# Advanced Soil Mechanics

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## Lecture – 7
## Stress Invariants

Welcome back all of you. So in this lecture, we will discuss about stress invariants. In the last lecture we have seen transformation of a coordinate axis. We have seen the importance of transformation. We have a given Cauchy stress tensor with stress components in it and we have seen this stress components can be transformed to any set of orthogonal axis. The first reference axis was x, y, z. It can be transformed to any other set of orthogonal axis.

We have also seen the importance of this transformation as one specific transformation gives a stress tensor with only diagonal elements. What is the significance of these diagonal elements? The significance is these diagonal elements represent principal stresses because all the shear components in the stress tensor is 0. Now there are wide variety of use of these principal stresses in geomechanics.

It is quite convenient in soil and rock mechanics to use principal stresses why because this stress tensor alone is able to capture the entire effect of the external loading that is happening on a body. So you need to know only 3 components and those are principal stresses. We have also seen the process of diagonalization. We have also seen the characteristic equation based on which you have the parameters $I_1$, $I_2$, $I_3$.

And we term these as invariants or rather to be very specific invariants of stress tensor. Now we will continue with our discussion on stress invariants because there are certain aspects which we need to discuss more and the importance of these stress invariants, its geometrical representation so that we will see in this lecture.

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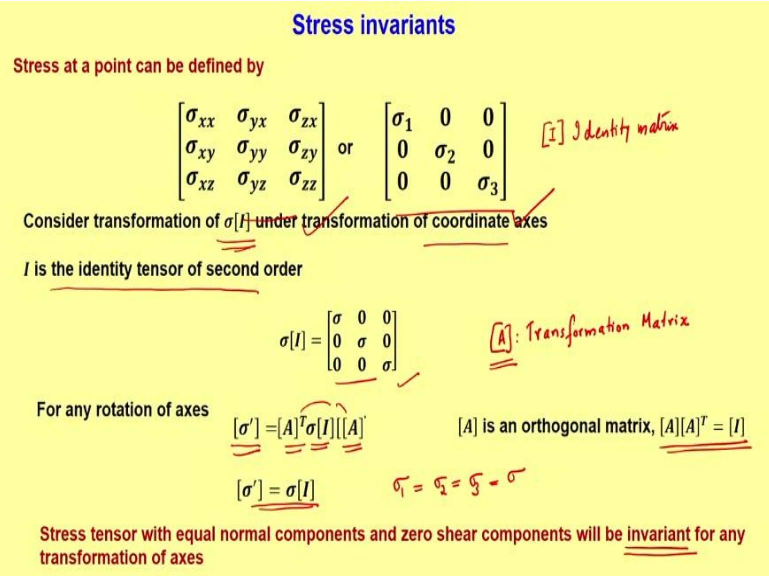


Let us start again with stress at a point. There is no harm in doing such repetition because for those who are exposed to this for the first time it is very important for them to understand. So let us start with stress at a point. Cauchy's stress tensor can also represented in terms of principal stresses. So, they represent the same stress effect. So, consider transformation of a given stress tensor which is represented by $\sigma[I]$, I is identity matrix.

So we are going to consider transformation of $\sigma[I]$ under transformation of coordinate axes. Now transformation of coordinate axes is very well known to us from the last lecture. Now we just want to see what will be the transformation of a given stress tensor which is represented by $\sigma[I]$ , I is the identity tensor of second order.**.** So $\sigma[I]$ gives

$$[\sigma] = \begin{bmatrix}\sigma & 0 & 0\\ 0 & \sigma & 0\\ 0 & 0 & \sigma\end{bmatrix}$$

We are going to transform this particular matrix. Now what do we have to know if we want to transform this into some other axis? First of all, we should know some reference axis and we should know the direction cosine matrix which we call it as transformation matrix that is matrix A. So, we need to know matrix A which is the transformation matrix. So once the transformation matrix is known for any rotation of axis, this can be to any axis depending upon the components of A.

So we can write the new set of stress tensor that we get will be σ '. Let us assume is equal to A transpose. $\sigma[I]$ is the stress tensor that gets transformed and A this comes from our previous lecture? Now one can note that A is an orthogonal matrix because A contains the normal vector. Its direction cosines, so it is A orthogonal matrix and it holds this particular law

$$[A][A]^T = [I]$$

Now you come to this particular expression you have A transpose and A so that will give you identity matrix. So what we are left with, we are left with $[\sigma'] = \sigma[I]$ That is when you do the transformation you are getting the same stress tensor, we started off with $\sigma[I]$ and we ended up in the transformation it is equal to $\sigma[I]$.

Now we need to notice that in $\sigma[I]$ all the 3 diagonal elements are same or they are equal,

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma$$

so this is the condition. So what does that mean? If you have a given stress tensor where the diagonal elements only exist and all the diagonal elements are equal., then the transformation of that gives the same matrix. Transformation means you are mapping it to some other axis.

That means rotation of axis is not going to have any effect on this particular matrix. So that is what is summarized here. Stress tensor with equal normal components and 0 shear components will be, this is the most important part, invariant for any transformation of axes. So if you have a stress tensor with diagonal elements and all diagonal elements are equal, then this happens to be an invariant as it is already proved.

Because whatever transformation you try to do, whatever be the value of A, it is not going to change and hence you can conveniently understand that such a stress tensor will be an invariant and it is not affected by the transformation of axes. This has again some relevance which we will see in the next slide

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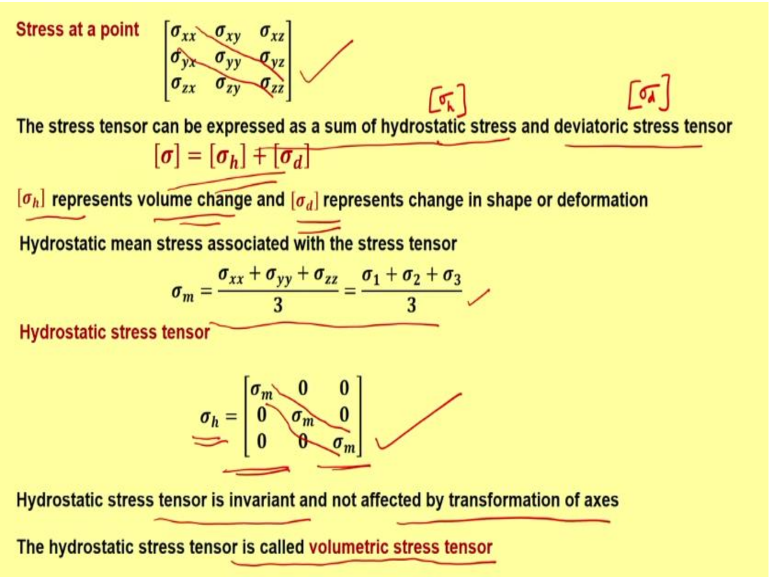

Again, we will start with stress at a point the same stress tensor. This stress tensor now, now we have known about Cauchy's stress tensor, we have exclusively discussed the various aspects of it. Now we are going to introduce another concept which is very important in geomechanics which is the stress tensor can be expressed as a sum of something known as hydrostatic stress and deviatoric stress.

Hydrostatic condition we know they are equal in all direction the something what we know right from the beginning. When we started off fluid mechanics, we know hydrostatic pressure it acts equally in all direction rather isotropic and hence we can represent the whole of this stress tensor as a sum of hydrostatic stress and deviatoric stress which is given by this particular expression. So $\sigma = \sigma_h$ where hydrostatic stress is $\sigma_h$ and $\sigma_d$

So combining these two you will get the stress acting at a point. Now in this, $\sigma_h$ that is the hydrostatic stress is associated with the volume change of a given material or a body. The $\sigma_h$ represents the volume change that is happening and $\sigma_d$ represents the change in shape or deformation. So these two aspects have been decoupled. The stress tensor as you know it is a sum effect of all these.

But we are decoupling these two and writing stress tensor as a sum of hydrostatic and deviatoric stress where hydrostatic stress part represents the volume change that happens in the body, only volume change, and the other part that is the deviatoric stress that represents the volume change that is happening. Now what does this mean? This means that if you are discussing about shear component or the deformation. What causes deformation?

Essentially the shear causes deformation. That deformation is captured by deviatoric stress. So we should understand that the component of deviatoric stress is very important in soils and rocks. We already know because in triaxial testing this component or this stress is very familiar to us, the deviatoric stress. So what causes shearing? It is the deviatoric stress or the unequal stress condition that causes shearing.

So deviatoric stress is very important in soils and rocks. Now the hydrostatic means stress is associated with the stress tensor, how? Now as I told this stress can be decoupled, so let us see how. The hydrostatic mean stress is given by this expression that is the summation of the diagonal elements divided by 3 okay, so it is the mean stress or average stress or hydrostatic mean stress associated with the stress tensor is given by ($\sigma_{xx} + \sigma_{yy} + \sigma_{zz}$) / 3.

And it is same as ($\sigma_1 + \sigma_2 + \sigma_3$)/ 3 why because both of them are same. The entire stress tensor can be represented by principal stresses. So the hydrostatic stress tensor can be very conveniently written as

$$[\sigma_h] = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}$$

So one part of the stress tensor we have obtained, one part of the stress tensor is the hydrostatic stress tensor $\sigma_h$, where $\sigma$ m is the hydrostatic stress.

So we can very easily write based on our previous slide whatever we have discussed we can easily write now this diagonal components are equal. So we can very well write that hydrostatic stress tensor is invariant and it is not affected by transformation of axes. So one invariant again we are stressing upon is hydrostatic stress tensor. So hydrostatic stress tensor is also called volumetric stress tensor.

So these are different terminologies which one should keep in mind. Mean stress, hydrostatic stress, means stress tensor, hydrostatic stress tensor, volumetric stress tensor all represents the same thing that is given by $\sigma_h$and $\sigma_h$  is an invariant, so this we have to keep in mind.

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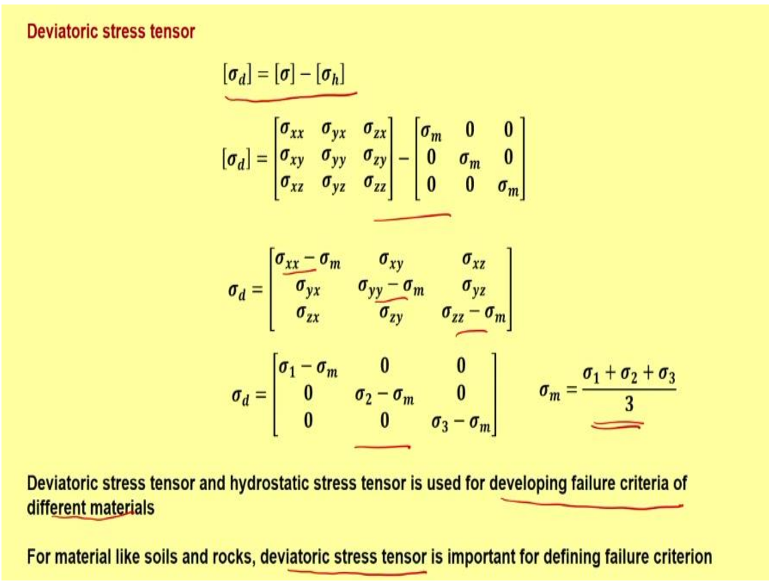

So the next part is deviatoric stress tensor. Now what is deviatoric stress tensor? It is given by σ d which is equal to the total stress tensor that is stress tensor minus the hydrostatic stress tensor, it is very easy. So one is the summation, so $\sigma_d$ happens to be the difference of stress tensor and the hydrostatic stress tensor this is what you get, again I do not need to explain it. So what you ultimately get is

$$[\sigma] = \begin{bmatrix} \sigma_{xx} - \sigma_m & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} - \sigma_m & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} - \sigma_m \end{bmatrix}$$

So this takes the form of the original stress tensor itself, only thing is the diagonal element changes or in the principal stress notation one can write we just replace it that is also pretty straightforward. Now you can write σ m is already known to us, we have already discussed.

So deviatoric stress tensor and hydrostatic stress tensor is used for developing failure criteria of different materials. Now it is very convenient to represent the failure criteria in terms of deviatoric as well as the hydrostatic stress tensor, but to be very specific for materials like soils and rocks deviatoric stress tensor is very important for defining the failure criteria. We will not be dealing with the failure criteria, yield criteria in this particular course.

The whole idea is to let you know that these components or this knowledge is important for understanding or for defining the failure criteria okay. So what we are concerned about is deviatoric stress, the reason is soil is a granular material and it basically fails in shearing and shear is represented by the deviatoric stress component because that influences the deformation. So that is why deviatoric stress tensor becomes important in the development of failure criteria for soils and rocks.

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**Deviator stress invariants $J_1, J_2, J_3$**

$$\sigma_d = \begin{bmatrix} \sigma_1-\sigma_m & 0 & 0 \\ 0 & \sigma_2-\sigma_m & 0 \\ 0 & 0 & \sigma_3-\sigma_m \end{bmatrix} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

$$J_1 = S_1 + S_2 + S_3 = (\sigma_1-\sigma_m) + (\sigma_2-\sigma_m) + (\sigma_3-\sigma_m)$$
$$= (\sigma_1+\sigma_2+\sigma_3) - 3\sigma_m$$
$$= 3\sigma_m - 3\sigma_m$$
$$= 0$$

$$J_2 = -(S_1S_2 + S_2S_3 + S_3S_1)$$
$$= \frac{1}{2}(S_1^2 + S_2^2 + S_3^2)$$
$$= \frac{1}{6}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]$$

$$J_3 = S_1S_2S_3$$

Now we have already seen in the last lecture what are the invariants of stress tensor? We have seen the characteristic equation. Now this deviatoric stress is very much similar to the stress tensor itself. So for the same hydrostatic stress tensor one can easily find out what are the invariants. The same logic of characteristic equation one can use and obtain the invariants $J_1$, $J_2$, $J_3$. When I discussed $I_1$, $I_2$, $I_3$ I have specified this point, what is that?

$I_1$, $I_2$, $I_3$ are invariants of stress tensor whereas here $J_1$, $J_2$, $J_3$ are invariants of deviatoric stress tensor, so we need to specify that and the process of obtaining $J_1$, $J_2$, $J_3$ is same as that of what we have adopted for $I_1$, $I_2$, $I_3$. So, I will not go into characteristic equations here, rather we will go ahead and write $J_1$, $J_2$, $J_3$. So here $\sigma_d$ is $\sigma_1 - \sigma_m$, so this we have already known and this is written in another form which is equal to $S_1$, $S_2$, $S_3$.

So $S_1 = \sigma_1 - \sigma_m$, similarly $S_3 = \sigma_3 - \sigma_m$. So the expression for $J_1$ is $S_1 + S_2 + S_3$, $I_1$ is $\sigma_1 + \sigma_2 + \sigma_3$. So $S_1 + S_2 + S_3$ can be written in this form, we can see that ultimately $J_1$ becomes equal to 0. so $J_1 = 0$, it comes from this expression. $J_2$ is expressed as $- S_1 S_2 + S_2 S_3 + S_3 S_1$. There is a lot of importance of this stress invariant $J_2$. I mean to say $J_2$ is the invariant of deviatoric stress tensor.

This $J_2$ has got a lot of importance in failure criteria or in geomechanics, so please pay a lot of attention on $J_2$. It is also expressed as half $S_1^2 + S_2^2 + S_3^2$. This expression is very important which is

$$J_2 = \frac{1}{6}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]$$

You will see that this particular invariant $J_2$ keeps repeating in various other forums.

Please note that $J_2$ is very important and we will try to see these expressions again as part of assignments of this particular course. We will not discuss that right in this lecture, but will try to include this in the assignment the expressions. Similarly, $J_3 = S_1 S_2 S_3$. So these are the three invariants of deviatoric stress tensor.

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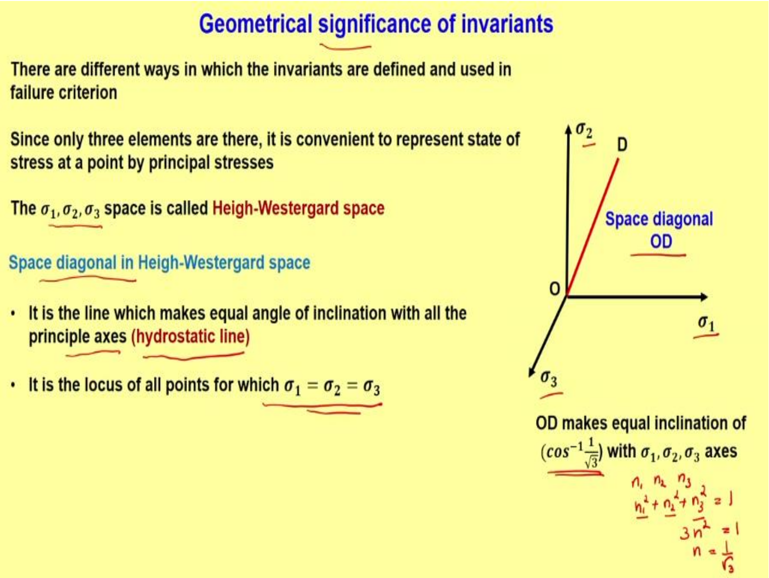


Now we will go to the geometrical significance of invariants. What do you mean by these invariants? Does it have any meaning? Does it represent something? So that is what we will see here. There are different ways in which the invariants are defined and used in failure criterion. So whatever we are discussing in this lecture it is not conclusive, there are different ways by which the invariants are defined in the literature.

So, we will see only very specific and the basic ones. Since only three elements are there, it is always convenient to represent state of stress at a point by principal stresses, we have already discussed this quite a number of times. Now the $\sigma_1$, σ, $_2$ $\sigma_3$ space is called Heigh-Westergard space. It is just a terminology which we would like to include why because there are certain other terminologies and the geometrical significance which we have to discuss.

So first of all we will understand $\sigma_1$, $\sigma_2$, $\sigma_3$ axis and the space is known as Heigh-Westergard space. Now we are going to see what is a space diagonal in Heigh-Westergard space. So space diagonal you can see that now this is the orthogonal axis where $\sigma_1$, $\sigma_2$, and σ

$_3$ is discussed. There is a line OD, it is any such line OD in space, so it is the line which makes equal angle of inclination with all the principal axis.

So OD is a line which makes equal inclination with $\sigma_1$, $\sigma_2$, $\sigma_3$ axis. We can always find one such line which is equally inclined with $\sigma_1$, $\sigma_2$, $\sigma_3$. This line is known as space diagonal OD. So what is space diagonal? It is same as hydrostatic line. So a space diagonal or hydrostatic line is a line which makes equal inclination with $\sigma_1$, $\sigma_2$, $\sigma_3$ axis.

So OD makes equal inclination means the inclination is $\cos^{-1}\frac{1}{\sqrt{3}}$ with $\sigma_1$, $\sigma_2$, $\sigma_3$ axis. You will be wondering how $\frac{1}{\sqrt{3}}$ comes because the normal vector in $\sigma_1$, $\sigma_2$, $\sigma_3$, if it is $n_1$, $n_2$, $n_3$ we know the relationship $n_1^2 + n_2^2 + n_3^2 = 1$ So we know these are same and equal, $n_1 = n_2 = n_3$. So you have $3n^2 = 1$, so we can write n = $\frac{1}{\sqrt{3}}$, so that is how it comes, so it is $\cos^{-1}\frac{1}{\sqrt{3}}$.

So it is the locus of all points for which $\sigma_1 = \sigma_2 = \sigma_3$. This also is a very important information. So on space diagonal or hydrostatic line you have also an information that $\sigma_1 = \sigma_2 = \sigma_3$. So what is the geometrical significance? So we will come to that again okay.

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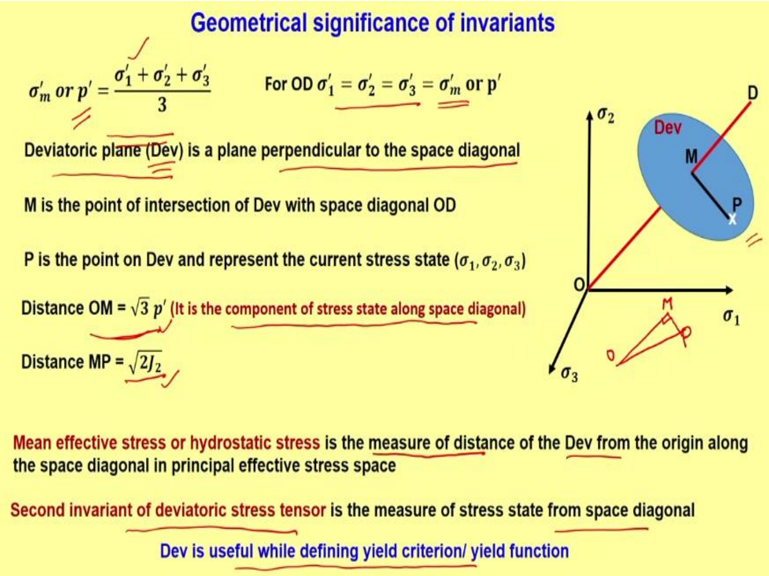


So what is the geometrical significance of invariants? Now we know $\sigma_m$ ' that is the mean stress or hydrostatic stress which is also denoted by p', I have specifically introduced this letter p' why because in most of the textbooks you will see that the mean stress is written in terms of p', so $\sigma_m$' is not that common usage but p' is, you have p q plot so p' that is what p ' represents.

So it is $(\sigma'_1 + \sigma'_2 + \sigma'_3) / 3$. Only thing is that effective stress component has been specifically mentioned here, it can be in total stress as well. So $(\sigma'_1 + \sigma'_2 + \sigma'_3)/ 3$. Now for OD we know that it is $\sigma'_1 = \sigma'_2 = \sigma'_3$ that is equal to $\sigma'_m$ or p'. So OD is a line which represents what $\sigma'_m$.

Now let us introduce the terminology what is known as deviatoric plane Dev in short. This is a plane which is perpendicular to the space diagonal. So any plane that is perpendicular to the space diagonal which is given as Dev is the deviatoric plane. Now let us say M is the point of intersection of deviatoric plane with the space diagonal OD. Now P is a point on the deviatoric plane which represents the current stress state. What is the current stress state?

The current stress state is $\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$. So this is the current stress state. This current stress state is represented by the point P and that is in terms of $\sigma_1$, $\sigma_2$ and $\sigma_3$. So now we have distance OM, we are talking about the geometrical significance. So we are just trying to see whether there is any sort of significance of these parameters $\sigma'_m$ or $J_2$ for that matter and what is that.

So we will first see what is the distance OM. So OM = $\sqrt{3}$ p' or $\sqrt{3}\sigma_m$. So it is the component of stress state along the space diagonal, what is meant by that? If I draw it here OM and this is P, so what is the component of OP on OM direction, so that this particular component is what has been talked about. So it is the component of stress state along the space diagonal and distance OM =$\sqrt{3}$ p'.

This distance is given by $\sqrt{3}$ p' and distance MP = $\sqrt{2J_2}$. So both p' or $\sigma_m$ or the hydrostatic stress and the second invariant of deviatoric stress has got some geometrical meaning. So first one is a measure of distance of OM and MP or the $J_2$ is the measure of distance of MP that is the distance of the current stress state from the intersection of space diagonal and deviatoric plane.

So mean effective stress or hydrostatic stress is the measure of distance of the deviatoric plane from the origin along the space diagonal in principle effective stress space. Please understand that it is specifically mentioned it is measure of distance, it is not equal to the distance why it is in terms of $\sqrt{3}$p', we will again see how $\sqrt{3}$p' in the assignment problems when we will discuss that. The second invariant of deviatoric stress tensor it is the measure of stress state from the space diagonal.

It is the distance of the current stress state from the intersection of space diagonal and deviatoric plane. So both $J_2$ and p' has got some meaning. So deviatoric plane is useful while defining yield criteria or yield function. Again the yield criteria and yield function will not be discussed in this particular lecture or in this particular course. The idea is the knowledge of deviatoric plane is important for one to understand what is yield criteria and how it is defined.

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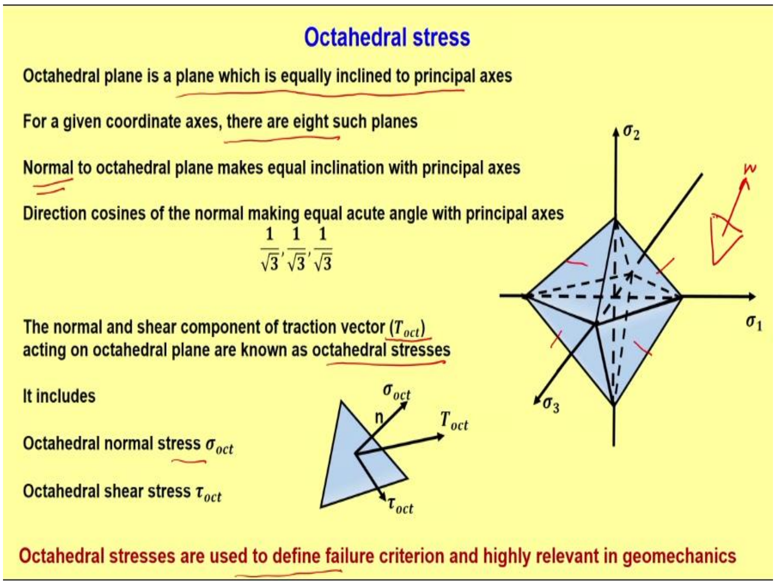


Now the next aspect of invariants is octahedral stress. Now what is octahedral stress? Now to understand octahedral stress one need to understand what is space diagonal that is just what we discussed. Octahedral plane is a plane which is equally inclined to principal axis. Can you relate this statement with our previous discussion? Now octahedral plane, now plane cannot have any inclination, what can have?

The normal to that particular plane can have an inclination. So octahedral plane is a plane which is equally inclined to principal axis. For a given coordinate axis there are eight such planes which is given by in this figure. You can see that this makes 8 planes and all the 8 planes together it is called as octahedral plane and this is $\sigma_1$, $\sigma_2$, $\sigma_3$.

So each of the plane there is you can draw a line which is normal to this plane. That particular normal when you draw this plane we can always draw normal to this plane, now this makes equal inclination with principal axis. it is same definition as that of space diagonal. Normal to octahedral plane makes equal inclination with principal axis. Now this normal is nothing but the space diagonal that we discussed in the previous slide.

So the direction cosines of the normal making equal acute angle with principal axis is $\frac{1}{\sqrt{3}}$, se have again seen this in the previous slide. Now what is of importance is we have seen before any plane, any stress acting on a plane is defined in terms of normal stress and shear stress. In the same manner, the stress acting on an octahedral plane is defined in terms of normal and shear component acting on the octahedral plane of the traction vector.

Now whenever we want to find normal and shear component, we need to know what is the traction which is acting on that particular surface. So normal and shear component of traction vector that is I am marking it as T octahedral. T octahedral is the traction which is acting on the octahedral plane. So normal and shear components of traction vector is the normal stress and shear stress acting on octahedral plane and these are known as octahedral stresses.

So what is meant by octahedral stresses? The normal and the shear component of traction vector which is acting on an octahedral plane is called octahedral stresses and whenever we say stress acting on a plane it is normal and shear stress, so this is what it means. So this is the typical octahedral plane with traction T octahedral acting on it. Now it includes the normal stress, σ octahedral.

This is the normal to this plane and the stress acting is the normal stress, normal octahedral stress, σ octahedral and octahedral shear stress which is tau octahedral. Now octahedral stresses are again used in some of the failure criterion and they are highly relevant in geomechanics problems.

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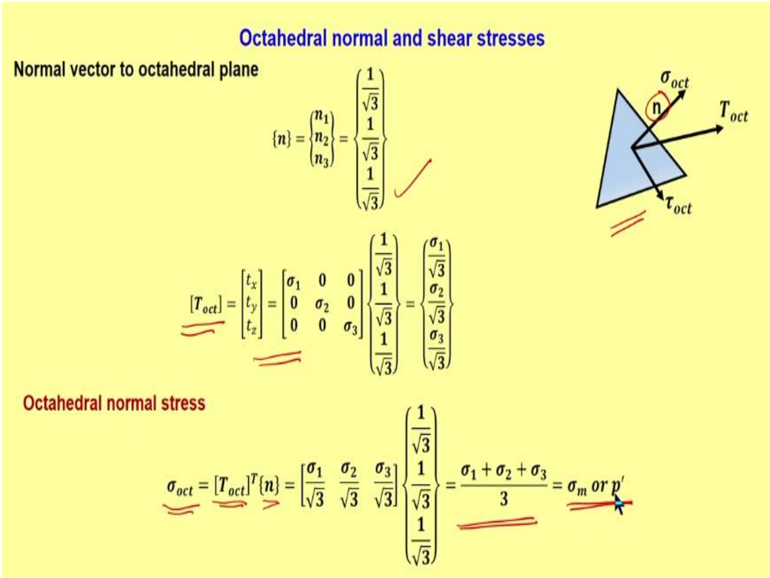


Now let us talk in detail about octahedral normal and shear stress. Again the same plane is noted here. We have normal vector to octahedral plane, this particular aspect normal vector n is given by $n_1$, $n_2$, $n_3$ and this can be arranged in this manner

$$[n] = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

this is coming from the previous slides. So one can always find out what is the traction component of this particular representation.

So you have T octahedral, where does this comes from? This comes from the Cauchy's formula. So the components of the traction $t_x$, $t_y$, $t_z$ it can be written in terms of the stress tensor, earlier we discussed this in terms of Cauchy stress tensor, here we are replacing it by principal stress tensor into the direction cosine So you get what is the traction component

$$[\tau_{oct}] = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} \frac{\sigma_1}{\sqrt{3}} \\ \frac{\sigma_2}{\sqrt{3}} \\ \frac{\sigma_3}{\sqrt{3}} \end{bmatrix}$$

So one can find out the octahedral normal stress as again T octahedral transpose into normal vector. You are mapping T octahedral onto the normal, so that is given by σ octahedral. So octahedral normal stress is obtained as T octahedral transpose into n and that is given by

$$\sigma_{oct} = [T_{oct}]^T\{n\} = \left[\frac{\sigma_1}{\sqrt{3}} \quad \frac{\sigma_2}{\sqrt{3}} \quad \frac{\sigma_3}{\sqrt{3}}\right] \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = \frac{\sigma_1+\sigma_2+\sigma_3}{3} = \sigma_m \; or \; p'$$

Finally you will get ($\sigma_1 + \sigma_2 + \sigma_3$) / 3 which is nothing but the mean stress or the hydrostatic stress. So σ octahedral is same as the mean stress.

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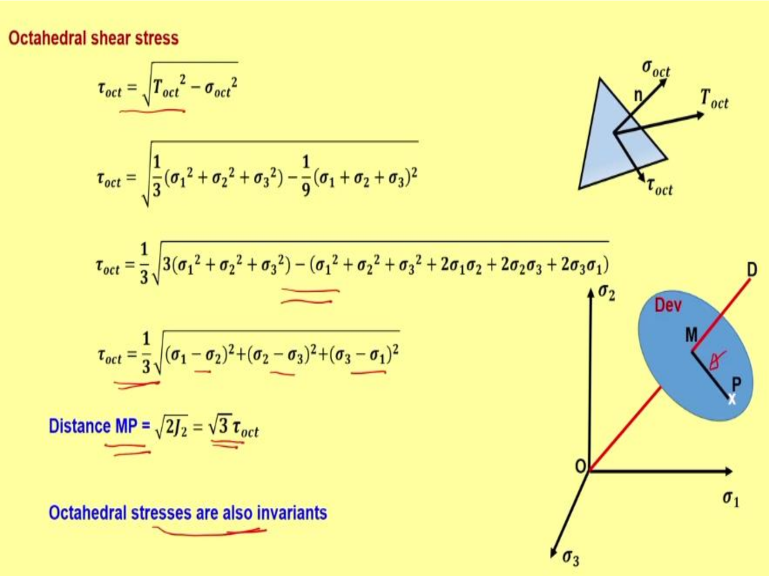


Now we have discussed one part of it, we will see what is octahedral shear stress. We know once we obtain the $\sigma_{oct}$ and we know the traction we can always find out the magnitude of octahedral shear stress and that is what is written here. $T_{oct}^2 - \sigma_{oct}^2$, both are known so one can obtain $T_{oct}$ and this is given by

$$\tau_{oct} = \sqrt{\frac{1}{3}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \frac{1}{9}(\sigma_1 + \sigma_2 + \sigma_3)^2}$$

This again both the expressions it comes from the previous slide. Again if you expand this you will get in this particular expression, I strongly advise all of you to work it out and see for yourself, it is a straightforward exercise so I am not spending time. So ultimately, we get the expression

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

Now this closely resembles the expression that we got for $J_2$. So here distance MP we have written it as $\sqrt{2J_2}$, we have already seen in the previous slide. Now this closely resembles $J_2$, hence it can also be written equal to $\sqrt{3}\,\tau_{oct}$. So both $\sqrt{2J_2} = \sqrt{3}\,\tau_{oct}$ . So please try to derive this, we will again try to discuss this as part of assignment.

So here this is the situation deviatoric plane M, the current stress rate P, so octahedral stresses are also invariants. So we have discussed the relevance of tau octahedral with respect to this particular distance. Also the $\sigma_{oct}$ is a measure of the distance OM. So hence we can also conclude that octahedral stresses are also invariants.

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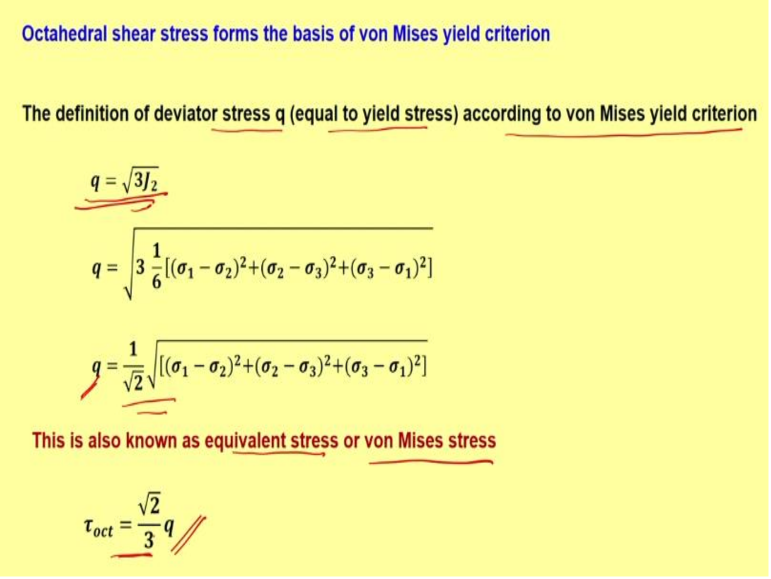


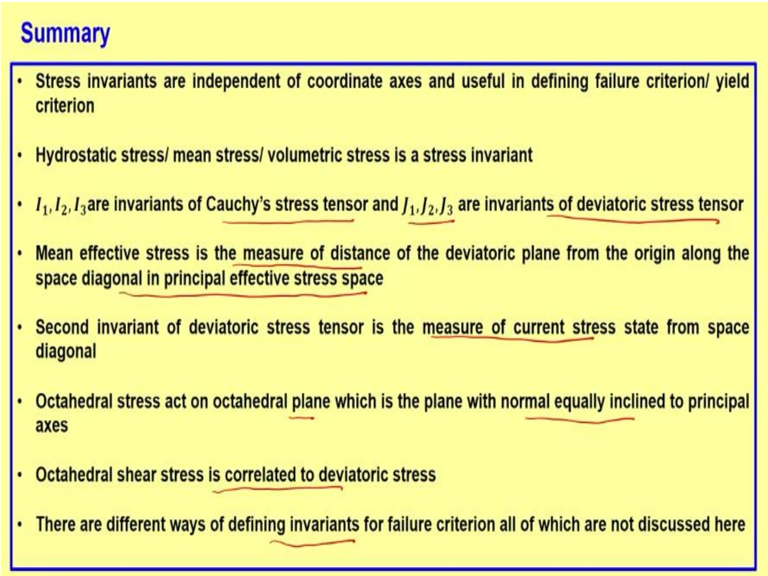

Now octahedral shear stress forms the basis of von Mises yield criteria. I have thought of introducing this why because there are certain definition for deviatoric stress. The definition of deviatoric stress q which you can see in certain textbook equal to yield stress according to von Mises yield criteria because this form of expression for q which I am going to discuss is rather popular. So $q = \sqrt{3J_2}$. This is defined in this manner $q = \sqrt{3J_2}$ according to von Mises yield criteria.

So if you follow this, $J_2$ is known, then one will get the expression for q.

$$q = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

This form of expression of q is important and you will see this in some textbooks. So that is why the relationship of q and octahedral shear stress just wanted to highlight because this particular expression comes from this definition and that is according to the von Mises yield criterion.

So this is also known as equivalent stress or von Mises stress, what q is known as equivalent stress or von Mises stress. So one can easily find out the relationship between tau octahedral and q because you have the expression for q in terms of $J_2$, you also have the expression for tau octahedral in terms of $J_2$, substituting both one can always obtain $\tau_{oct} = \frac{\sqrt{2}}{3} q$ **(Refer Slide Time: 39:13)**




So we will summarize what we have seen till now related to stress invariants. The stress invariants are independent of coordinate axes and useful in defining failure criterion or yield criterion. For any materials that you encounter in geomechanics for soils and rocks stress invariants they are very important and it is a very convenient way of defining a given problem. Hydrostatic stress or mean stress or volumetric stress is a stress invariant.

$I_1$, $I_2$, $I_3$ are invariants of Cauchy's stress tensor and $J_1$, $J_2$, $J_3$ are invariants of deviatoric stress tensor. Mean effective stress is the measure of distance, please underline measure of distance it is not equal to distance, of the deviatoric plane from the origin along the space diagonal in principle effective stress space, along the space diagonal means hydrostatic line. Second invariant of deviatoric stress tensor again it is the measure of current stress state what is the given current stress state from the space diagonal.

So that is the second invariant means $J_2$. Octahedral stress act on octahedral plane which is the plane with normal equally inclined to principal axes. Octahedral shear stress is correlated to deviatoric stress. Now there are different ways of defining invariants for failure criterion which you will see in geomechanics now all of which are not discussed here. So I want to make this point very clear the discussion on stress invariants is not conclusive.

There are a lot of other aspects which one can discuss, but in this particular course it is a bit difficult to go into the details of stress invariants. So whatever bare minimal understanding one should have to start continuum mechanics or geomechanics problems has been discussed here. So that is all for now. We will see in the next lecture.