Plates and Shells
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Module No # 09
Lecture No # 26
Analysis of Spherical Dome

Hello everybody, today I am delivering lecture 2 of the module 9. In the last class I have introduced the main theory of surface of revolution. And the surface of revolution is formed by rotating plane curve around an axis of rotation, and this plane curve may have any nature. It can be an arc of a circle. Or it can be a part of a parabola or ellipse, and accordingly, the surface of evolution is named.

So today, I will discuss a surface of revolution, which is formed by rotating an arc of a circle, and it is commonly known as spherical dome. So, today our discussion will be membrane analysis of spherical dome subjected to gravity loading along the surface of the shell and also the loading which is spreading over the horizontal plane of the shell. So 2 types of loading we will consider because, in the latter case, this snow loading is applicable. Snow loading is generally specified in terms of loading over the horizontal surface.

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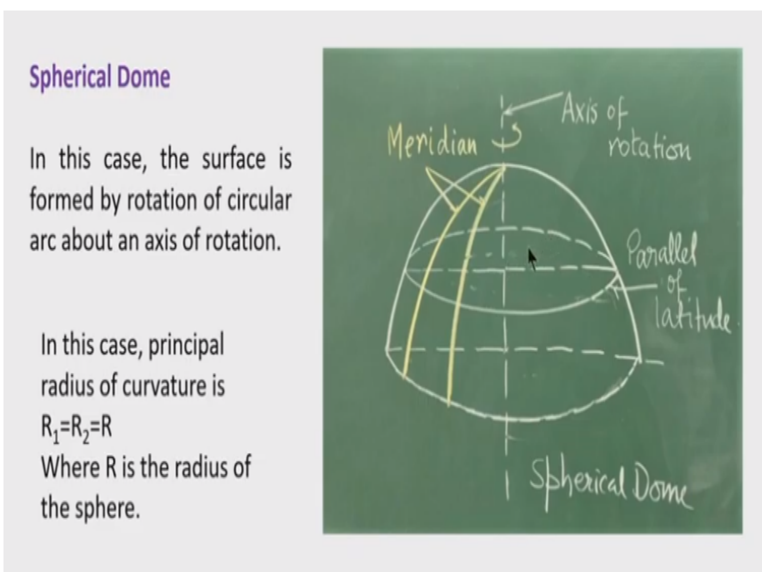

Now this spherical done is formed by rotating a part of a circular arc around an axis of rotation that you are seeing here. And this arc is known as meridian by. As result of this circular arc rotating about an axis of rotation, the curvature along 2 principal directions are same. And the radius of curvature of the shell that is the principle radius of curvature R_1, R_2 is now equal to the radius of the sphere. And you can see at any level we have a parallel of latitude that is also a circular in plan.

And it has a radius which is different from the radius of the sphere, but this radius can be related to the radius of the sphere by trigonometrical relation.

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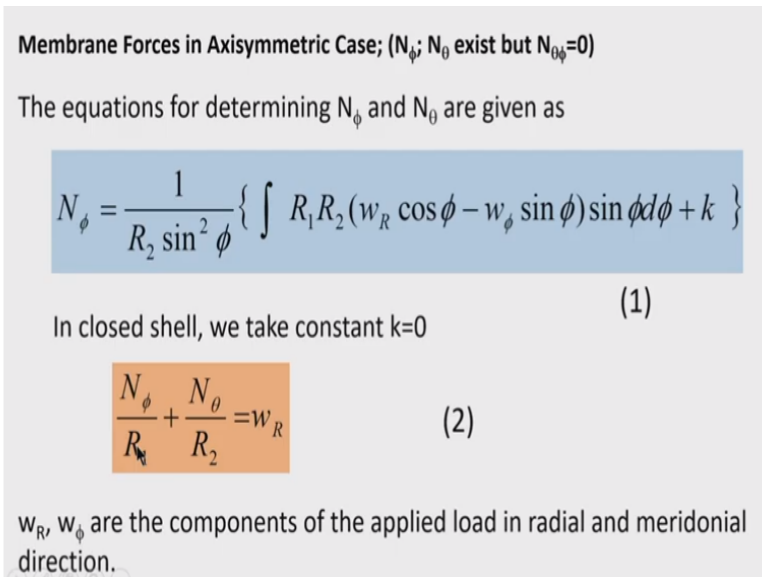

Now in case of membrane analysis of shell, we are basically dealing with the 3 quantities one is N_ϕ, and another is N_θ and another $N_{\theta\phi}$. In general, $N_{\theta\phi}$ also exists for any type of loading, but when the axis symmetrical cases consider that means loading is symmetrical about an axis of rotation, then we neglect $N_{\theta\phi}$. So, the only quantities that have to be found out are N_θ, N_ϕ.

N_ϕ is the membrane stress along the meridional direction, and N_θ is the membrane stress along the tangent to the parallel circle. So that is the circumferential stress so for axis symmetrical cases, if we recall, we arrive at 2 equations of equilibrium. In general, there are 3 equations of equilibrium. But because of $N_{\theta\phi} = 0$ second equation becomes meaningless, and we now have to deal with the first 2 equation.

So, first equation gives after integration this value of N_{ϕ}, and third equation that; is actually an algebraic equation. So once the N_{ϕ} is found for the given loading and given geometry of the shell N_{θ} can be calculated. You can see this equation here it is second equations which relates $N_{\theta}\ to\ N_{\phi}$ by our radius of curvature of the shell, and this is the third equilibrium equation equilibrium condition is taken here along the normal direction normal to the tangent plane at the meridian any point in the meridian.

And w_R is the component of the load along the normal direction along the radial direction, so we call w_R as the radial load. That means component of this load that is acting over the surface of the shell along the radial direction. And there are other components that w_{ϕ} along the meridian direction, whereas w_{θ} is taken as 0. So, these are the cases we have to consider there, and you can see this integral .

$$N_{\phi} = \frac{1}{R_2 \sin^2 \phi}\left\{ \int R_1 R_2 (w_R \cos\phi - w_{\phi} \sin\phi) \sin\phi d\phi + k \right\}$$

Then integration, we have to do $R_1 R_2 (w_R \cos cos\ \varphi\ - w_{\varphi} \sin sin\ \varphi\ ) \sin sin\ \varphi\ d\varphi\ +\ k$ is a constant of integration. Generally, it is a definite integral, so put a; consent of integration and when for a closed shell the limit of integration become 0 to any level of latitude that ϕ then constant become 0. So, we take constant $k\ =\ 0$, so, therefore, we find the integration of this expression with a limit 0 to 5 if this shell is closed, and we can obtain the value of N_{ϕ}.

Now, remember that for spherical shell this $R_1 = R_2$ and R_2 and R_1 now becomes the radius of the sphere.

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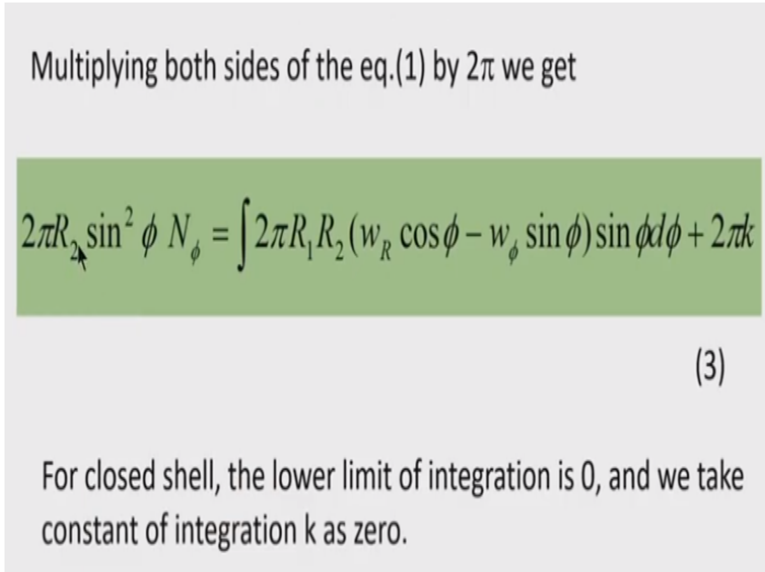

So, multiplying both sides of the equation 1 that is the equation 1 I multiply both sides of the equation 1 by 2π. So, multiplying both sides of the equation by 2π then we can get this equation as $2\pi R_2 \phi\, N_\phi$ that is R_2 I have taken here equal to integration of this $2\pi R_1 R_2 (w_R \cos cos\, \varphi\ - w_\varphi \sin sin\, \varphi\,) \sin sin\, \varphi\, d\varphi\, +\, 2\pi k$ is again the question previous constant was k, and it was multiplied by 2π.

So now the constant stands at 2π however, the constant of integration here is taken as 0 for closed shell.

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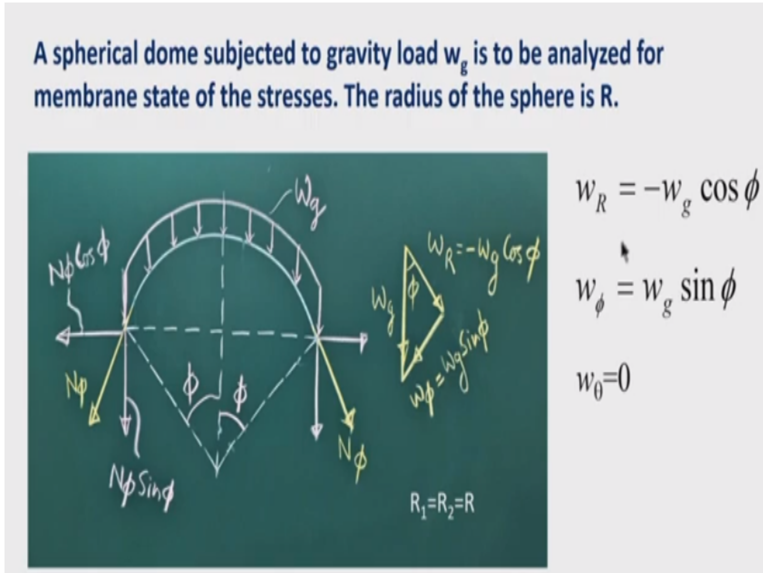

Now you can see here a spherical dome that has to be analyzed for the gravity load w_g consist of a self-weight of the shell. And live load specially, if there is a live load on the shell that is for maintenance purpose if the shell is used for roof, then for maintenance purpose some live load will be there, but it will be very nominal. So, w_g includes self-weight as well as live load here. And it is distributed over the surface of the shell.

Now, if you see that equilibrium equation have been obtained by resolving the forces into 2 directions. One is the ϕ direction that is the tangent to the meridian and second direction is tangent to the parallel circle and third direction is normal to the parallel circle. So, with the 3 directional components and when these are added and equated to 0, we get the equilibrium equation.

Now w_g is your load that is acting vertically downward, but we have to dissolve these forces along 2 directions that is w_R w_ϕ because w_θ is not taken here. So, equilibrium along the tangent to the parallel circle here for axis symmetrical condition is not applicable. So we take only the component along the radial direction that is w_R and along the meridional direction that is w phi. So this phi is the angle that you are seeing here, which is measured from the vertical axis of rotation.

And accordingly, this angle will be ϕ and therefore, component $w_R =- w_g \cos cos\,\phi$ the negative sign is taken here. Because the normal direction outward to the surface of this shell is taken positive, so therefore it is towards the inwards side. So therefore, the negative sign is taken, so $w_R =- w_g \cos cos\,\phi$. And the component along the meridional direction is w_ϕ is equal to $w_g \sin sin\,\phi$. So, we write w_R equal to $w_g \cos cos\,\phi$ and $w_\phi = w_g \sin sin\,\phi$ that up to be used in our first equation that I have shown in the integral form and w_θ is 0.

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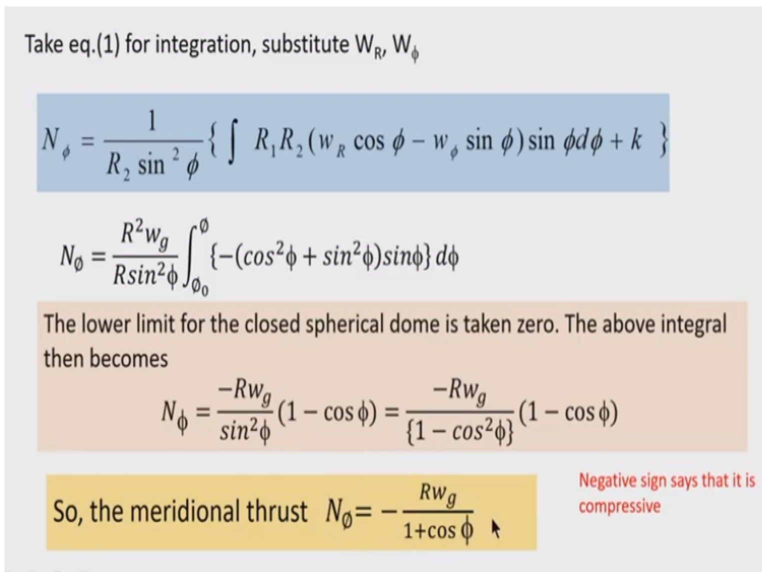

Now coming to this equation that we have to now evaluate $N_\phi = \frac{1}{R_2\phi}\{\int R_1 R_2 \left(w_R \cos cos\,\phi - w_\phi \sin sin\,\phi\right) \sin sin\,\phi\, d\phi + k\}$. Of course, we will take k = 0 because the shell is closed shell. And under axis symmetrical loading condition, there will be no membrane shear force. There is a $N_{\theta\phi}$. So now, substitute the value of w_R, w_ϕ as we have obtained in the earlier cases by resolving w_g into respective directions.

What is the respective directions? one direction is along the normal direction, and radial direction and another direction is along the meridional direction. So, taking $w_R =- w_g \cos cos\,\phi$ and $w_\phi = w_g \sin sin\,\phi$, then we take here w_R is substituted in terms of

w_g and $\cos cos\ \phi$. And similarly, w_ϕ is substituted in terms of w_g and $\sin sin\ \phi$, and one is with minus sign, so, therefore, we take minus common it becomes $\cos cos\ \phi\ +\ \phi$, and this $\sin sin\ \phi$ is already there.

So, we write it as $\sin sin\ \phi \times d\phi\ +\ k$ and other quantity that is R_1, R_2 equal to R_2 so therefore it becomes R^2 we take it outside the integral sign. Because the radius of this sphere is a constant quantity and in numerator R is already there, and ϕ is there. So, cancelling this R^2 with R that is R^2/R we get R, and then this term is 1, $\phi\ +\ \phi\ =\ 1$.

Then remains only the integration of $\sin sin\ \phi\ d\phi$, and with the limit ϕ_0 to ϕ now for a closed shell, the lower limit is taken as 0. So 0 to ϕ we have to integrate so now $\int \sin sin\ \phi$ will be $\cos cos\ \phi$ and limiting 0 to ϕ so after substituting the limit in the integral then we get this N_ϕ equal to $N\ = \frac{-Rw_g}{sin^2}(1\ -\ \cos cos\ \phi)$. This expression can be simplified this ϕ can be written as $1\ -\ cos^2$, and this numerator is already there $(1\ -\ \cos cos\ \phi)$, then after factorizing this that $1 - \phi$ can be written as $(1\ +\ \cos cos\ \phi\ )(1\ -\ cos\phi)$.

Then $1\ -\ cos\phi$ and $1\ -\ cos\phi$ is cancelled, and then we only get $N_\emptyset = -\frac{Rw_g}{1+\cos cos}$. Now N_ϕ is coming as negative, so negative sign is compressive force, and positive sign will be denoted for the tensile force. So N_ϕ we got as in terms of the radius of this sphere and the w_g that is the intensity of the vertical load that is acting on the shell surface. And in terms of the meridional phi so you can see the nature of the N_ϕ is compressive.

So, for this type of shell even if we construct it with reinforce concrete it remains compression for any loading that is vertical loading, and meridional angle ϕ for any meridional angle for the shell surface that we assign in the design it will remain compressive. So that will give advantage to the design of concrete because concrete is very weakened structure, so if it remains in compression that we have the advantage of designing this structure.

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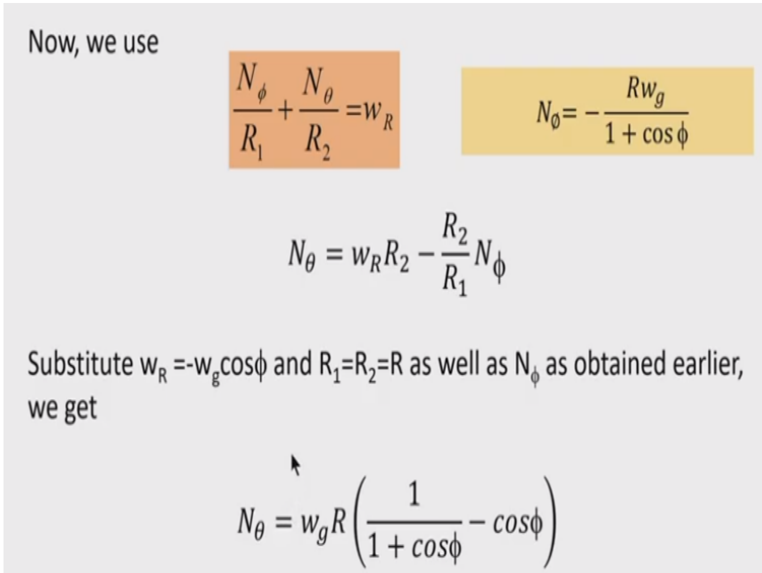

So, once we obtain this N_ϕ then we substitute here, then we get N_θ now again note it that $R_1 = R_2$. Therefore, N_θ from this equation we get $w_R R_2 - \frac{R_2}{R_1} N$. $\frac{R_2}{R_1} = 1$, so therefore $N_\theta = w_R R_2 - \frac{R_2}{R_2} N$ now substitute N_ϕ what is w_R? w_R is the component of the load in the radial direction it is $-w_g \cos cos\, \phi$. So, this expression is needed here so substituting w_R here and $N_\emptyset =- \frac{Rw_g}{1+coscos}$ into this expression.

And $R_1 = R_2$, then we get $N_\theta = w_g R\left(\frac{1}{1+cos} - cos\right)$ so this expression for the circumferential stress N_θ may be compressive may be tensile depending on the value of ϕ that we have to find out.

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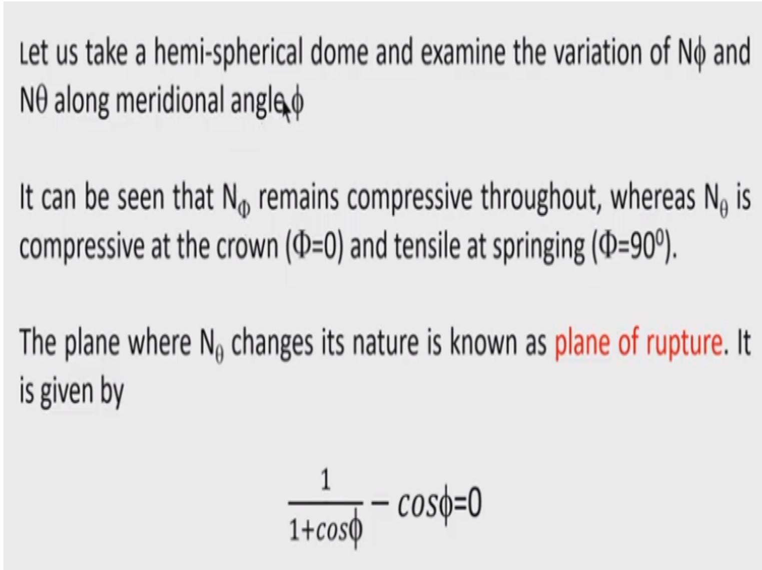

Now let us see we take a hemispherical dome examine the variation of N_ϕ and N_θ along the meridional angle ϕ. Now we already noted that N_ϕ the meridional stress is always compressive for any value of ϕ. However, the value of N_θ changes for certain value of ϕ nature of the N_θ may change with the value of ϕ. So, therefore, we have to see at what value of ϕ this angle this magnitude of this nature of the N_θ changes.

That means the nature is reverse previously, say it has compression, now it will be tension so this angle we have to find out. Now plane at where now N_θ changes it is nature or sign is known as plane of rupture. We generally intend to avoid the tensile force in the concrete structure, so therefore if we know the plane of rupture, we can limit our dimension in such a way that the ϕ does not exceed a certain critical value so that tension is not developed in this shell surface.

So N_θ is 0 if this quantity $\frac{1}{1+cos} - cos$=0 because w_g and R is non-zero constant. So, equating these 2, 0, we can solve for cos cos ϕ .

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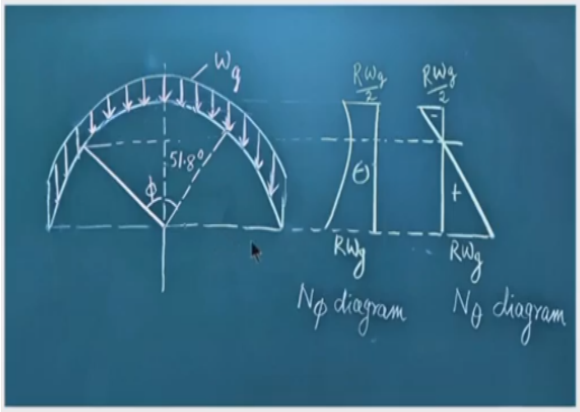

$$cos^2\phi + cos\phi - 1 = 0$$

$$\text{So, } cos\phi = \frac{-1 \pm \sqrt{5}}{2} = 0.61803 \text{ Hence } \phi = 51.82^0$$

So, these angle gives a quadratic equation in $\cos cos\,\phi$, and the value of $\cos cos\,\phi$ now becomes 1 value is 0.61803, and this only the value that can be accepted that value to be not relevant to the problem in question. So, therefore, the $\cos cos\,\phi$ is found as 0.61803, and ϕ is $51.82°$. So now we get the limit of the angle phi up to which this shell remains under compressive force only. So up to this level, this showing here remains in compression.

So, but the N_{ϕ}, the meridional stress is compression throughout the surface of the shell from the springing to the ground. This is the springing level, and this is the ground level, so here you can see the variation of N_{ϕ} if I draw from 0 to say ϕ here, it is ϕ, and here it is $\phi_{90°}$.

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For an elementary strip, area of the surface = $(2\pi R sin\alpha)Rd\alpha$

Hence Total surface area of the spherical dome A = $\int_0^{\phi} 2\pi R^2 sin\alpha d\alpha = 2\pi R^2(1-\cos\phi)$

Now total load at level ϕ, $W = w_g \times 2\pi R^2(1-\cos\phi)$ (9)

Total vertical component of N_ϕ, $F_V = N_\phi \sin\phi\,(2\pi R\sin\phi)$ (10)

Equating (9) and (10), we get again,

$$N_\phi = \frac{Rw_g(1-\cos\phi)}{sin^2\phi} = \frac{Rw_g}{(1+\cos\phi)}$$

So that you can see in this expression, if you put $\phi = 0$ means it will be N_ϕ will be $-Rw_g/2$. Because cos $cos\ 0$ is 1, so $Rw_g/2$ will be the value of $N_\phi = 0$ and when $\phi = 90°$ that is cos $cos\ \phi$ is 0 then N_ϕ will be $-Rw_g$. In both the cases N_ϕ compressive that is coming with a negative. So, therefore, variation of N_ϕ we can see now $Rw_g/2$ to Rw_g because ϕ is measured from the vertical axis.

So here the ϕ is 0, and at this point it is 90°, and because of symmetric, this same stress will be generated here also. Now, if I see the variation of N_θ this is the expression for N_θ, so at $\phi = 0$ here, you are getting this $\frac{1}{2}$ because cos $cos\ 0$ is 1. So, $1/2$ -1 that is -1/2 so at $\phi = 0$ we are getting this as $-\ Rw_g/2$. So that is the magnitude of N_θ at $\phi = 0$ now, let us see at $\phi = 90°$.

So, when ϕ is 90°, cos $cos\ 90 = 0$, so therefore N_θ is w_g/R, and it is coming as positive quantity. So therefore, tensile, so here you are getting tensile forces are developed after this plane, and this plane is nothing but the plane of rupture, and it is located at $\phi = 51.8°$.

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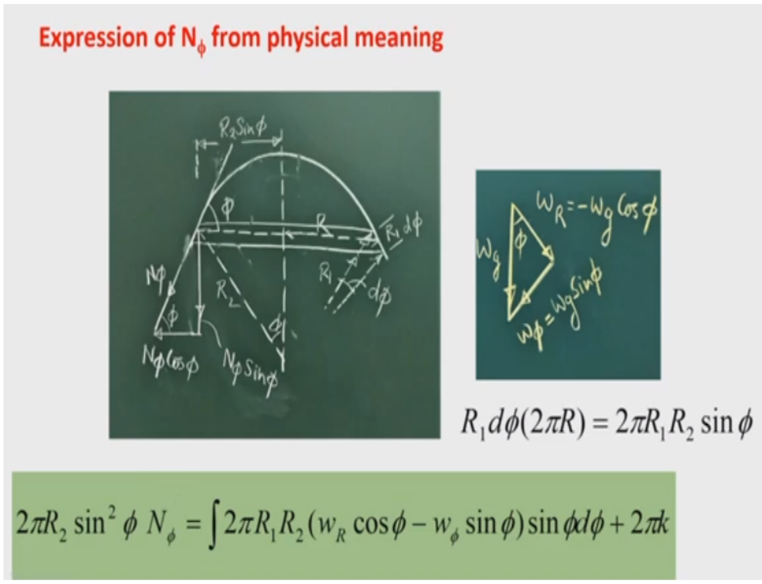

Now this expression this integral you are getting here

$$2\pi R_2 \phi\, N_\phi = \int 2\pi R_1 R_2 \left(w_R \cos cos\, \phi\ - w_\phi \sin sin\, \phi\right) \sin sin\, \phi\, d\phi\ +\ 2\pi k$$

as some physical meaning. So if I apply the physical meaning in some problem we can readily obtain the expression without going for lengthy integration in some cases. So now, here we will try to give you the physical meaning of this expression.

Now, if you see the take a small step of the shell whose width is $R_1\ d\phi$ and $d\phi$ is the small angle here meridional angle $d\phi$ and ϕ is here so this small angle is $d\phi$. And this radius of this sphere is $R_2 = R_1$, and R is the radius of the parallel circle which can be related to R_2. So, R can be written as $R = R_2 \sin sin\, \phi$. Now, you can see here this the direction of N_ϕ is along the meridian tangent to the meridian, so this is N_ϕ.

And this angle is phi again because this is ϕ this angle will be ϕ, so vertical component of N_ϕ if you see the $N_\phi \sin sin\, \phi$ and horizontal component is $N_\phi \cos cos\, \phi$. Now from this expression this you are seeing that $N_\phi \sin sin\, \phi$ term is included here. So $N_\phi \sin sin\, \phi$ means vertical component of N_ϕ here per unit length then the circumference of this spheroid circle that is $2\pi R$ whereas $R = R_2 \sin sin\, \phi$.

So R 2 sin phi 2 pi R is the circumference of this parallel circle so that means if I multiple the vertical component of the N phi that is meridional force N phi vertical component is N phi sin phi with the circumference that is the total length over which it is acting now at this level. So you will get total vertical force at this level. So total vertical force at this level is $2\pi R_2 \phi\, N_\phi$. Now, this total vertical force at any level must be balanced by the external load.

The N_ϕ is the reactive force that is in response to the external load that is gravity load. Now, if I see the right-hand quantity, which is under the integral sin will be able to find out the relation between this total vertical meridional force around the circumference. And the total vertical load acting about that level, so here you can note this quantity that if you take a small strip here for the shell surface $R_1\, d\phi \times 2\pi R$ is the area of the strip.

So, $2\pi R \times R_1 d\phi$ into the area of this strip, and since R can be written as $R_2 \sin sin\, \phi$, so total area of this strip is $2\pi R_1 R_2 \sin sin\, \phi$. So, under the integral sign you will get this terms are there $2\pi R_1 R_2 \sin sin\, \phi$. So that means total surface is there for the strip now inside the parenthesis, the load component let us examine. So load component w_R is $w_g \cos cos\, \phi$ and load component w_ϕ is $w_g \sin sin\, \phi$.

Now here, you can see the $w_R \cos cos\, \phi$ is the component of w_R component of the vertical direction. And similarly, here, $w_\phi \sin sin\, \phi$ is the component of w_ϕ along the vertical direction. So the quantity inside the parenthesis indicates that it is nothing but the vertical load. So effective vertical load multiplied by the area of the strip because it is axis-symmetrical loading.

So, 0 to ϕ is taken at the limit of the integration, so that means this quantity the left hand is nothing but the vertical component of this meridional force total vertical component of the meridional force at level ϕ. And it is equal to the total vertical load acting on the shell surface above that level. So using this physical meaning, we can solve many complicated problem without going to integrate the expression.

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For an elementary strip, area of the surface = $(2\pi R sin\alpha)Rd\alpha$

Hence Total surface area of the spherical dome A = $\int_0^{\phi} 2\pi R^2 sin\alpha d\alpha = 2\pi R^2(1-\cos\phi)$

Now total load at level ϕ, $W = w_g \times 2\pi R^2(1-\cos\phi)$ (9)

Total vertical component of N_ϕ, $F_V = N_\phi \sin\phi\,(2\pi R\sin\phi)$ (10)

Equating (9) and (10), we get again,

$$N_\phi = \frac{Rw_g(1-\cos\phi)}{sin^2\phi} = \frac{Rw_g}{(1+\cos\phi)}$$

Now let us see whether we can get the same result we have obtained by integration earlier. Using the integration, we obtain this N_ϕ, and N_θ is this, so let us see whether we get this by means of physical interpretation. So, for an elementary strip area of the surface is suppose this, for example, this small angle is taken as $d\alpha$ is taken only for avoiding the confusion in the integral because the limit we are taking 0 to ϕ and integration is done with respect to ϕ.

So, we are taking another variable α instead of ϕ but the expression will be same so if I take this angle as $d\alpha$ and this strip which is $R_1\ d\alpha$ then total area of this strip will be $R_1\ d\alpha \times 2\pi R$. So that is we get this $2\pi R\ sin\alpha\ Rd\alpha$ so $R\sin sin\ \alpha$ is this can be taken because this $R_2 \sin sin\ \alpha$ and R is nothing but R. R_2, R_1 is equal for this spherical shell, so we get this quantity as the area of this surface.

So total surface of this spherical dome is now found after integration from 0 to phi, and after integration, we get it as $2\pi R^2(1-\cos cos\ \phi)$. Now total load at level ϕ is say w_g is the vertical load acting on that this w_g multiplied by this surface area is the total load. So, we get 1 quantity that in the right-hand side; without integration, we get this quantity appearing in the right-hand side by means of physical meaning.

Then the vertical component of N_ϕ that is $N_\phi \sin sin\, \phi$ is the component per unit length, and this is the circumference of the parallel circle $2\pi R \sin sin\, \phi$. So now, equating this F_V to W we get now $N_\phi = \frac{Rw_g}{(1+\cos cos\,)}$, so again this same quantity is obtained by means of physical meaning and which the integration is avoided. This technic is useful for some shell where discontinuity arises because of this cut some shell as some opening.

So, because of this discontinuity, we denote integrate this up to the limit therefore, it is not a closed shell, and constant is not 0. So, in that case, this application of this physical interpretation will be useful to obtain the stress resultant. So, once you obtain the N_ϕ then you can obtain the N_θ can be very easily obtained because we know the relation N phi by R 1 + N theta by R 2 = w R. Whereas this N_ϕ is obtained from the equation so N_θ can be obtained simply by substituting N_ϕ in the third equation of the equilibrium is this equation.

So, you can arrive at the same quantity that you have obtained by means of integration; this quantity is actually obtained by integration; other quantity is obtained by solving this algebraic equation $\frac{N_\phi}{R_1} + \frac{N_\theta}{R_2} = w_R$.

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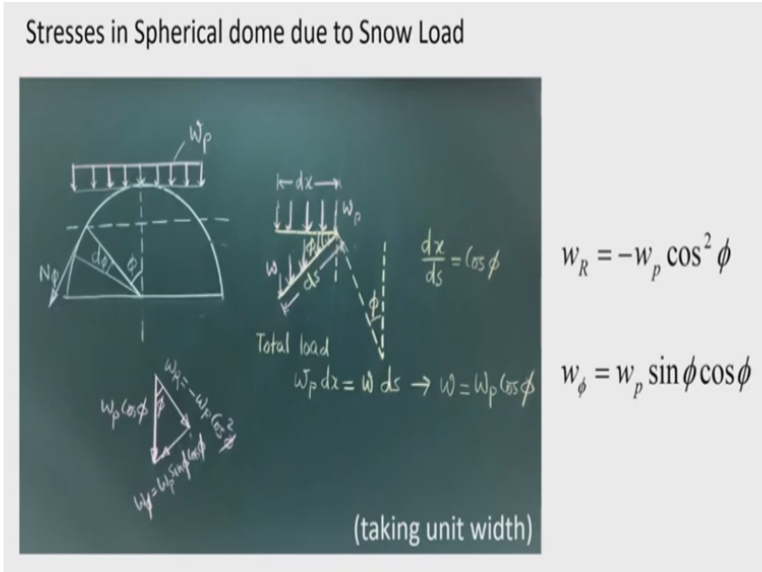

Now let us see how this shell response the load which is acting over the horizontal surface. Now previous, we have taken the load is acting along the surface of this shell is curved. But here, the road is taken on the projected surface that is the only horizontal plane. So, w_p is the load per unit area on the horizontal surface, and w if you see only w is load that is acting on the curved surface of the shell.

So how to obtain this stress resultant due to this type of load now. Here, if you see take the small length of the shell which is of length dx and the horizontal length is dx and we take the unit width of the shell. Then we can say that the total load that is w_p is the load acting along the horizontal position. So, $w_p ds = w \times dx$ because total load is equal, so by equating this, we get $w = w_p \cos cos\, \phi$ how it is $\cos cos\, \phi$?

Because dx/ds that w is dx/ds and dx/ds from this angle, you can get $\cos cos\, \phi$ so the load here along this surface vertical load along the surface of the shell we take $w_p \cos cos\, \phi$. And it is component along the radial direction is $-\ w_p \cos cos\, \phi\ \times\ \cos cos\, \phi$. So $-\ w_p \phi$ and along the meridional direction it is $w_p \cos cos\, \phi\ \sin sin\, \phi$. So, $w_p \cos cos\, \phi\ \sin sin\, \phi$ is written here as the component of the load along the meridional direction.

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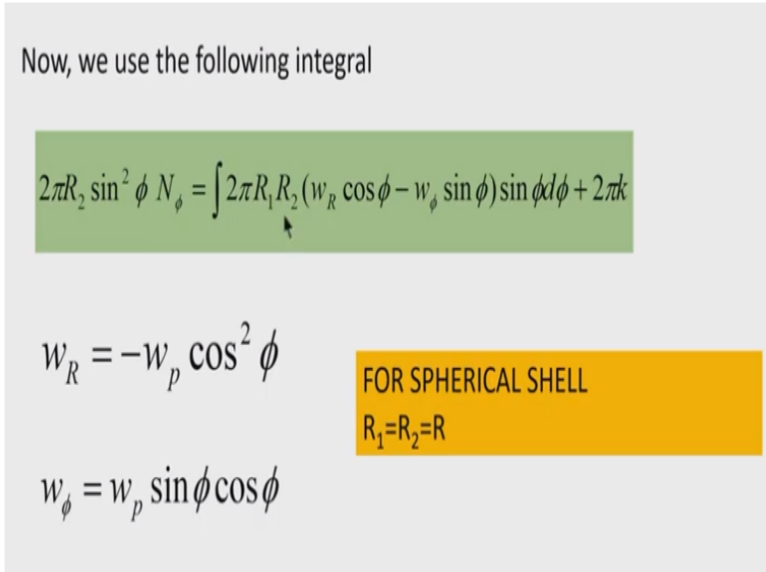

Now we use the same equation; let us do it by integration from the first principle so the membrane stress along the meridional direction is given by this integral, and that integral is found from the first differential equation. That differential equation was found by equating this sum of the forces along the direction of ϕ along the tangent to the meridian that is we call it ϕ direction. So, this integral is obtained, and we now used this w_R and w_ϕ that we obtain for the projected load.

So, by using this quantity $w_R =- w_p \phi$ and $w_\phi = w_p \cos cos\, \phi\, \sin sin\, \phi$, we now write this equation.

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$$N_\phi = \frac{R^2}{R\sin^2\phi}\int_0^\phi (-w_p\cos^3\phi - w_p\sin^2\phi\cos\phi)\sin\phi d\phi$$

$$= \frac{w_p R}{\sin^2\phi}\int_0^\phi -(\cos^2\phi + \sin^2\phi)\cos\phi\sin\phi d\phi$$

$$= -\frac{w_p R}{2\sin^2\phi}\int_0^\phi \sin 2\phi d\phi$$

$$= \frac{w_p R}{4\sin^2\phi}(\cos 2\phi - \cos 0)$$

$$= \frac{w_p R}{4\sin^2\phi}\{\cos^2\phi - \sin^2\phi - \cos^2\phi - \sin^2\phi\} = -\frac{w_p R}{2}$$

And then, we obtain this N_ϕ by systematic integration of the quantity I am showing you the full integration. So, $\frac{R^2}{R\phi}$ and then integration 0 to ϕ and this quantity actually I have substituted here for the radial component and the component along the ϕ direction and after substituting this component is the expression is converted in this form. So, taking this cos cos ϕ common from this parenthesis, we can now write say $\frac{w_p R}{\varphi}\int_0^\varphi -\ (\varphi\ +\ \varphi)\cos cos\ \varphi\ \sin sin\ \varphi\ d\varphi$.

So this quantity is 1 now, cos cos φ sin sin φ if I multiply it by 2 and then divided by 2 then we can write it sin sin 2φ and this divided by 2 that is factor 2 is coming in the denominator here. So now integrate this expression sin sin 2φ it will be $-\frac{\cos cos\ 2\phi}{2}$ and then substituting the limit of the integration we now get this quantity as $\frac{w_p R}{4\varphi}$. And after carrying out this simplification there, cos cos 2ϕ can be written as φ − φ.

And this cos cos 0 is 1, but 1 I have substituted as ϕ + ϕ, but it is under the minus sign. So, therefore, φ − φ so we can now write this ϕ we get cancelled here and then ϕ it will be ϕ so 2 is divided here with 4 and then the quantity this expression for N_ϕ becomes $-w_p R/2$. So, this expression is for small snow load, and membrane force N_ϕ for snow load now becomes $-w_p R/2$.

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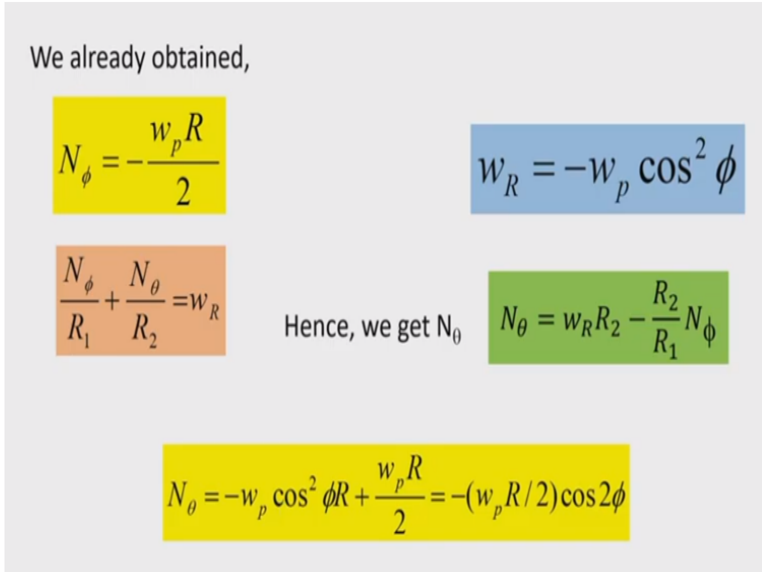

Now N_ϕ is obtained, we can now obtain N_θ the expression is this N_ϕ/R_1 is this $\frac{N_\theta}{R_2} = w_R$. And w_R as we have resolved earlier it is $-\ w_p \varphi$ so after substituting all this values here the N_θ is calculated as $-\ w_p \varphi\ R\ + \frac{w_p R}{2}$. And after simplification, we can write it –w p R by 2 Cos 2 phi. Now we can see the variation of N_ϕ and N_θ with the change of angle ϕ.

Now you can see here the N_ϕ do not depend on the meridional angle ϕ for this type of loading and it is a constant quantity and it is always compressive. So this is a typically a result that we have obtained in case of the load that is acting on the shell surface; we are getting the variation of N_ϕ with respect to ϕ; of course, that time also we want this N_ϕ as compressive throughout the variation of ϕ.

Here also, N_ϕ is compressive, but it is a uniform quantity N_θ, of course, varies with ϕ and let us see how it various and where N_θ changes its sign. So it can be readily verified from this expression, so when $\phi\ =\ 45°$ then this $\phi/4$ the N_θ from compression changes to tensile.

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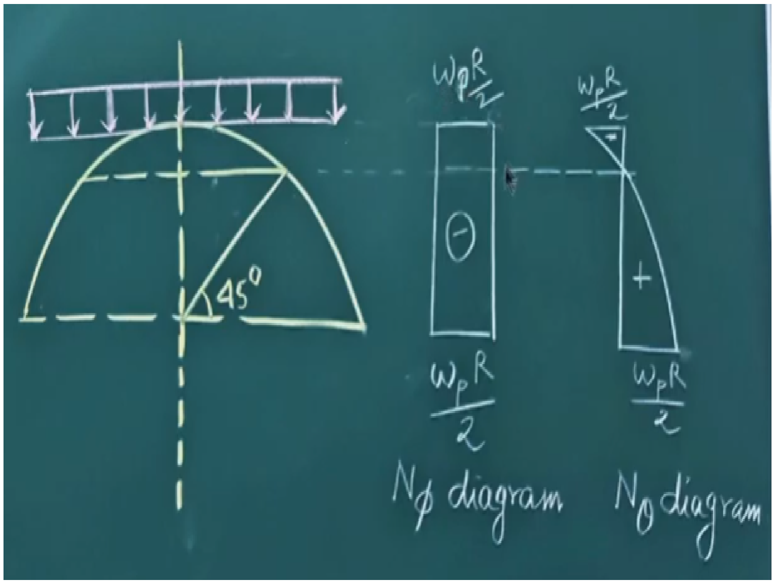

So, the variation of this quantity will be like that the N_ϕ diagram always compressive, and this N_θ will be compressive. That is, with negative sign up to 45° angle of ϕ and then it will be tensile and the variation is only here but here the variation is uniform.

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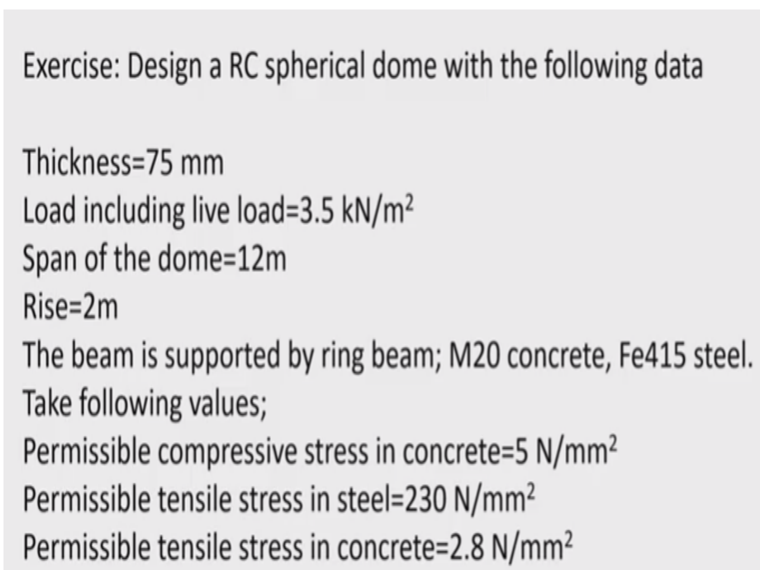

Exercise: Design a RC spherical dome with the following data

Thickness=75 mm
Load including live load=3.5 kN/m^2
Span of the dome=12m
Rise=2m
The beam is supported by ring beam; M20 concrete, Fe415 steel.
Take following values;
Permissible compressive stress in concrete=5 N/mm^2
Permissible tensile stress in steel=230 N/mm^2
Permissible tensile stress in concrete=2.8 N/mm^2

Let us one example which will clarify the theory that we have discussed. So how this membrane theory of shell can be used to design a spherical dome that will be illustrated with a numerical example. So, example is this a RC is spherical dome has to be designed, the thickness of the done is 75 mm load, including live load is $3.5\ kN/m^2$. Span of the dome is 12 meter rise is 2 meter, and the spherical dome is supported over the periphery by a ring beam that is the edge beam.

And the edge beam concrete is M20 guide shell concrete also, you can take M20 guide, and steel is Fe415, and also following input are given. Permissible compressive stress in concrete is phi Newton per mm square, and permissible tensile stress is $230\ N/mm^2$ permissible tensile stress in concrete as $2.8\ N/mm^2$. Let us see this solution of this problem.

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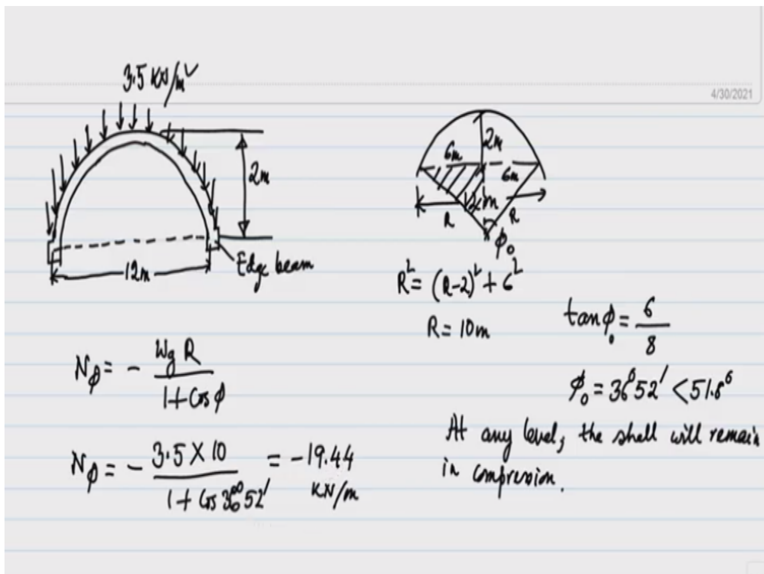

So, we have the thickness of the shell as 75 mm, and you can see that span of the shell is mentioned as 12 meter in relation to span thickness is very small. So it is a thin shell so let us find out this quantity we design quantity. We have this shell, it is a spherical shell of very small thickness but covering a large area. And it is supported over an edge beam at the edge; it is supported by a beam. The span of the shell is given as 12 meter, so this distance is 12 meter.

You can take it from center to center of the edge beam rise of the shell that is called rise, is given as 2 meters. Radius is not specified now radius can be found out very easily and shell is subjected to gravity load along it is surface, and this load is uniform load it is $3.5 kN/m^2$. So that is the geometry of the shell, and this is your edge beam. Now, if I see the geometry of the shell, let us draw a line diagram.

This is 12 meter, and this is 2 meter; the rise of the shell then radius of the shell is still unknown. We have to find this radius say this is the radius R now, we can see here that R^2 if I consider this angle then R^2 equal to this side is $(R - 2)^2$ + this distance is 6 meter this is also 6 meter, 6^2. So by solving this, we get this $R = 10\, meters$, so radius of the shell is 10 meters which will be used for finding the membrane stresses.

The membrane stress expression, you need the description or the value of the radius of the shell. Now let us examine whether the geometry of the shell permits any advantages for the RCC construction; that advantage I am implying that whether shell will be throughout compression or some tension will be developed. Now we have seen that if the ϕ is limited up to $51.8°$ then there will be no tension.

So let us find out what will be value of ϕ, so this is the value of ϕ here, and we can see here that $\tan tan\, ϕ$ can be calculated as 6; this angle is constant table say this is $ϕ_0$. So $\tan tan\, ϕ_0$ is 6 by total radius is 10 so 10 – 2 that is 8 hence we get $ϕ_0$ equal to after calculation we get $36°52'$. So, it is less than $51.8°$, so hence there is no chance of any tensile stress in the shell so therefore, at any level, the shell will remain in compression.

So that is our conclusion by seeing the angle, so now we can readily find it the expression for $N_ϕ$ is your $-w_g R/(1 + \cos cos\, ϕ)$. So $N_ϕ$ integers with ϕ so the maximum value of ϕ is $36°52'$ so substituting this $N_ϕ$ you will get $w_g\, is\, 3.5$, radius of the shell is 10, and this is $1 + \cos cos\, 36°52'$. By calculating this, we get this $N_ϕ$ as -19.44 kilo Newton per meter. So shell remains in compression, and maximum value of compression that is $N_ϕ$ is $19.44\, kN/m$.

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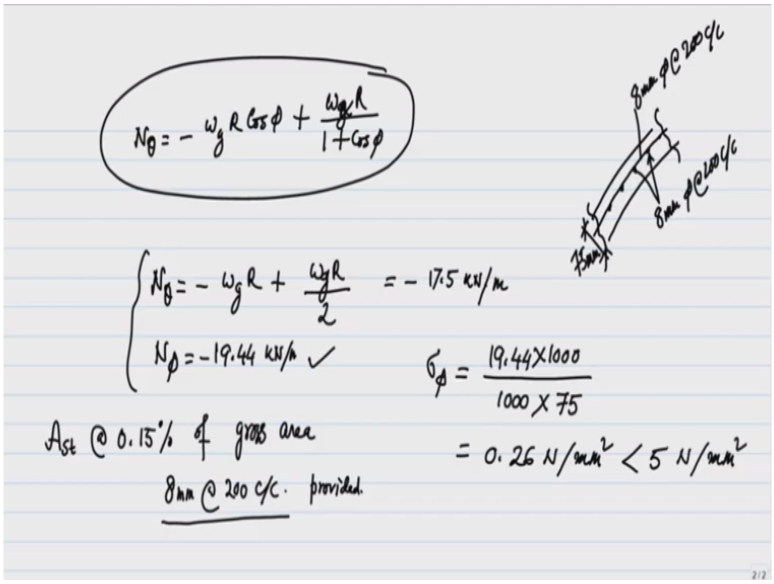

Now calculate N_θ the expression for N_θ we already derived, and this found out as $w_g R \cos cos\, \phi\ +\ w_g R/(1\ +\ \cos cos\, \phi\ )$. So, this is the value of the expression for N_θ . Now it is to be noted that N_θ decreases with ϕ. So maximum value of N_θ obtained when ϕ is 0, so now maximum value of N_θ is obtained as $-w_g$ if I put ϕ as 0, $-\ w_g R\ +\ \frac{w_g R}{2}$.

So that means the maximum value of N_θ is now obtained as $-\ 17.5\ kN/m$, so we have got the maximum value of membrane stress N_θ is this $-\ 17.5\ kN/m$. And also, the N_ϕ is 19.44 in minus sign is for compression kN/m. So, with these 2 values now, we can go for design the shell. Now, if it is of reinforce concrete, then we can compare the stress developed in the section with the permissible stress in concrete.

Now the maximum stress is this N_ϕ , so therefore $\sigma_\phi\ is\ 19.44\times1000$ that will be Newton and then if I divide it 1000 again, it will be converted a Newton per millimeter. So, 1000 and thickness of the shell is 75, so now we get $\sigma_\phi =-\ 0.26\ N/mm^2$, and this is less than the permissible stress of concrete of grade 20 that we used in this design calculation. So, therefore, the shell is safe maximum compressive stress they developed is 0.26, and permissible stress is phi.

Now since the more tension is developed in the shell in principle, there is no necessary for any reinforce there is no need of any reinforce. But you have to provide nominal reinforcement, so nominal reinforcement at the rate of 0.15% has to be provided. So A_{st} at the rate of 0.15% of gross area is required, and you can see this gross area is found as thickness into $1000mm^2$ at this rate the reinforcement 8mm at the rate of 200 center to center is adequate so it is provided.

So how this shell reinforcement is provided; suppose, if I show you a section of shell surface reinforcement is provided at the middle surface. So, this is 8 mm, ϕ $at\ the\ rate\ of$ 200 center to center, and this is also same reinforcement distribution style if provided. And thickness of shell is 75 mm how about the thickness may require to the increased at the edges. Because they are the membrane stress of stress is not so much applicable so let us see the condition of edge bean.

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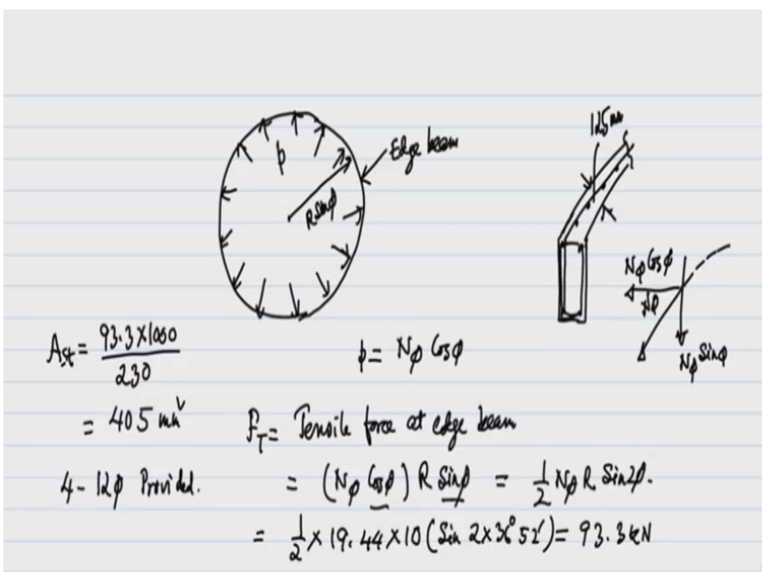

So if we see the plane of the roof dome, there is a circular, and this is your edge beam. Now you can see here that the edge beam near the edge beam thickness is increased, so the thickness here was 100 how here thickness is increased see increase thickness may be 125 mm. So I am proposing the thickness at this edge is 125 mm compared to thickness as 75 mm at the other portion. So reinforcement is here there up to be brought inside this edge beam now; let us see what will be the dimension of the edge bean and reinforcement of the edge beam?

Now you can see the membrane stress N_{ϕ} at this level of this shell have 2 component 1 is $N_{\phi} \sin sin\ \phi$, and other component that is horizontal component is N_{ϕ} $\cos cos\ \phi$. So $N_{\phi} \cos cos\ \phi$ component will give the tensile stress in the edge beam that has to be taken into consideration for design of edge beam for checking this safety of the edge beam. So this pressure is p in the edge beam p is here $N_{\phi} \cos cos\ \phi$ this radius is here 6 meter because the span of the shell was 12 meter.

So now we can get the tensile force on the edge beam as just like your hook stress that you consider it is tensile stress tensile force at edge beam will be $N_{\phi} \cos cos\ \phi$ into this radius is here your this R this will be your this $R \sin sin\ \phi$. Now $\cos cos\ \phi$ $\sin sin\ \phi$ can be combined, and this can be written as $0.5\ N_{\phi}\ R \sin sin\ 2\phi$, so we can now get this total value of tension half N_{ϕ} at this level 19.44 and this radius of this shell is 10, and this $\sin sin\ 2\phi$.

So sin 2 into 36 degree 52 minutes this is equal to 93.3 kilo Newton so this, the ring beam has to designed for this value we are taking this. So Ast for ring beam is now found out the total tensile force is $\frac{93.3\times1000}{230}$ so this area becomes $405\ mm^2$. So, therefore, we can say that the A_{st} of $405 mm^2$ that means if I provide $4\ -\ 12\phi$ that will be adequate in addition to stirrup.

So $4\ -\ 12$ bar as to be provided, and stirrup reinforcement will be there this size of the beam can also be found out from the tensile stress criteria.

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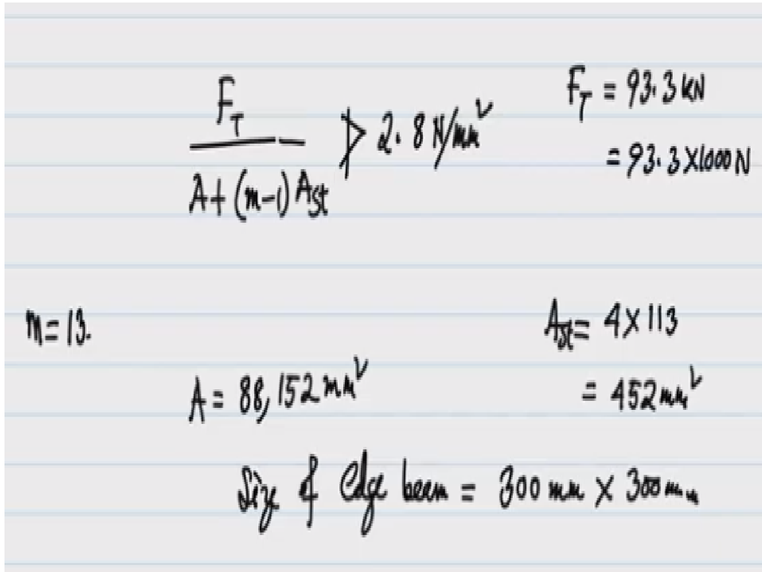

And we know that total tensile force divided by the area transformed area into $A + (m-1) \times A_{st}$ should not exceed the tensile stress permissible tensile stress in concrete given. Now m is taken for $F_T \frac{N}{mm^2}$ is 13 A_{st} already we have found as = $4 \times 12 mm\ dia\ bars$, 113 is the area of 1-12 remember. So that is $452 mm^2$ A_{st} hence we get from this equation that quantity A because F_T is found out earlier.

F_T was found out as $93.3kN$ that means $93.3 \times 1000\ N$, so A is found out as $88152\ mm^2$; from this equation minimum A is required. So that means this size of edge beam, which is a ring-type that is a circular beam, edge beam is now taken as root over this that is we take 300 mm by 300 mm site. So, design is completed, we found the size of the edge beam reinforcement of the edge beam, the thickness of the shell that was adequate to resist the maximum compressive force, and we got the reinforcement in shell. Thank you very much