frac{a^2}{2b^2}\right)$. So, from this expression, it is clear that N_ϕ is always positive that means N_ϕ is always tensile in nature. However, N_θ can reverse its nature. So, $N_\theta = pa\left(1 - \frac{a^2}{2b^2}\right)$, pa quantity which are positive but this quantity may be positive or may be negative depending on this $\frac{a}{b}$ ratio.

Now, it can be seen that N_θ becomes negative if $\left(1 - \frac{a^2}{2b^2}\right) < 0$. Now, this is possible when $\frac{a}{b} > \sqrt{2}$, that is 1.414. So, it becomes negative at the equator if $\frac{a}{b} > \sqrt{2}$, that is 1.414. So, that means your this N_θ changes sign.

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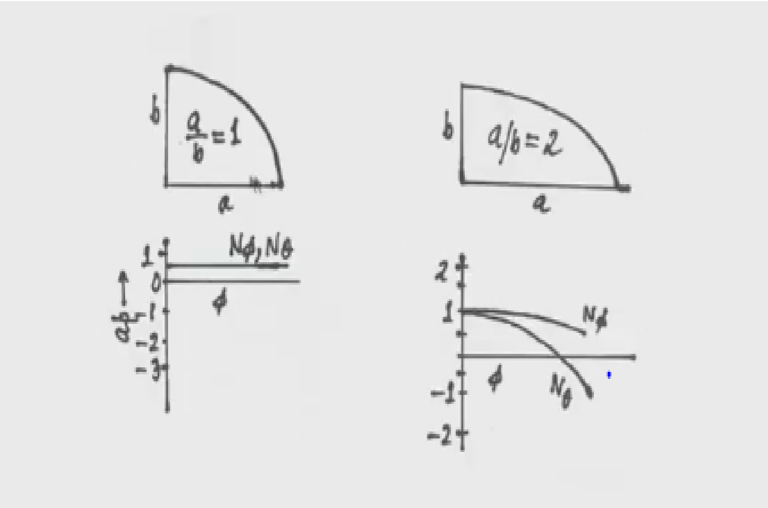

Variation of N_ϕ and N_θ for a/b=1 (sphere) and a/b=2

That is illustrated with an example, say we take the ratio of semi-major axis to semi-minor axis as 1. So, there is the case of a sphere and we know that in case of sphere these stresses are same and it is $\frac{pa}{2}$, if a is the radius of the sphere here it is denoted as semi-major axis. So, the stresses are $\frac{pa}{2}$, that is $0.5pa$. Now here you can see for a/b ratio N_ϕ, N_θ is constant magnitude and it is always tensile. Now when $\frac{a}{b} > 1.414$ that I have told it will change the nature of the forces from tension to compression.

Now here the plot is shown with $\frac{a}{b} = 2$ that is the ratio of semi-major axis to the semi-minor axis is greater than 1.414 that is $\sqrt{2}$ and $N_\phi = \frac{pr_2}{2}$. So r_2 if you put here r_2 is a positive quantity, so if you put here, you will get always the positive forces here, but it will vary according to the level of the point that is at which meridional angle it is located. So, depending on the value of ϕ it will vary but you will see that N_θ changes signs.

So, N_θ for some value it will be some value of N_ϕ it will be positive and then it will cross the axis and it will go to the compression region. So, when compressive forces are developed in this shell especially in the elliptical vessel which is used as a head closer, so in the knuckle region the compressive stresses are harmful actually, compressive stresses in a metallic pressure vessel and in the form of a closure head may cause some buckling.

So, this is given as an evidence from some research paper and which is carried out experimentally to test the behaviour of the closure head under the action of internal pressure and the model test is carried out.

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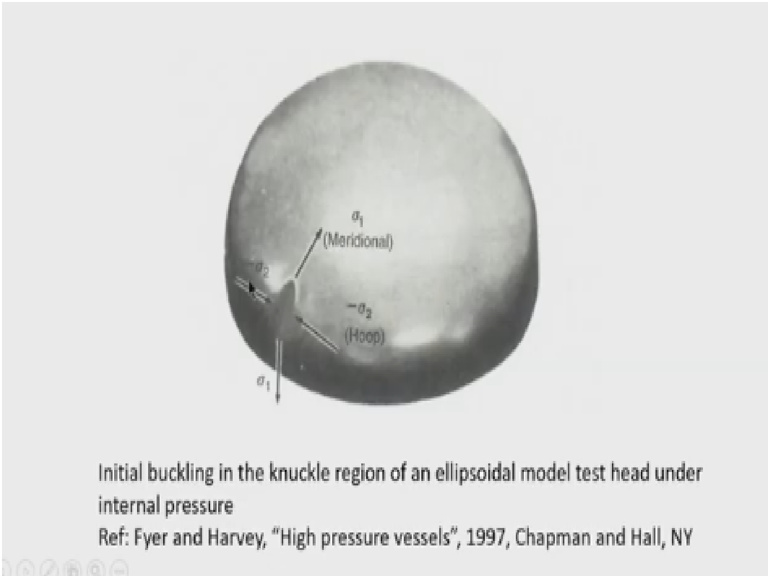

Initial buckling in the knuckle region of an ellipsoidal model test head under internal pressure
Ref: Fyer and Harvey, "High pressure vessels", 1997, Chapman and Hall, NY

I will show a picture of this the knuckle region of the ellipsoidal shell where the buckling is taking place due to compressive stresses developed. Compressive stresses is developed in this circumferential direction when a by b ratio is greater than $\sqrt{2}$. So, this test results or test picture is taken from a book that is written by Fyer and Harvey on high-pressure vessels. So initial buckling is observed here in knuckle region of the head closer.

So today I have discussed the pressure vessels of types that is circular cylindrical type circular conical type and spherical vessels and vessels in the form of ellipsoidal of revolution. So next day I will start this vessels in the form of torus. Thank you very much.