Advanced Soil Mechanics
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Lecture-55
CSSM Problems

Welcome back, all of you. So, with the last lecture we have completed more or less all the required concepts for critical state soil mechanics which I planned for this particular course. That does not mean that we have completed everything related to critical state soil mechanics, there are a lot more to learn further. But what is required to start with and for this particular course as advanced soil mechanics we have finished.

And we have touched upon the stress, the plastic behaviour as well as the strain. But the very next stage is what is actually needed for practicing it in for solving the actual problems in geomechanics, so that part we are not getting on to in this particular course. Now based on the concepts that we have learned, we will work out some problems. So, that whatever concepts you have gone through it will help you to understand better.

So, today we will be solving some problems relate which will use the concepts that we have discussed in the past few lectures**.** So, let us move on to the first problem.

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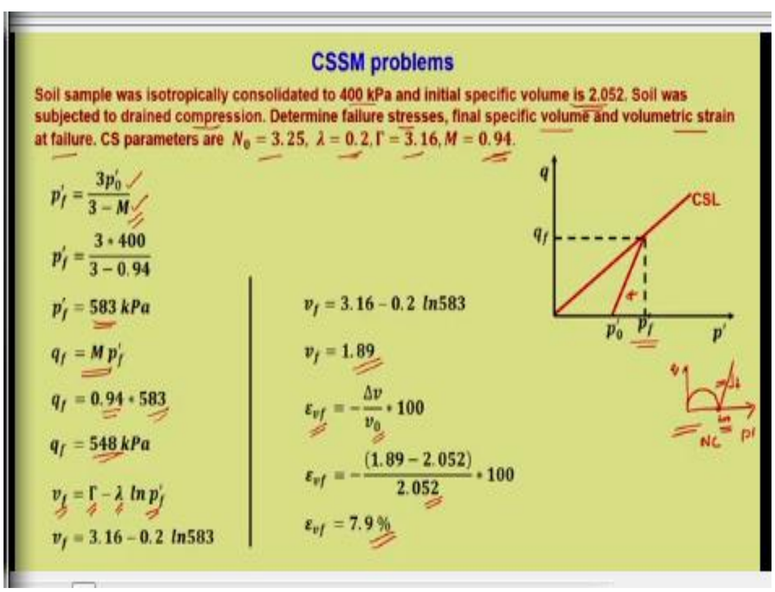

The problem is soil sample was isotropically consolidated to 400 kilopascal and the initial specific volume is 2.052. So, this initial specific volume corresponds to this particular isotropic consolidation. The soil was subjected to drained compression, determine failure stresses, final specific volume and volumetric strain at failure? Critical state parameters $are\ N_o\ =\ 3.25, \lambda =$ $0.2, \gamma =\ 3.16, and\ M$ 0.94, so what has been given?

There is a soil sample whose critical state parameters are known, then we have been given with the initial state, that it has been isotropically consolidated. Now knowing the initial state and the critical state parameters we will demonstrate or that is what we started off with, why do we need a critical state framework? For predicting the behaviour of soil now what do you mean by actually prediction? We have not underlined and specifically stated what actually it is.

Even though we have shown the figures we have shown from initial state how the stress path will move, how it will yield and how it will fail, we have discussed this. But then this problem is a very good demonstration of you can use critical state concept for predicting the behaviour of soil or where it will yield, where it will fail depending upon the known initial condition. And obviously the initial condition will be known, and based on the known critical state parameters for that particular soil.

That is what the question is all about**.** So, now it is given, it is isotropically consolidated, that means it is having the maximum yield stress. So, for example if I draw q p', what I mean is that? This is the yield curve, this is 400, so it is already at it is yield point, it is not unloaded, unloaded means we are creating lightly over consolidated to heavily over consolidated state**.** Here in this case it is sheared under drain condition soon after isotropic consolidation.

So, that means it is a normally consolidated point, so please keep this in mind. So, 400 kilopascal here it refers to NC normally consolidated state. So, isotropically consolidated but it is normally consolidated. Now from here if there is a shearing which is done under it is given drained compression. So, drained compression we know that it will move like this, so it is at a inclination of 3, this line is actually marked here, this is the one.

So, here I have not shown the yield curve rather but it will look like this, so that is we please keep in mind when you read the question you should be able to visualize or rather you should be drawing sketching the required details. And please understand about the aspects of there are 3 aspects one is whether you have to draw it in q p' v ln p', it is better you draw it on both because that will give you additional understanding.

Second thing is the effective stress path, how it will be whether it is drained or undrained? Placing of the yield curve, now for all practical purpose let us consider modified Cam Clay and hence you can draw an ellipse. So, half ellipse you can draw, so that will give the entire picture of that particular problem. So, p'_f can be directly determined knowing

$$p'_f = \frac{3p'_O}{3 - M}$$

refer back to the lectures for knowing this. This expression has come from the geometry of the figure. So, if you substitute

$$p'_f = \frac{3 * 400}{3 - 0.94}$$

$$p'_f = 583\ kPa$$

Now once you know p'_f, we can obtain q f, because we know the relationship

$$q_f = Mp'_f$$

So, substitute for p'_f we will get

$$q_f = 0.94 * 583$$

$$q_f = 548\ kPa$$

So what has been asked, determine the failure stresses? So, one part is over, failure stresses can be obtained based on the critical state framework. Then we also have the relationship

$$v_f = \Gamma - \lambda \ln p'_f$$

So, substituting we have

$$v_f = 3.16 - 0.2 \ln 583$$

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$$v_f = 1.89$$

So you can see that it is a drained compression and hence the specific volume will reduce, so it has come to 1.89. So, the final specific volume has been determined. Now what is the volumetric strain at failure? At failure the volumetric strain ϵ_{v_f}

$$\epsilon_{vf} = -\frac{\Delta v}{v_o} * 100$$

Change in void ratio, so it change in specific volume upon initial specific volume, initial specific volume is given 2.052, Δv can be determined because the final one is known so 2.052 is the initial minus final that will give the change in specific volume.

$$\epsilon_{vf} = -\frac{1.89 - 2.052}{2.052} + 100$$

$$\epsilon_{vf} = 7.9\%$$

So, this how we can use a very simple example of how we can use the critical state framework for predicting the final conditions. And here the conditions can be failure condition or may be final specific volume. Now here it is the case of drained compression, so we know that it undergoes volume change and initial specific volume will be different from the final specific volume ok**.**

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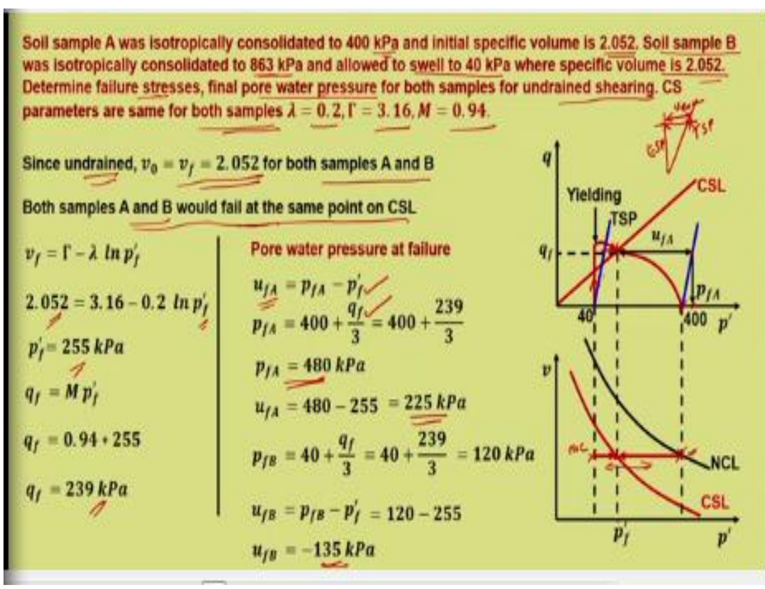

So, let us move to the next example. Soil sample A was isotropically consolidated to 400 kilopascal and the initial specific volume is 2.052. So, sample A is that from the previous discussion we know that this is a normally consolidated state. Soil sample B was isotropically consolidated to 863 kilopascal, and then it is allowed to swell to 40 kilopascal where the specific volume is again 2.052.

Please note here, sample B has been subjected to a very high stress 863, it is allowed to swell to a very low stress say 40 kilopascal. So, without looking anywhere you can find, you can see that this particular soil sample B will be heavily over consolidated, there is no doubt about it. Determine failure stresses, final pore water pressure for both samples for undrained shearing? So, now earlier example was drained shearing, here we have undrained shearing along with that we have both the samples NC and OC.

Now very important aspect which you have to keep in mind is that the initial specific volume of both the samples are same, sample A and sample B which has 2.052. Critical state parameters are same for both the samples $\lambda = 0.2, \gamma = 3.16, and\ M = 0.94$. Now the very first condition which what we know is, that it is undrained. So, the initial specific volume will be same as final specific volume, so $v_0 = v_f$ which is equal to 2.052 for both samples A and B.

So, this both samples 2.052 initial condition means it gives us a lot of clue about how this test would take place and that is the beauty of the idealized critical state, we will see how? Both samples A and B would fail at the same point on critical state line, how? If you have understood the critical state concept clearly this sentence it is not very difficult to understand. But for that it is always better that we sketch the diagram.

So, it is q p’, v p’, the first sample is 400, so now this is a normally consolidated point, so this is the starting point, this is the required point. Now the next sample it has been unloaded to 40 kilopascal from 863. So, for us loading to 863 is not very important, it will be somewhere down and from there it will be unloaded. So, that it reaches the final state of on the unloading curve, so this unloading curve comes from somewhere down.

So, I am not drawing it here but we need to understand that. So, here along the 40 kilopascal we know that the initial specific volume is 2.052. So, whatever is the specific volume for NC the same should be the specific volume for HOC as well, so somewhere here, so 2.052, so it will be somewhere in this line. Now we know that this is an undrained test, so there is no change in the specific volume, so initial equal answer.

So, the path will move in this direction and fail on the critical state line. Now same horizontal line will be the HOC response also, you can see that this is the starting point of HOC. So, this is HOC and this is NCL, now this is in the same horizontal line, so it will fail at this particular point. And that is what is meant by both samples A and B would fail at the same point on critical state line, why it is so?

It is so, because the starting specific volume is same for both the samples which is equal to 2.052. Here the failure stress will also be same, so this is the failure stress p'_f, p'_f is the failure stress that also remains same, if p'_f is same then q_f is same for both the samples. And it is all possible because we know that the initial specific volume is same q_f, p'_f it is all same for both.

Now here this is the kind of effective stress path for sample A, and what will be the sample B? See if we do not know the concept of yielding what we will do is, we will draw something like this, this is also possible or it will be drawn like this. But now we know that it has to yield before it has to fail. So, with that understanding and we know that there is an undrained shearing, so A will be equal to 1/3 till it yields.

So, it is all possible and it is in this direction because we know the concept. So, that is the point of yielding, I am not showing the yield curve here separately. So, yielding has happened but it comes and fails at the same point. Now we are asked to find out the final pore water pressure as well. And for that any stress condition on the TSP minus the homologous condition on ESP will give us the excess pore water pressure even at failure or even at whatever the yield point of wherever we need we will get that.

For example in this particular case, let us say this is ESP and this is TSP. Now if I draw a horizontal line this point and this point they are called homologous points. So, the difference between this will give the u excess, this is the pore water pressure. So, that is what we need to find out and hence we need to have the total stress path, and it is very clear total stress path will be at an inclination a slope of 3, that also we know.

So, at both the initial points at 400 as well as 40 we need to find out the final pore water pressure. Now here this difference, this is the failure point, so from here are this difference will give u_f of A, that is the final pore water pressure of the sample A. I cannot mark for B because it is very clumsy, but it is already discussed in the lecture. It is the point between this and this, if you draw a horizontal line, so it will be this total minus effective.

So, again the same like what we have for u_{fA}. So, if that is the case we have the expression

$$v_f = \Gamma - \lambda \ln p'_f$$

And we also know $v_f = v_o$, unless we know these conditions it will be difficult to solve this problem. So, substituting the initial specific volume for v_f, we have

$$2.052 = 3.16 - 0.2 \ln p'_f$$

$$p'_f = 255\ kPa$$

$$q_f = Mp'_f$$

$$q_f = 0.94 + 255$$

$$q_f = 239\ kPa$$

So, we have determined the failure stresses. Now we are left with final pore water pressure at failure. So,

$$u_{fA} = p_{fA} - p'_f$$

so we need to determine what is p_{fA}? Now p_{fA} is this, that is a point, so this is the point which we need to find out, so what is p_{fA}?

$$p_{fA} = 400 + \frac{q_f}{3} = 400 + \frac{239}{3}$$

$$p_{fA} = 480kPa$$

$$u_{fA} = 480 - 255 = 225\ kPa$$

$$p_{fB} = 40 + \frac{q_f}{3} = 40 + \frac{239}{3} = 120\ kPa$$

So, once we know p_{fA} we know p'_f, so u_{fA} can be determined,

$$u_{fA} = p_{fB} - p'_f = 120 - 255$$

$$u_{fB} = -135\ kPa$$

And this is very correct because it is an HOC, so the final pore water pressure will be negative, so that is exactly what we are getting and that is equal to -135 kilopascal. So, we have in the previous problem we predicted final condition of the specific volume, volumetric strain and the failure stresses. So, in this problem since it is an undrained test, we have determined the failure stresses as well as the final excess pore water pressure.

So, we should keep in mind using critical state concept, we can even determine the pore water pressure at failure, at yield or at any point intermittent point from the starting to the failure. So, these are all possible, provided we follow critical state framework, we know critical state parameters and we know what is the initial state of the sample.

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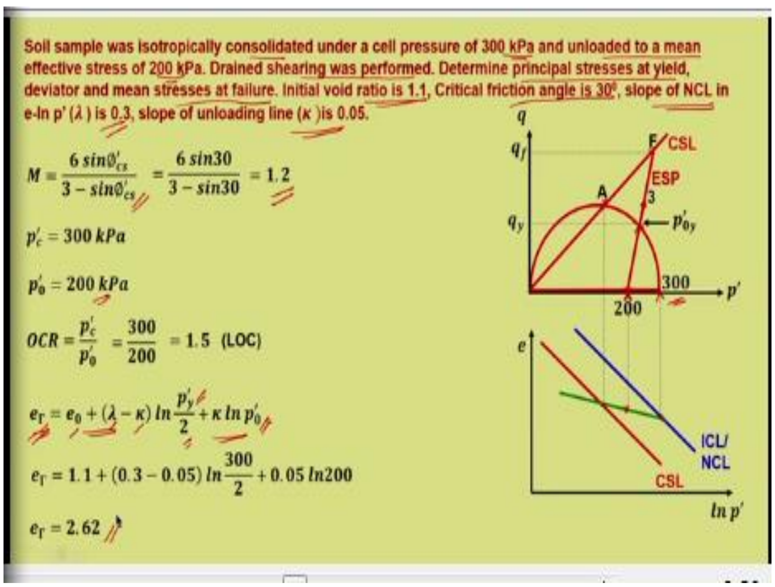

So, let us move on, now most of the things have been covered, now what is not covered? We can see we have not determined the yield conditions. So, now we will demonstrate a problem wherein we try to determine the yielding. Soil sample was isotropically consolidated under a cell pressure of 300 kilopascal and it is unloaded to a mean effective stress of 200 kilopascal. So, now from the question itself we can make out it is not a normally consolidated, it can be LOC or HOC.

Drained shearing was performed, so volume change is going to take place. Determine principal stresses at yield? So, we need to understand what is yield, at what point it will yield and we need to find out what are the principal stresses, it is not q and p', it is principal stresses at yield. Now for finding out principal stresses at yield, we need to determine what is the q and p' value at yield, deviator and mean stresses at failure, that is q_f and p'.

The conditions are initial void ratio is 1.1, now initial void ratio of 1.1 means it corresponds to the unloaded portion, it is not the isotropically consolidate, it is unloaded now. So, 1.1 corresponds to unloaded portion, critical friction angle is given 30 degrees, slope of NCL in e-ln p' is given as λ which is 0.3. So, here we need to understand that $\lambda\ or\ \kappa$ is the slope, that we know, that corresponds to either NCL or the unloading-reloading line.

Now we have discussed most of the things in terms of specific volume and p'. But we can the same thing can be plotted in e versus p', only thing is there will be the scale Difference of 1 + e. So, here please note in the question it is given λ and κ corresponds to the slope in e ln p' plot. So, you can go ahead with y axis taken as e instead of specific volume v, so I would just like to show you this as well.

So, you need to see the question properly and λ and κ do not go by that blind understanding that, it is always related with specific volume v, no. Whatever is the slope if it is defined in the question λ corresponds to this, please go ahead with that. The slope of the unloading line is 0.05, so all the details are given. So, M we know that it is

$$M = \frac{6\sin\phi'_{cs}}{3 - \sin\phi'_{cs}}$$

Now here the critical friction angle is already given and that is equal to 30 degrees, if you substitute we will get M the value to be 1.2,

$$M = \frac{6\sin 30}{3 - sin30} = 1.2$$

so that is known, p'_C, what is p'_C? We need to understand here, the significance of p'_C, $p'_C$is the maximum isotropic consolidation. So, that is the maximum yield stress or pre-consolidation pressure, we need to understand, that is the extreme point of the major axis of the ellipse, it is unloaded to 200 kilopascal.

$$p_c' = 300\ kPa$$

$$p_o' = 200\ kPa$$

$$OCR = \frac{p_c'}{p_o'} = \frac{300}{200} = 1.5\ (LOC)$$

OCR is less than 2 we can consider this to be a lightly over consolidated sample, so it is an LOC, so this is what it means. So, this is the yield curve, ellipse yield curve for modified Cam Clay. So, this is the extreme yield stress or the pre-consolidation pressure 300 unloaded to 200, then it is subjected to drained shearing. So, this the effective stress path fails at point F, so this is the point of interest to us.

Because we need to determine principal stresses at yield. So, this q_y and p_{oy}' we need to determine. So, same thing is shown here, this corresponds to 300, here is the unloaded point 200 and this is the point where the unloading line crosses the critical state. And according to the definition of MCCM and the elliptical yield curve, we know that this is the midpoint. So, now we do not have the gamma parameter given, so we can compute gamma parameter rather in this case it is e_Γ.

$$e_\Gamma = e_o + (\lambda - \kappa)\ln\frac{p_y'}{2} + \kappa \ln p_o'$$

So, e_Γ is given, the expression we have already derived, please look back you will find in the lecture that we have derived this particular equation. p_y' is the yield stress of pre-consolidation pressure, λ is given, κ is given, e_o is given, the initial void ratio is 1.1.

$$e_\Gamma = 1.1 + (0.3 - 0.05)\ln\frac{300}{2} + 0.05\ln 200$$

$$e_\Gamma = 2.62$$

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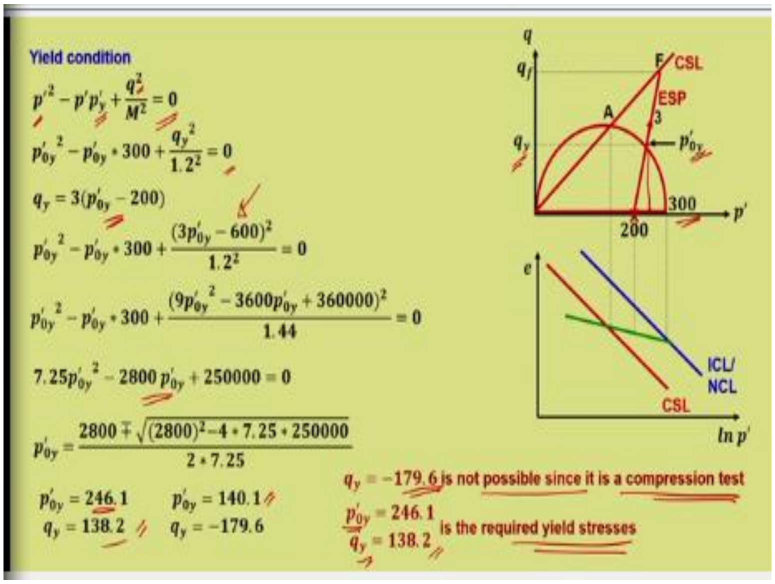

So, now we can determine the yield condition. We know that the effective stress path meets the yield curve at this particular point. So, we can always substitute q y and p'_{oy} in the elliptical yield curve equation. So,

$$p'^2 - p'p'_y + \frac{q^2}{M^2} = 0$$

$$p'_{oy} - p'_{oy} * 300 + \frac{q_y^2}{1.2^2} = 0$$

So, q_y from the geometry we know that it is

$$q_y = (p'_{oy} - 200)$$

$${p'_{oy}}^2 - p'_{oy} * 300 + \frac{(3p'_{oy} - 600)^2}{1.2^2} = 0$$

$${p'_{oy}}^2 - p'_{oy} * 300 + \frac{(9{p'_{oy}}^2 - 3600p'_{oy} + 360000)^2}{1.44} = 0$$

Again expanding we will be left with a quadratic equation. So, here we will have 2 roots

$$7.25{p'_{oy}}^2 - 2800p'_{oy} + 250000 = 0$$

$$p'_{oy} = \frac{2800 \pm \sqrt{2800^2 - 4 * 7.25 * 250000}}{2 * 7.25}$$

One of the root for the p'_{oy} is 246.1. And if you substitute for q_y will be 138.2

$$p'_{oy} = 246.1$$

$$q_y = 138.2$$

The another root of p'_{oy} is 140.1 and q_y is -179.6

$$p'_{oy} = 140.1$$

$$q_y = -179.6$$

Now q_y= -179.6 is not possible rather it is not useful for us because it is a compression test.

We know that it is a compression test and hence there is no possibility of q_y, it achieving an negative value. So, even though the root is there that is not the useful term that we have to use. So, we will go with the final value of p'_{oy} and q_y, 246.1, 138.2 is the required yield stresses.

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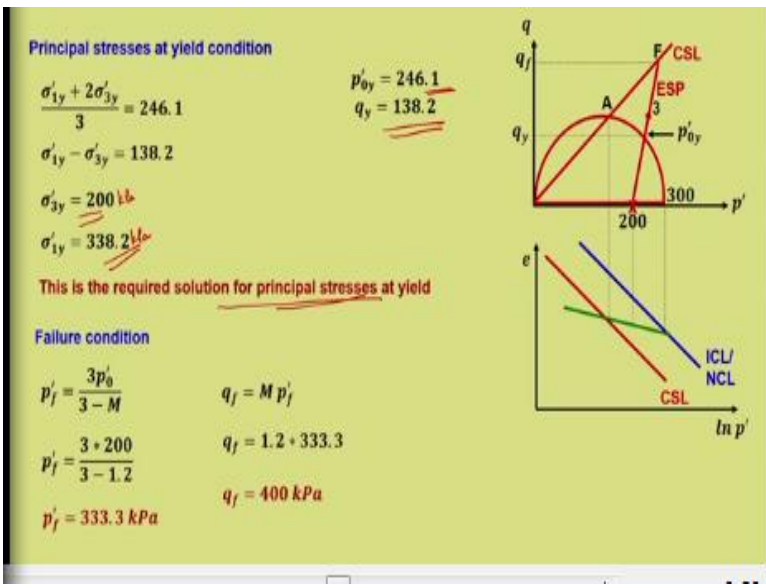

Now we can easily find out the principal stress at failure, why? Because p'_{oy} and q_y is known, substitute the expression. So, principal stresses at yield condition, we have determined, this will give

$$\frac{\sigma'_{1y} + 2\sigma'_{3y}}{3} = 246.1\ kPa$$

$$\sigma'_{1y} - \sigma'_{3y} = 138.2 \text{ kPa}$$

Solving these 2, we will get

$$\sigma'_{3y} = 200 \text{ kPa}$$

$$\sigma'_{1y} = 338.2 \text{ kPa}$$

$$p'_f = \frac{3p'_o}{3 - M}$$

So, this is the required solution for principal stresses.

Now the failure condition, we are asked to find out the q_f and p'_f which we already know. So, it is the same solution

$$p'_f = \frac{3p'_o}{3 - M}$$

$$p'_f = 3 * \frac{200}{3 - 1.2}$$

$$p'_f = 333.3kPa$$

$$q_f = Mp'_f$$

$$q_f = 1.2 * 333.3$$

$$q_f = 400kPa$$

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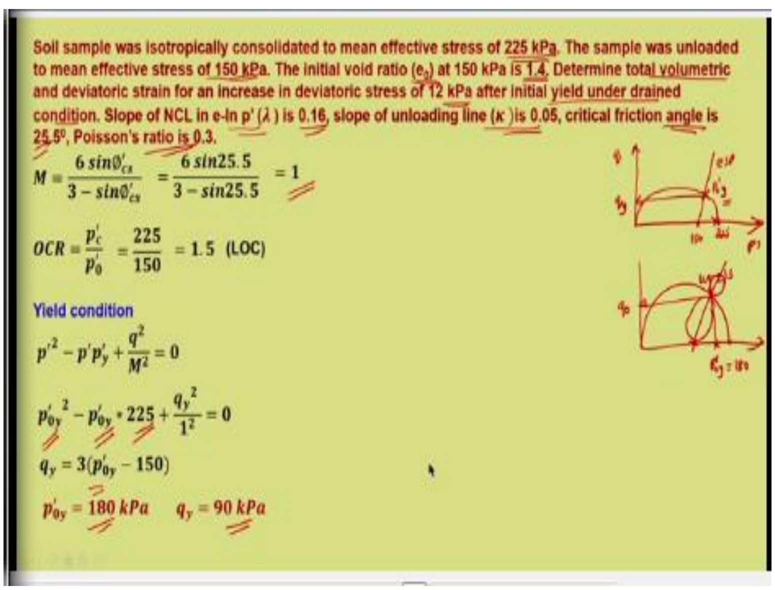

So, now we have completed more or less the predictions of stresses. Now we also have found out we can obtain the stiffness from the critical state framework. We can also determine the strain from the critical state framework. So, we will do a problem which will demonstrate that particular aspect.

Soil sample was isotropically consolidated to mean effective stress of 225 kilopascal, sample was unloaded to mean effective stress of 150 kilopascal. The initial void ratio e_o at 150 kilopascal is

1.4, so the initial void ratio is given, determine total volumetric and deviatoric strain for an increase in deviatoric stress of 12 kilopascal after initial yield under drained condition. So, what we know? What we know is that it is loaded to a maximum or the pre-consolidation pressure of 225 kilopascal. Now there is a elliptical yield curve corresponding to this and the major principle **x** major axis of the ellipse will be 225 kilopascal.

It is unloaded to 150 kilopascal and the initial void ratio is given. Now what is asked, we need to find out what is the total volumetric strain and deviatoric strain for this given problem, when the effective stress path moves beyond the initial yield curve. And that how much it is given? It is given there is an increase in deviatoric stress of 12 kilopascal. So, when the deviatoric stress moves 12 kilopascal from the initial yield curve. I mean to say the effective stress path what will be the total volumetric and deviatoric strain? So, we need to count from the initial point to the point where effective stress path goes past beyond the initial yield curve by an increment in deviatoric stress by 12 kilopascal. So, you will come to know better when we show it on a diagram. Slope of NCL in e-ln p' λ is given 0.16, slope of unloading line is 0.05, critical friction angle is 25.5 degrees, Poisson's ratio is 0.3.

So, now we need to use the stiffness concept, we need to use the strain concept. M we can find out there is no need for explanation,

$$M = \frac{6sin\phi'_{cs}}{3 - sin\phi'_{cs}} = \frac{6sin25.5}{3 - sin25.5} = 1$$

$$OCR = \frac{p'_c}{p'_o} = \frac{225}{150} = 1.5\ (LOC)$$

Yield condition, so we need to have the yield first, we will come to the figure a bit later, let us first resolve what is yield of the effective stress path. So, what I mean to say is, this is q p', we have 225, so once we have 225 we can always draw the yield curve and then it is unloaded to 150. So, here this is the point which I want to say and when there is a drained shearing. So, we need to obtain, since it is given total volumetric and deviatoric, so if you want to have volumetric strain it is understood that it has to be drained. So, here this is the ESP, now this is the point which we are talking about where we need to find out q_y and this is p'_{oy}.

So, this is what we need to determine and that we have already shown it in the last problem. So, just substitute it

$$p'^2 - p'p'_y + \frac{q^2}{M^2} = 0$$

$$p'^2_{oy} - p'_{oy} * 225 + \frac{q^2}{1^2} = 0$$

$$q_y = 3(p'_{oy} - 150)$$

$$p'_{oy} = 180\ kPa \quad q_y = 90 kPa$$

So, to be very precise, this is if I draw it again on a bigger scale, this is 150, this is set to 3, so $p'_{oy} = 180\ kPa$ and $q_y = 90kPa$. Now we are asked if it moves past if this effective stress path moves by this is 12 kilopascal, deviatoric stress is 12 kilopascal. So, this is the point which it corresponds to when it deviates by this what will be the kind of strain, total strain that it has undergone. So, this will include the strain due to this elastic portion and the strain due to this increment, so both we need to find out, so that is what we will solve now.

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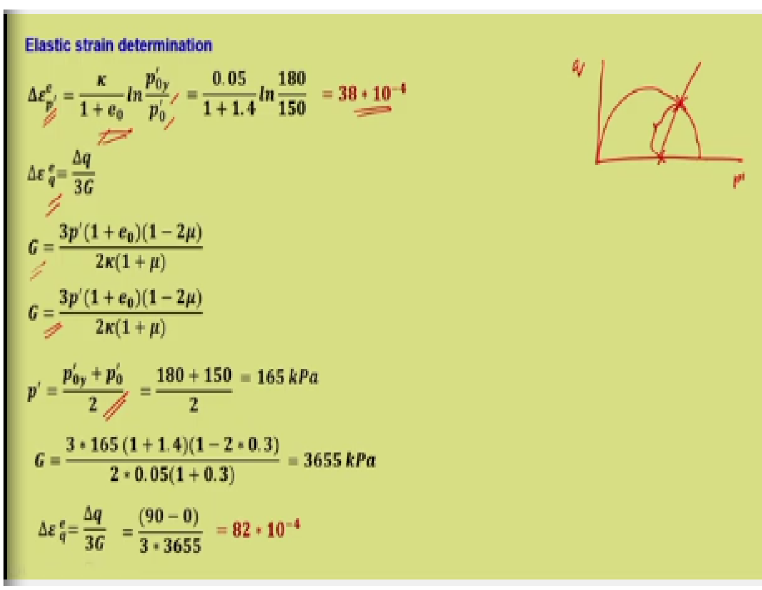

So, first let us do the elastic strain determination. So, $\Delta\epsilon^e_{p'}$ we have already derived this, please refer back strain from critical state that particular lecture wherein it is clearly given $\Delta\epsilon^e_{p'}$. That means the elastic part of the volumetric strain and for the initial yield we do not have any plastic strain. Till it reaches the yield point it exhibits completely a volumetric elastic strain.

So, that is what we have found out here, that is

$$\Delta\epsilon^{e}_{p'} = \frac{\kappa}{1+e_o} \ln\frac{p'_{oy}}{p'_o} = \frac{0.05}{1+0.4} \ln\frac{180}{150} = 38 * 10^{-4}$$

Which is the elastic volumetric strain when the effective stress path moves from the initial point to the first yielding. And $\Delta\epsilon^{e}_{q}$that is the elastic deviatoric strain is

$$\Delta\epsilon^{e}_{q} = \frac{\Delta q}{3G}$$

Now what is Δq and for G we need to find out we have the expression for G

$$G = \frac{3p'(1+e_o)(1-2\mu)}{2\kappa(1+\mu)}$$

μ is the Poisson's ratio, e_o is known, p' please remember here since it is a line starting from q = 0 to q = q_y better to take the average of the stresses corresponding to this. That is p' will be the average value, so

$$p' = \frac{(p'_{oy} + p'_o)}{2}$$

$$= \frac{180+150}{2} = 165\ kPa$$

So, instead of using 180, use the average stress condition. So, that will give

$$G = \frac{3 * 165(1+1.4)(1-2*0.3)}{2*0.05(1+0.3)} = 3655\ kPa$$

So, then we can find out what is the deviatoric elastic strain till the first yield point, that is

$$\Delta\epsilon^{e}_{q} = \frac{\Delta q}{3G} = \frac{90-0}{3*3655} = 82 * 10^{-4}$$

So, both the elastic strain deviatoric and volumetric, till the first yield we have determined. I mean to say the first yield means q_p', when the effective stress path starts from here. So, this is the first yield, so by the time it reaches here what is from here for this stretch what is the elastic strain it has undergone? Both volumetric and deviatoric strain we have determined.

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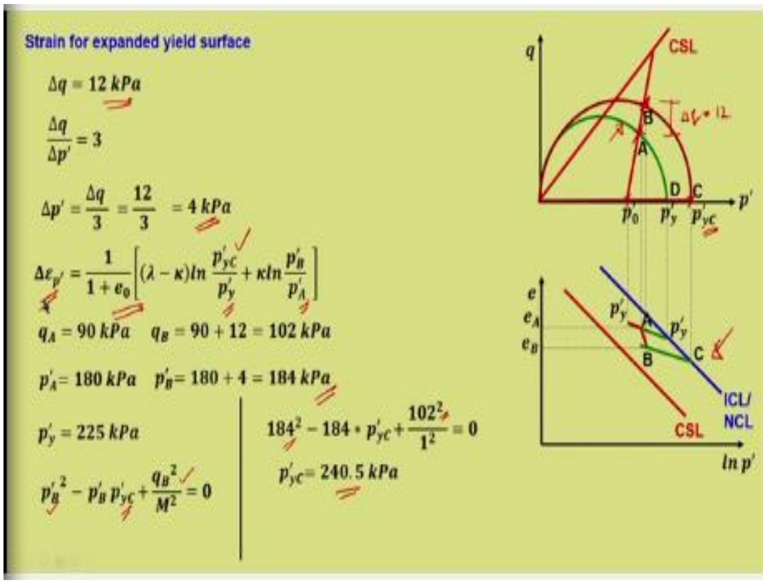

Now we need to determine the strain for expanded yield surface or when the effective stress path crosses the first yield point, then we have both. Both means plastic strain as well as elastic strain, now this is an elastoplastic response. It has yielded at the first yield point, it is moving by 12 kilopascal of deviatoric stress.

So,

$$\Delta q = 12kPa$$

$$\frac{\Delta q}{\Delta p'} = 3$$

we need to find out if Δq = 12 kilopascal, what will be delta $\Delta p'$?

$$\Delta p' = \frac{\Delta q}{3} = \frac{12}{3} = 4kPa$$

Now this is the final form of the figure, so this is our initial, the green one is our initial yield curve and that is p'_y. So, we have already done, now effective stress path moves from p'_o , this is the first yield point the conditions we have everything it is clear. Now it expands from A to B because there is an increment of Δq = 12.

Now corresponding increment in delta p' for this point B is known which is 4 kilopascal. And now we need to find out this particular point, what is the maximum yield stress corresponding to the expanded yield curve? Remember the soil sample has not failed, it is intermittent, so this is an intermittent yield circle. The same thing is shown please refer back to the lecture for more explanation.

Now the total volumetric strain which is inclusive of both elastic as well as plastic is given. So, we have already derived this, we know this how to obtain,

$$\Delta\epsilon_{p'} = \frac{1}{1+e_o}\left[(\lambda-\kappa)\ln\frac{p'_{yC}}{p'_y} + \kappa\ln\frac{p'_B}{p'_A}\right]$$

Now what is p'_{yC}? p'_{yC} is the extreme maximum yield or pre-consolidation pressure, p'_y is the previous pre-consolidation pressure, p'_B is the new stress state that is after yielding and p'_A is the initial yielding.

So, all these characteristics we need to know, we need to find out p'_B is known, p'_A is known, p'_y C is not known. So, we need to find out p'_{yC},

Known are

$$q_A = 90\ kPa; \qquad q_b = 90 + 12 = 102\ kPa$$

$$p'_A = 225\ kPa; \quad p'_B = 180 + 4 = 184\ kPa$$

So, what is the stress rate of B is now known. Now if we substitute for the bigger yield curve. So, what we can write is

$$p'^{\,2}_B - p'_B p'_{yC} + \frac{q_B^2}{M^2} = 0$$

$$184^2 - 184 * p'_{yC} + \frac{102^2}{1} = 0$$

$$p'_{yC} = 240.5 kPa$$

We get p'_{yC} which is equal to 240.5 kilopascal. So, p'_{yC}, so all what is needed for $\Delta\epsilon_{p'}$ is now determined.

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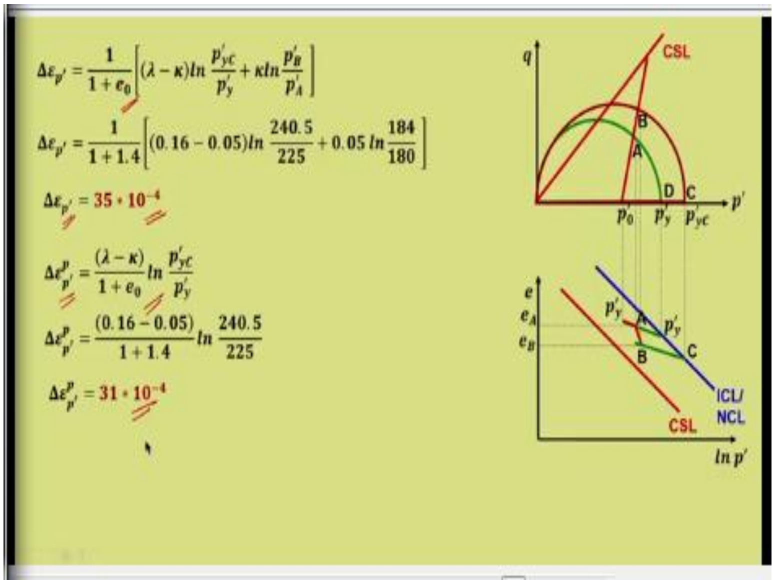

So, just we need to substitute these values in this equation, once we substitute it we will get

$$\Delta\epsilon_{\mathrm{p}'} = \frac{1}{1+e_o}\left[(\lambda-\kappa)\ln\frac{p'_{yC}}{p'_y} + \kappa\ln\frac{p'_B}{p'_A}\right]$$

$$\Delta\epsilon_{\mathrm{p}'} = \frac{1}{1+1.4}\left[(0.16-0.05)\ln\frac{240.5}{225} + 0.05\ln\frac{184}{180}\right]$$

$$\Delta\epsilon'_p = 35 * 10^{-4}$$

Please remember here this is the total volumetric strain when the effective stress path moves from A to B. And this is inclusive of both elastic and plastic strain, only for that incremental portion. Now we need to find out the q component that is ϵ_q what is the ϵ_q when it moves? So, both elastic as well as plastic we need to understand.

$$\Delta\epsilon^p_{p'} = \frac{\lambda-\kappa}{1+e_o}\ln\frac{p'_{yC}}{p'y}$$

So, we just need to substitute it here, all the parameters are known.

$$\Delta\epsilon^p_{p'} = \frac{0.16-0.05}{1+1.4}\ln\frac{240.5}{225}$$

$$\Delta\epsilon^p_{p'} = 31 * 10^{-4}.$$

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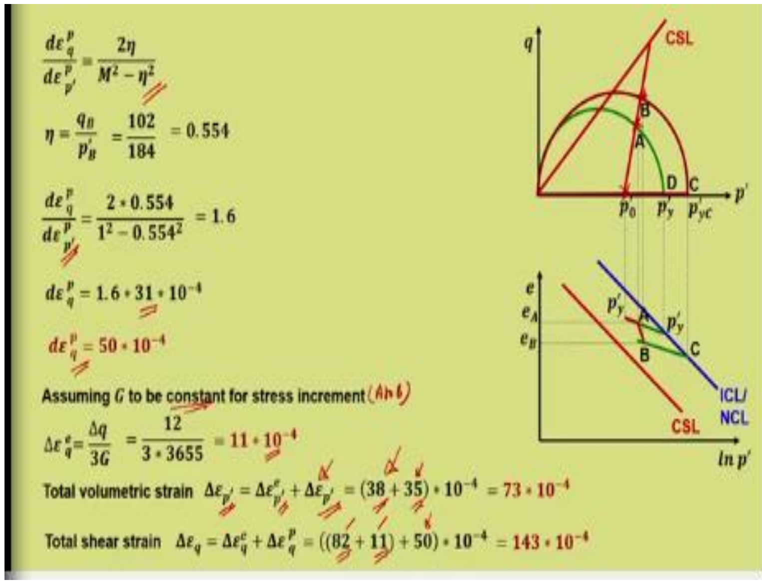

So, we have determined the total volumetric strain for the incremental portion, knowing that we have determined what is the plastic volumetric strain for the incremental portion. Now we are left with deviatoric stress, so

$$\frac{d\epsilon_q^p}{d\epsilon_{p'}^p} = \frac{2q}{M^2 - \eta^2}$$

So, $\eta = \frac{q_B}{p_B'}$ because we are considering the point B.

$$\eta = \frac{q_B}{p_B'} = \frac{102}{184} = 0.554$$

M is known, substitute

$$\frac{d\epsilon_q^p}{d\epsilon_{p'}^p} = \frac{2 * 0.554}{1^2 - 0.554^2} = 1.$$

So,

$$d\epsilon_q^p = 1.6 * 31 * 10^{-4}$$

$$d\epsilon_q^p = 50 * 10^{-4}$$

Please remember, we have obtained the plastic part of the deviatoric strain, that is $d\epsilon_q^p = 50 * 10^{-4}$. We need to obtain the elastic part of the deviatoric strain for the incremental portion of A to B. So, assuming G to be constant for the stress increment, that is from A to B, we can write

$$\Delta\epsilon_q^e = \frac{\Delta q}{3G} = \frac{12}{3 * 3655} = 11 * 10^{-4}$$

So, all what is needed we have computed now. So, total volumetric strain that is from the initial point to the point B is $\Delta\epsilon_{p'} = \Delta\epsilon_{p'}^{e}$ that is from the starting point to the point A to $\Delta\epsilon_{p'}$ which is the total volumetric strain, both elastic as well as plastic it is inclusive for the volumetric strain up to the point from A to B.

$$Total\ volumetric\ strain\ \Delta\epsilon_{p'} = \Delta\epsilon_{p'}^{e} + \Delta\epsilon_{p'} = (38 + 35) * 10^{-4} = 73 * 10^{-4}$$

Now total shear strain Now the plastic portion comes only from for the portion A to B, the remaining is elastic strain. Please note, here in this particular case we could obtain this 35 directly, but in the case of a deviatoric strain we have obtained the plastic strain and the elastic strain separately for the portion A to B. So, that is why 82 is the elastic deviatoric strain which undergoes when it starts from the initial to point A, 11 is the elastic strain which it is subjected to when it moves from A to B. So, the total elastic strain will be

$$Total\ shear\ strain\ \Delta\epsilon_{q} = \Delta\epsilon_{p'}^{p} + \Delta\epsilon_{q}^{p} = \left((82 + 11) + 50\right) * 10^{-4} = 143 * 10^{-4}$$

So, that is what, here in this particular case this issue did not come up because we are getting the total volumetric strain from A to B. So, all the concepts with respect to critical state we have tried to cover in these problems, I would advise you strongly to go through more such problems for practice and that will clear all your concepts. So, that is all for this lecture, thank you.