Plates and Shells

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Module No # 09
Lecture No # 27
Some examples of Axi-symmetrical Cases in Surface of Revolution

Hello everybody today I am delivering you the third lecture for module 9 in the module I was discussing about the surface of evaluation and in that respect I have found the differential equation of equilibrium. And then I reduced it to a axi-symmetric problem by with the special condition that $N_{\phi\theta} = 0$ for axi-symmetrical loading on this surface of evaluation. And on the basis of that I have shown you how the analysis of a spherical dome which is very common in application can be done with the help of membrane theory using axi-symmetrical condition.

So today I will give you some further examples of surface of evolution under the action of axi-symmetrical loading. So only I will consider the axi-symmetrical loading case so that you can the most of the cases in practical field are of axi-symmetrical nature except the wind loading in our civil engineering structure that is the dome. That wind loading becomes an asymmetrical case that is it is not symmetrical with the axis of rotation.

So in that case formulation will be of different type but so far we are discussing only the axi-symmetrical loading and in that situation or the formulation that we have derived or demonstrated in case of spherical dome will be applicable.

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AXISYMMETRIC LOADING CONDITION

For axisymmetric condition, $N_{\phi\theta}=N_{\phi\Phi}=0$, and deformation and loading of the shell is independent of θ,. Thus following two equations need to be considered

$$\frac{d}{d\phi}(RN_{\phi})-R_1N_{\theta}\cos\phi+RR_1w_{\phi}=0$$

$$\frac{N_{\phi}}{R_1}+\frac{N_{\theta}}{R_2}=w_R$$

Now we know that under axi-symmetrical condition N phi theta equal to N theta phi is 0 so that means deformation in loading of the shell is independent of theta and in that case the 3 equation of equilibrium that we have obtained 1 along the tangent to the meridian. That is the resolving the forces along the tangent to the meridian and equating them to 0. Again resolving the forces the along the tangent to the parallel circle and taking some of the forces and equating to 0 we get second equation of equilibrium.

And third equation was obtained by considering the some of the component of forces to the 0 along the normal to the tangent plane at the meridian. So this 3 equations we have obtained for in any general kind of loading but when we consider the excess symmetrical condition that $N_{\phi\theta}=0$. Therefore all the stress in deformation external loading are independent of θ.

So therefore equations have been reduced to number 2 and we have seen these are the 2 equation's d $\frac{d}{d\varphi}(RN_{\varphi})-R_1N_{\theta}\cos\varphi+RR_1w_{\varphi}=0$. So it indicates that it is the equation of equilibrium obtained by summing up all the forces along the tangent to the meridian. Because the component of the external load along the tangent to the meridian is w_φ. So that equations you can see it consist of only consist of 2 variables 1 is N_ϕ and another is N_θ.

But the third equation that I have told the equations of equilibrium obtained along the normal direction I mean here that the direction along the normal to the tangent at meridian. So these equations fortunately an algebraic equation known derivative expression of derivative quantities are involved in the third equation that is the equilibrium equation along the radial direction. So therefore it is a simple equation $\frac{N_\varphi}{R_1}+\frac{N_\theta}{R_2}=w_R$.

Now if you recall R_1, R_2 are the principle radii of the curvature of the shell surface whereas R is the radius of the parallel of latitude. So however R is related to R_2 by trigonometrical relation that we have seen earlier also and here also we will use it w_R is the component of the load along the normal direction that we call in the radial direction. Now this equation after integration before integration you want 1 more step that step is that N_θ is now substituted from this equation in terms of N_ϕ.

Because this integer plain equation $\frac{N_\varphi}{R_1}+\frac{N_\theta}{R_2}=w_R$ this is the plain algebraic equation. So therefore N_θ can be expressed in terms of N_ϕ and we can substitute back here in the first equation. So ultimately the first equation can be converted into a equation of single variable unknown variable N_ϕ

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After integrating the first equation, between limits $\phi = \phi_0$ and ϕ = constant

$$N_\phi = \frac{1}{R_2 \sin^2 \phi} \int_{\phi_0}^{\phi=const} R_1 R_2 (w_R \cos\phi - w_\phi \sin\phi) \sin\phi d\phi$$

$$2\pi R_2 \sin^2 \phi N_\phi = \int_{\phi_0}^{\phi=const} 2\pi R_1 R_2 (w_R \cos\phi - w_\phi \sin\phi) \sin\phi d\phi$$

The physical interpretation of this integral is,

"The vertical component of N_ϕ acting around the circle of latitude ϕ is balanced by the total vertical force above that level"

So that is done and after integrating the first equation one can get the equation of

$$N_\varphi = \frac{1}{R_2 \sin^2 \varphi} \int_{\varphi_0}^{\varphi=const} R_1 R_2 (w_R \cos\varphi - w_\varphi \sin\varphi) \sin\varphi d\varphi$$

So when we take the definite integral of this equation then constant terms disappears. So in between these limits phi into phi naught and phi as a other angle we can find out the integral expression of this N. Therefore N_ϕ is completely known given the geometry of the surface and loading on the surface we can obtain all the parameters. And ultimately after integrations we can get N_ϕ now this integration as some physical meaning. We have seen earlier also now just to recapitulate this same this thing I am taking it again.

If I multiply this left hand slide and right hand side by $2\pi R_2$ then will get here

$$2\pi R_2 \sin^2 \varphi N_\varphi = \int_{\varphi_0}^{\varphi=const} 2\pi R_1 R_2 (w_R \cos\varphi - w_\varphi \sin\varphi) \sin\varphi d\varphi$$

So now what happens that this term $2\pi R_2 \sin^2 \varphi N_\varphi$ can be interpreted.

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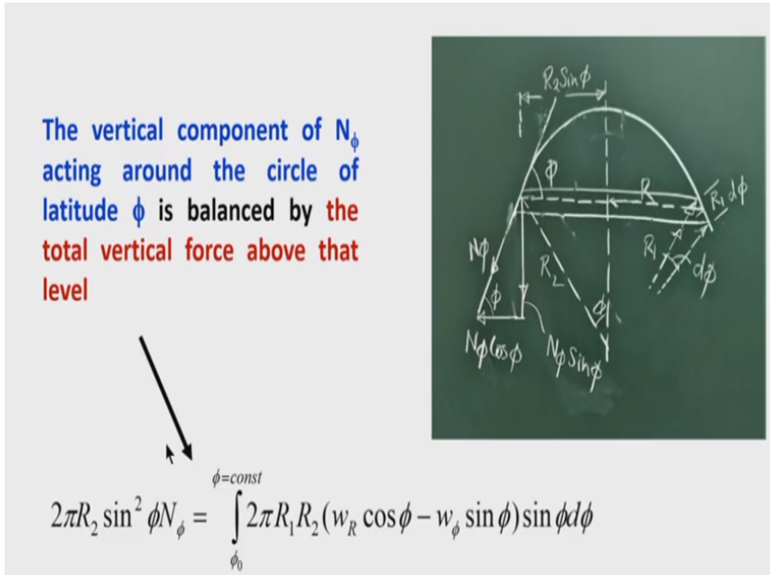

Because when we see the shell surface say under the action of load and this meridional stress that is N_ϕ and N_θ that is developed in the meridional and circumferential direction we can see that vertical component of N_ϕ is can be obtained as $N_\phi \sin\phi$ And total vertical component that is this N_ϕ is acting per unit length. And if I see this circumference of this parallel circle then it will be $2\pi R$ that is the circumference of parallel circle where R is the radius of the parallel of latitude or a circle.

So in that case this $2\pi R N_\phi \sin_\phi$ will be the total vertical component N_ϕ acting at this part. However R can be expressed in terms of R_2 that is one of the principle radii that R can be written as ϕ_0. So $R = R_2 \sin\phi$ so therefore this expression is developed putting this value of $R = R_2 \sin\phi$. So therefore we get a complete expression to interpret this integral expression physically what is this interpretation?

If I see this left hand side it indicates that total vertical component of N phi and if I look at the right hand side quantity then it represents this you can see $w_R \cos\varphi - w_\varphi \sin\varphi$ is the net vertical load on the shell surface. And when it is multiplied by this surface area of the small strip you can see here $R_1 d\phi$ is the width. And if you see circumference then $2\pi R$ and R can be expressed in terms of $R_2 \sin\phi$

So these are involved in the inside the integral expression so that means here we are getting that total load acting on the shell surface at this level in the right hand side. In the left hand side we are getting the total vertical component of the meridional force N_ϕ. So this means that the total vertical component of N_ϕ acting at a level should be balanced by the total vertical load actin above that level. So based on that we can obtained several important results and we can solve very too many practical problem with this.

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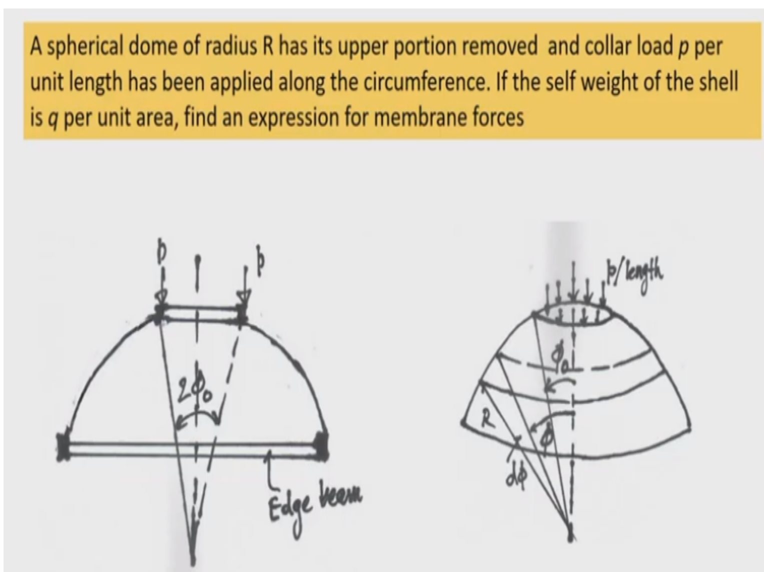

So let us consider one of the, such problem that sometimes a shell having discontinuity means a shell is having an opening at the top. That is sometimes used or a adopted for entry of light and a structure for example a light structure is constructed over this and the load distance is by a edge beam. So this is the edge beam on the shell and this load you can see here I have drawn in a 3D view that load p that is the line load acting along the circumference.

So the extra load that is put here at the edge of the opening is p multiplied by the length of this circumference. Now this is the extra load pool but this has some discontinuity here and therefore the integration expression we want to avoid. So just want to use this physical interpretation of expression to solve this problem. So our intention is to find the membrane forces in this shell one is the membrane forces are found then we can easily obtain the membrane stress.

Because the forces are expressed as a quantity per unit length so if I divide the membrane forces say N_ϕ by thickness of the shell we will get the membrane stress. Similarly, if I divide this N_θ by thickness we will get the other membrane stress that is the circumferential stress acting on the shell. So if I consider a strip here for example I consider a strip here of width say this width of the strip will be say $Rd\phi$. R is the radius of the sphere. Since it is a sphere at spherical dome part of sphere so therefore this $R_1 = R_2 =$ radius of this sphere.

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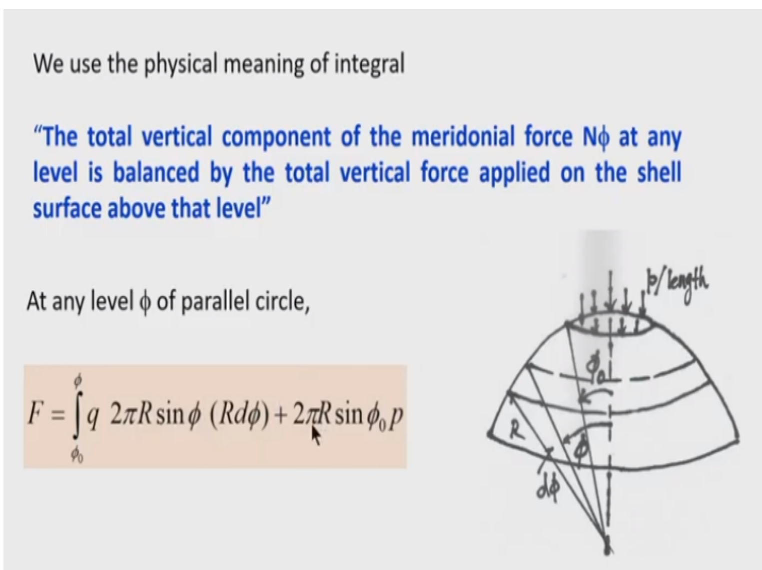

So we use the physical meaning of the intregal says that the total vertical component of the meridional force N_ϕ at any level is balanced by the total vertical force applied on the shell surface above that level. So if I consider now a level here so this level is defined from this is the edge of the opening is at a angle ϕ_0. So ϕ_0 to ϕ so this is at any level say at level ϕ we are interested to calculate this membrane forces.

So at this level let us see what will be the total force? One of the component of the total force is the line load that is I call it a collar load on this shell at the opening a collar load is applied around the periphery. So due to this collar load, you can see the radius of the sphere is R so the radius of this small opening will be $R\sin\phi_0$. So $2\pi R\sin\phi_0$ is the circumference multiplied by P, this is the total collar load this strip loading acting on the shell above the level that we have considering.

Now let us calculate the surface load that is due to self-weight or gravity load so if q is the gravity load including self-weight and light load. Then at this level the radius of the parallel of the latitude or parallel of circle is $R\sin\phi$. So $2\pi R\sin\phi$ is the circumference of the parallel circle at this level ϕ and $Rd\phi$ is the width of the strip. So $2\pi R\sin\phi \times Rd\phi$ is the area of the strip and these areas multiplied by q will represent the vertical load acting on the strip.

Now if I integrate it between the limit ϕ_0 to ϕ. I will get we put a load acting on the shell surface from ϕ_0 to ϕ.

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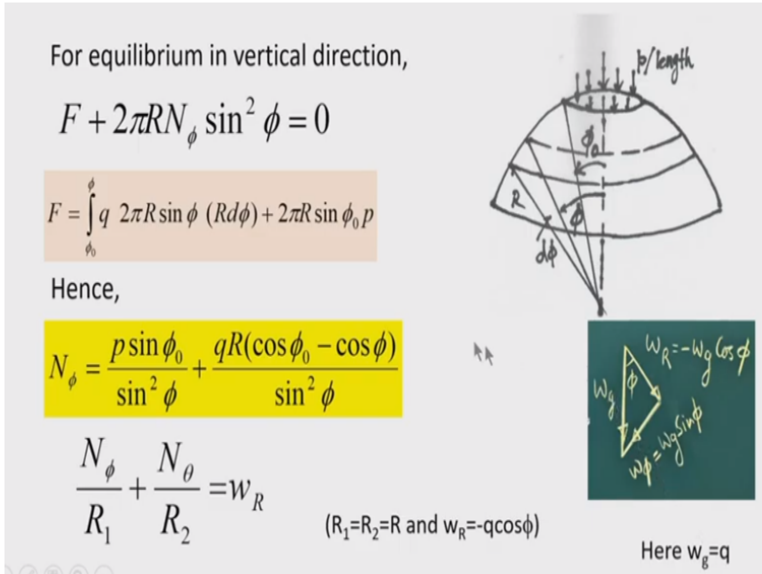

So F is found in this way so after evaluating this integral we can find F and then the F has to be balanced by the total vertical component of the meridional force at this level that I have shown in the first picture. That is total vertical component of the N_ϕ at any level ϕ will be $N_\phi \sin\phi$ multiplied by circumference of the parallel circle at this level. So based on that I calculate that $N_\phi \sin\phi$ will be the total vertical component of the meridional force per unit length and when we multiply it by circumference that is $2\pi R\sin\phi$

So it represents the total vertical component of N_ϕ at the level ϕ so this is to be balanced by F. So I write this equilibrium equation in vertical direction. So F we have obtained earlier in this integral form. Now the task remain is to complete this integral so this integral is completed and it is added with this and then N_ϕ can be found out this integral can be completed and it is substituted in this equation and then N_ϕ can be calculated.

$$N_\varphi = \frac{p\sin\varphi_0}{\sin^2\varphi} + \frac{qR(\cos\varphi_0 - \cos\varphi)}{\sin^2\varphi}$$

So N_ϕ is found as an expression in moving this p that is the collar load then ϕ_0 that is the angle at the edge of the opening and qR and q is the vertical load acting on the surface R is the radius of this sphere. So and ϕ is the level at which we are interested to find N_ϕ so we get this expression of meridional stress due to opening. By simple consideration of your, this physical meaning of the integral without integration of this expression to find out N_ϕ .

So now use the third equation of equilibrium that is the equilibrium condition along the normal condition. So that is given by $\frac{N_\varphi}{R_1}+\frac{N_\theta}{R_2}=w_R$. w_R is the component of the load in the radial direction or direction along the normal to the tangent at the meridian. So if I see the triangle the width vertical side is represented by load W_g so therefore radial component will be $w_R=-w_g\cos\phi$ minus sign is taken because the positive direction of $\cos\phi$ it is in the negative direction of the ϕ .

So therefore this radial load is taken with a negative sign. $w_\phi=w_g\sin\phi$ this component so now we substitute this w_R into this expression we substitute this w_R . And therefore we get a relation $N_\phi+N_\theta=w_R R$ because $R_1=R_2=R$. So if I multiply both sides by R then the equation becomes simply $N_\phi+N_\theta=w_R R$. Now substitute $w_R=-w_g\cos\phi$. Here $-W_g$ is q we have taken symbol q to represent the vertical load in the shell.

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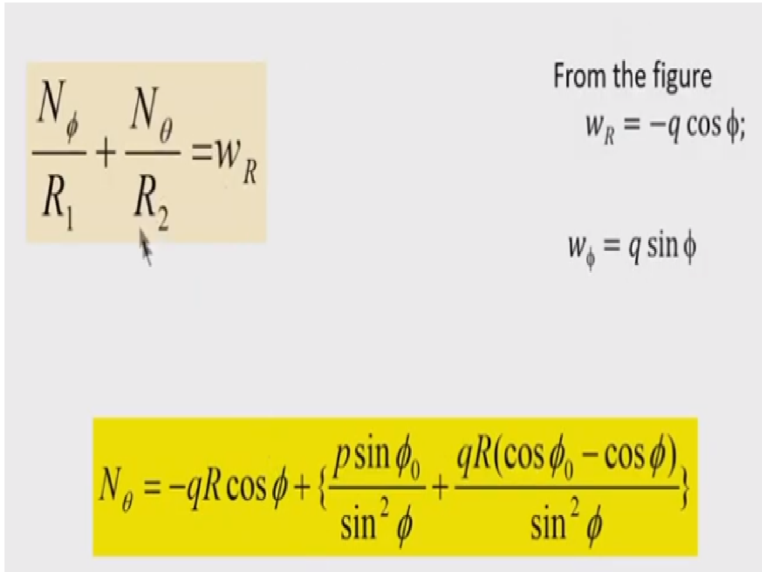

So after substituting this value w R as q Cos phi and we ultimately get

$$N_\theta = -qR\cos\varphi + \{\frac{p\sin\varphi_0}{\sin^2\varphi} + \frac{qR(\cos\varphi_0 - \cos\varphi)}{\sin^2\varphi}\}$$

So when the opening is not there then phi naught is 0 then we get this expression for the shell which is closed shell. So there was no opening in the shell.

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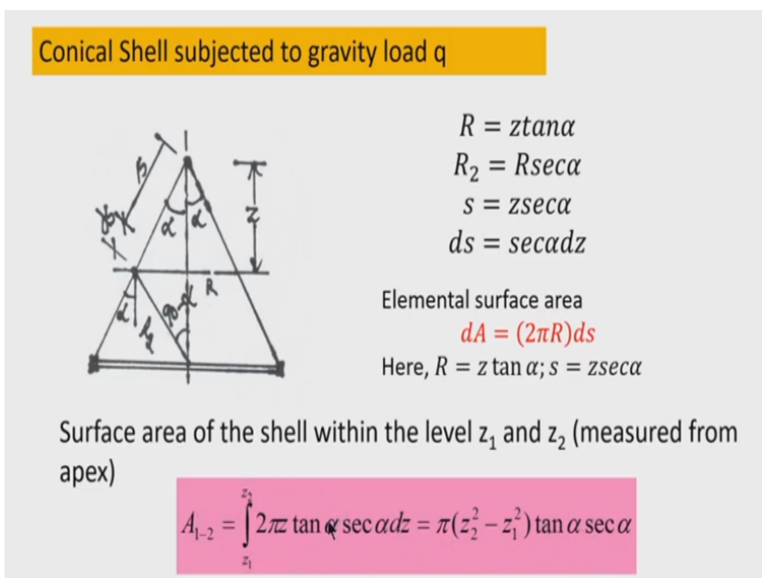

Second problem let us consider a conical shell and we consider there the gravity load that is the vertical load acting on the surface of the shell. Conical shell is also common because this conical type of roof that is umbrella type roof is also common for small shed and other applications also. And conical vessel is also there for storage of liquid or other gaseous substance. So therefore we need to know the analysis of conical shell.

Now one important characteristics of conical shell is that it is a surface of evolution having 0, Gaussian curvature. So that statement I have told the 0 Gaussian curvature how can I say that it is 0 Gaussian curvature. We know that Gaussian curvature is nothing that the product of 2 principle curvatures now here you can see how this shell surface of the evolution is found shell of surface of evolution is formed by rotating a plane curve around a axis of rotation.

Now here the curve is a straight that is meridian to a straight line so it is rotated around the integer straight line inclined and it is rotated around the axis of rotation to form the conical shell. Now because of straight line meridian it is principle radius of curvature that is one principle radius of curvature R_1 that is the major principle radius of curvature is principle radius of curvature because it is straight line.

So principle radius of curvature is infinity so therefore we have very special condition here that is R_1 tends to infinity whereas R_2 is finite R_2 is this radius one principle radius R_2. So we now find the expression for R_2 and other geometrical parameters. Now here from the geometry if you see the 2α is the apex angle you can tell and is the semi apex angle. Now z is the level which is defined from the apex so x is the distance measured from, the apex, from any level say this level I am showing here.

Now here the parallel of latitude or parallel of circle is having radius R now you can see that R can be related with z by simple trigonometrical direction $R = Z \tan\alpha$ that means $2\pi r$. So $R = Z \tan\alpha$ now R_2 is one of the principle radius of the curvatures so R_2 now can be related to R now we can see that R this R divided by R_2 is nothing but $\sin 90-\alpha$. So $\sin 90-\alpha$ is $cos\alpha$ so therefore R2 becomes $\frac{R}{\cos\alpha}$ alpha that is R $\sec\alpha$.

Now let us consider this slant side or slanting side because the generator or direct is meridian in case of conical shell is a straight line. So it is a sloping line that we call slanted line so we now measure the distance S and find the distance as S in terms of other geometrical parameters because S is now can be related to α and Z by a simple relation $s = z\sec\alpha$. So ds if I take an elementary surface of the shell here of width ds.

So ds can be found out at $ds = \sec\alpha dz$ because α is a constant quantity so elemental surface area da if I want to calculate da will be $2\pi r$ that is the circumference of the parallel circle $2\pi r \times ds$. We can substitute ds here so $2\pi r\sec\alpha dz$ so this is the elementary area now total surface area of the cone now can be calculated from 0 to this level or from any level from apex to any other level.

So here say I defined a level say by distance Z_1 at one plane another plane it is z_2 so therefore I take an integral with a limit Z_1 to Z_2 to find out the area of the strip between any 2 level. If one level is say the reference point that is Z_1 is 0 then we get the surface area measured from the fx. Now here you can see that I have substituted d A and then I integrated. So, dA is $2\pi z\tan\alpha\sec\alpha dz$ because R is substituted as $z\tan\alpha$ and ds is substituted as $ds = \sec\alpha dz$.

So one side we get this expression then after integration we find this area of the surface of the conical shell between the level Z_1 and Z_2 equal to $\pi(z_2^2 - z_1^2)\tan\alpha\sec\alpha$. so this is the area of the shell between the 2 level Z_1 and Z_2.

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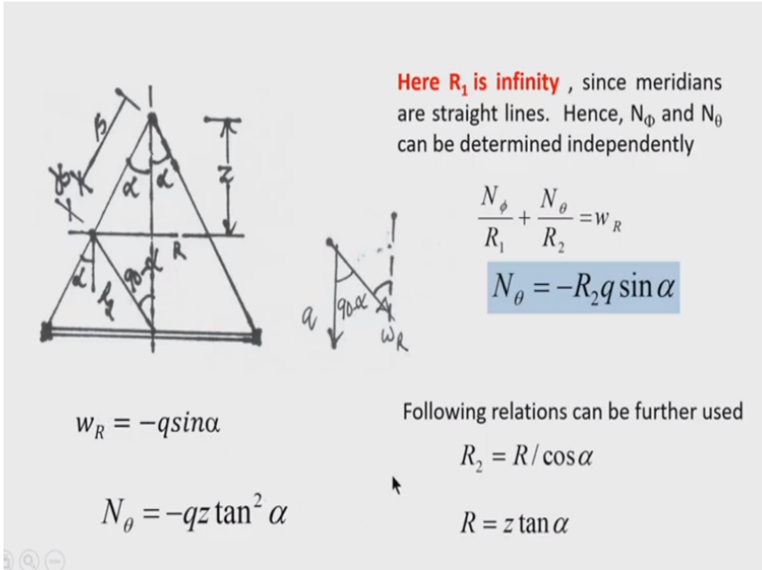

Now because of infinite radius of curvature R_1 we now have a very simple equation $\frac{N_\varphi}{R_1}$ will be 0. Because if I put R_1 as the infinity here in the denominator this term become 0 so therefore we get only $N_\theta=-R_2 q\sin\alpha$. So w_R is this component of the force and q is the vertical load so after dissolving this q along the radial direction we get $-q\sin\alpha$. So N_θ is found $-R_2 q\sin\alpha$ and you can substitute $R_2=R/\cos\alpha$ and $R=z\tan\alpha$

So ultimately this N_θ is expressed in terms of vertical load and the distance from the apex the level where we want to calculate N_ϕ we can put the level as z or distance as z and the $\tan^2\alpha$ is the semi apex angle.

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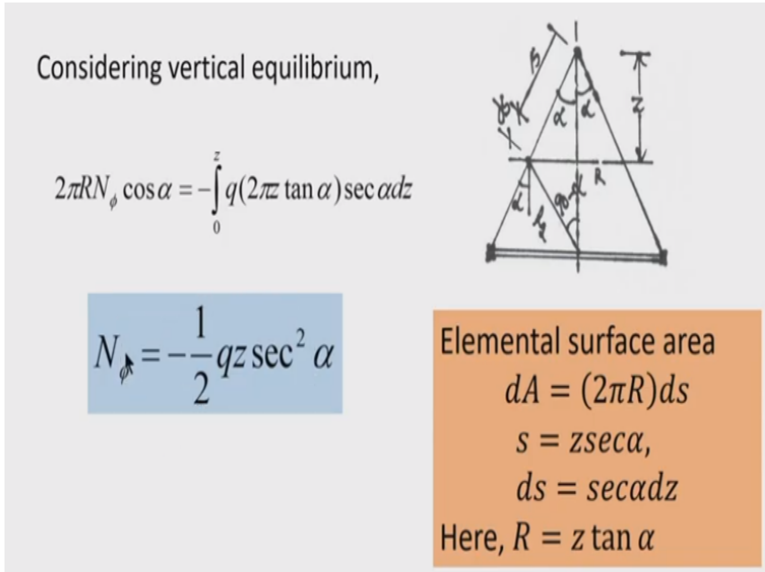

So one the membrane stress is known second membrane stress is N_ϕ as to be now calculated and use the physical meaning of the expression. Now at this level if I consider the vertical equilibrium we can see that here the N_ϕ will be along this slanting side. So therefore component of N_ϕ along the vertical direction will be N phi Cos alpha. So $N_\varphi \cos\alpha$ is the component; of this meridional force of the vertical direction and it is multiplied by this circumference of the parallel circle.

Circumference of the parallel circle is $2\pi R$ so therefore we get the total vertical component of N_ϕ. Then it is equated to the vertical force above that level so vertical force about that level q multiplied by surface area. So surface area calculated from 0 to Z earlier we have calculated from the Z_1 to Z_2 any other level the upper limit lower limit is taken as 0 and upper limit is Z. So after integrating this expression and simplifying it becomes that $N_\varphi = -\frac{1}{2} qz \sec^2\alpha$.

So this is the expression for N_ϕ negative sign indicates compression under the action of vertical load and these are quantities that we have used in finding the relation between these N_ϕ and other shell parameters in using the load. So, 2 expressions of the meridional force N_ϕ and N_θ are calculated.

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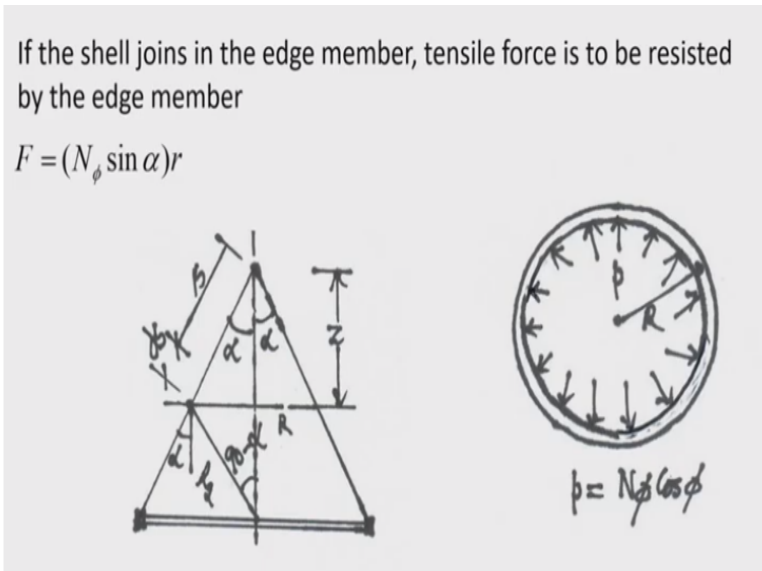

Next let us consider the support of the conical shell because of the shell is supported at the edges by a ring beam. This is a ring beam or peripheral beam so how the forces will be transferred at the ring because if I see the component of N_ϕ is along the slanting side so it has got 2 components 1 along the vertical direction one along the vertical direction. So vertical component is N phi sin alpha and vertical component will be $N_\phi \cos\alpha$ horizontal component will be $N_\phi \sin\alpha$.

So these horizontal components will now act here and will give a pressure on this ring beam. So this ring beam now will be subjected to this tension due to this pressure. So therefore if I know the pressure on the ring beam I can find out this tension in the ring beam and accordingly I can design the beam. So F here $F=(N_{\varphi}\sin\alpha)r$. Here $\phi=90-\alpha$ so therefore $N_{\phi}\sin\alpha\times R$.

Now in the case of sphere we use this ϕ and θ. And ϕ is measured from the vertical axis of rotation. Now here mainly in the case of conical shell the cone angle will be specified. So therefore the relationship is expressed in terms of α instead of ϕ. However α and ϕ can be related very easily with the help of this triangle. So therefore all trigonometrical relation that was expressed in terms of alpha can also be converted in case of phi if want to specify the angle with respect to vertical axis of rotation.

Instead of this angle and this angle is measured from the vertical axis of rotation towards the shell surface. Now here also here this slanting side is originated from apex. So therefore the measurement is different from the 2 respective reference points hence we generally take the apex angle of α semi apex angle α in case of shell. Whereas in; case of this sphere spherical dome we generally take the ϕ instead of α.

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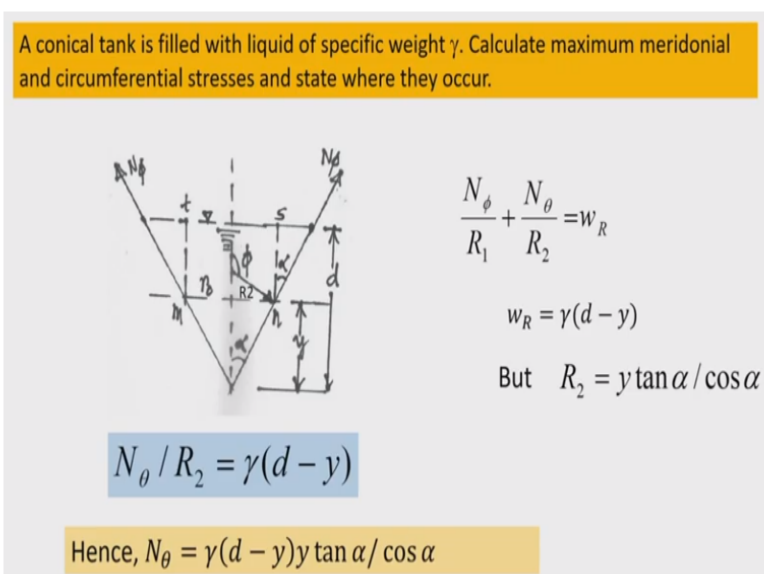

Now let us consider the third problem a conical tank is filled with liquid of specific weight γ . Calculate the maximum meridional and maximum stresses and state where they occur. A conical stress is common in many cases for example we have a water tank which is combination of cylinder sphere and cone. That means this type of tank is generally known as Intex type of tank. Here Intex (34:50) type of water tank is the elevated water tank you will find the roof cover is the spherical dome of small rise.

Then the tank wall will be a cylindrical shell then it will have a slanting wall that is like a conical shell like a first term of a cone. And then it has a spherical bottom instead of flat bottom it will have a spherical bottom. So all types of surface of revolution involved in case of this type of elevated water tank generally known as Intex type of water tank. So, conical shell analysis is also necessary when we apply it to this liquid detaining structure.

So here an example of conical tank filled with liquid the density of liquid is γ and water is filled with up to depth d. This semi cone angle is α and total cone α is 2α now here we first use this expression $\frac{N_\varphi}{R_1}+\frac{N_\theta}{R_2}=w_R$. w_R is the load along the radial direction now you can see the water pressure here is the w_R and water pressure acts equally in all direction according to Pascal load.

So therefore w_R equal to $w_R = \gamma(d - y)$. so if y is the distance measured from the apex in this level m n and d is the total depth of the water. So hydrostatic water pressure at this level is $\gamma(d-y)$ and this is acting outwards towards the surface. So therefore it is taken positive as per sin convention. So w_R is this so immediately we can get N_θ as $w_R \times R_2$ but what is R_2?

R_2 is this distance this radius of curvature now R_2 can be expressed as because if I see this triangle small triangle these distance is $y\tan\alpha$. And this angle is again can be converted in terms of α. So $\frac{y \tan \alpha}{\cos \alpha}$ will be your R_2 so second principle curvature is obtained like this. So subsisting this R_2 as $\frac{y \tan \alpha}{\cos \alpha}$ we now get

$$N_\theta = \gamma(d - y) y \tan tan\, \alpha / \cos cos\, \alpha$$

This is the stress in the circumferential direction we generally call it as a hoop force or hoop tension. And we want to calculate where the maximum value occurs.

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$$N_\theta = \gamma(d - y) y \tan \alpha / \cos \alpha$$

For a given cone angle, the maximum value of N_θ will occur when

$$\frac{dN_\theta}{dy} = 0$$

It gives $y = \frac{d}{2}$, Therefore, substituting y=d/2 in the expression of Nθ, we get

$$N_{\theta,max} = \gamma \frac{d^2 \tan \alpha}{4 \cos \alpha}$$

For calculating the maximum value for a given cone angle we differentiate this quantity N_θ with respect to d y. So will get the depth where the maximum this circumferential force is circumference stress will occur. So differentiating $\frac{dN_\theta}{dy} = 0$, you can get here say it will $dy - y^2$ and these are other quantities are constant. α is a constant so $\tan \alpha$ is a constant so $\cos \alpha$ is a constant γ is a constant.

So if I differentiate this $dy - y^2$ with respect to y and equate to 0 then we get y = d /2. So therefore d/2 is the position or a plane that is half the depth of the cone where the maximum circumferential stress will occur. Let us calculate the maximum value of this circumferential stress substituting y = d/2 here in this expression and after simplification you will get $N_{\theta, max} = \gamma \frac{d^2 tantan\, \alpha}{4 coscos\, \alpha}$

So one component of this membrane stress is obtained that is N_θ and its maximum value can also obtained now let us find the other component.

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Equating P and F

$$\frac{1}{3}\pi\gamma y^2 \tan^2\alpha\{3d - 2y\} = N_\phi \cos\alpha(2\pi y \tan\alpha)$$

Hence, we get N_ϕ as

$$N_\phi = \frac{\gamma y \tan\alpha(d - \frac{2}{3}y)}{2\cos\alpha}$$

For maximum value, $\frac{dN_\phi}{dy} = 0$ which gives $y = 3d/4$, hence substituting y in the expression of N_ϕ we get

$$(N_\phi)_{max} = \frac{3\gamma d^2 \tan\alpha}{16\cos\alpha}$$

Other component is N_ϕ which will now be obtained using this physical meaning of the integral expression. Now at this level the level is defined with a distance y from the apex. So total vertical force F say total vertical force F at this level M N what are the total vertical force acting on this level is the load of the water in the cone and load of the water on the cylindrical part here. So that means weight the width in the conical part M N O plus the weight of the fluid is cylindrical part that is M N S T.

So now and γ is the density of the fluid so let us calculate this component of this weight and then find the total vertical force. So F is the volume of the cone is the one third πR^2 × height. So $\frac{1}{3}\pi\gamma$ (γ is the density of the fluied) $y\tan\alpha$ is the radius at this level. So this square of $y\tan\alpha$ × y this is the volume of the cone and multiplied by γ we get the weight of the liquid here.

$$F = \frac{1}{3}\pi\gamma(y\tan\alpha)^2 y + \pi\gamma(y\tan\alpha)^2(d-y)$$

Then adding the weight of the cylindrical portion of the water that is the radius of this circle is $y\tan\alpha$. So $\pi R^2 \times \gamma \times (d-y)$. So after simplification we get the total weight as

$$F = \frac{1}{3}\pi\gamma(y\tan\alpha)^2 y + \pi\gamma(y\tan\alpha)^2(d-y) = \frac{1}{3}\pi\gamma y^2 \tan^2\alpha\{3d-2y\}$$

Now this weight at this level should be balanced by the total vertical component of N_ϕ . So, total vertical component of N_ϕ if is see the vertical component of N_ϕ is $N_\phi \cos\alpha$ and radius of this circle here is $2\pi y\tan\alpha$.

$$P = N_\varphi \cos\alpha(2\pi y\tan\alpha)$$

So we get this total vertical component to N_ϕ so equating this P with F we get the value of N_ϕ . So equating P and F we now write an equation

$$\frac{1}{3}\pi\gamma y^2 \tan^2\alpha\{3d-2y\} = N_\varphi \cos\alpha(2\pi y\tan\alpha)$$

From where we can extract N_ϕ so N phi is here

$$N_{\varphi} = \frac{\gamma y \tan\alpha (d - \frac{2}{3} y)}{2\cos\alpha}$$

For maximum value again we differentiate N_ϕ so N_ϕ is differentiated with respect to y. Because you know where the maximum value will occur and accordingly want to calculate the maximum value. So after differentiating and equating to 0 you will get at y = 3d/4 we get the maximum value of N_ϕ. So substituting y as 3d/4 this expression but we now get the maximum value of N_ϕ.

So, $$(N_{\varphi})_{\max} = \frac{3\gamma d^2 \tan\alpha}{16\cos\alpha}$$

So you have understood how the different problem of surface of revolution under axi-symmetrical loading can be handle by using the physical meaning of the integral or directly using the integral also. Because in this case of the axi-symmetrical loading your 2 equations of equilibrium are only needed and the second equation of the equilibrium will your, this algebraic equation.

So N_θ can be expressed in terms of N_ϕ so first equation can be written in terms of only 1 unknown quantity and after integration we get the desired result.

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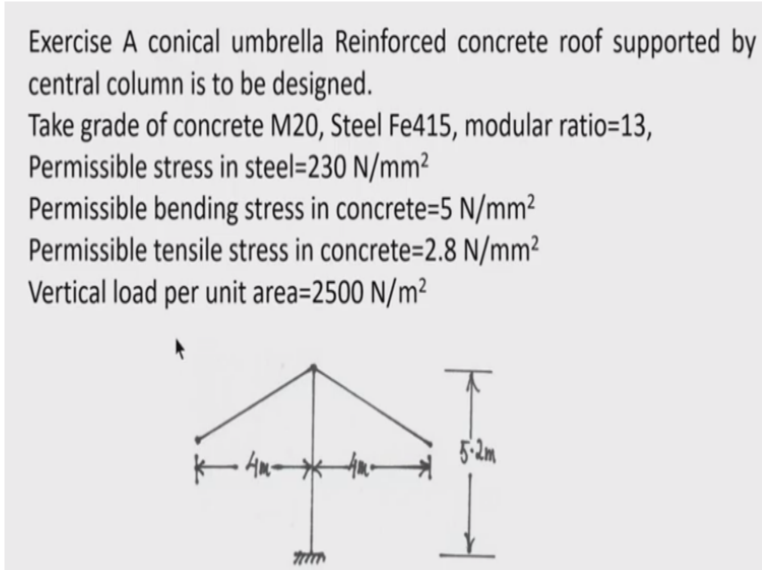

So let us see an exercise for example a conical umbrella roof type roof is to be designed it is made up reinforce concrete and it is supported at the central column. So now let us do this problem take grade of concrete as M20 steel Fe415 and other data for concrete modular ratio is 13 permissible stress in steel is 230 N/mm^2. Permissible bending square in concrete is 5 N/mm^2 permissible tensile stress in concrete has 2.8 N/mm^2.

And vertical load per unit area is 2500 N/m^2 so we can take the vertical load as 2.5 kN/m^2. So problem I want to solve it so we have seen that a conical umbrella roof has to be designed.

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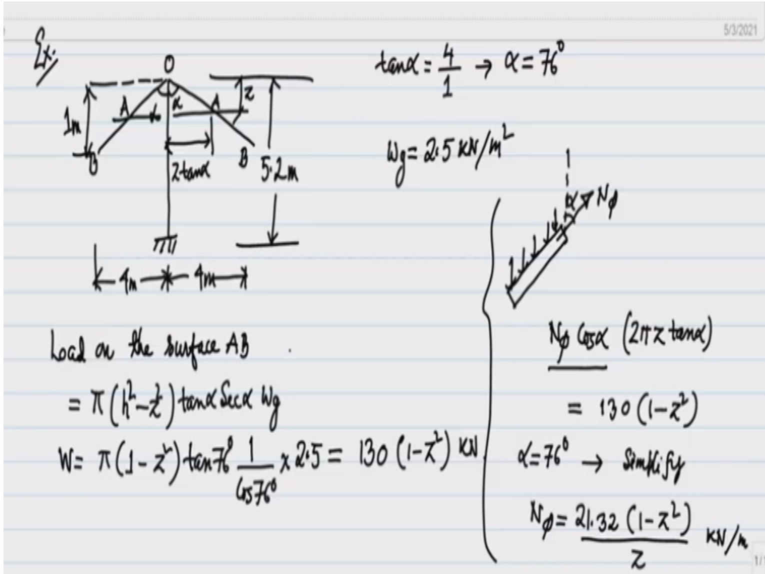

And it is supported on the central column the height of the column is given in the problem as 5.2 meters the base of the cone is 8 meter. The distance from the apex up to free edge of the cone is 1 meter. So the semi cone angle α is here and α we can calculate now so given this data we can calculate $\tan\alpha = \frac{4}{1}$. So from where α is equal to 76^0 now we take a level here and suppose the apex is denoted by the symbol O and here the plane is A at the free edge it is BB.

We have to calculate load on the surface and then we use the physical relation of this integral expression to get the expression for N_ϕ. Of course the one radius of one principle radius is infinity so $\pi(h^2 - Z^2)\tan\alpha\sec\alpha w_g$ can be directly found out. Now load on the surface is AB we have seen that in m earlier slides $\pi(Z_2^2 - Z_1^2)$, 2 levels are there. Now here 1 level is say h, h is the total height of the cone minus at any level say that level is defined as Z^2 now here Z_2 is h and Z_1 is z multiplied by $\tan\alpha\sec\alpha$ multiplied by component of the load in the vertical direction.

Therefore Load on the surface AB is $\pi(h^2 - Z^2)\tan\alpha\sec\alpha w_g$

Now only we have the vertical load is applied so vertical load w g is 2.5 $\frac{kN}{m^2}$. So after substituting h as 1 we get

Total $W = \pi(1-Z^2)\tan 76^0 \frac{1}{\cos 76^0} \times 2.5 = 130(1-Z^2)kN$

Now this is now used we consider a free body diagram of this portion A B. If I consider the free body diagram of portion A B the load acting here say A whatever load is acting is now calculated. And this is the direction of your N_ϕ and the free edge this edge is free so therefore no stress is there. And you can see that total vertical component of the load N_ϕ as to be balanced by the vertical load acting on the portion. Now using this $N_\phi \cos\alpha$ is the total vertical component of N_ϕ multiplied by this circumference of the circle at this level.

So circumference of the circle at this level is 2π and you can see this distance is $Z\tan\alpha$ this distance is $Z\tan\alpha$. So this is equal to $Z\tan\alpha$ and it has to be equated to the total vertical load that we have obtained. So total vertical load we have obtain is $130(1-Z^2)$. So from that quantity we can now after substituting the value of α is 76^0. And simplify we get $N_\phi = 21.31\frac{(1-Z^2)}{Z}kN/m$

So N_ϕ we have obtained now we go for obtaining N_θ.

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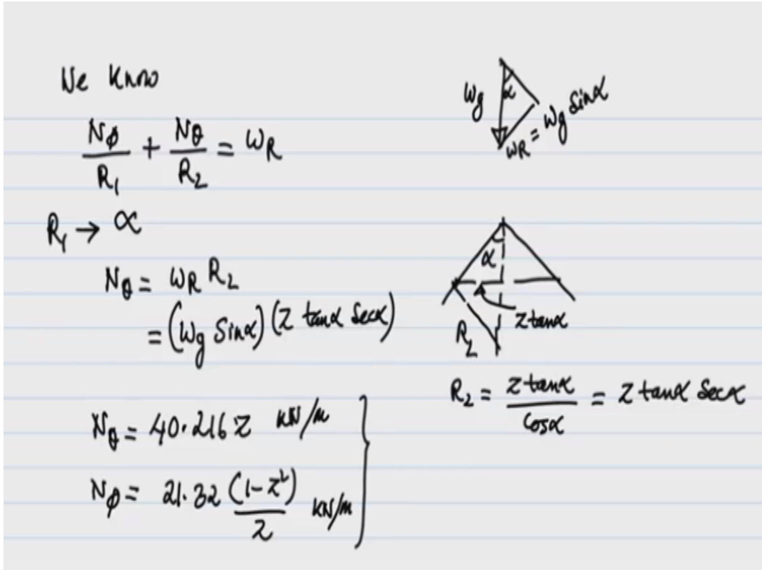

We know that $\frac{N_\varphi}{R_1} + \frac{N_\theta}{R_2} = w_R$

Now R_1 is infinity because the straight is revolved. So the straight line having the radius of curvature has infinity. If I see the component of the load this is a total vertical load say w_g. And this is your angle α so naturally this w_R that we calculate is $w_g \sin\alpha$. So we now get his component of the load as w_R is this so $N_\theta = w_R R_2$, Now w_R is $w_g \sin\alpha$ and R_2 as we have seen earlier R_2 is one of the other principle radius of curvature.

Let us express this in terms of α and other dimensions if this angle is α at this level this is R_2 and this distance is your $Z \tan\alpha$. So naturally $R_2 = \frac{Z \tan\alpha}{\cos\alpha}$. so we can write R_2 as $R_2 = Z \tan\alpha \sec\alpha$. So $N_\theta = w_R R_2 = w_g \sin\alpha (Z \tan\alpha \sec\alpha)$. After putting the value of w_g, as 2.5 $\frac{kN}{m^2}$ and the value of α as 76^0 we now get $N_\theta = 40.216 Z \frac{kN}{m}$. So this is the value of N_θ and it is $\frac{kN}{m}$ run and it is compressive.

So 2 expressions we have obtained 1 expression for N phi that too as given as $N_\phi = 21.31\frac{(1-Z^2)}{Z} kN/m$ it was tensile. So got; these 2 expressions so now we can go for designing this shell. Now one thing you can note at z = 0 the N_ϕ is unbounded that is at the apex where this shell is defined with the central column. Then there will be generation of bending moment so generally the shell is monolithically built with the column structure.

So bending moment will be produced and therefore remain theory will not be adequate at the junction of the shell with the column. So therefore away from this junction of this shell with the column we have to calculate the forces.

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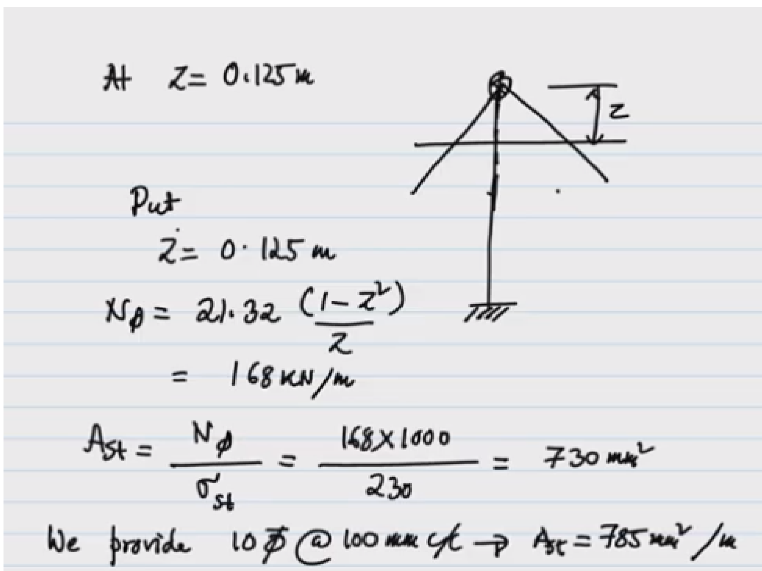

So let us say at say z = 0.125 meter just little away from the apex we calculate this value z = 0.125 meter. That if I see this self-structure z is measured from here and it is the center column this is the center here it is difficult that is at this we have infinite magnitude of membrane stresses especially N_ϕ. So therefore let us calculate at distance z = 1 to 5 meter put this z = 0.125 meter in this expression $N_\phi = 21.31\frac{(1-Z^2)}{Z} kN/m$ that we have got earlier. And we get this value as $N_\phi = 168\frac{kN}{m}$. So similarly this z can be this N theta can be calculated but N_θ you can see N_θ is 0 here and N_θ is again maximum at the free edge. But free edge you know that there should not be any stress so therefore again the membrane theory fails to predict the stresses at the free edge as well as at the edge whereas joins with the column.

So we will omit this 2 points now based on the calculation of N_ϕ we now calculate the area of steel required . $A_{st} = \frac{N_\phi}{\sigma_{st}} = \frac{168\times1000}{230} = 730mm^2$

So we provide 10ϕ tor steel at the rate of 100 millimeter c/c that will give you A_{st} slightly more than this 785 millimeter square per meter.

Now if I use 8 mm bar I have seen that spacing becomes around 60 that will give congestion in this structure. So therefore to avoid the congestion I will give the spacing of the reinforcement so A_{st} we have found. Now we have to check the thickness of the shell is at based on the maximum tensile stress that is given and we consider that the tensile stress should be omitted.

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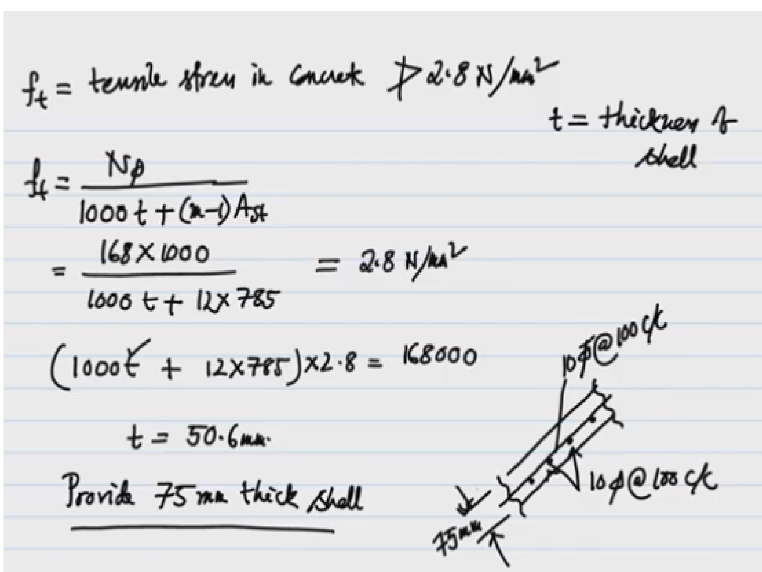

So f_t is the tensile stress in concrete and that should not exceed the given limit 2.8 $\frac{N}{mm^2}$. Now;

f_t can be calculated as $f_t = \frac{168 \times 1000}{1000t + (12)785} = 2.8 \frac{N}{mm^2}$ 't' is the thickness of the shell

So substituting this N_ϕ as 168 ×1000 in Newton, m is 13, A_{st} is 785 we get

$$f_t = \frac{168 \times 1000}{1000t + (12)785} = 2.8 \frac{N}{mm^2}$$

And this should be limited to maximum 2.8 $\frac{N}{mm^2}$. Now from this equation the 't' can be calculated found out as 50.6 millimeter. So for the design purpose we provide slightly more thickness so provide 75 mm thick shell. So in this way we utilize the membrane theory to find the reinforcement and thickness of the shell structure.

So a section of the shell if I draw for the conical shell reinforcement is put here in the middle and then the distribution still also maybe of same steel. So we have provided this 10ϕ @ 100 c/c also the distribution steel is provided as 10ϕ @ 100 c/c thickness of the shell is now prescribed as 75 mm thank you very much.