Plates and Shells
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Lecture-23
Classification of shell structure

Hello everybody, today I am starting the lecture 2 of module 8. In the first class that is the module 8 lecture 1 of shell, I introduced what is shell and then I define the shell as a stressed skin and based on that I have shown a differential equation of a stress membrane, very popular differential equation with the Laplacian form and I discuss the solution of this membrane with a uniform air pressure.

So, that analogy is useful for evaluating the membrane stresses because shell is acting like a stressed skin. And therefore membrane theory is mostly applicable and still it is used in design office, the membrane analysis is popular and used for design of shell. So, today our discussion will be on the classification of shell structure. We have first defined this shell as a stressed skin, but how shell can be grouped into different categories mainly on the geometry.

So, geometrical classification has been adopted universally and it is also included in our Indian standard code of practice for reinforced concrete shell and folded plate. So, today I will discuss how the classification is done for the shell structure. Mainly the application of shell in civil engineering construction or of RCC, the material is reinforced concrete. And in many mechanical engineering application or aerospace engineering application shell are of metal structure.

So, in that case the difference is the material but behaviour is same, behaviour that shell acts transmit the load in in-plane direction, that is applicable whether shell is constructed with the different materials say RCC or this steel, even the aluminum or composite shells are also common in various application of aerospace structure. So, before starting the classification of shell we have to first discuss the geometry of the shell. The shell you can see it is actually characterized by the curved surface.

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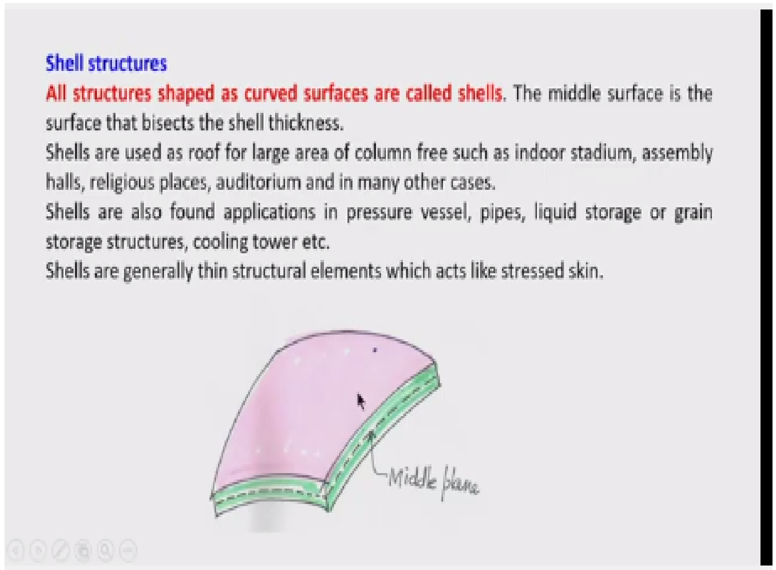

And surface is the main geometrical element or plane; it is the important geometrical element of the shell. Now, all the stresses and that strains etcetera whatever we discuss are referred to the middle plane. So, what is middle plane? Just like our plate middle plane is the plane on middle surface, is a plane which bisects the thickness of the shell, say this is the thickness of the shell and middle plane is bisecting the shell.

The shells are used as a roof for large area of column free space that is required for indoor stadium, assembly halls, religious spaces, auditorium and in many other cases. And also the shells are applied or shells are used adopted in pressure vessels, pipes, liquid storage or grain storage structures, cooling tower etcetera. And different geometrical forms are given to the shell structure to resist the load. So, geometry is most important thing for the shell structure and shell is classified based on the geometry.

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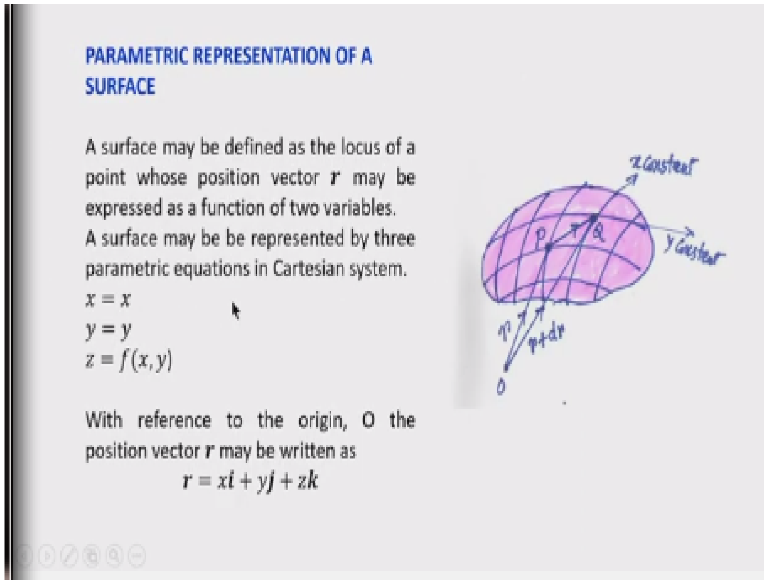

Now as I have told the shell element is characterized by a surface. So, let us see what are the fundamental properties of the surface? And how the surface can be represented with the parametric equations? So, here you can see a surface and the curved lines you are seeing in both direction orthogonal direction in x direction as well as in y direction. Suppose this is one direction that is aligned along the x axis.

So, in that case the y is constant and if this line is aligned along the Cartesian y axis then x is constant along this line. Now in the parametric form, the surface of the shell is represented by this function of 2 variables. So, x and y are the variable which governs the shell geometry. So, if the z is the coordinate which defines the elevation of the shell at any point x and y, then in the parameter form the shell surface can be represented as $x = x$, $y = y$ and z is a function of x and y.

Now, say 2 points are there say P and Q, such that these distance the position vector of P with refer to origin O is r and position vector of Q with reference to the origin O is $r + dr$. So, these are the position vector of these 2 points P and Q and this length PQ is the dr. Now with reference to the origin, the position vector of point P can be represented in vector form in 3 dimensional space and this is given by r equal to, r is a vector here.

The position vector of P, you can see it is a radial line from the origin O and the direction is from O to P that is the positive direction of the vector r. So, the position vector of r is nothing but if the coordinate of P is x, y, z for example and origin coordinate is 0, 0, 0 then position vector r will be x - 0 into unit vector along the direction of the radial vector r, then y - 0 into j, j is a unit vector along the Cartesian direction y.

In this case i is the unit vector along the Cartesian direction x that is the along the x axis. So, we generally know that i, j, k are the unit vectors along the x, y and z axis, r is the vector which is directed from O to P and magnitude of r is nothing but $\sqrt{x^2 + y^2 + z^2}$. So, the position vector of P can be represented by this vector $r = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$.

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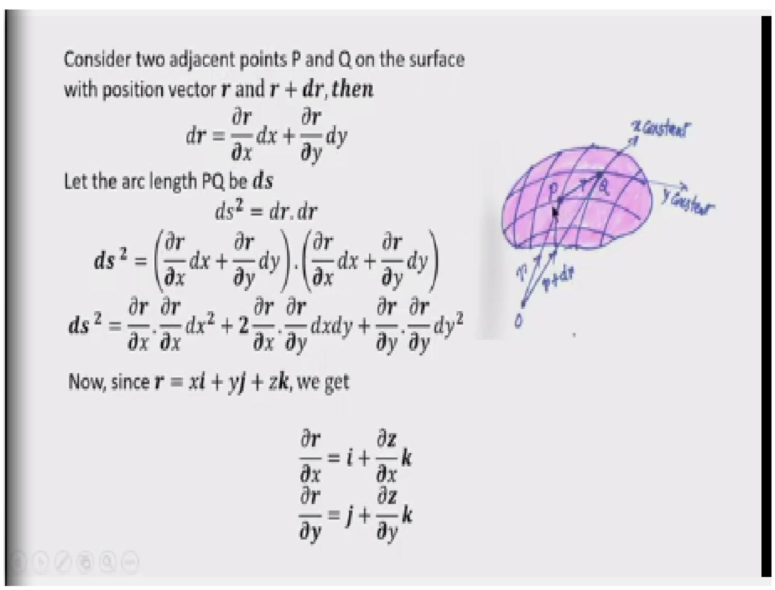

Now, let us see if I can express the dr the small distance from P to Q with the help of the derivative of the position vector then we will be able to find the arc length. So, we can write the vector $dr = \frac{\partial r}{\partial x}dx + \frac{\partial r}{\partial y}dy$, where the vector r is differentiated that is the partial differentiation is done here with respect to y and here the vector r is differentiated with respect to x.

Remember that r is a vector which contains 3 variables in 3 dimensional space that variables are x, y, z. Now let the arc length PQ is ds. Now, here PQ is shown as a straight line, it is a chord actually but arc is ds. Now we can represent that $ds^2 = dr.dr$, that is the dot product of this vector PQ and PQ that is 2 vector when this is making a dot product of the 2 vector, then it becomes a scalar quantity.

So, $dr.dr$ is a scalar quantity which is nothing but ds^2. Now let us substitute dr as $\frac{\partial r}{\partial x}dx + \frac{\partial r}{\partial y}dy$. Here partial differentiation that is with respect to r, we use the symbol ∂. So, ∂ here represents the partial differential operator and here dx is the small length of the element along the x direction and dy is the small length of the element along the y direction. So, dot product is done here.

And now if I expand this product, that means term by term multiplication if I carry out, then I will get $\frac{\partial r}{\partial x}.\frac{\partial r}{\partial x}$ and dx, dx these are scalar quantities, so dx^2 plus here you will get $\frac{\partial r}{\partial y}.\frac{\partial r}{\partial y}dy^2$. But when you multiply or make a dot product with this and this $\frac{\partial r}{\partial x}.\frac{\partial r}{\partial y}$ and $dx\ dy$. Similarly, again when you make a dot product with this term $\frac{\partial r}{\partial y}$ with this $\frac{\partial r}{\partial x}$ then again this term will be coming.

And you know that by the property of the dot product that is a scalar product $a.b = b.a$. So, therefore we can whatever way you multiply it, then it will be this $\frac{\partial r}{\partial x}.\frac{\partial r}{\partial y}$ is also equal to $\frac{\partial r}{\partial y}.\frac{\partial r}{\partial x}$. But this is not true if I take the cross product of the vector. So, now since $r = xi$, $\boldsymbol{i}$ is the unit vector, yj, $\boldsymbol{j}$ is also a unit vector along the y axis and zk, $\boldsymbol{k}$ is a unit vector along the z axis.

Then x, y, z is the coordinate of P, then if I differentiate this because I have to use this derivative of this r in this expression. Therefore, we should find out the derivative of this expression r and we want to substitute here to find the complete expression of ds^2. Now if you differentiate this r

vector with respect to x then it becomes say here it $\boldsymbol{i}$, because if you differentiate with dx by this with respect to x this becomes 1. And therefore this coefficient is $\boldsymbol{i}$, $\boldsymbol{i}$ is the unit vector along the x axis, the magnitude of $\boldsymbol{i}$ is 1 and direction of $\boldsymbol{i}$ vector is along the x axis. Then if I differentiate this quantity with respect to x, then this differentiation will be 0 because the y is here to be treated as a constant, when it is differentiated with respect to x. Then this becomes $\frac{\partial z}{\partial x}k$ because z is a function of x and y, so when we differentiate we treat this y is a constant and therefore we differentiate $\frac{\partial z}{\partial x}$.

Similarly, when we differentiate this r vector with respect to y, we will be getting this $\frac{\partial r}{\partial y}$ equal to say here again x if I differentiate this quantity with respect to y, then this quantity will be 0. And therefore only this quantity will be coming say $\boldsymbol{j} + \frac{\partial z}{\partial y}k$ because z is a function of x and y. So, we are now getting these 2 vectors here $\frac{\partial r}{\partial x}$ and $\frac{\partial r}{\partial y}$. So, this can be substituted now here to again perform the dot product and then get the full expression.

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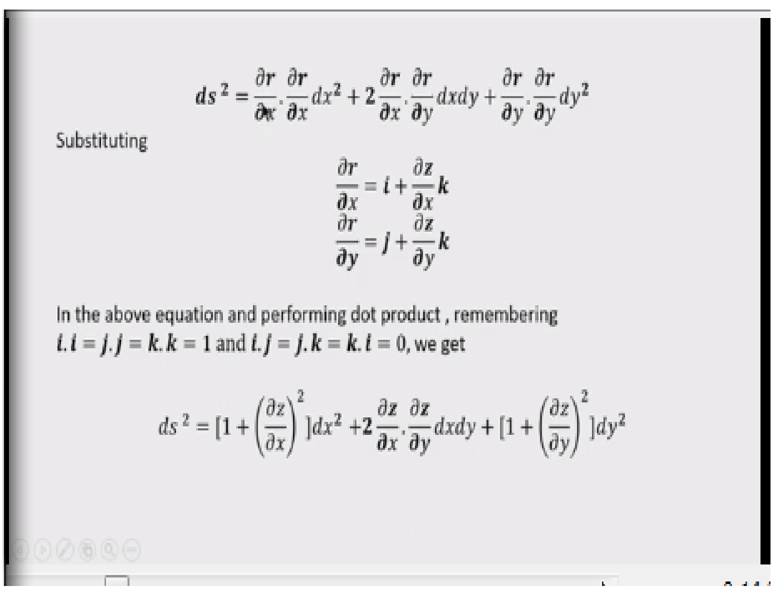

So, now this $\frac{\partial r}{\partial x}$ this is the expression that I have shown, here we have obtained by carrying out term by term multiplication and then substituting this derivative in this expression. So, when we substitute this say $\frac{\partial r}{\partial x}$ that is $i + \frac{\partial z}{\partial x}k$ dot again $i + \frac{\partial z}{\partial x}k$. So, we have to make the dot product of this quantity again with this quantity, that is $\frac{\partial r}{\partial x}.\frac{\partial r}{\partial x}$.

So, here what I mean that we have to use the property of the dot product because $i.i = j.j = k.k = 1$. Because by definition of dot product or scalar product, suppose 2 vectors are considered for dot product, say a vector and b vector lying in a plane and the angle between the vector a and b is say θ. Then we get by definition of dot product that $a \cdot b = |a||b|$ × cosine of the angle between these 2 lines.

Now since this $\boldsymbol{i}$ vector is along the x direction or a line parallel to the x direction, similarly here also. So, when we make a dot product say with $i.i$, then the angle between them will be 0. So, cos0 is 1 and magnitude of $\boldsymbol{i}$ vector is 1, so therefore this quantity is 1, similar is the $j.j$ and $k.k$. So, when we carry out the dot product of $i.i$ or $j.j$ or $k.k$, this product will be 1.

Now when we carry out the dot product of $\boldsymbol{i}$ and $\boldsymbol{j}$ vector, so $i.j$. Now what is this? $\boldsymbol{i}$ vector is along the x axis and $\boldsymbol{j}$ vector is along the y axis, so angle between them is 90°. So, by definition of dot product that 2 vectors magnitude will be there and then cosine of the angle between the 2 vectors. Now cos90 is 0, so therefore $i.j$ is 0, similarly $j.k$ and $k.i = 0$.

Now using this property, we can now come to this expression, $ds^2 = [1 + \left(\frac{\partial z}{\partial x}\right)^2]dx^2 + 2\frac{\partial z}{\partial x}.\frac{\partial z}{\partial y}dxdy + [1 + \left(\frac{\partial z}{\partial y}\right)^2]dy^2$. So, this is the arc length relationship ds^2. And this is found from the vector relation that I have discussed in the previous slide. And this is a scalar quantity now, where $\frac{\partial z}{\partial x}$ represents the slope of the curve along the x direction and $\frac{\partial z}{\partial y}$ represent the slope of the curve along the y direction.

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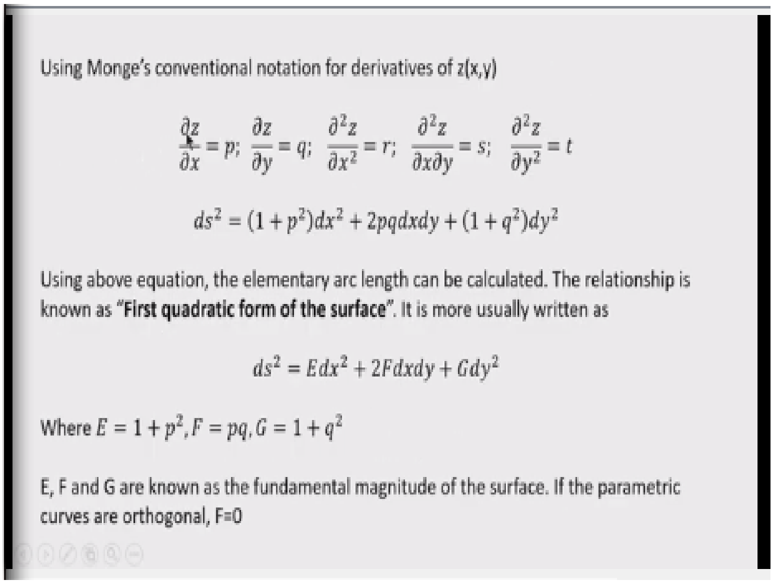

So, further denoting this $\frac{\partial z}{\partial x}$ by p, there is a convention that is used by one author Monge's in Shell structure and this convention is followed in differential geometry to represent the parametric form of the surface. So, we generally represent the slope by p, so $\frac{\partial z}{\partial x}$ as p, $\frac{\partial z}{\partial y}$ the slope of the curve along the *y* direction as q and this is the curvature of the curved line that is $\frac{\partial^2 z}{\partial x^2}$ and it is denoted by *r*. And curvature along the *y* direction $\frac{\partial^2 z}{\partial y^2}$ is denoted by *t* and $\frac{\partial^2 z}{\partial x \partial y}$, it is the twist curvature that is denoted by *s*. So, therefore ds^2 that we previously obtained as $[1 + \left(\frac{\partial z}{\partial x}\right)^2]dx^2 + 2\frac{\partial z}{\partial x}.\frac{\partial z}{\partial y}dxdy + [1 + \left(\frac{\partial z}{\partial y}\right)^2]dy^2$, now it can be written as $\left(1 + p^2\right)dx^2 + 2pqdxdy + \left(1 + q^2\right)dy^2$. So, expression in this form is known as first quadratic form of the surface.

So, ds^2 is written sometimes using certain notations say $Edx^2 + 2Fdxdy + Gdy^2$, where $E = 1 + p^2$, $F = pq$, $G = 1 + q^2$. *E*, *F*, *G* are known as the fundamental magnitude of the surface and if the parametric curves are orthogonal then we can see that *F* will be 0. So, defining a point on the curves, we can find the fundamental magnitude of the surface.

Fundamental magnitude of the surface is *E*, *F* and *G* and *E* is related to the slope in the *x* direction that is $1 + p^2$, *F* is related to the product of the 2 slopes, that is slope in the *x* direction multiplied by slope in the y direction. And G is again related to the product of the slope in the y direction that is $1 + q^2$. So, now we define the slope and curvature as well as twist curvature of the surface at any point.

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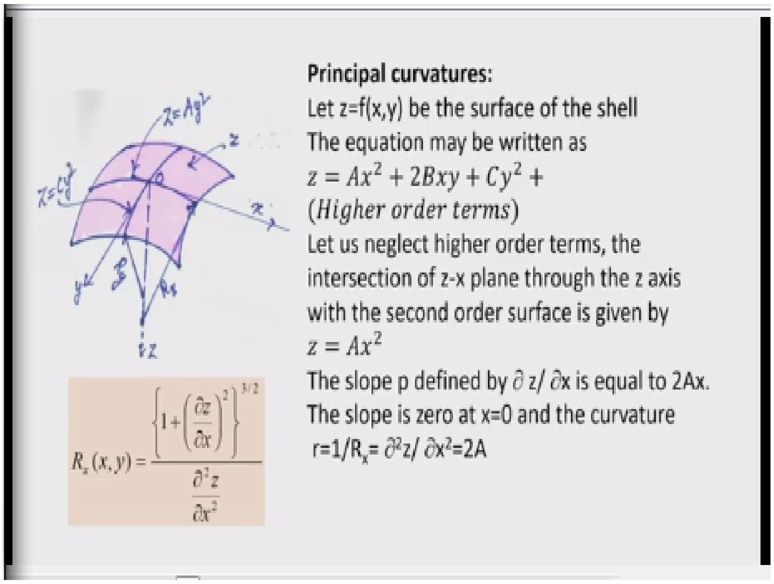

Based on that, now let us define the principal curvatures. The principal curvatures will be useful to classify the shell. Now let $z=f(x,y)$ be the equation of the surface of the shell or the equation of the shell. Actually we will not call this as a plane because it is a curved structure, so therefore it is called as a surface and this is a curved surface. So, $z=f(x,y)$ if it is expressed in the polynomial form, say generally it is a curve, so it will be a nonlinear function.

So, let us express this in terms of polynomial, say first I am writing a quadratic polynomial say $Ax^2 + 2Bxy + Cy^2$. Then I can also add higher order terms to that, so z is represented here with the help of polynomial. Now let us neglect for simplicity the higher order terms and then proceed to calculate the principal curvatures. Now you can see the radius of curvature at any point x, y on the surface of the curve say which is parallel to the x direction can be found out by this formula.

That is well known in the differential calculus $\left\{1+\left(\frac{\partial z}{\partial x}\right)^2\right\}^{3/2} \Big/ \frac{\partial^2 z}{\partial x^2}$. So, this gives the radius of curvature of the curve that is aligned along the x direction. Similarly, we can find this radius of curvature along the y direction. So, in that case we have to substitute here $\frac{\partial z}{\partial y}$ and here $\frac{\partial^2 z}{\partial y^2}$.

Now let us neglect for simplicity the higher order terms, the intersection of z-x plane, what is z-x plane? z-x plane, this is the x axis and this is z axis, so z-x plane you can see that this plane is z-x plane and it intersects the surface. Now, in this z-x plane the y coordinate is 0, so therefore this equation of the surface is represented by $z = Ax^2$ if we consider only this quadratic forms. Now for that plane the slope p of the curve is defined as $\partial z / \partial x$ and it is equal to if I differentiate this quantity $\partial z / \partial x$ will be $2Ax$.

Now you can see because this is O is a crown that is the apex of the shell structure, here the slope is 0 along the x direction, similarly along the y direction it is also 0. So, these 2 curves are orthogonal here, so therefore you can see the slope at x direction is 0. That is here we can say at $x = 0$ at origin, if we substitute here x, 0 then the slope is 0, so slope is $2Ax$. So, if I substitute $x = 0$ in this equation, then $\partial z / \partial x = 0$, that means p is 0.

Let us now find the curvature. So, curvature is nothing but reciprocal of the radius of curvature, so if the radius of curvature is denoted by capital R_x, then curvature r that is along the x direction is defined as $1/R_x$. And $1/R_x$ is nothing but, if we consider the small quantity that is the slope is small then this quantity can be neglected $\partial z / \partial x$ and then we get this $r = \partial^2 z / \partial x^2$. So, $\partial^2 z / \partial x^2$ is nothing but $2A$, so we are getting this r.

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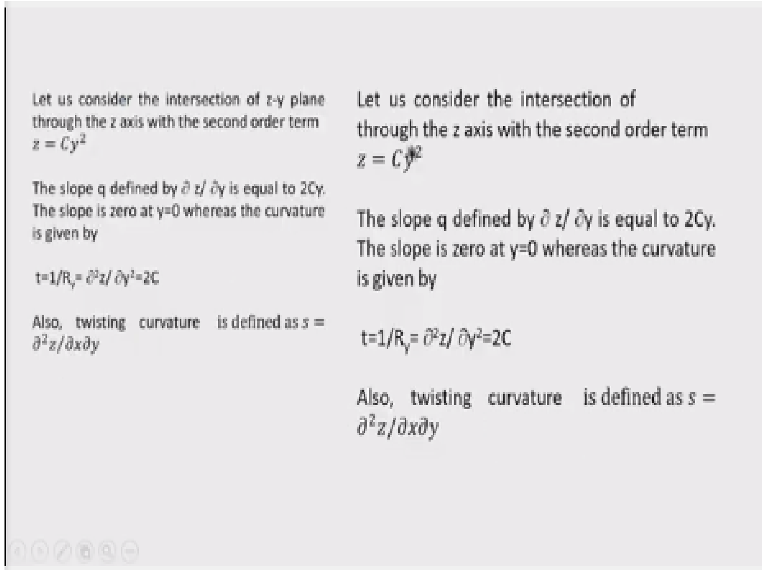

If I now consider the *z-y* plane through the *z* axis, that *z-y* plane is *z* and *y* along the *z* axis. Then in this plane the *x* is 0, so therefore here the equation will be represented by $z = Ay^2$ instead of *x* it will be y^2. And therefore the *q* that is the slope along the *y* direction will be just $1/R_y$ and it will be nothing but this. Instead of here equation will be $z = Cy^2$, Cy^2 will be the equation and therefore slope will be 2*Cy* and at 0 that at O, the slope is 0 and therefore curvature will be 2*C*. So, we get slope and curvature in both the orthogonal direction.

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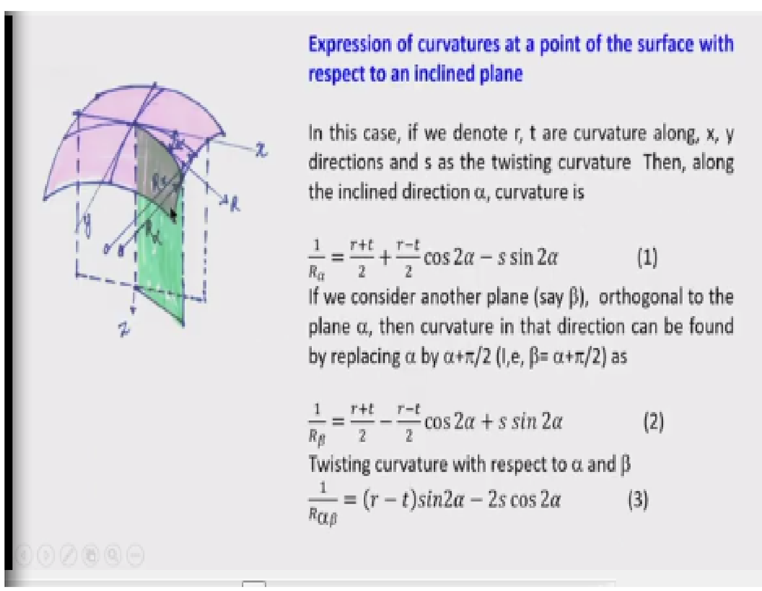

Now based on that we now go to find the expression for curvature with respect to an inclined plane. Let us consider the inclined plane which is at in an inclination of α. First we have considered this plane say *z-x* plane and we have found out and then we consider the *z-y* plane, we have found out the slope and curvature. Now consider the inclined plane that is shown by this green colour and the plane is inclined at an angle α with the original, this is the plane this *x-z* plane that is the original plane *x-z* plane. Now here the axis is oriented along the other direction, so this orientation here axis maybe *x'* axis. So, in this case if we denote *r*, *t* as the curvatures along *x*, *y* directions and *s* as the twisting curvature, twisting curvature is $\frac{\partial^2 z}{\partial x \partial y}$. Then along the inclined direction α, curvature is $\frac{1}{R_\alpha} = \frac{r+t}{2} + \frac{r-t}{2}\cos cos\, 2\alpha\ - s \sin sin\, 2\alpha$, this equation comes from the transformation of the derivative along the new direction.

So, new direction is the direction α, so therefore if this direction is denoted by say *n* direction, we can find we know the operator $\frac{\partial}{\partial n}$ in terms of cosine of the angle and sine of the angle and with respect to the differential operator with respect to *x* and *y* coordinate. So, when we know this operator then again when it is used to differentiate the slope $\frac{\partial z}{\partial x}$ or $\frac{\partial z}{\partial y}$ or here actually we have to differentiate with respect to new direction.

So, if new direction is α, that means we have to find the slope along the direction α, so $\frac{\partial z}{\partial \alpha}$ and $\frac{\partial^2 z}{\partial \alpha^2}$. So, therefore making this transformation of the derivatives in other directions, we can get this quantity $\frac{1}{R_\alpha} = \frac{r+t}{2} + \frac{r-t}{2}\cos cos\, 2\alpha\ - s \sin sin\, 2\alpha$. Now if I consider another plane say β, what is β plane? β plane is a plane which is perpendicular to the plane that is at an angle of α with the original *x-z* plane.

So, in that case if I just substitute β with $\alpha + \pi/2$, that is β is a new plane but new plane is orthogonal to this inclined plane. So, this quantity can be directly found out if I substitute α with

$\alpha + \pi/2$. Now you can see when the α is substituted with $\alpha + \pi/2$, then it becomes $\cos(2\alpha + \pi)$, so $\cos(\pi + 2\alpha)$. $\cos(\pi + 2\alpha)$ is nothing but - $\cos(2\alpha)$, so this term is appearing with negative sign.

Similarly, this $\sin(\pi + 2\alpha)$ and therefore this is also appearing with negative sign and then here it becomes positive. So, we have got this R_α and R_β, β is a direction which is perpendicular to the direction of α. Now twisting curvature similarly can be found out and it is denoted by $\frac{1}{R_\beta}$ is nothing but $(r - t)sin2\alpha - 2s\cos cos\ 2\alpha$.

So, knowing the value of this curvature along any arbitrary direction α and a direction normal to the direction, previous direction α that is the new direction β, we can find the direction of principal curvatures. And here of course R_β is the twisting curvature. Now we know that in the direction of the principal curvatures the twist curvature must be 0.

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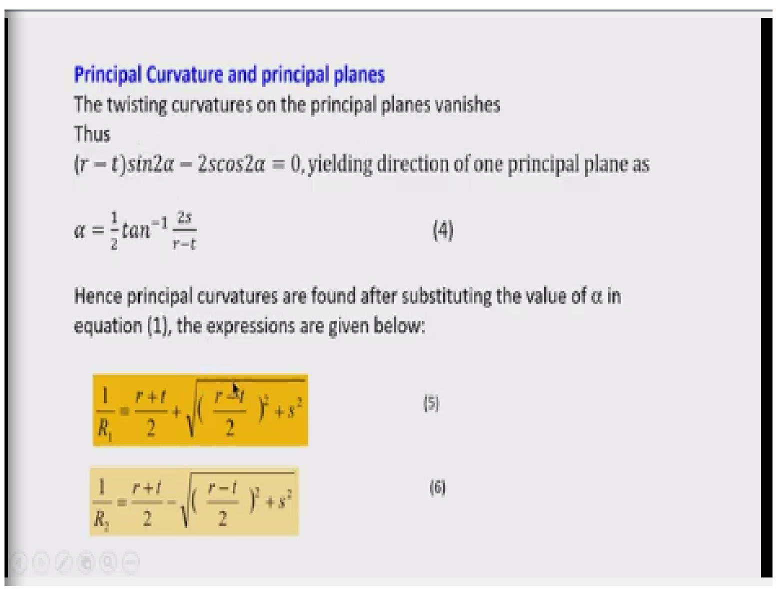

So, based on that twist curvature on the principal planes vanishes and therefore we can write this expression or we can equate this expression to 0. So, $(r - t)sin2\alpha - 2scos2\alpha = 0$, if I make this equation then I will get $tan2\alpha = \frac{2s}{r-t}$. So, that is $\alpha = \frac{1}{2}tan^{-1}\frac{2s}{r-t}$. So, α is a direction that is the desired direction of the principal planes where there will be no twist curvature.

This is analogous to the principal stresses that we have seen in the plate plane stress problem. So, now substituting α in this expression we can find the principal curvatures. Substituting α here and after simplification we can find 2 principal curvatures $1/R_1$ and $1/R_2$. $1/R_1$ is the major principal curvature that is $\frac{r+t}{2}+\sqrt{\left(\frac{r-t}{2}\right)^2+s^2}$ and $1/R_2$ will be $\frac{r+t}{2}-\sqrt{\left(\frac{r-t}{2}\right)^2+s^2}$.

So, 2 principal curvatures are found and one of the principal plane contain this curvature and other principal plane will be orthogonal to this plane and will contain the another principal curvature and there will be no twist curvature in the principal planes.

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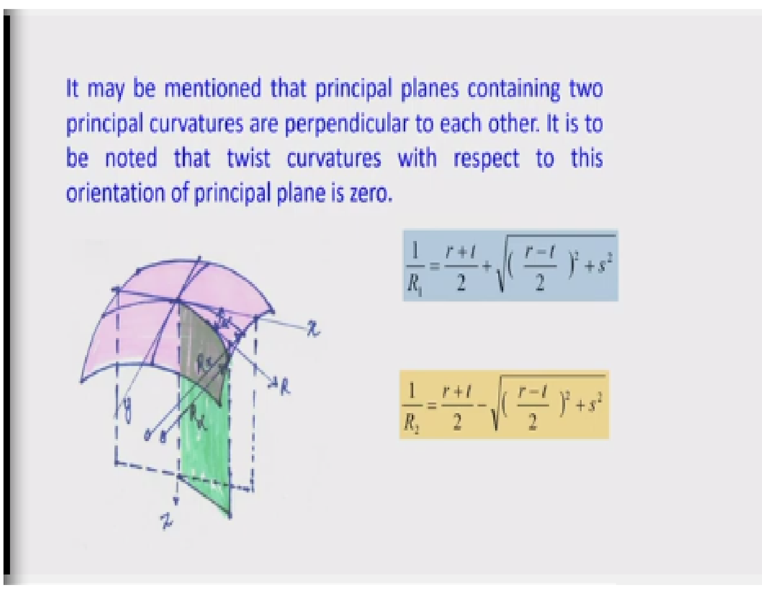

It may be mentioned that principal planes containing 2 principal curvatures are perpendicular to each other. And it is to be noted that twist curvature with respect to this orientation of principal plane will be 0. So, it is clear here if α is the principal direction, then this plane will contain no twist curvature and another principal plane will be orthogonal to this plane which contains other principal curvature.

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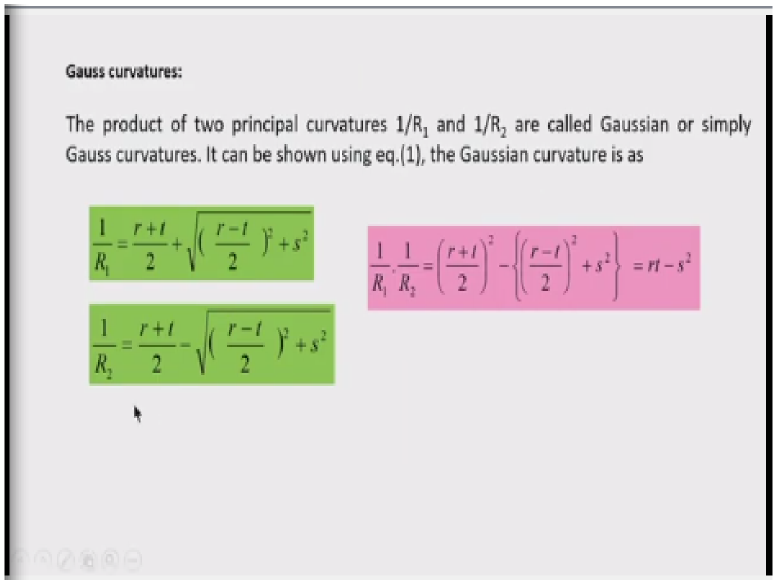

Now let us define a term based on the principal curvature applicable to the shell surface. If I see that $1/R_1$ is a principal curvature and $1/R_2$ as another principal curvature and get the product of the 2 principal curvatures which is called as Gaussian curvature or simply Gauss curvature. So, if I make a product of $1/R_1$ and $1/R_2$, I will get this $\left(\frac{r+t}{2}\right)^2 - \left\{\left(\frac{r-t}{2}\right)^2 + s^2\right\}$. So, after simplification you will get $rt - s^2$. So, Gaussian curvature is nothing but $rt - s^2$. What is *r*? *r* is actually the curvature along the *x* direction and *t* is the curvature along the *y* direction and *s* is the twist curvature. So, knowing this we can find the Gauss curvature. Now Gauss curvature is an important parameter which is used to classify the shell in different categories.

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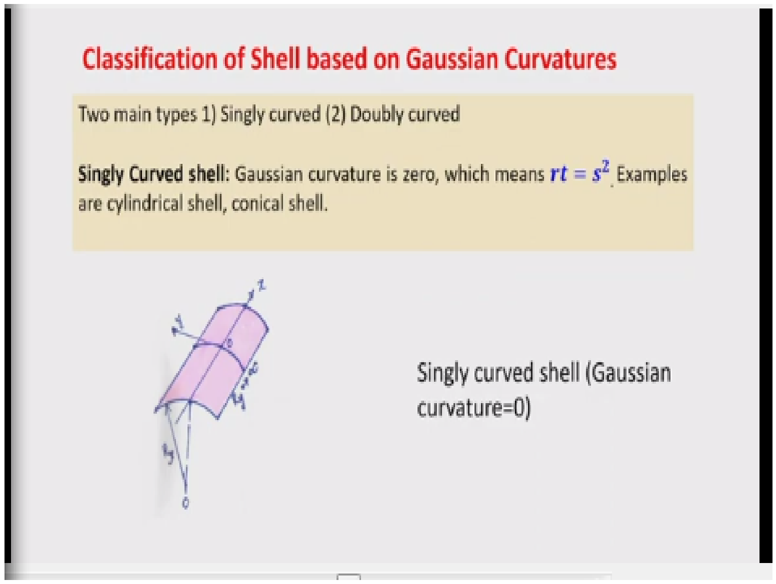

So, we shall now discuss the classification of shell based on the curvature and specially the parameter Gauss curvature is used to classify the shell. Now broadly the shell is classified as singly curved shell and doubly curved shell. What is singly curved shell? Singly curved shell has a curvature in one direction; in other direction the curve has no curvature. So, that means like a say it is a cylindrical surface, in the direction this y direction you will get the curvature but along the x direction you get this, this is a straight line. So, therefore 1 curvature because this is the straight line, so radius of curvature is infinity, so hence the curvature is 0. So, therefore for such shell that is which is singly curved shell like a cylindrical shell, the Gaussian curvature is 0 that means $rt = s^2$. Now we have found out this Gaussian curvature this is $rt - s^2$, so if I make this quantity to be 0, then $rt = s^2$. That means product of the curvature in the 2 orthogonal direction is equal to the square of the twist curvature. So, this is an example of singly curved shell, Gaussian curvature is 0.

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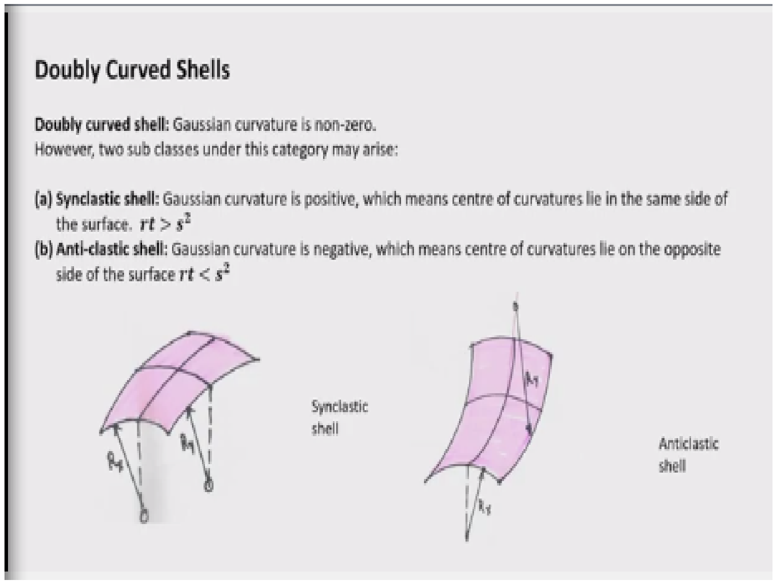

Now let us come to the doubly curved shell. doubly curved shell the Gaussian curvature is non zero. Now if I see the expression of this Gaussian curvature, there is $rt - s^2$, that is the result of this product $1/R_1 \times 1/R_2$. Now in one case $rt = s^2$, that is the Gaussian curvature is 0, we classify the shell as singly curved shell. So, example I have shown that it is a cylindrical shell or it may be a conical shell also.

So, here if $rt - s^2$ is non zero quantity, 2 cases may arise, in one case $rt - s^2 > 0$, in another case $rt - s^2 < 0$. So, let us discuss what happens to the type of shell in these 2 cases? So, in one case first let us consider $rt > s^2$. So, in that case we will find that Gaussian curvature is positive which means that centre of curvature lie in the same side of the surface, say this is the shell surface. So, the shell surface has curvature in both the direction and therefore surfaces like that this radius of curvature or centre of curvature of the radial line lies in the same direction. So, it may be either convex or it may be either concave, such type of shell is known as synclastic shell.

So, now let us come to the other type where rt is less than s square. So, what is *rt*? *rt* is the product of curvature in 2 principal directions and s is that twist curvature. So, if this quantity $rt < s^2$, that means Gaussian curvature is negative in that case. Then we have a shell like that, that means for curve in one direction, we have the centre lying in one side and for the curve in other direction, the centre will be lying on the opposite side. So, this type of surface is known as anti-clastic shell.

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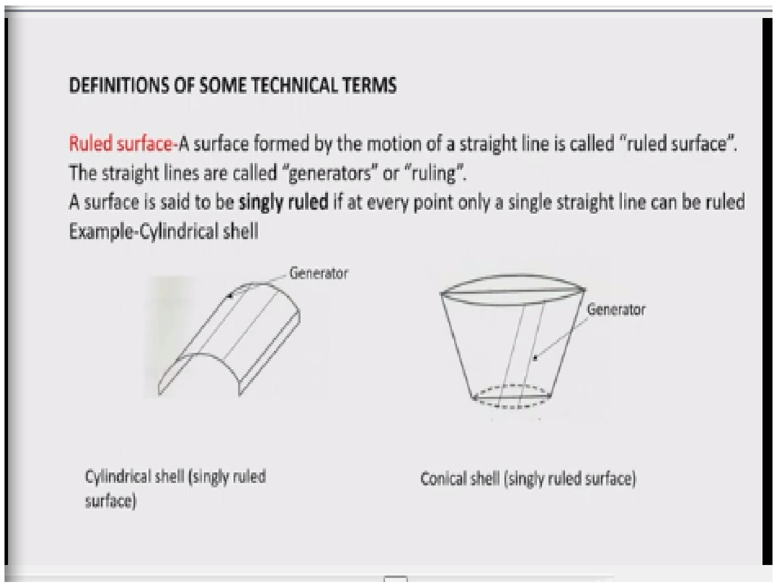

Now these 2 are the main classification of shell but there are other terms which are also used to group the shell into different categories but this will also fall under these 2 categories. These 2 categories that I told singly curved and the doubly curved shell are the 2 main types. Now let us define some technical terms which will be used to define or group the shell into different types. First let us define ruled surface; a surface is formed by the motion of straight lines over a motion of straight line along a curve.

So, this is a curve for example and this curve is called as directrix and these straight lines are called generator. So, when the generator moves over the directrix, a surface is formed and that surface is known as ruled surface. So, straight lines are called generators or ruling, so straight line has to be moved or straight line has to be ruled along the curve. So, this is the example of ruled surface that is a cylindrical shell which is also a singly curved shell, falling under the category of singly curved shell.

Now here you see a frustum of a cone that is also shell of singly curved group, why it is a singly curved group? Because the curvature along the one direction is only important because in the other direction it is a straight line, so generator is also a straight line which is moved revolved around the circle to form a cone. So, therefore it is also a singly curved shell and single straight line is used to form the ruled surface, therefore this type of shell is known as singly ruled surface.

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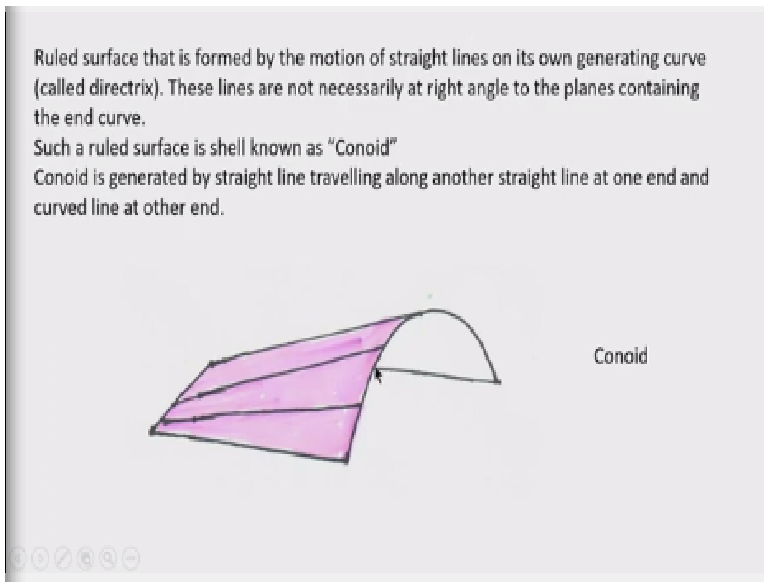

Ruled surface that is formed by the motion of straight lines on its own generating curve is known as directrix. These lines are not necessarily at right angles to the planes containing the end curve. So, there will be 1 end curve, and there will be 2 end curves are there for the shell and the generating straight lines may not be at the right angle. In the previous case we have seen that generating curves are at right angles to the end curves.

But in that case the generating curves are not at right angles to the end curves. And such type of shell is known as a conoid, this is also ruled surface. But one end of the generator lies on a straight line and the other end rules over a curve. So, conoid is generated by straight line, travelling along another straight line at one end. That is at one, there should be 1 straight line and at the other end, there will be 1 curved line. So, that type of shell is known as conoid, it is also a singly curved shell.

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The surface is called doubly ruled, if at every point 2 straight lines can be ruled, examples are a hyperboloid of revolution. For example, this type of shell is specially seen in case of cooling tower. And at every point you can see the 2 straight lines can be ruled. So, 1 straight line is in this direction and 1 straight line in this direction. So, these forms surface which is known as hyperboloid of revolution, so that is doubly ruled surface.

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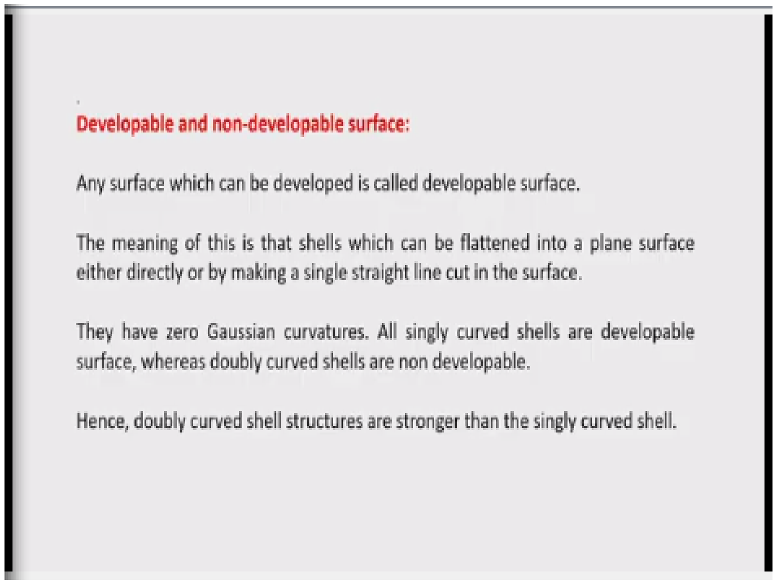

Now there are other classes which are known as developable and non developable surface. Now definition of developable surface is that, any surface which can be developed is called developable surface. The meaning of this is that, shells which can be flattened into a plane surface either directly or by making a single straight line cut in the surface. For example, you take a cylindrical shell and you make a cut along the length and then you apply pressure, the cylinder will be flattened.

So, this type of shell has Gaussian curvatures, say all developable shells have Gaussian curvatures. And all singly curved shells are falling into the category of developable surface. Whereas doubly curved shells are non developable, doubly curved shells cannot be flattened like that. So, hence doubly curved shell structures are stronger than the singly curved shell.

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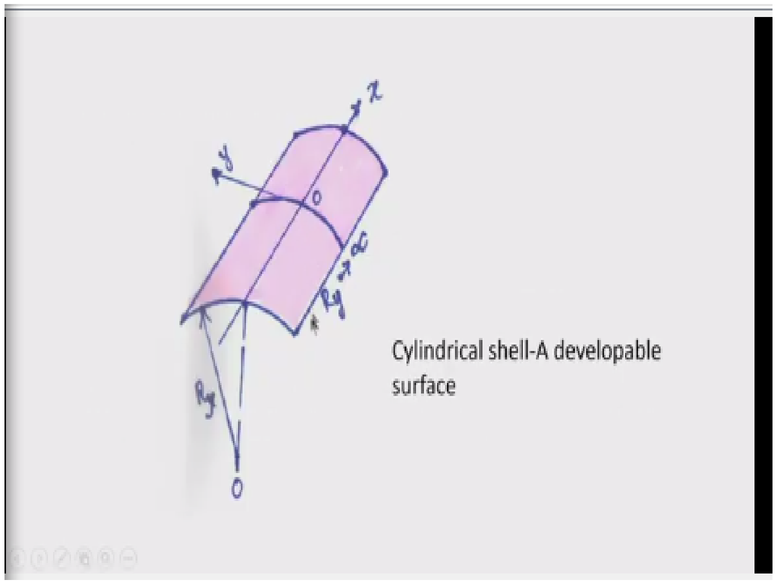

So, cylindrical shell is an example of developable surface.

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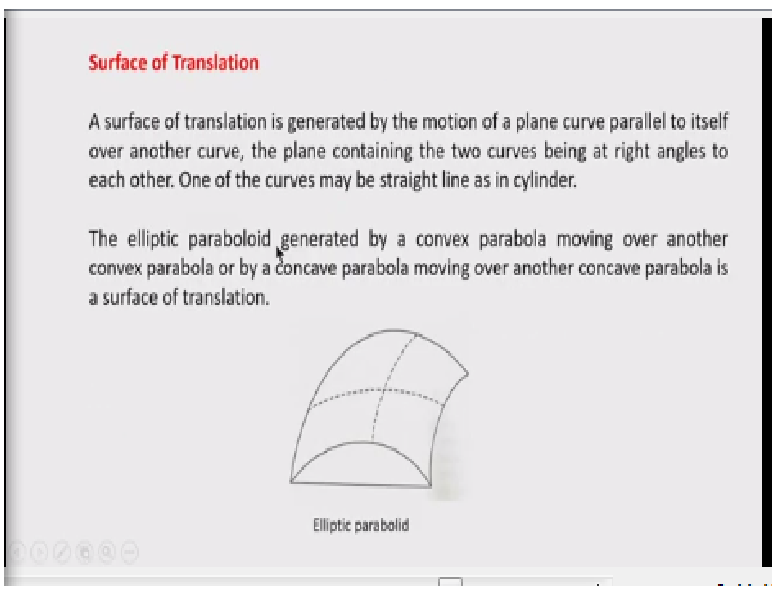

Now let us see another class of shell, is a surface of translation, but this is also falling under the head of doubly curved shell, that is the 2 main categories singly curved and doubly curved. Now ruled surface and surface of translation, there is some difference. Ruled surface is formed by ruling of a straight line, but here surface of translation is generated by the motion of a plane curve parallel to itself over another curve, the plane containing the 2 curves being at right angles to each other. Here that special case may also arise that one of the curves may be straight line as in cylinder. So, surface of translation is also included in the ruled surface, if one of the curve is a straight line. So, ruled surface is a special category of surface of translation, where 1 curve is a straight line. But, here in this example I have shown that 2 curves are used to form the surface of translation. That is 1 curve is moved over the another curve, this surface is generated by the motion of a plane curve parallel to itself over another curve. The plane containing the 2 curves being at right angles to each other and such type of curve you are seeing here, that elliptic paraboloid generated by a convex parabola. Say this is a convex parabola, moving over another convex parabola. So, this parabola is moving over another convex parabola is forming the elliptic paraboloid. So, this is falling under the surface of translation.

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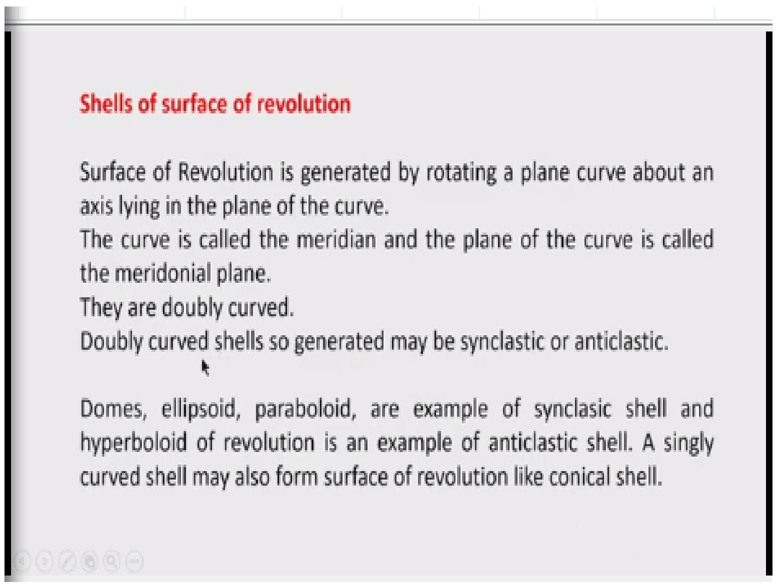

Now there is a very common type of shell which is known as shell of surface of revolution. Surface of revolution is generated by rotating plane curve about an axis lying in the plane of the curve.

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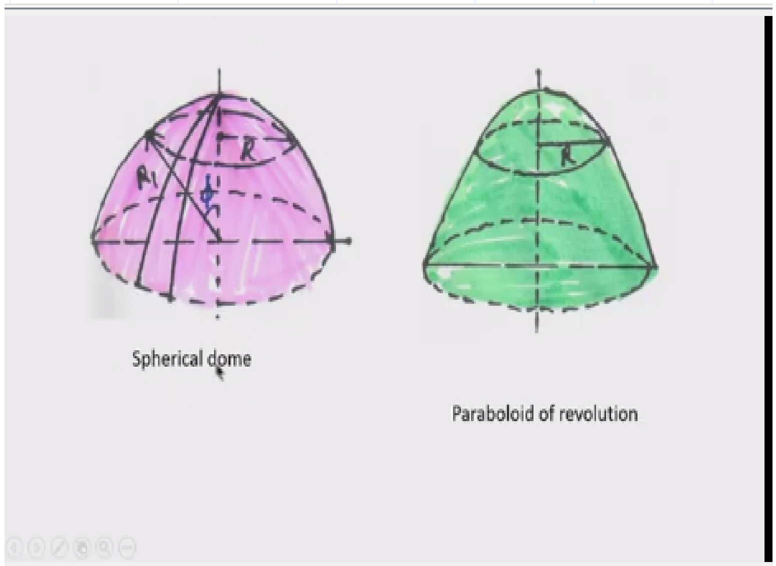

So, that means if I see here, this is 1 example of surface of revolution which is a spherical dome. So, this is the plane curve, it is an arc of a circle and this is rotated about an axis of revolution. Axis of revolution is this and then the surface formed is known as the surface of revolution. So, this is a shell of surface of revolution and it is formed by revolving a part of this circle, the plane curve which is rotating around this axis of revolution is known as meridian.

So, this is a meridional curve and such type of surface is known as spherical dome because this will be a part of a sphere. But on the other hand if a parabola is rotated, suppose this is a parabola, plane curve is parabola and it is rotated about the axis of revolution. Then the name of the shell will be paraboloid of revolution, so that is the difference. Similarly, the ellipsoid of revolution may be also there, if elliptical meridian is revolved around the vertical axis then the ellipsoid of revolution is formed.

The curve is called the meridian that is the plane curve according to which the name of the surface is given. So, for example when the plane curve is a part of a circle, the name of the shell will be this spherical dome, actually it is called spherical dome. And when the plane curve is a parabola, then the surface is known as paraboloid of revolution, when the plane curve is an ellipse then surface is known as ellipsoid of revolution.

So, these surface of revolution are always doubly curved shells and it may be synclastic or it may be anti-clastic also. Domes, ellipsoid, paraboloids are examples of synclastic shell and hyperboloid of revolution that we have seen in case of cooling tower, that example I have shown you. This hyperboloid of revolution, this is anti-clastic shell. So, the doubly curved shell that is surface of revolution maybe synclastic or maybe anti-clastic. Now here the picture that I have shown you are all synclastic shell.

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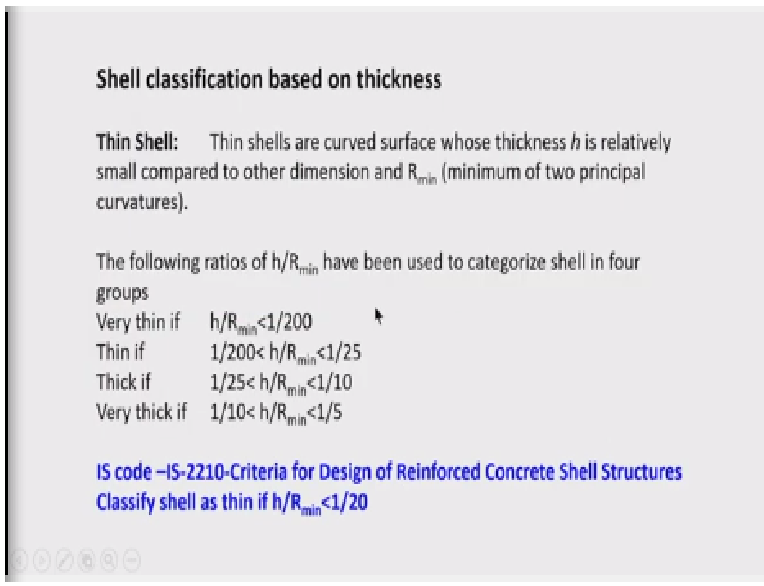

Now some authors prefer the shell classification based on thickness, but the shell classification based on geometry is mainly on curvature. Shell classification based on curvature is mainly recognized. Based on thickness, the shell is classified into 4 categories, very thin, thin, thick and very thick. Now how the categories are found? These categories are found based on this thickness to this minimum of the 2 principal curvatures.

So, R_{min} is the minimum of the 2 principal radius of curvature, is taken to find the ratio. So, if $h/R_{min} < 1/200$, we call this shell as a very thin shell. And when h/R_{min} lies between 1/200 to 1/25, then we call this as a thin shell. And then h/R_{min}, when it lies between 1/25 to 1/10 we call it thick shell. And this ratio h/R_{min} when it is lying between 1/10 to 1/ 5 we call it very thick shell.

The Indian code that is IS code, IS 2210 criteria for design of the reinforced concrete shell structure, classify shell as thin, if h/R_{min} is less than 1/20. So, in our theory of shell, we will mainly discuss the mechanics of the thin shell that deformation, strains and stresses, all will be related to the thin shell.

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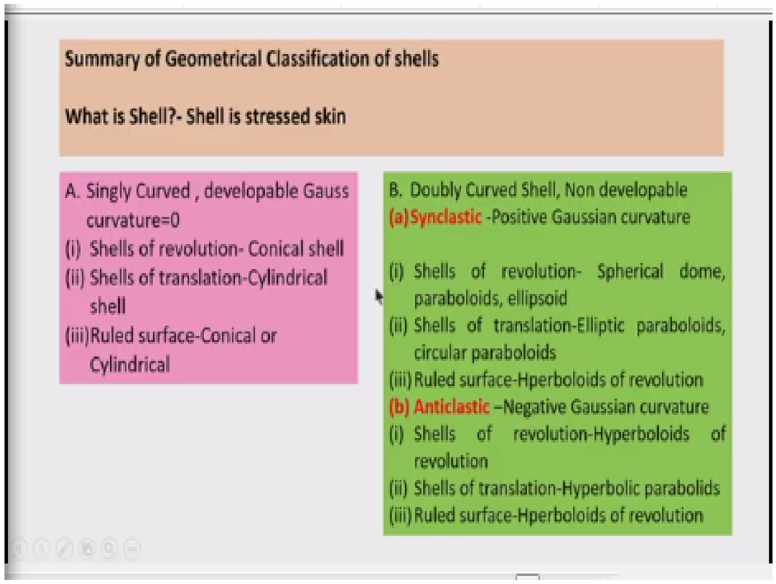

Let us summarize the classification of the shell. Now in this class, shell is classified based on the curvature, so geometrical classification I have given and shell I have defined as a stressed skin. Broadly 2 categories of classification, 2 classes singly curved and doubly curved. In singly curved shell, the Gauss curvature is 0, doubly curved shell Gauss curvature is non zero. But it may be positive, it may be negative, when it is positive, it is known as synclastic shell and when it is negative, it is known as anti-clastic shell.

So, in the singly curved shell, the surfaces that are formed are developable surface and Gaussian curvature is 0, so that is characteristics. In doubly curved shell the surface is non developable and the Gaussian curvature may be positive or maybe negative. Shells of revolution that is 1 type, conical shell or spherical dome all are shells of revolution. Shells of translation that is formed by moving a straight line over a plane curve.

That is straight line is called the generator and the plane curve is known as directrix. That type of shell I have given the example of cylindrical shell. Then ruled surface that is conical shell is also a ruled surface and this cylindrical shell is also ruled surface, so these are some of the examples. In the doubly curved shells, synclastic nature is found in case of spherical dome, paraboloid and ellipsoid.

Shells of translation is found in case of elliptic paraboloid, circular paraboloid. And ruled surface is found in case of hyperboloid of revolution. Anti-clastic shell, negative Gaussian curvature and shells of revolution maybe there in case of anti-clastic nature and these are hyperboloid of revolution. Shells of translation maybe there that is hyperbolic paraboloids and ruled surface is another category of anti-clastic shell that is found in hyperboloids of revolution.

So, today I have given the classification of shell based on geometry. I have introduced the geometrical parameters of the shell mainly the curvatures and how to find out the principal curvatures based on the curvature in 2 orthogonal direction and twist curvature. And then I use this one important parameter that is the product of 2 principal curvature known as the Gaussian curvature to classify the shell. So, 2 broad categories of shell are found, one is singly curved and another is doubly curved.

And singly curved shells are all developable surface and in singly curved shell we have seen that shells of revolution, shells of translation and ruled surface. Doubly curved shell may be of synclastic nature and anti-clastic nature and we have found in case of synclastic nature, shells of revolution, shells of translation, ruled surface. Anti-clastic nature which is characterized by negative Gaussian curvature, the examples are hyperboloids of revolution, hyperbolic paraboloid, hyperboloids of revolution. So, all doubly curved shells are non developable, thank you very much.