Advanced Soil Mechanics
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Lecture – 9
Principle Stresses and Eigen Vectors

In the last few lectures rather we have discussed about stresses acting at a point, transformation of coordinate axes, determination of principal stresses and the eigen vectors which represents the direction cosine metrics which will be useful for transformation. In this lecture, we will try to see a small example demonstrating this.

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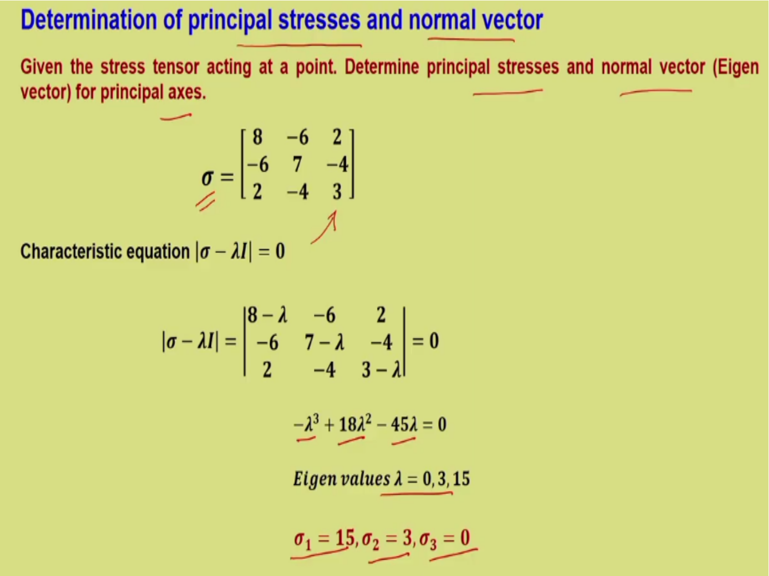

So we are discussing about determination of principal stresses and normal vector. So given a stress tensor acting at a point which is σ and the components are given. We are asked to determine the principal stresses and the normal vector that is eigen vectors for the specific principal axis. Now we know the characteristic equation is $|\sigma - \lambda I| = 0$ which would diagonalize the given stress matrix.

If you write that $|\sigma - \lambda I|$ will give you lambda I, I is an identity matrix. So

$$= \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$$

Now if you expand this, you find the determinant, you will get

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0.$$

The last component becomes 0. I strongly suggest you to go through and you do it by yourself.

So solving this equation you will get the eigen values λ = 0, 3 and 15. Now what are these eigen values? These eigen values are nothing but the principal stresses and you can arrange this in a diagonal matrix form. So this is σ_1 = 15, σ_2 = 3 and σ_3 = 0 is the principal stresses or the eigen values. It is an eigen value problem which we have obtained.

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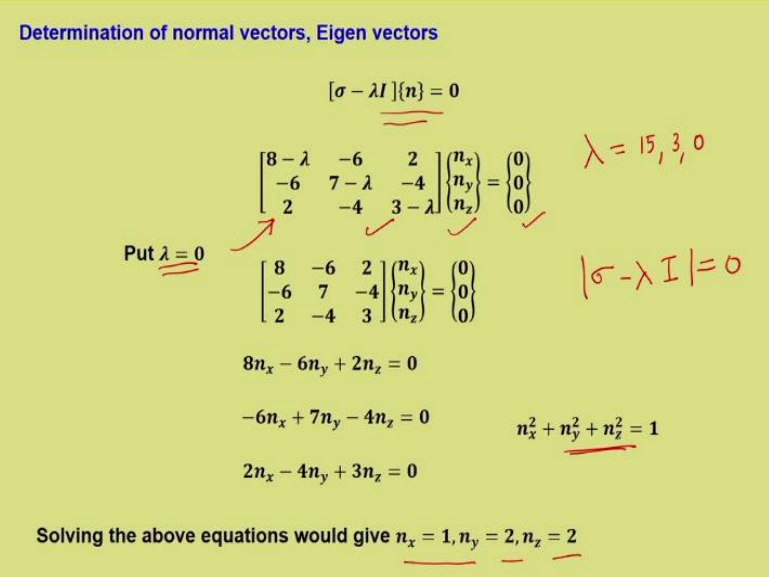

So now we are left with determination of normal vectors or eigen vectors. Now if you use this equation, this also we have explained during the lecture. $|\sigma - \lambda I|\{n\} = 0$ is the equation. Now what we have to do? We have $|\sigma - \lambda I|$ which is this, the normal vector that is n_x n_y n_z that is equal to 0. Now for the first we have the eigen values lambda which is already there which is 15, 3, and 0.

So first let us put λ = 0 in this equation and you will get same matrix n_x into 0. You will get a system of equations where $8n_x - 6n_y + 2n_z = 0$. Similarly, there are other 3 equations. Now we already know that the determinant $|\sigma - \lambda I| = 0$. This means that the row or the columns they are not independent; hence these equations may not be independent equations.

So we may have to use or not we may, we have to use this additional equation

$$n_x^2 + n_y^2 + n_z^2 = 1$$

So this we already know. So using these equations, we can solve for n_x, n_y, n_z by putting λ = 0. This will give n_x = 1, n_y = 2, n_z = 2.

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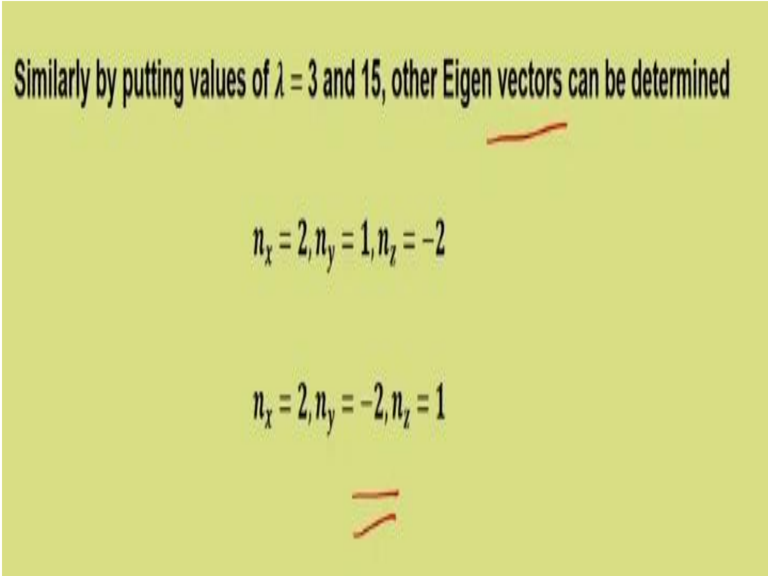

Similarly, we have to put λ = 3 and 15, the other eigen vectors can be determined. So it will give you $n_x = 2$, $n_y = 1$, $n_z = -2$ and $n_x = 2$, $n_y = -2$, $n_z = 1$. So this is how we find the principal stresses and the corresponding transformation matrix okay. So, with this we finish this part of the problem. We will see in the next lecture. Thank you.