Plates and Shells
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Lecture - 18
Buckling Load of Rectangular Plate with Navier's Boundary Condition

Hello everybody, today I am continuing the buckling of thin plate and it is the lecture number 2 of the module 6. So, today I will start the buckling of thin plate. You are very much familiar with the buckling of columns in your undergraduate courses, but there is some difference between the buckling on column and the plate that I have told earlier also. Because when the column buckles that means buckling load up column is the failure load that is taken in design.

But in case of plate, we have seen that once they plate buckles the pole buckling strength is also higher. So, another important observation in buckling of plate is that when the column buckles into a waves that is when the critical load is reached. For example, a column with (())(01:31)), when the critical load is reached, then only one-half wave is formed just to transform the state configuration to bend configuration.

In that critical stage one, this half wave is form, but in the plate, it will be shown that due to its two way action it is not necessary that the number of half wave that should be formed at critical load is equal in two directions. So, that is the peculiarity of the buckling of plate and let us see the detail.

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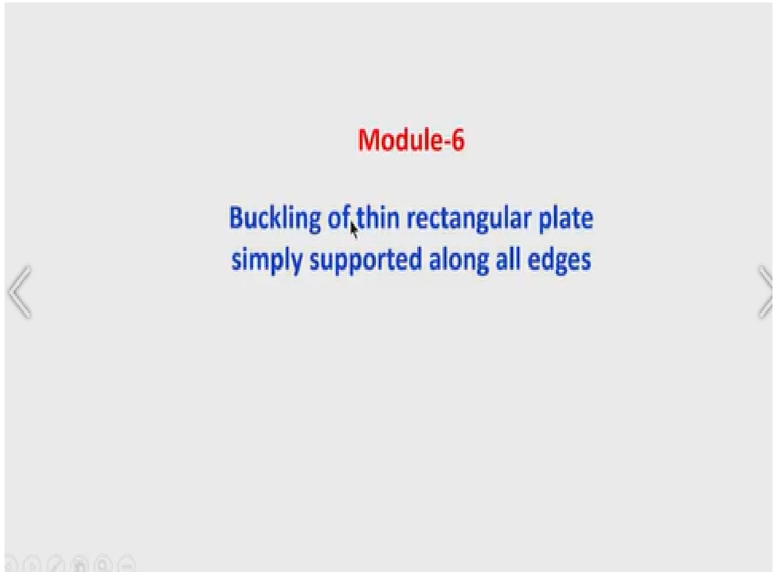

So, buckling of rectangular plate we have taken and for first case I have taken a simply supported boundary condition along the all edges. Now, when the rectangular plate is simply supported along all edges, that boundary condition is popularly known as Navier condition and Navier proposed a series solution for that. And therefore, the solution become very easy according to if we take the help of Navier double trigonometrical series.

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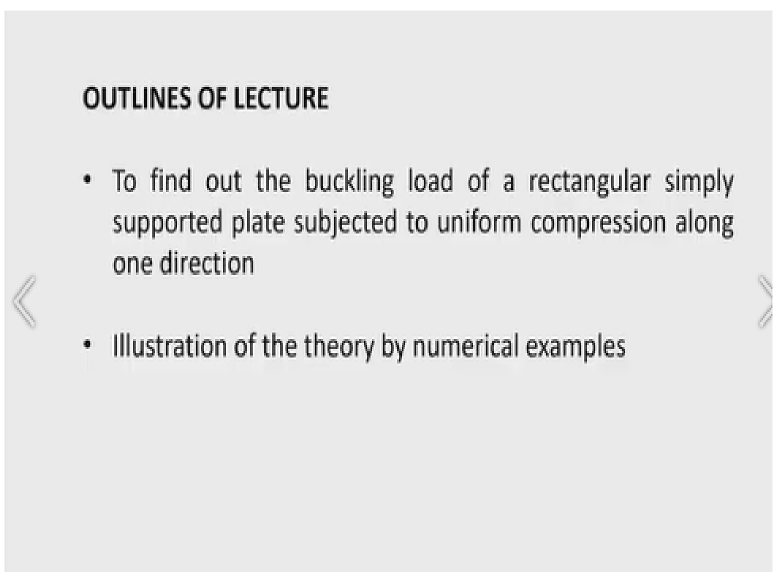

So, today's lecture, we will find out the buckling load of a rectangular simply supported plate subjected to uniform compression along one direction. Illustration of the theory by numerical examples and I will also demonstrate one problem when the plate is subjected to biaxial compression also. Although the illustration is done with this uniform compression, but the general formulation that I have derived in the earlier class can be used to solve a problem of rectangular plate with the biaxial compression as well as the shear loading also, membrane shear loading.

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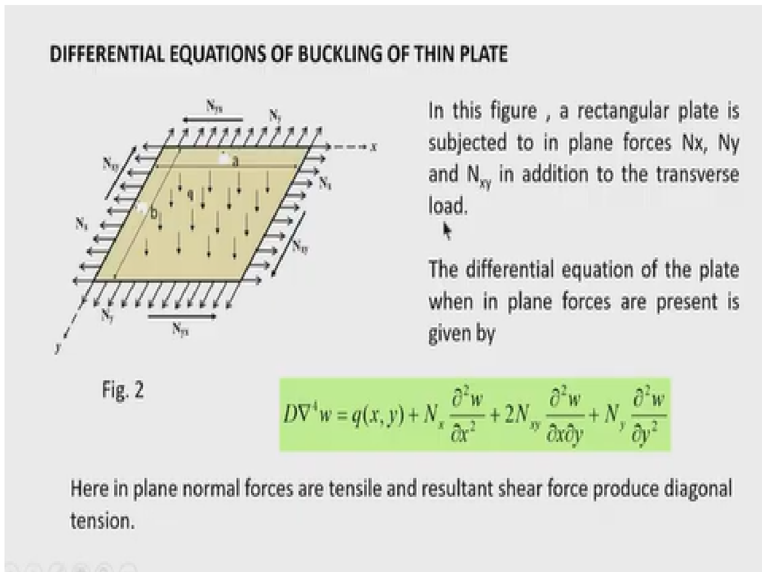

So, here you are seeing in this figure a rectangular plate is subjected to in plane forces N_x in the x direction, N_y in the y direction and N_{xy} is the membrane shear force acting along the edges. So, these are the membrane forces N_x, N_y and N_{xy} and the equilibrium equation connecting the membrane forces is known to you and it is similar to this plane stress equilibrium equation. Now, if I consider the action of in plane load edge as a fictitious load on the plate.

Then we write the equation of the plate subjected to simultaneous bending and stretching. That is the simultaneous action of this transverse load and in plane forces, then we can write it

$$D\nabla^4 w = q(x,y) + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}$$

Now here ∇^4 is the bi-harmonic operator that you will know it and then the again I am telling it. It will be $\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$

Now, here in plain normal forces are tensile therefore, we get this equation and resultant shear force produce diagonal tension. But in case of buckling, compression has to be there. So, therefore, we reverse the sign of in plane forces and we get the equation of buckling provided the transverse load is taken edge zero.

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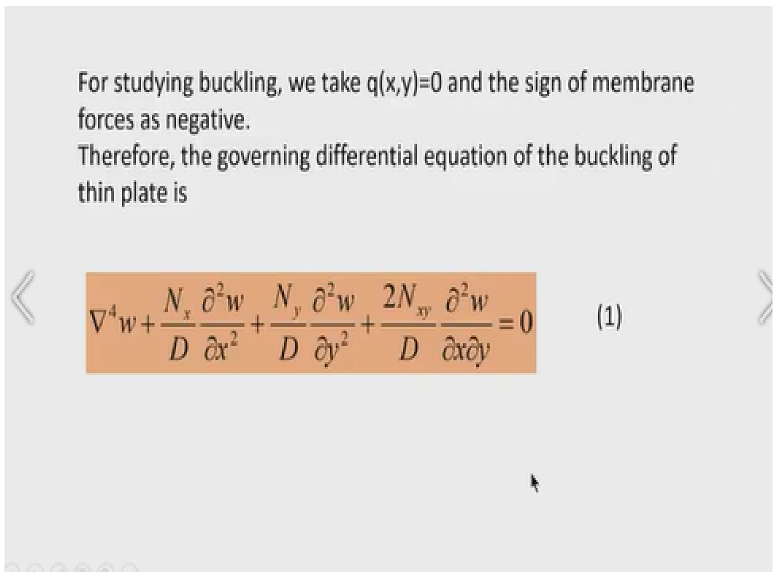

So, now, the equation for buckling is done, by taking q (x, y) = 0 in the differential equation, only a differential equation and the sign of the membrane forces are reversed. Previously it was taken positive now, it is taken negative. Therefore, the governing differential equation of the buckling of thin plate becomes like that $\nabla^4 w + \frac{N_x}{D}\frac{\partial^2 w}{\partial x^2} + \frac{N_y}{D}\frac{\partial^2 w}{\partial y^2} + 2\frac{N_{xy}}{D}\frac{\partial^2 w}{\partial x \partial y} = 0$

You can see in the earlier equation here in the right hand side N_x, N_{xy} and N_y were appearing. Now, if I take the sign of N_x, N_{xy} and N_y is negative, then it will be transferred as a positive quantity on the left-hand side. And if I divide both sides by the flexural rigidity of the plate D, then we get this equation. So, this equation is the equation of buckling of a thin rectangular plate of course, we have assumed the thin plate assumptions.

Now, to find the critical load on the plate. We shall first use this analytical method using the Navier series because Navier series is suitable for a plate which has all four edges is simply supported. Of course, there is another classical boundary condition or classical approach that is known as Levy's boundary condition that can also be applied to a plate which has all the four edges simply supported.

But in case of Levy's solution in the intermediate stage, you have to again solve a differential equation, fourth order differential ordinary differential equation to find the unknown function. Now, in the Navier's method in the intermediate stage, no differential equation has to be solved. So, this becomes easier compared to Levy's method but Navier's method, the double Fourier series is used.

Whereas, in; Levy's method, single Fourier trigonometrical series is used. Now, let us see how we can solve it.

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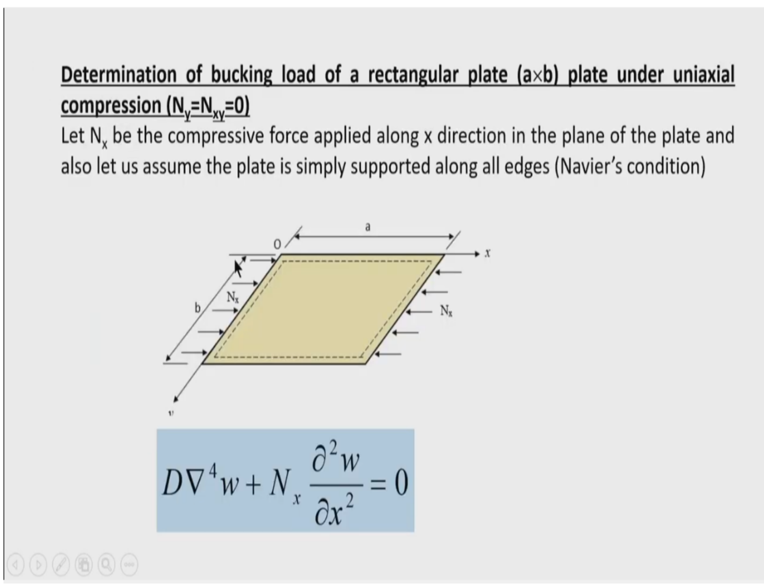

We have taken a problem case, where rectangular plate of dimension 'a' and 'b', 'a' is the length of the plate and 'b' is the width of the plate. So, that a/b is the aspect ratio, and it is subjected to uniaxial compression. Now, the forces compression so, previously I have derived the equation of governing differential equation with the tensile force you can see and we have got this equation. Now, in this buckling analysis.

We have used a compressive force N_x and for uniaxial compression, I assume that N_y and N_{xy} are 0. So, differential equation of the plate now becomes $D\nabla^4 w + N_x \frac{\partial^2 w}{\partial x^2} = 0$ …….. (2)

Now, N_x is a uniform compression along the edge and you can see this is taken as a constant it is not a function of these variables space variable x and y. So, therefore, this differential equation is a partial differential equation of fourth order with a constant coefficient.

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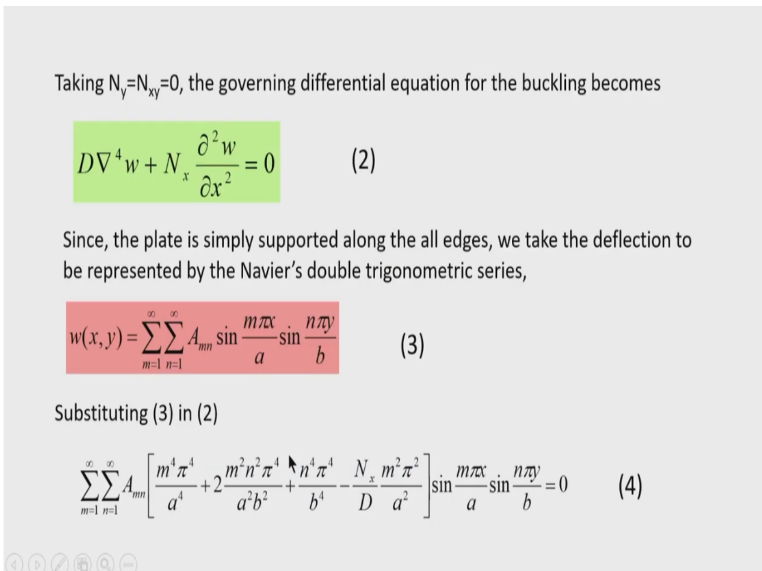

Now, taking the help of Navier double trigonometric series as expressed here as w (x, y)

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad …….. (3)$$

So, this is the double trigonometrical series, suggested by Navier for a solution of the plate, which has boundary condition along all edges as simply supported. So, this equation is taken to represent the deflection of the plate.

And it represents or it satisfies the boundary condition applicable in simply supported condition. So, let us see how it satisfy the boundary condition in simply supported plate. In simply supported plate and x = 0 if x = 0, edge is simply supported then w becomes 0. Again, if y = 0 edge is simply supported then w is 0. Now, for bending moment to be 0 at x = 0 or x = a or y = 0 or y = b.

We should take the second derivative of this equation with respect to x also with respect to y, then we can write the bending moment function as

$$M_x =- D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$

Then, if we substitute the second derivative of the function, again you will find that the terms $sin\frac{m\pi x}{a}$ $and\ sin\frac{n\pi y}{b}$ are appearing. So, naturally at x = 0 and x = a or y = 0 y = b the bending moment vanishes. So, this condition is exactly satisfied by Navier's double trigonometrical series. So, we take it as a solution of the differential equation of the plate for the buckling. Now, you can note here for buckling analysis we have not considered any transverse load because this is a homogeneous solution of the differential equation that will yield the buckling loads.

And it will after imposing the boundary conditions the buckling equation, the characteristic equation that forms is a homogeneous type of equation. And most of the cases you will find the equation is formed in the form of trigonometrical functions or hyperbolic functions and these are called the transcendental equations and solving of this transcendental equation yield multiple values of roots. So, that we can get the buckling loads different modes.

But the lowest buckling load will give you the critical load and that is taken for the design. Now, substitute the equation 3 equation 2, if we substitute this equation into equation 2. First $\nabla^4 w$ has to be expanded if we expand it, you will find it consists of three terms. One is $\frac{\partial^4}{\partial x^4}$, second term is $2\frac{\partial^4 w}{\partial x^2 \partial y^2}$ and third term is $\frac{\partial^4}{\partial y^4}$.

If we expand this and then substitute here, then you will find the equation as $\frac{m^4\pi^4}{a^4}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$. This term will be there and this term will be coming due to differentiation of this function four times with respect to x. Then we go for the second term that is $2\frac{\partial^4 w}{\partial x^2 \partial y^2}$ and if I take the mix derivative of this function with respect to x and y two times.

Then we will get $2\frac{m^2 n^2 \pi^4}{a^2 b^2}$. So, this term is obtained after taking the cross derivative or mix derivative. What is mix derivative? That is there in the derivative function both differentiation with the x and y appears. So, in that case the mixed derivative is to be taken as del 4 w by del x square del y square. So, w is differentiated 2 times with respect to x and after differentiation, the result is again differentiated with 2 times with respect to y or vice versa.

Then substituting (3) in (2) that equation is obtained

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\left[\frac{m^4\pi^4}{a^4} + 2\frac{m^2 n^2 \pi^4}{a^2 b^2} + \frac{n^4 \pi^4}{b^4} - \frac{N_x}{D}\frac{m^2\pi^2}{a^2}\right]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} = 0$$

You can see this equation that with $N_x \frac{\partial^2 w}{\partial x^2}$ the second derivative of this function is necessary. So, it is taken and $\frac{m^2\pi^2}{a^2}$ term is coming. The same procedure can be applied when there is say load in the y direction, here the in plane forces in the x direction. But in some problem the in-plane forces in the y direction or there is a combination of in-plane forces in x and y direction as well as in membrane shear forces.

Then also this function can be substituted in the original differential equation and we can get a relation like that. Now, we get here

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\left[\frac{m^4\pi^4}{a^4} + 2\frac{m^2 n^2 \pi^4}{a^2 b^2} + \frac{n^4 \pi^4}{b^4} - \frac{N_x}{D}\frac{m^2\pi^2}{a^2}\right]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} = 0 \quad \text{………… (4)}$$

Now, you see this equation number (4), this equation is summation of different terms and ultimately this sum renders to be 0 quantity. So, that means, if I take say m = 1 n = 1, then I will get

$$A_{11}\left[\frac{\pi^4}{a^4}+2\frac{\pi^4}{a^2b^2}+\frac{\pi^4}{b^4}-\frac{N_x}{D}\frac{\pi^2}{a^2}\right]\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}=0$$

There is the a 11 then again if I add another term say a 12, a 12 is what? Say m = 2 n = 1. Then we will get

$$A_{21}\left[\frac{16\pi^4}{a^4}+2\frac{4\pi^4}{a^2b^2}+\frac{\pi^4}{b^4}-\frac{N_x}{D}\frac{4\pi^2}{a^2}\right]\sin\frac{2\pi x}{a}\sin\frac{\pi y}{b}=0$$

So, like that you can take a different combination of terms say A_{mn} and then you can form the series. Now, you can see here the right hand side of the equation must be 0 and you are getting infinite number of terms in the left hand side which are summed up and the result is 0. Now, since $Sin\frac{m\pi x}{a}$ $Sin\frac{m\pi x}{b}$ for any integer integral value of m and n should not become 0. Then each of the terms associated with the coefficient A_{mn} should be 0.

Another possibility is there $A_{11,}$ $A_{12,}$ A_{13} etc. should be 0 that means coefficient A_{mn} is 0, but this is not possible because if A mn is 0, then you the solution that I will get is a trivial solution. So, it will not serve the purpose of finding the buckling load. So, therefore; non trivial solution only possible when the quantity inside the bracket that you are seeing here is 0.

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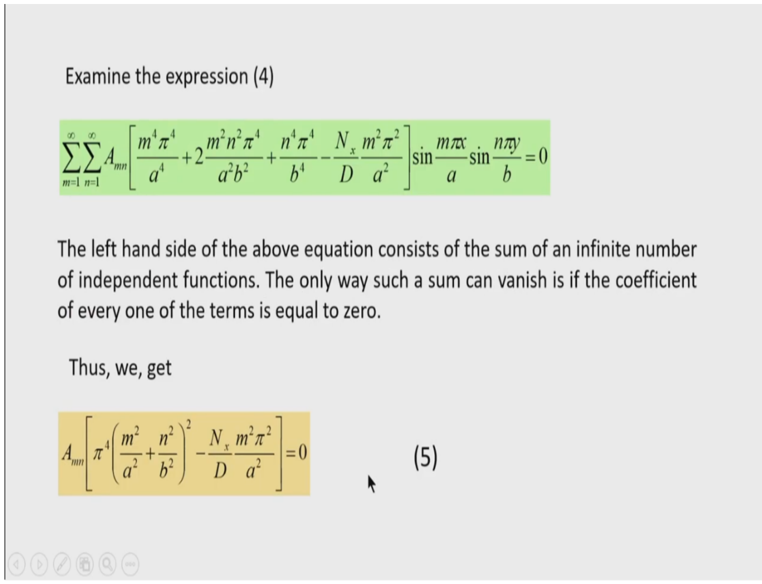

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\left[\frac{m^4\pi^4}{a^4}+2\frac{m^2n^2\pi^4}{a^2b^2}+\frac{n^4\pi^4}{b^4}-\frac{N_x}{D}\frac{m^2\pi^2}{a^2}\right]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}=0$$

So, according to that, we write the quantity inside the bracket third bracket = 0. And you can see this these three terms $\frac{m^4\pi^4}{a^4}+2\frac{m^2n^2\pi^4}{a^2b^2}+\frac{n^4\pi^4}{b^4}$ can be combined to give a term say $\left(\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right)^2$. Now if I take π^2 outside then I will get $\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$. And this term remains as it is - $\frac{N_x}{D}\frac{m^2\pi^2}{a^2}$. Hence the overall equation can be written as

$$A_{mn}\left[\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2-\frac{N_x}{D}\frac{m^2\pi^2}{a^2}\right]=0$$

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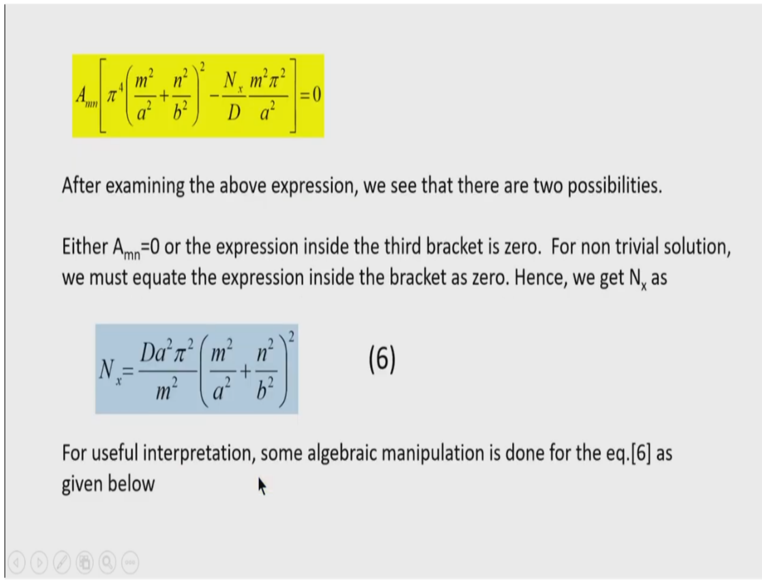

So, we ultimately get that N x equal to since mn is not 0 mn cannot be 0. So, therefore, because two possibilities that there, one is a - 0 or the quantity inside the bracket is 0, but if a - 0, then the solution is trivial solution. So, therefore, the quantity inside the bracket is 0. So, taking this quantity to be 0, and then our target is to find the N x. So, N x become

$$N_x=\frac{Da^2\pi^2}{m^2}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 \quad \text{...............(6)}$$

So, this is the in-plane force, force N_x that we want to find and we want to find what will be the critical load in the x direction. So, what is the critical value of N_x? Now, we have to examine several conditions for finding the critical load. So, equation 6 can be erased with some algebraic manipulation. So, that useful interpretation can be done.

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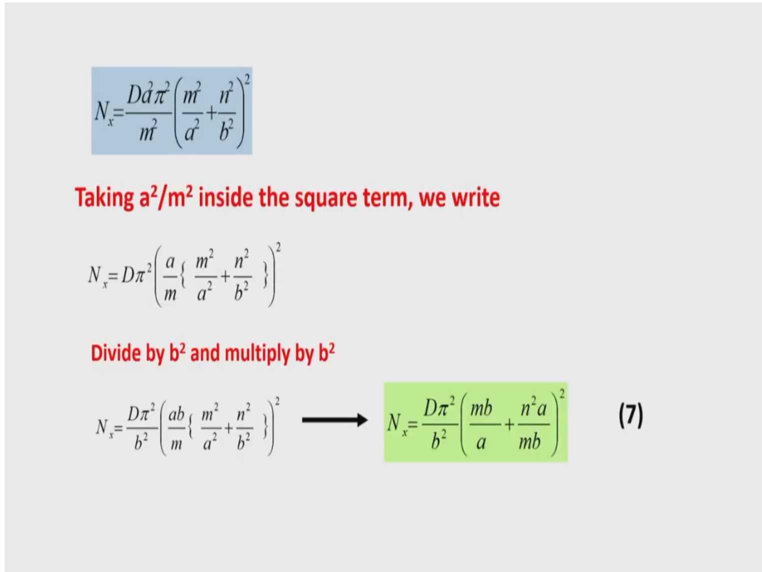

Now, this N_x that is $N_x = \frac{Da^2\pi^2}{m^2}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$. So, this term can be arranged in such a way that we can get a clear cut idea that to what have wave number or what this aspect ratio the buckling takes place. The boundary condition is simply supported along the all edges and you should mind it that if the boundary condition is changed the buckling load is also changed.

That is solution of the differential equation will be different. Now, taking $\frac{a^2}{m^2}$ inside the bracket.

Now, if I take $\frac{a^2}{m^2}$ this square term then I will multiply this term inside the bracket by $\frac{a}{m}$, because when I take $\frac{a}{m}$ outside the back end then it will be $\frac{a^2}{m^2}$. So, taking $\frac{a^2}{m^2}$ inside the bracket here I get $D\pi^2$.

And now I multiply the term inside the bracket with $\frac{a}{m}$, i.e $\frac{a}{m}\left\{\frac{m^2}{a^2}+\frac{n^2}{b^2}\right\}$ and then the square of this term inside the bracket.

$$N_x = D\pi^2\left(\frac{a}{m}\left\{\frac{m^2}{a^2}+\frac{n^2}{b^2}\right\}\right)^2$$

Now, divided by b^2 these quantities divided by b^2 and multiplied by b^2. So, then the equation is not changed. So, divided by b^2 and multiplied by b^2. So, what is done here you can see

$$N_x = \frac{D\pi^2}{b^2}\left(\frac{ab}{m}\left\{\frac{m^2}{a^2}+\frac{n^2}{b^2}\right\}\right)^2$$

So, $\frac{D\pi^2}{b^2}$ is here and then we have multiplied by b^2. So, when b^2 is going inside the bracket the bracket is containing the squared term. So, naturally b^2 will be b. So, therefore, $\left(\frac{ab}{m}\left\{\frac{m^2}{a^2}+\frac{n^2}{b^2}\right\}\right)^2$ Now, multiply each term inside the bracket. So, we are getting $\frac{D\pi^2}{b^2}$ which is the coefficient and then here we are getting say if I multiply $\frac{m^2}{a^2}\frac{ab}{m}$.

Then I am getting $\frac{mb}{a}$. Similarly, if I multiply the second term $\frac{n^2}{b^2}\frac{ab}{m}$ then I am getting $\frac{n^2 a}{mb}$. So, this term is obtained.

$$N_x = \frac{D\pi^2}{b^2}\left(\frac{mb}{a}+\frac{n^2 a}{mb}\right)^2$$

Now, this expression is very much useful or it will be used to find the buckling load.

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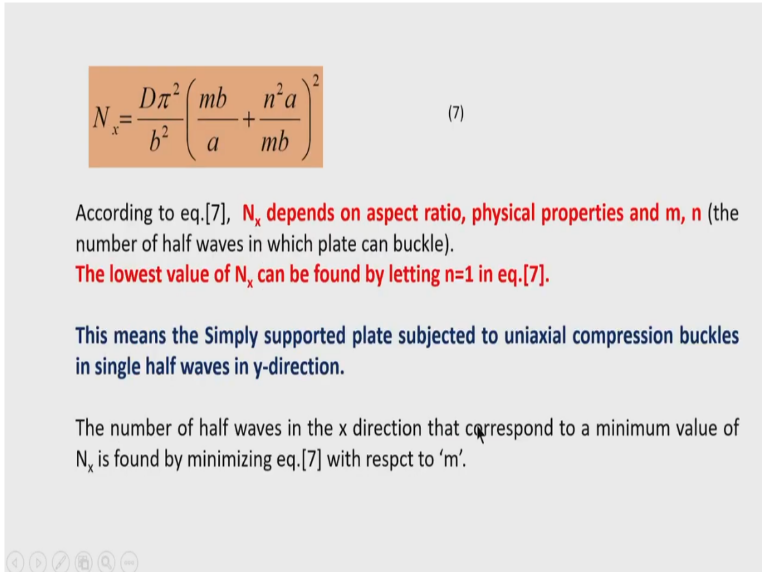

The equation 7, if we examine very carefully, then we will find that N_x will be the lowers when n is 1 because the minimum value of n can be 1 only, and we want the lowers value of the buckling load that will give you the critical load. So, according to equation 7, N_x depends on aspect ratio physical properties and m and n, m and n what these are integer, but this is also some physical meaning, that is the number of half waves in which the plate can buckle.

So, you can see here the last load N_x for the half wave in x and y direction is possible when n is 1. So, it is by inspection it is possible to tell that n = 1 gives the buckling load, but still, we do not know what is that m number of half waves in the x direction. So, lowest value of buckling load in the simplest supported plate that is in the unidirectional compression N_x can be found by letting n = 1.

So, that is shear from that expression and it is certain that by putting n, the lowest value of n here is 1. Then it is the integer so, 1 2 3 like that, so, 1 is the lowest value. So, therefore, if I put n = 1. I will get the lowest value of N_x, for n. But m is not known here. So, we have to find ways to calculate the value of number of half waves in the x direction to find the lowest load that is the critical load.

So, this is too for simply supported plate. Simply supported plate subjected to either axial compression buckles in single half wave in y direction provided the compression is in the x direction. So, in the plate you have taken a plate, say one metre by 0.5 for example. And the compression is applied along the direction of one metre that is along the edges which have length of 0.5 m.

So, in that case the shorter edge will buckle in one half wave. That is always true for rectangular plate having simply supported condition along the four sides. But number of half waves that is required for buckling in the lowest mode has to be found out. So, how it can be found because the N_x should be lowest for critical load. So, therefore, we differentiate this expression with respect to m and apply the condition of minimum value for m.

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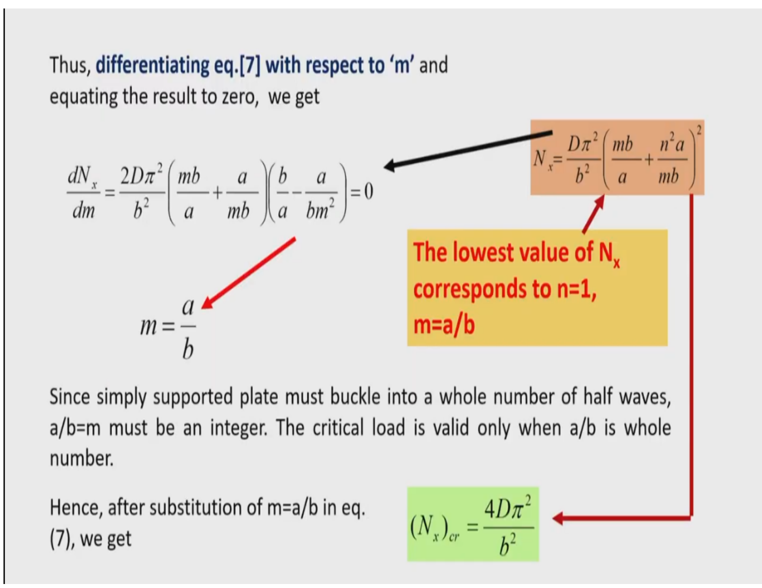

So, now, differentiating equation 7 equation 7 is this and n value = 1 is put here that means we are getting m $\left(\frac{mb}{a}+\frac{a}{mb}\right)$. So, this equation is differentiated with respect to m. So, $\frac{dN_x}{dm}$ is differentiated and equated to 0, to find the condition for which the buckling or half wave numbers in the x direction can be found out. That is we want to find m. Now, after differentiating, we get this $\frac{2D\pi^2}{b^2}\left(\frac{mb}{a}+\frac{a}{mb}\right)\left(\frac{b}{a}-\frac{a}{bm^2}\right)$.

How m^2 is coming? Because, 1/m here it is 1/m because here if you differentiate first you differentiated this quantity inside the brackets. So, 2 is coming and you are getting this quantity as it is. Then you are differentiating this quantity inside the bracket and therefore, first parameter that is $\frac{mb}{a}$ becomes $\frac{b}{a}$, after differentiation with respect to m, then second term becomes n = 1. So, $\frac{a}{b}$ and 1/m differentiation is $-\frac{1}{m^2}$. So, this value is obtained.

$$\frac{dN_x}{dm}=\frac{2D\pi^2}{b^2}\left(\frac{mb}{a}+\frac{a}{mb}\right)\left(\frac{b}{a}-\frac{a}{bm^2}\right)=0$$

Now, this is equated to 0 and obviously, this is valid if this quantity is 0, either this quantity is 0 or this quantity is 0. That means $\left(\frac{mb}{a}+\frac{a}{mb}\right)=0$ that is one possibility. Other possibilities $\left(\frac{b}{a}-\frac{a}{bm^2}\right)=0$. Both possibilities can be there, but thing is that if I take $\left(\frac{mb}{a}+\frac{a}{mb}\right)=0$, I will get the absolute value of m which becomes a negative and negative integer is not possible, A_{mn} are positive integers.

So, the second quantity that is $\left(\frac{b}{a}-\frac{a}{bm^2}\right)=0$. So, equating these to be 0 we get m = a/b. So, now, it is clear that number of half waves in the plate is a/b that is the aspect ratio. So, since simply supported plate must buckle into a whole number of half waves a/b must be an integer, so, a/b is an integer now, because m is integer we will saw so, it cannot be there other than integers.

So, n = 1 that we have earlier found so, for a number of half waves in y direction is one that is decided number of half waves in m direction according to our formulation. We have found it a/b So, it is a m = a/b. What is a? a is the length of the plate and b is the width of the plate. Now, question arises whether the aspect ratio is always an integer value? The answer is no, because the plate length maybe 2 metre width maybe say 1.5 metre or plate length maybe 3 metre width maybe 2 metre.

So, integer value is not always possible. There may be a non integer value also for this aspect ratio. So, therefore, we have to further investigate what should be the value of m if a/b is not an integer. So, let us proceed towards that. So, what do we get it? After substitution m = a/b suppose m = a/b is your number of half waves required for buckling then you can substitute this m = a/ b here n = 1. So, what do you will get? If you substitute m = a/b then this function

$$N_x = \frac{D\pi^2}{b^2}\left(\frac{mb}{a} + \frac{n^2 a}{mb}\right)^2 \quad \text{..........(7)}$$

this term is 1 and if you again substitute m = a/ b this is also 1 of course, n is 1. So, 1+1 is 2, 2 squared is 4. So, therefore, we get the buckling load for such a case is N_x is the uniaxial compression that we have in our problem it is existing and is critical value for a/b, m = a/b

$$(N_x)_{cr} = \frac{4D\pi^2}{b^2}$$

So, this is true when the length of the plate and width of the plate, the ratio is such that.

It becomes a integer. For example, the length of the plate is 6 metre, the width of the plate is 3 metre, then m = 2. So, this is a integer so, these value can be used. So, suppose the length of the plate a is say 1 metre and the width of the plate b say 0.5 metre then also it is an integer. So, this value can be used, but for non-integer value of a by b we have to further analyse it.

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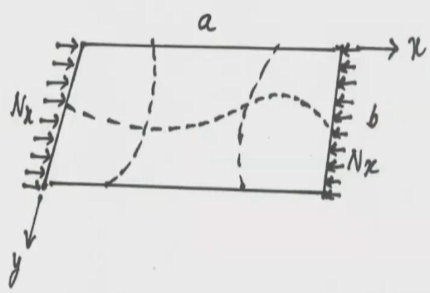

Fig. 3 Simply supported rectangular plate with uniform compression in X direction

So, let us see. Now, it is decided that in case of simply supported plate simply supported mean all edges are simply supported all 4 edges are simply supported. And this the in-plane forces or N_x there is the force in the x direction is applied at the boundary this compression and it is constant. So, number of half waves that is required to be formed for finding the critical load of such a plate is 1. So, plate buckles in y direction in single half wave that is clear.

But number of waves that; is required in this x direction in the longitudinal direction on lengthwise direction that is equal to a/b. So, a/b is the aspect ratio and this is a and this is b. This is an integer then we can get suppose a = 4 and b = 2, 4 metre and b = 2 metre. In that case, the two half waves are form. But this is not always true. So, let us see what we can do?

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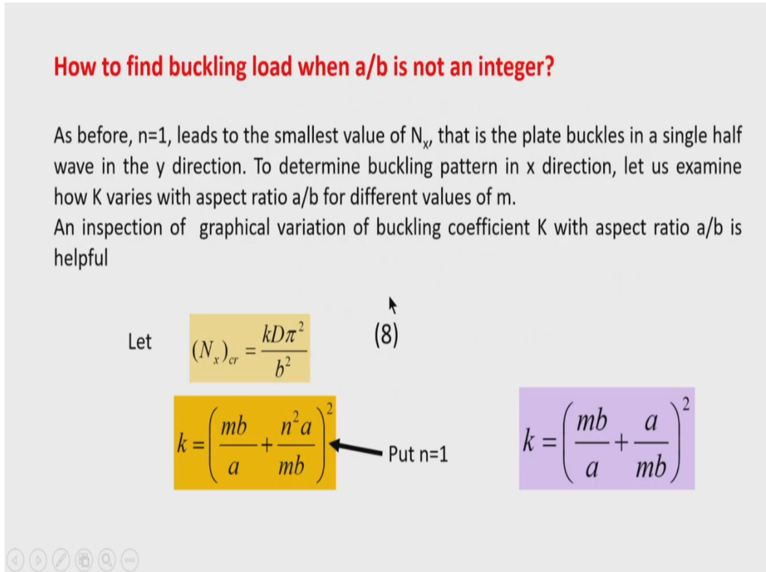

So, how to find the buckling load when a by b is not an integer? So, as before n = 1 leads to the smallest value of N_x, that is the plate buckles in a single half wave in the y direction. That is sure. To determine the buckling pattern in x direction let us examine how k varies with aspect ratio a by b, k is the buckling stress coefficient, k is the buckling stress coefficient. What is buckling stress coefficient?

If we see equation no (7) $N_x = \frac{D\pi^2}{b^2}\left(\frac{mb}{a} + \frac{n^2 a}{mb}\right)^2$

These value $\frac{D\pi^2}{b^2}$ and if I take this $\left(\frac{mb}{a}\right)$ plus n is 1 n is always 1 for rectangular plate simply supported no all edges and compression is in the x direction then n is 1. So, buckling coefficient we can find out and buckling load critical load will be $\frac{kD\pi^2}{b^2}$. Now, you can see here the critical load of the plate depends on the width of the plate. If it is compressed in the lengthwise direction and there is no dependency on the length.

This is the difference between the buckling of plate with the column. Because we have seen that in column the buckling load depends or you know the Euler's buckling formula, the critical load depends on the length of the column. But here it is not it is depending on the width of the plate. So, number of half waves in the x direction is required now, for any non-integer value of a/b.

So, buckling coefficient case introduced here and critical value of N_x is written as $(N_x)_{cr} = \frac{kD\pi^2}{b^2}$ where k is given as $k = \left(\frac{mb}{a} + \frac{n^2 a}{mb}\right)^2$ and n=1. That is, we already determined.

So, actually k is nothing but $k = \left(\frac{mb}{a} + \frac{n^2 a}{mb}\right)^2$, this is the value of k. Now, we required to find m.

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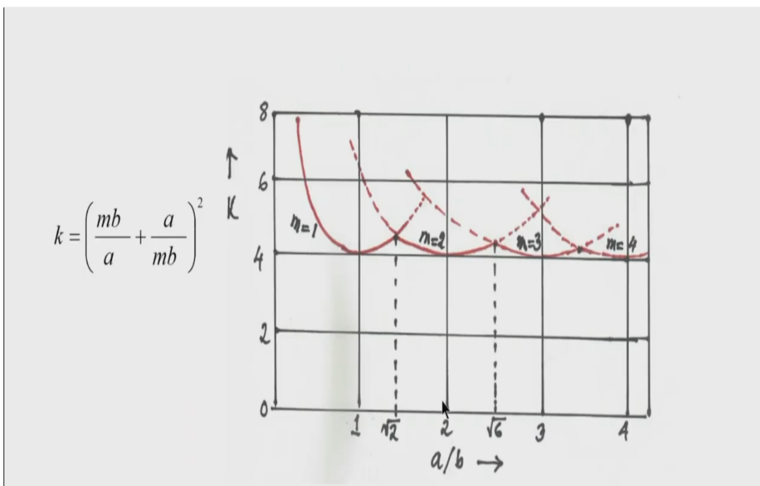

Fig. 4 Buckling stress coefficient k with a/b ratio

We present graphically how k varies with the aspect ratio. So, in the x direction we plot the a/b ratio in the y direction it is the value of k. So, for m = 1 for different values of aspect ratio 1 2 3 m any non-integer value also we have plotted the curve for k. So, this is the curve for me equal to 1. The curve is 4 k, so, m = 1, the lowest buckling load is for a coefficient is 4. So, that means, when half wave number in the x direction is 1.

Then again, we can find the buckling load of this by taking this critical load by taking k = 4, n is already 1 that we have determined earlier. Now, for m = 2. If I plot k against a/b ratio, I will get a curve like that for m = 2, this curve is for m = 2. Plotted here by substituting m = 2 and bearing a/b ratio. So, a/b ratio, we are varying here along the x axis and we have gone up to a higher value say for very long plate a by b ratio is high, and up to four were plotted.

So, now, you can see that two branches of the curve for which m = 1 is taken for first curve. Buckling coefficient and m= 2, is taken for the second curve of buckling coefficient. They intersect at some point. Now, similarly for other m we can draw it and we can get the curve neglecting the upper portion of the curve. So, say the buckling curve is now taken like that, then this is like that. So, these are the buckling curves which are formed by joining the lower branches of the curve.

Now, you can see that the m is 1 this is the curve for buckling coefficient when m is number of half waves in x direction is 1. So, this curve is valid for a/b ratio less than $\sqrt{2}$. That is 1.414. So, a/b ratio is less than 1.414, you can use this curve. That means m=1, that means you have to put the value of m in the buckling coefficient as 1. For example, here if you get, say for example, here it is 1.2 aspect ratio 1.2.

So, in that case the buckling stress test coefficient critical stress coefficient will be slightly higher than 4. At exactly at one it is 4. Now, if I see a value for example, buckling m = 2, for any integer value of these m we have seen that critical test coefficient of buckling is stress coefficient is full. So, here again it is conforming to the result. Now, any other value in this curve. So, m = 2. This is the curve for a m = 2 buckling coefficient for m = 2.

And this curve has to be considered for finding the critical stress for the plate within the aspect ratio $\sqrt{2}$ to $\sqrt{6}$. Like that it will go and at a very higher value of a/b ratio you will get that this buckling stress coefficient is almost nearly 4.

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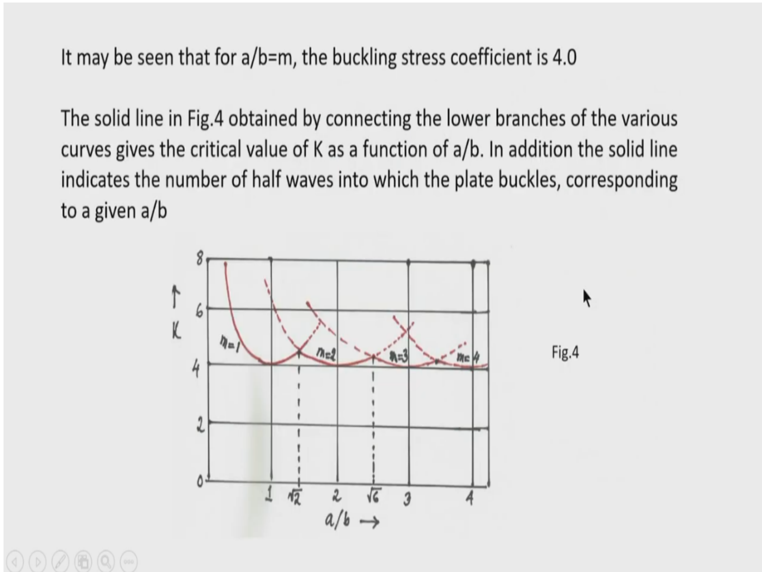

Now, let us see. So, it may be seen that for $\frac{a}{b}$ =m the buckling test coefficient is 4 that we have calculated. And here this solid line obtained by connecting the lower bunches of the various curves gives the critical value of k as a function of $\frac{a}{b}$. In addition, the solid lines indicate the number of half waves into who is the plate buckles corresponding to a given $\frac{a}{b}$ ratio.

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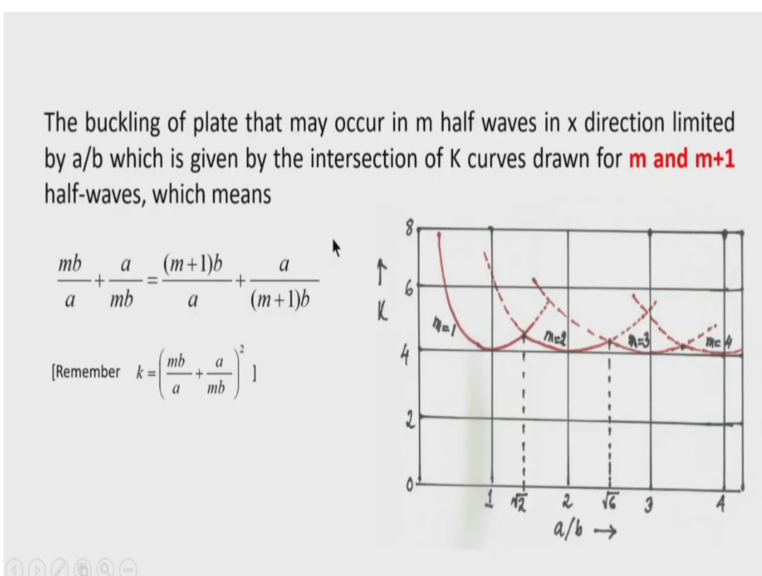

Now, the buckling of the plate that may occur in m half waves in the x direction limited by $\frac{a}{b}$ which is given by the intersection of K curves these are the K curves. These are the curve or buckling coefficient key drawn for m and m + 1. You have seen that any conjugative value of m that we take say m is = 2, m = 3, and here we find the point up to which the buckling coefficient can be found considering m = 2.

Then again from here to here, intersection of this m = 3 and m = 4. We will find that buckling coefficient is valid for m = 3. So, that pattern is seen from the graphical representation. So, now, if I see that m^{th} curve intersects with $(m + 1)^{th}$ at curve. So, we can write an equation $\frac{mb}{a}+\frac{a}{mb}$ That is a buckling stress coefficient without square that is the next buckling curve is $\frac{(m+1)b}{a}+\frac{a}{(m+1)b}$. Remember that k is $\frac{mb}{a}+\frac{a}{mb}$

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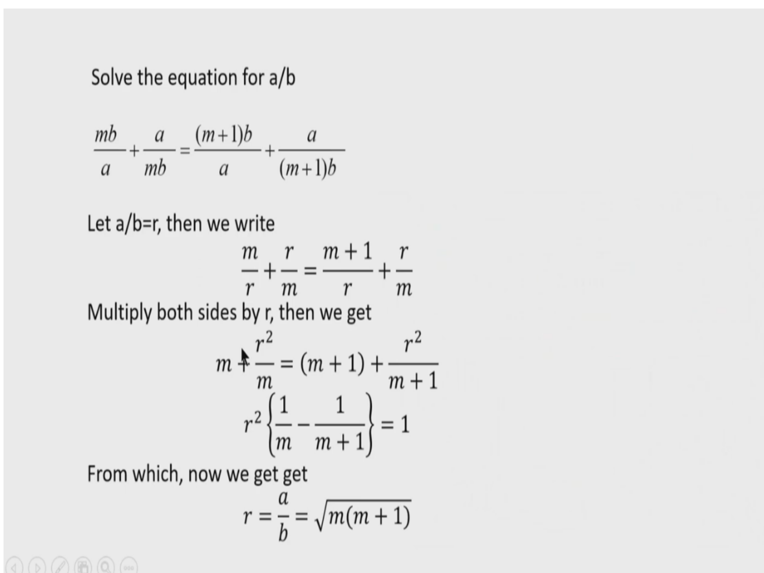

Now, let us solve for $\frac{a}{b}$. So, let $\frac{a}{b}=r$ then we can write here

$$\frac{m}{r}+\frac{r}{m}=\frac{m+1}{r}+\frac{r}{m}$$

Multiply both sides by r, then we get,

$$m + \frac{r^2}{m} = (m + 1) + \frac{r^2}{m+1}$$

So, these quantities form now, we want to solve for this r. How it is r is solve this equation very simple equation from which the r^2 is taken out and hence the equation becomes

$$r^2\left\{\frac{1}{m} - \frac{1}{m+1}\right\} = 1$$

‘r’is the aspect ratio $r = \frac{a}{b} = \sqrt{m(m + 1)}$. So, this is the value of aspect ratio for any non-integer aspect ratio which corresponds to m and this may be non-integer also.

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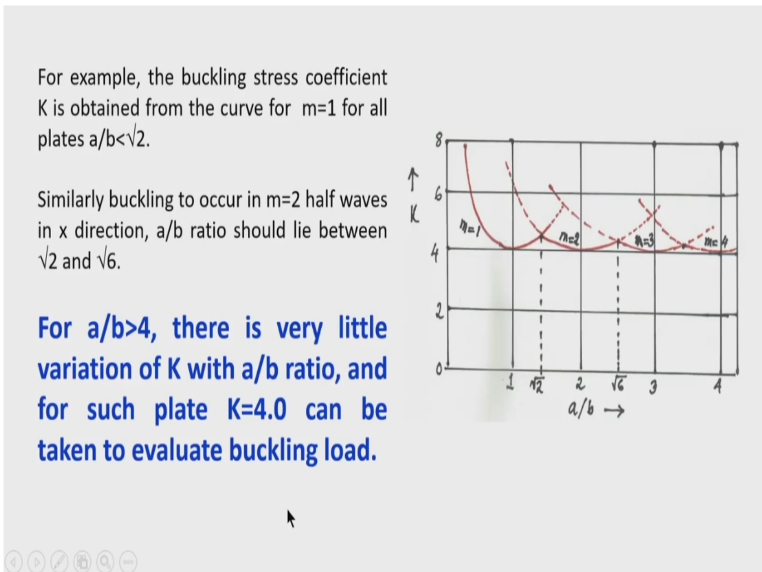

So, for example, the buckling stress coefficient ‘k’ is obtained from the curve for m = 1 for all plates $\frac{a}{b} < \sqrt{2}$. So, for $\frac{a}{b} < \sqrt{2}$. buckling stress coefficient can be obtained for m = 1.

Similarly buckling to occur in m = 2 half waves in x direction $\frac{a}{b}$ ratio should lie between $\sqrt{2}$ and σ_x. Now, one thing you should note that the number of half wave that we have taken in y direction is always one for this condition.

That is boundary condition is that all edges are simply supported and the plate is compressed in the direction of length. So, that condition is when this condition is valid, then we get one half wave in y direction y direction is the width direction that is the shorter dimension the longer dimension is a. Now, for $\frac{a}{b}$ > 4 there is little variation of k with $\frac{a}{b}$, there is not much variation.

And for the purpose of calculation, the cuticles test coefficient for such a plate can be taken as 4 without any hesitation and you can find the buckling load.

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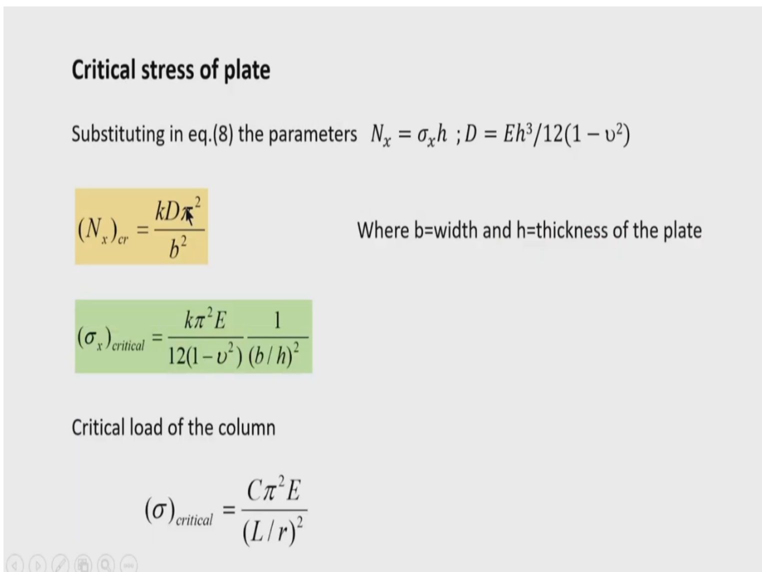

So, critical stress we have found the buckling load N_x. Now, let us see what is critical stress? The N_x is the force per unit length. So, when we divide this N_x by h because all the quantities in the plate are expressed as whether it is a force bending moment etc. It is expressed per unit length. So, N_x is also force per unit length that means, if σ_x is the stress along the x direction normal stress. So, σ_x into h into 1 h is the thickness of the plate will give you the N_x.

$$(N_x)_{cr} = \frac{kD\pi^2}{b^2}$$

Substituting D as $D = \frac{Eh^3}{12(1-\upsilon^2)}$ where υ is the Poisson ratio, we can find the critical stress.

So, therefore, these σ_x which is that critical stress in the x direction can be found as

$$(\sigma_x)_{critical} = \frac{k\pi^2 E}{12(1-\upsilon^2)} \frac{1}{(b/h)^2}$$

So, the critical stress here you can see it depends on the ratio of the width to thickness of the plate. So, critical stress in the play there is a in the thin plate rectangular thin plate depends inversely proportional to b by h ratio square ratio. So, if we compare the critical load of the column with the critical load of the plate.

We can see that critical load of column is $2 \times 10^5 \frac{N}{mm^2}$

In both the cases you can see that critical stress depends on the material properties, that is E. E is the most important parameter, which influences the critical load. Then in case of plate you are finding these non dimensional parameter which is $(b/h)^2$. But in case of column, it is found is $(L/r)^2$.

Because r is the radius of gyriation; which is coming from the moment of inertia term. If you recall the critical load of the column is nothing but $\frac{\pi^2 EI}{L^2}$, where is the second moment of area. And that can be expressed as $I = Ar^2$, where A is the cross sectional area. Then we get this critical load of the column as $(L/r)^2$. So, these differences you can note the critical load of the plate does not depends on the length, it depends on the width of the plate.

On the other hand, the critical load of the column depends on the length of the plate. Of course, the boundary condition is the most influencing factor for both cases, whether you calculate the critical load for this column or critical load for plate the boundary condition is the most important factor. Because you are now; considering the boundary condition which is taken as simply supported along the four edges.

If I altered the boundary condition suppose for example, boundary condition is fixed along the all edges naturally buckling load will be different. So, similarly in the column which has the pin pin end then the buckling load is the standard Euler load, $\frac{\pi^2 EI}{L^2}$ and in the critical stress parameter it can be written $(\sigma)_{critical} = \frac{C\pi^2 E}{(L/r)^2}$ where C is actually the constant term. This constant term actually is dependent on the boundary condition.

For as the other condition of boundaries a fixed fixed boundary this will be different. For one and fixed other in free that C will be also different. But here you are finding that this expression is only valid for the boundary condition which has simply supported condition along the four edges. No other boundary condition is to be applicable for that plate because K is here is taken according to that condition.

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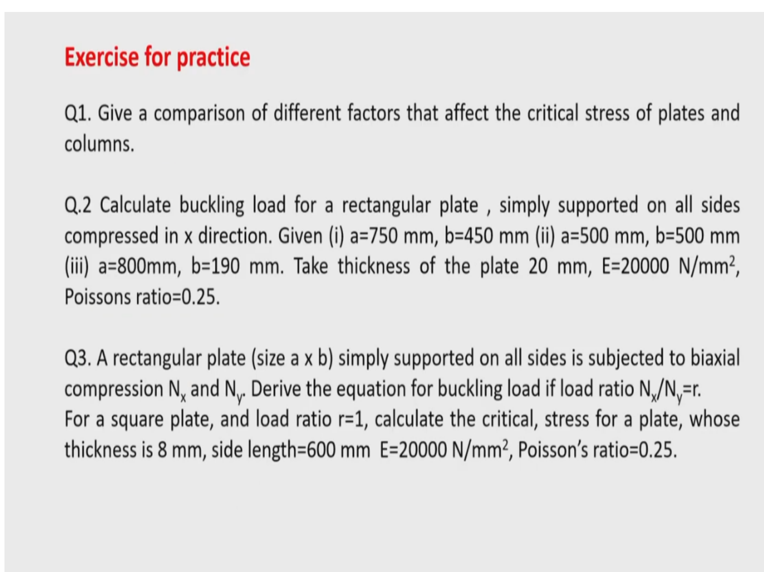

Now, let us see some exercise chapter that I have now discussed. So, first is comparison of different factors that affect critical stress of plate and column. So, that factors I have discussed also in between the lectures and you have noted down also, I will give these important points for that question also. Second question is the buckling load calculation for a rectangular plate simply supported on all sides compressed in the x direction for different say length and width.

So, a is 750 mm b is 450 mm in one case, second case a is 500 mm b is 500 mm, third case a is 800 mm b is 190 mm. Now, one thing we can see that here in the second case a is 500 mm b is 500 mm. So, a by b ratio is one that means, aspect ratio is one. For any integer aspect ratio, we have found that critical stress coefficient is 4. So, here we can use the coefficient 4 but, in that case, when we get say a = 800 mm b = 190 mm.

Here also we have found that a by b ratio is greater than 4 and this is termed as a long plate for long plate this a by ratio is detailed as greater than 4 and again here the cuticles test coefficient is 4. So, we can calculate the critical stress coefficient for that cases. Then another case is the rectangular plate size a × b there is a general dimension of the plate simply supported on all sides is subjected to by biaxial compression.

Here instead of any uniaxial compression. I am considering the compression in the both directions x and y. So, we have to derive the equation for buckling load if load ratio $\frac{N_x}{N_y}$ is r. Load ratio mean ratio of the compression along the x direction and along the y direction is the ratio. And we have to find for a square plate and load ratio is one the buckling load below given the numerical parameter like the thickness of the plate is 8 mm side length is 600 mm and is 2 into 10 to the power 5 Newton bar mm square Poisson ratio is taken 0.25.

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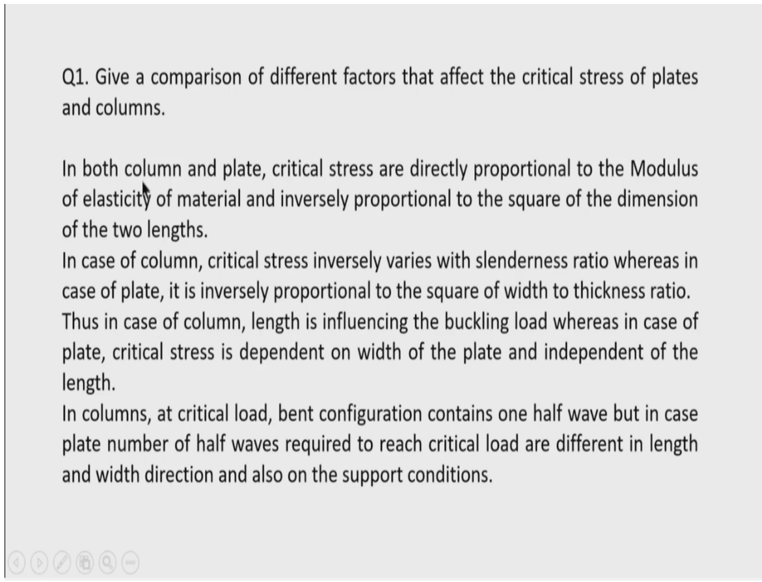

Now, let us see the comparison that I have given in the first case in both column and plate critical stress are directly proportional to modulus of elasticity of the material that you have observed and inversely proportional to the square of the dimension of the two length. In case of plate the two length or width of the plate n thickness if the compression is along the length. Then in case of column the dimension that the ratios of two dimensions are one is the length of the column and radius of gradation.

So, this difference you have noted. Now, in case of column critical stress is inversely proportional to the slenderness ratio whereas, in case of plate it is inversely proportional to the square of the width to thickness ratio that we have observed from the derivation. That in case of column length is influencing the buckling load whereas, in case of plate critical stress is dependent on width of the plate and independent of the length.

In columns at critical load bend configuration contains one half wave that is the buckling takes place from straight to bend configuration just when the state configuration says this is the point, we take it for critical load. So, in the bend configuration one half wave is formed in case of column for simply supported cases the half wave base clearly the $\sin\frac{m\pi x}{L}$. But, in case of plate you have seen the number of half waves depends on the aspect ratio. So, that are the difference that we have noted from our analytical work.

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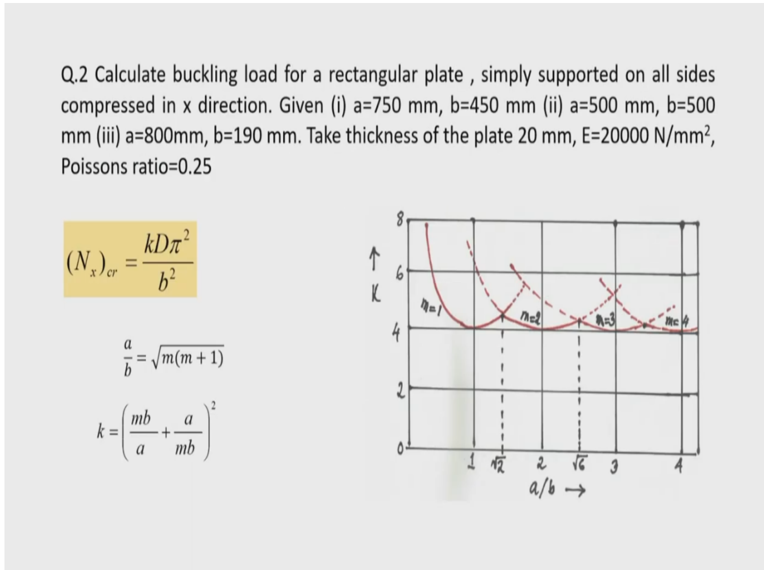

Now, let us see the second problem. Second problem is calculate buckling load for the rectangular plate simply supported on all sides compress the x direction. Given length of the plate 750 width of the plate is 450 in one case. Another case 500 by 500 plate dimension thickness of the plate in all cases are say thickness of the plate we can take is thickness of the plate 20 mm. So, this modulus of elasticity this $2\times10^5 \frac{N}{mm^2}$.

So, these are the things that we derived in this class. The critical value of the buckling stress is $(N_x)_{cr} = \frac{kD\pi^2}{b^2}$ and a/ b equal to in general $\frac{a}{b} = \sqrt{m(m+1)}$ And $k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$

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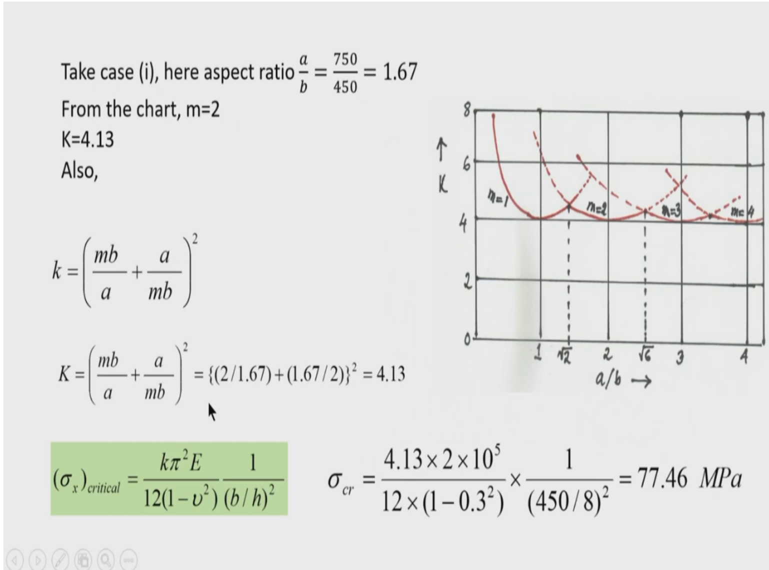

Now, let us see the first case. Here the thickness of the plate instead of 20 mm I have taken for calculation purpose 8 mm because 20 mm plate is too thick and the thick plate behaviour may come. So, therefore, for thin plate buckling I have taken change the thickness of the plate as 8 mm. Now, take case one, case one aspect ratio is a/ b is 750/450 so, it is 1.67. From the chart we can find that 1.67 a/b ratio.

This is 1.414 i.e $\sqrt{2}$. So, if it is 1.67 then m is 2. It is obvious that m is 2. So, therefore, substituting m is 2 in this expression, we can get this k equal to 4.13 or from graph also we can read it. If the graph is do not accurately. So, once the k is known then buckling critical stress can be found with the help of this formula $(\sigma_x)_{critical} = \frac{k\pi^2 E}{12(1-\upsilon^2)}\frac{1}{(b/h)^2}$, υ we have taken 0.3 into this $\frac{1}{(b/h)^2}$ so b is 150 and thickness of the plate I have taken 8 mm.

So, after calculation this comes as 77.46 MPa. So, this is the buckling load of the plate. The third problem that is the biaxial compression, let me solve it and show you the solution step by step.

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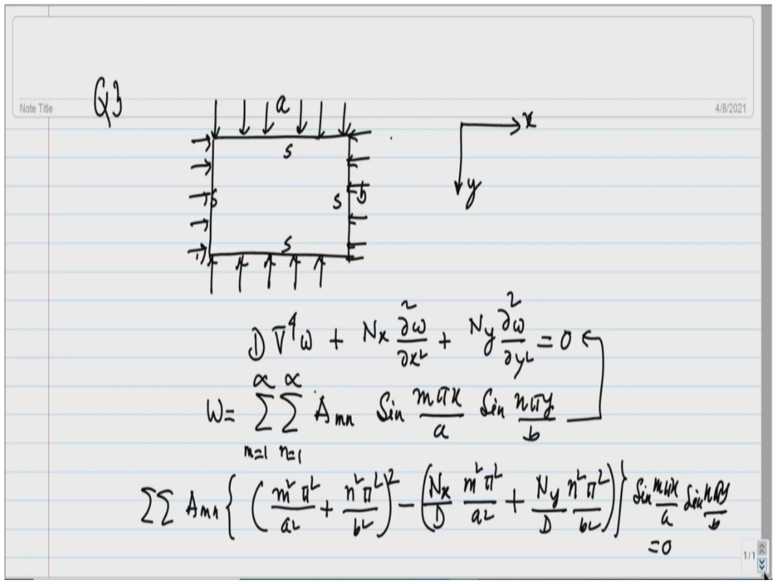

In the third question we have a plate rectangular plate dimension is a and b and both all sides are simply supported. So, this is simply supported this is also simply supported I am giving the symbol s instead of drawing the dotted line and it is compressed. So, this is our x axis and this is y axis and also in the y direction this compression exists. From the discussion done in today's class and also the knowledge that you have gathered earlier in the theory of plates that I have discussed.

I think you will be able to understand the procedure for solving this problem. So, plate equation is

$$D\nabla^4 w + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} = 0 .$$

It is the equation for the buckling of the plate. Now, for this plate which is simply supported on all edges we use this the Navier series $w = \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} Sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$

A_{mn} are integer it may varies from one to infinity. Now, substituting this equation is substituted here and after substitution we get this value that is again double summation

$$\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\left\{\left(\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right)-\left(\frac{N_x}{D}\frac{m^2\pi^2}{a^2}+\frac{N_y n^2\pi^2}{Db^2}\right)\right\}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}=0$$

So, after substituting these we get arrive at this equation.

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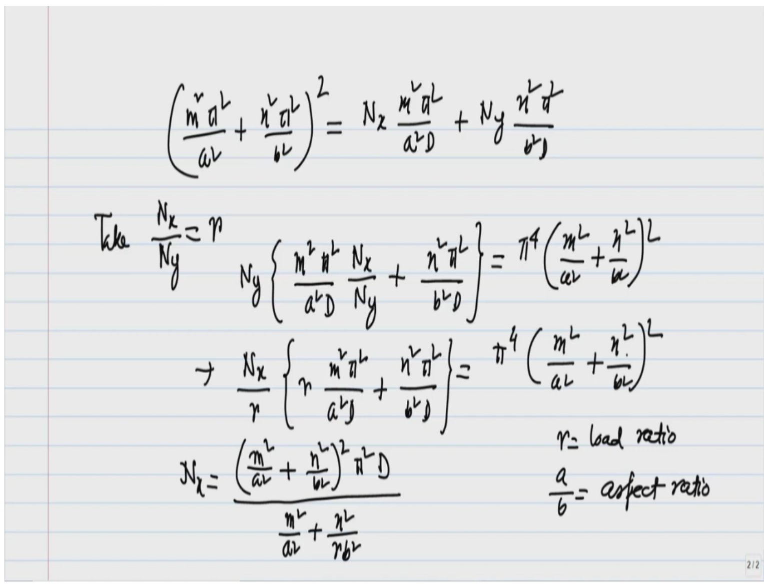

Now, naturally for non trivial solution of this buckling load we have to obtain that

This equation is obtained for non trivial solution. Now, from that equation if I take say take N_x/N_y that is the load ratio take this is equal r.

So, if I take this is r, then I can write

$$N_y\left\{\frac{m^2\pi^2}{a^2D}\frac{N_x}{N_y}+\frac{n^2\pi^2}{b^2D}\right\}=\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$$

Now taking this $N_x/N_y = r$ then I can transform this equation to

$$\frac{N_x}{r}\left\{r\frac{m^2\pi^2}{a^2D}+\frac{n^2\pi^2}{b^2D}\right\}=\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$$

Now, our target is to find N_x. So, from that equation N_x is found as after simplification you can see also

$$N_x=\frac{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2\pi^2D}{\left(\frac{m^2}{a^2}+\frac{n^2}{rb^2}\right)^2}$$

So, this is the value of N $_x$ in terms of load ratio r is the load ratio and a by b will be the aspect ratio. Now, we have to solve the problem of square plate. So, let us take this.

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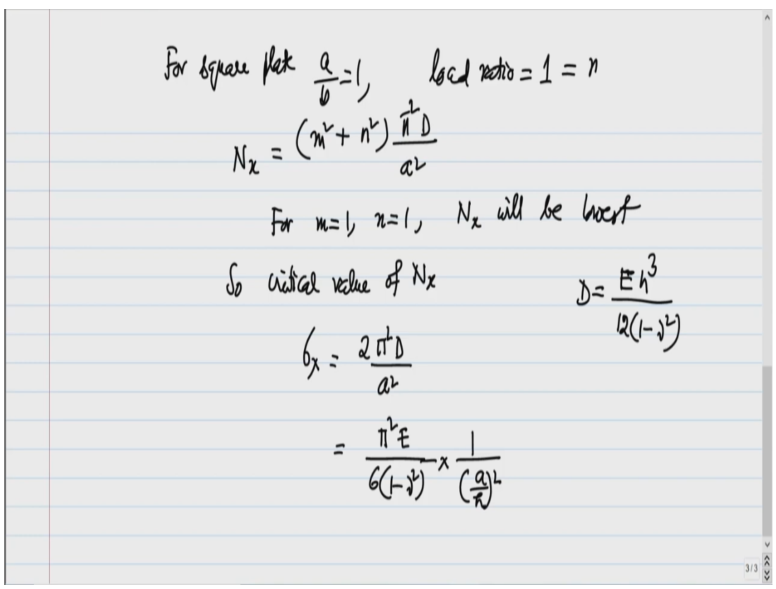

For square plate a/b = 1 and we have taken load ratio is one for this problem = 1. There is r = 1 load ratio means r. Now, after substituting this we in the previous expression that for N_x we now get

$$N_x=\left(m^2+n^2\right)\frac{\pi^2D}{a^2}$$

So, simple expression is now found for N_x. Now, this is lowest. What will be the lowest value of N_x for what m and n for m = 1 and n = 1, because these are constant parameter the N_x will be lowest. So, critical value of N_x is now obtained by substituting m = 1 and n = 1, we get $\frac{2\pi^2 D}{a^2}$.

Substituting D as if I take $D = \frac{Eh^3}{12(1-\upsilon^2)}$, where υ is the Poisson ratio. We can get now

$$\sigma_x = \frac{\pi^2 E}{6(1-\upsilon^2)} \times \frac{1}{\left(\frac{a}{h}\right)^2}$$

So, this is that critical load this is not the critical stress, I should call it critical stress.

So, critical stress is obtained like that and now, this for substituting the numerical value one can calculate the critical stress for this plate. So, thank you, I think you have understood how to proceed to calculate the critical load of a thin plate subjected to uniform compression as well as biaxial compression, but I have taken a special case where the load ratio is 1 and aspect ratio is also 1 corresponding to a square plate. Thank you very much.