Advanced Soil Mechanics
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Lecture-52
Prediction of Soil Behaviour From MCCM 2

Welcome back all of you, today's lecture is the continuation of last lecture which is basically prediction of failure conditions from MCCM. In fact in the last few lectures what we have seen is, we have got the concept of strength, we have got the concept of stress path then we integrated yielding and yield curve with these concepts. And then saw how a soil state from it is initial state reaches to the critical state.

So, in the process it gets yielded and then it reaches the critical state. So, both the yielding conditions and the failure conditions can be obtained from the given critical state models. In some sense by knowing the critical state parameters which we have defined all the while in the last few lectures, and by knowing the geometry by substituting the required parameters into the equation of ellipse which is the yield curve and corresponding to the failure conditions.

So, this much we have seen, now today's lecture we will be adding a delta information to what we have already seen. That we will try to integrate how this looks like, the whole of the failure looks like from the stress strain response and the volume change or pore water pressure, depending upon whether it is a drained or undrained test. So, that is what we will be seeing in today's lecture.

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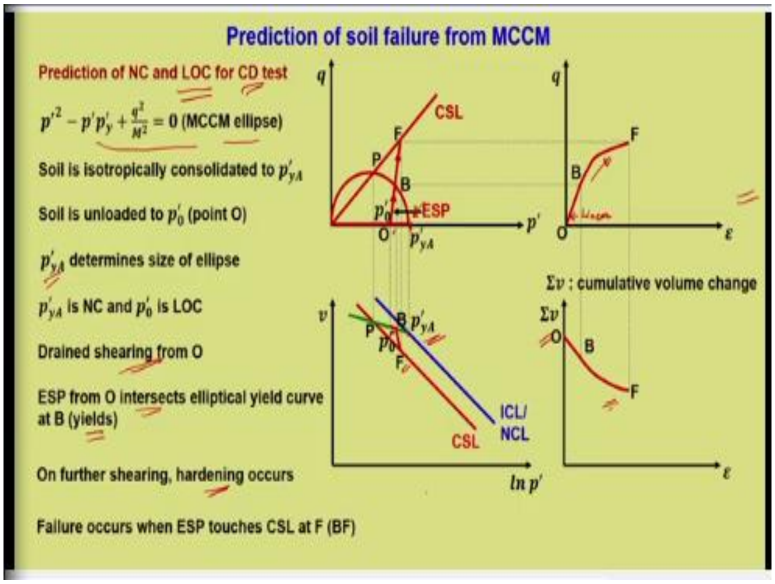

So, it is about prediction of soil failure from MCCM. So, first let us see prediction of normally consolidated and LOC for CD test. It is more or less the same, much of these aspects we have covered in the last few lectures. So, it should not be a problem, so it will be more like a revision come understanding. So, I think it should be easy for you to consolidate what all you have understood till now with this lecture.

Let us start with, because in the critical state model what we will be dealing with will be MCCM that is the ellipse. So, the equation of the ellipse

$$p'^2 - p'p'_y + \frac{q^2}{M^2} = 0$$

Now for NC or LOC because both of them exhibit more or less similar behaviour and we have seen that OCR less than 2 can be considered as LOC. And when it is greater than 2 in our previous lectures we have defined it as 4.

So, this is not a very strict definition of OCR mostly we will come to know by it is response and the type of the soil. So, OCR what is the actual definition for LOC and HOC, that would differ and it is not a very strict rule of thumb. So, soil is isotropically consolidated to p'_{yA}, so this is q_p' plot. So, this is the maximum yield point and that comes from the isotropic consolidation.

Soil is now unloaded to p_o', that is point O, why? To create a LOC state, so this is p_o'. And the yield curve corresponding to this will be this, so that is the maximum point is p'_{yA}. Now critical state line has been marked and according to MCCM it passes through the center, so it is p. Now we have v ln p' plot, so this is the isotropic consolidation point p'_{yA}. Let us say it is unloaded from p'_{yA} to generate p_o', so this is point p and here you can see it is p_o', so it is unloaded to p_o'.

So, this is very essential, now to find out to understand this sequential steps. Now in our previous lectures these were not very strictly followed, I will tell you one more aspect which we did not follow very strictly. But now things are very clear when we integrate the yield curve along with the stress path, so now p_o' is on the unloading curve. As I told we will try to also integrate q versus ϵ behaviour and volume change.

Because now it is a CD test, so it is volume change $\Delta \Sigma v$ which is cumulative volume change. So, these many details we are going to understand for a typical NC, LOC in a CD test. So, now we are ready with the initial point which is p_o'. Now we know from our previous discussion also p'_{yA} determines the size of the yield curve. And we have also seen that these are the state parameters based on which size of the yield curve gets determined, this one and q.

Now p'_{yA} determines the size of ellipse, so the major axis is given by p' y A. Now this particular state p'_{yA} is NC and p_o' is LOC, it is very clear. Now let us do a drained shearing from O that is the initial point is fixed then it is drained shearing. Now we do not have to explain much, because now we know how ESP would look like in a CD test which will intersect the elliptical yield curve at B.

So, this is the condition like the ESP meets the initial yield curve at B, so that B is given. So, it is slightly difficult, because if you try to map it then the amount of space available is quite less. So, you can see that from p 0' it should move towards B, ESP from O that is O intersects the elliptical yield curve at B where it yields. Now you need to note one important thing from O to B it is elastic behaviour and hence it has to move along this green line, that is the loading, unloading line.

I come to this again, because we are going to have a bigger picture of this in the next slide. So, I will just tell that the path in v ln p’ is denoted by this small red line, which is difficult to see but it will be clear in the next slide. Now when you map this to q ϵ, you can see that up to B it exhibits a linear behaviour because it yields only at point B, so that is the point. So, O B this is linear behaviour, same, the volume change also it will not be much because it is in the elastic range**.**

So, the B point has been mapped in all the figures. Now when it is further sheared from here since it is NC and LOC, we know that strain hardening happens. So, failure occurs when ESP touches the critical state line at F that is B F, so this is the further extension of the path. So, it is very easy that we have already seen before and then it completes, so it fails at F, so from B to F it moves. Now what happens?

You can see that this is a hardening response after yielding and hence the volume change also will be more. In fact you are moving towards this particular point and then it comes down, so it is no longer on the loading, unloading line and hence the volume change is going to be high, so from B to F there is significant volume change.

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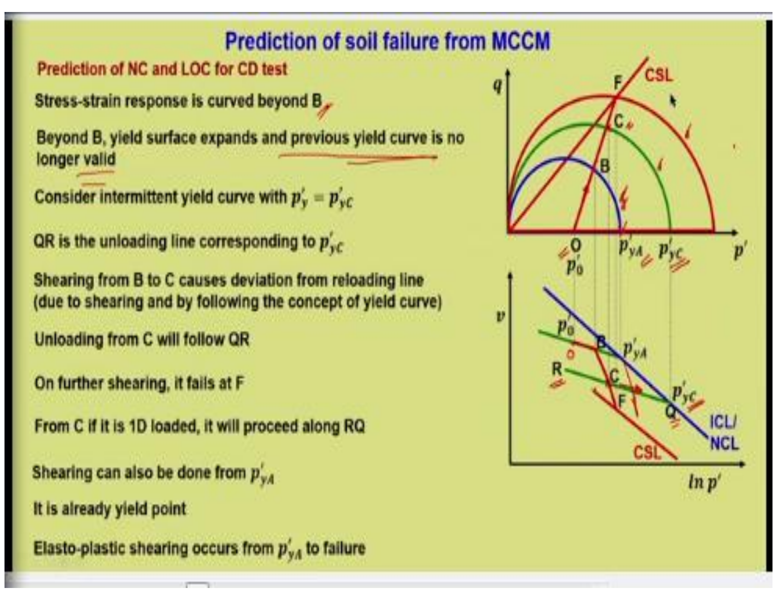

Now the same figure we will see in an enlarged version to understand a few more things which is very important. Now stress strain response is curved up to B it was elastic and beyond B it is curved. Beyond B, yield surface expands and then the previous yield curve is no longer valid, now this is very important aspect. The moment the stress path crosses the yield curve I mean to say the initial yield curve, then the previous yield curve which was there that is the initial yield curve now it is no longer valid, what is the significance of the statement let us see.

So, it is again q p'and v ln p', I have done this critical state line it is a smaller line you can see, this is mainly done to accommodate a bigger figure. The same figure that we discussed in the last slide, we are just going to repeat it. So, it is again p_y', the initial point is p_o', critical state line, and this is the effective stress path we have already seen point B, B F. Now this is an intermittent yield curve, this green one, before failure.

You can see that this particular yield curve it is not touching the point of intersection of ESP with critical state line, this we have discussed in our previous lectures that which yield curve defines the failure one, that yield curve beyond which there is no further expansion of yield curve. So, that yield curve which is passing through the point of intersection of effective stress path with the critical straight line, and that is this particular point F.

So, this intermittent point is given as C that is point of intersection of ESP with the green yield curve which is an intermittent yield curve and then the final yield curve is given as red one. So, what it means is that once it crosses point B, this yield curve is now no longer valid it goes with green and then comes to red where it stops. Now there is no further expansion of yield curve, and this we have already talked with respect to plastic potential and plastic strain vector.

p'_{yA} , we are just mapping it into v ln p' and from here the unloading happens, and then the starting point is p 0'. And when it is loaded that is from the point O, I can also write here it is point O. So, from point O, we know that up to here it is elastic response up to B. So, then if it is elastic response it cannot deviate from any but it has to move along the green line, why? Because we know that unloading, reloading line is an elastic response, so we can easily take it to be along O B, so that is what is marked.

And this is the same thing we have seen in the previous slide. Now you imagine a situation where we do not know the concept of yielding and the yield curve is not marked, and that is what we have done in our previous lectures. We did not consider yield curve into picture, when we discussed the first few lectures of critical state, you can see that we have not discussed the yield curve.

Now we have drawn the stress path to failure, we have also marked in v ln p'. If you remember from this particular point we would have just drawn a curve like this to failure to critical state line. That is because we have not introduced the concept of yielding, now once this particular yield curve comes into picture things have very clear. That means up to where exactly it will follow the unloading, reloading curve and from where it will deviate towards the critical state line.

Unless we have the yield curve in picture we cannot do this, and we have followed a line like this which goes to critical state. You can refer back and see for yourself, and this information is complete only when we draw the yield curve along with the stress path**,** so that is all I just want to add here. Now consider intermittent yield curve with $p'_y = p'_{yC}$, just because this is named as C, the extreme yield point is named as p'_{yC}, so that is given here.

Now let us draw a loading, unloading line from p'_{yC}, just for our understanding. Now QR this particular line, QR is the unloading line corresponding to p'_{yC},. Now shearing from B to C causes deviation from reloading line due to shearing and also by following the concept of yield curve. Now further shearing at point B, now it will deviate from elasticity and it will undergo hardening behaviour.

Now from B, it will deviate from point B initially it was along the unloading, reloading line which is the elastic response, but shearing at B it has to deviate. So, at this particular point B only it is deviating, initially we have drawn it as a curve, now it is coming along the unloading curve and then it is coming towards the critical straight line. So, this particular point from where it will deviate it comes from the concept of yield curve.

So, point C let us say that, that particular point is there on the unloading line QR because it is touching here. Now we are redefining the elastic limit, let us consider that the point is HC, so this particular point is the point C. Now the moment it deviates from point B or the moment it passes point B, we are defining a new yield curve. Now let us say that the stress path has reached up to point C, and it is at point C right now.

So, that particular point defines a new yield limit, and that is what is shown here at the point C that is QR is the new elastic response for this particular soil corresponding to point C. So, when it deviates it has come to point C, now it is at this particular point. Now let us say that you are unloading from point C, that is you have sheared, now let us consider that you are unloading from point C, so what will happen?

It will follow this particular unloading, reloading line because now it will become more like an elastic behaviour if you are unloading from point C. But if you continue to shear it will fail at F, that is normal response that we know and it is like this. So, you can see here now from the starting point it is following a curve like this. And initially without knowing this point we have done like this, you can see for yourself that how we have drawn the failure line from O to critical state line**.**

So, on further shearing it fails at F, now from C if it is 1 dimensionally loaded it will proceed along RQ, now this information is very important. Let us say from C for assumption, I am not telling it is already under shearing procedure. But then to understand how this path will follow depending upon the condition of loading that you apply that is what I mean to say. Now if from point C, let us say that it is one dimensionally loaded, not sheared, if that is the case then it will move along this line.

Because this is the one dimensional unloading, reloading curve. So, it instead of moving C F, it will move along this, provided it is one dimensionally loaded. But in this case we are shearing from C, so it will continue and it will fail at F, shearing can also be done from p'_{yA}. Now let us say that we have followed a lightly over consolidated point at O, but one can always load without unloading you can load from point p'_{yA} itself.

Now the beauty of this point is that it has already yielded, it is already at it is yield limit. So, it is already at yield point, so elastoplastic shearing occurs from p'_{yA} . So, if you start loading from here it will follow the path and it will move to the critical state line, it will go like this and it will fail on the critical state line because there is no yielding needed. So, whatever we have understood in our previous lectures it remains same.

But here one additional aspect is the information what the yield curve provides to us in terms of predicting the failure, so that we have to keep in mind.

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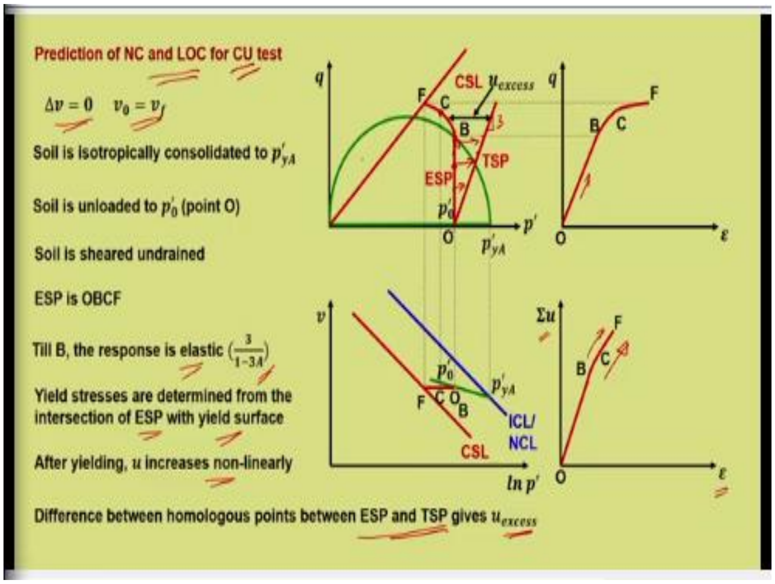

Now prediction of normally consolidated LOC for CU test, now initially it was CD test, now it is CU test, most of the discussions remain same only thing is the volume change is replaced by excess pore pressure generation. And this is again a prominent point we know, there is no volume change so $\Delta v = 0 \quad v_o = v_f$ so this is a very important clue for solving the problem.

Later lectures while we try to solve the problem that $v_o = v_f$ assumption is important for solving the problem. So, again it is the same response, the soil is isotropically consolidated p'_{yA} soil is unloaded to p'_o, that is point O. Now that is mapped on to v ln p', I do not think we need any further discussion on this. Soil is sheared and drained from point O, ESP is denoted as OBCF, so till B the response is elastic again.

I will not discuss this because the slope is $\left(\frac{3}{1-3A}\right)$; A is equal to 1/ 3. So, it takes the path upwards till it yields at point B. That is the point, same point O and B is in the same line, so O and B, so here also O and B is a point, same point. Then it fails to critical state line that is at F, so it is the curved response and this is an intermittent point C, that intermittent point is marked here. So, from O to F it is a horizontal line, all these things we have already seen before.

Yield stresses are determined from the intersection of ESP with the yield surface. Now you can see that this ESP meets the yield surface at this point. So, one can always substitute the values in the elliptical equation and obtain the value of q_y and p'_y. Similarly for F also we will get the value of p'_f and q_f . And that mostly you can get the results from the geometry of the figure itself.

Now B, you can see that this is a linear response, here also in the pore water generation also it is a smaller response. Because it does not have much of because it is an elastic loading, the pore water pressure generation is also more or less linear. Now F, from once it yields you can see that there is a non-linearity coming up and there is a sharp increase in the pore water pressure. And that is why there is a drastic reduction and it moves towards F.

After yielding u increases non-linearly, so you can see here this B F is a kind of non-linear response. Difference between homologous points between ESP and TSP gives u excess, so if you want to predict excess pore water pressure that is also possible. Because we know undrained test what is the total stress path. This is the effective stress path and this is the total stress path it is at an a slope of 3.

So, any difference between these homologous points, homologous point means the same point on ESP and TSP, that difference is going to give u excess. So, u access at yield u excess at failure, everything can be predicted merely looking at the figure and determining the total and effective stress corresponding to that. For example if you want to determine what is the u excess at this particular point that is yield point p'_y.

We need to determine p'_y, so we know p'_y here it is equal to p_o'. So, this point is known, one can also determine this particular point. So, this difference will give you u excess at yield, so it is very convenient to predict what is the kind of pore water pressure that gets generated, so that is what it means. So, based on the MCCM that is the elliptical yield curve and the effective stress path information jointly will be able to predict the yield point and the failure point. But obviously for all these we need to know what is the critical state parameters.

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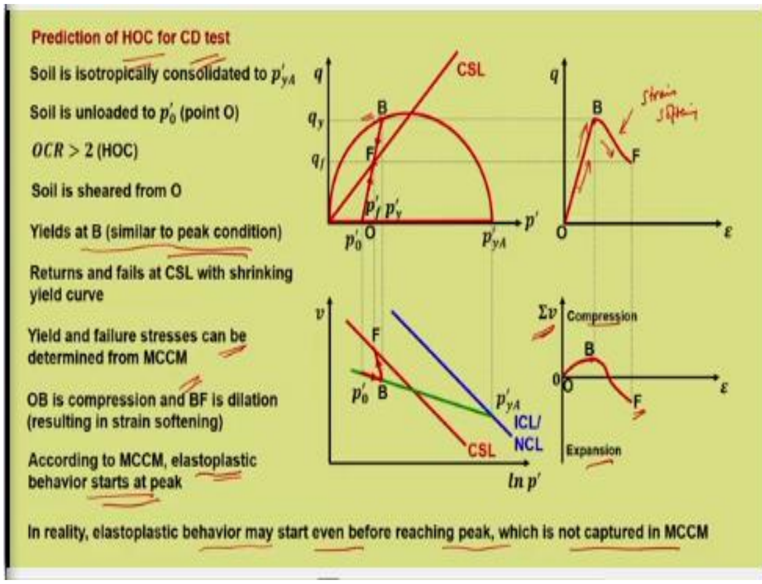

So, now let us see prediction of HOC for CD test. Soil is isotropically consolidated p'_{yA} and it is unloaded to p_o', the same procedure. But then now the unloading is a bit more, so that HOC state gets emerged and OCR is greater than 2, so that we already know. So, again the repetition of the same, now only thing is here it is volume change and we need to consider both compression and expansion.

Because we expect dilation to happen in heavily over consolidated soil. p'_{yA}, so this is the yield curve associated with p'_{yA} , the extreme isotropic consolidation point. The same is marked on v ln p', unloaded; you can see that it is very near to the q axis or v axis. So, OCR will be definitely greater than 2. Critical state line, so yields at point B, now it is a CD test, it is at an inclination of 3, so it is not meeting and failing here, we have already seen this, it has to yield first.

Now this point B is similar to the peak condition. Now from O to B it is elastic because it has to yield first. So, here also it has to move along the unloading, reloading line, so this line, green line it has to move because it is an elastic response. Now in our previous lecture, we have associated this 2 peak because the maximum point of q that you can get for OC is the peak point. Now here we are limiting it by yield, earlier we have limited this by a peak line; you can refer back and see in our earlier lectures we have done that.

Now here this peak and the yield point is analogous to each other, we are considering this to be more or less same. And what it will look like in q epsilon, so up to B it should be a in elastic response. Now in the case of MCCM we are forcing certain conditions here, when it reaches peak it is yielding, so that is what it means. So, yield point B is same as what the peak point, so here this is one imposing and we do not know like whether the soil is going to behave in that manner.

Whether the soil is going to yield at the peak point, this is not clear. But according to the explanation of critical state model specifically MCCM what we are dealing now what it states is that the soil will yield at it is peak. So, this is a kind of idealization one has to keep in mind. And hence up to peak the response is also linear; this is also an additional condition that gets imposed because we are considering a critical state framework.

So, you need to understand that many things gets idealized which may not be same in the real soil behaviour. So, from here, now another important aspect, like what is the cumulative volume change. Now up to here if it is yielding, it is more or less the peak behaviour is the end of their initial compression. These things we never bothered when we plotted the results in the previous lectures, we have done all these in our previous lectures right from module 2 onwards. But we never bothered how we are plotting it. So, here there are certain clear indications which we need to follow.

Now at this particular maximum point is more or less the end of compression and where it will start dilating. So, q_y is the yield point and q y is the yield stress which is same as q peak. So, this p'_y is the yield mean stress, returns and fails at critical state line with shrinking yield curve, shrinking yield curve also we have seen because this is a strain softening behaviour, so it comes and fails at this particular point.

So, from here it is dilating and that is why there is a sudden drop and that is the point F, the failure point. I am not getting into the details again because we know this we have discussed this, now that is the failure point p'_f. Now how does it look like in q epsilon plot? You can see that there is a strain softening, so this strain softening we have discussed before but now at what point it will start strain softening, that dilation happens this is adding more meaning to it.

So, by defining the yield curve you can see that we know along this it will move that is the elastic response and then it will move upwards or dilates and fails at F. Now these information becomes quite handy in the presence of yield curve, that is the yield curve supplements these knowledge along with our discussion. So, from here at this particular point B strain softening means there is a kind of dilation that is happening up to F where it fails.

So, yield and failure stresses can be determined from MCCM by substituting at these appropriate points. So, one can also get the failure and the yield stresses. Now OB is compression and BF is dilation, now this becomes quite strict in the MCCM model, resulting in strain softening. Now according to MCCM the actual elastoplastic behaviour starts at peak, now this is defined because it is explained in that manner.

In reality elastoplastic behaviour may start even before reaching peak which is not captured in MCCM, now that is what we need to understand. All the time the model may not represent the actual behaviour but it is a fair good approximation. By saying that the elastoplastic behaviour more or less sets at the peak. Because that is where the dilation would have started that is the yielding would have started.

Now exactly the peak may not be correct but it will be around that peak, so that is what I just need to point out those differences when you try to idealize the shearing behaviour of soil using MCCM.

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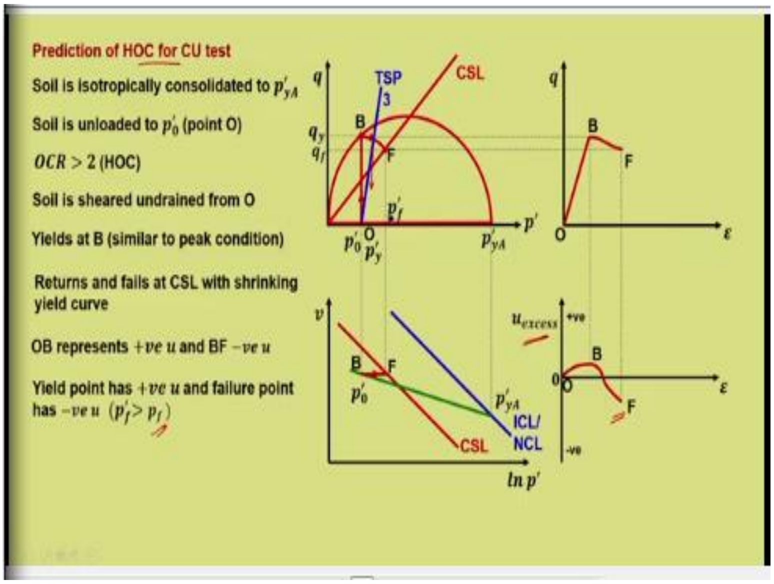

Now for CU test for HOC. So, we have defined all the four combinations that is NC LOC, CU CD, HOC CU CD. So, now we will discuss CU test. Soil is isotropically consolidated to p'_{yA} , again the same condition let us skip this, it is the same condition. So, u excess, now instead of cumulative volume change, now we have excess, excess pore water pressure. Same yield curve, everything remains same, loading, unloading curve, p'_o the point O, the initial point critical state line.

Now yields at B similar to peak condition, now only difference is the slope of the curve that is $\frac{3}{1-3A}$, we know that it is vertical. So, OB p'_o from O to B is the yielding, so here B and p_o' is at the same point. Now how does it look like in q ϵ plot and u excess plot, it is positive instead of compression. So, here the positive pore water pressure everything remains same, q_y is the point and p'_y.

So, $p_o' = p'_y$ according to this particular plot. Returns and fails at critical state line with shrinking yield curve, this also we have already discussed, so this is the failure point F**.** So, it fails at F and q_f *and* p'_f is the failure stresses, and strain softening behaviour in q ϵ plot. And you can see that it more or less becomes a negative pore water pressure at failure for OC sample.

Because of the tendency to dilate in a drain test is same as negative pore water pressure failure in OC. Now OB represents positive u, here it is positive pore water pressure and at BF it becomes negative pore water pressure. And this is very clear from the total stress path with an inclination of 3 which is drawn. Yield point it is on the left side of TSP, so at this particular point it is positive u, and at failure point here it will be negative u.

Because it is crossing the TSP and now it is on the right side. So, p'_f is greater than p_f, now p_f corresponding to failure line, this is the point F we can see that this is the particular point if you draw it down that is p_f. Now you can see that p'_f is greater than p_f

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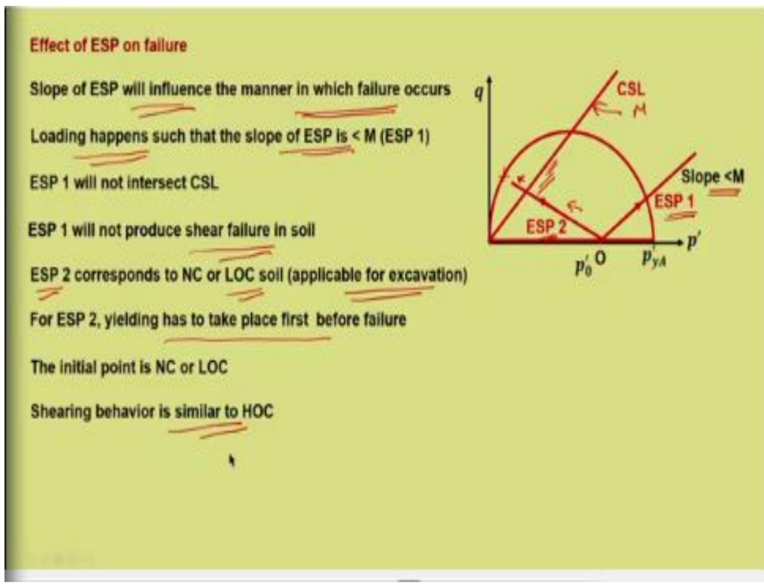

So, effect of that is all about heavily over consolidated and the prediction of q_f, p'_f, q_I, p_y will happen based on the geometry of the figure and by substituting the relevant equations in combination with the yield curve**.** So, we will come to that when we solve the problem. So, effect of ESP on failure. So, we just need to understand 2 important points here, slope of ESP like in what manner it is loaded.

Slope of ESP means in what manner it is loaded, what is the load combination that we follow. Now slope of ESP would influence how it will fail, in which manner it is going to fail? So, that is what we want to demonstrate here. So, it is the initial yield curve p'_{yA} , let us say that loading is done in such a manner that slope is less than M, now what is M? M is the slope of CSL.

Now let us say you consider a point which is slightly over consolidated at O. And the loading is done such that ESP 1, here it is ESP 1 is with a slope less than M, you can see this, what would happen in this case? ESP 1 is not going to intersect critical state line, so the loading can happen without failing; probably it is very close to isotropic loading. So, if ESP is progressing with a slope less than M, it is not going to touch the critical state line, in that case it is not going to fail.

The loading is not going to bring about failure; this is a very important information which comes from here. ESP 1 will not produce shear failure in soil. Now let us consider another case again the starting point is same and ESP 2 which corresponds to NC or LOC soil which is mostly applicable for excavation. So, this is the way in which the ESP 2 is progressing. There is a loading such that ESP 2 takes this particular slope, now this is a typical case of excavation.

For ESP 2, yielding has to take place first before failure, we have seen that in all our previous discussions for NC and LOC it yields first and it fails. But then it was very easy, it never cross the critical state line, when did it cross the critical state line? First the effective stress path crosses critical state line yields and then comes back means it is a typical HOC behaviour. But now our point is LOC, in all our previous discussion LOC and HOC it yielded first and then went ahead towards the critical state line.

But now if the loading such as that in excavation if it is done in such a manner that it follows ESP 2, then it will cross the critical straight line as you can see here, it crosses it goes and yields and then will come back to failure. So, that is what just wanted to highlight here, an LOC can give a behaviour close to HOC during shearing, provided the loading combination is in such a manner that it produces an effective stress path similar to ESP 2.

So, initial point is NC or LOC, but shearing behaviour is similar to HOC, why? Because it crosses critical state yield and then gets back to failure.

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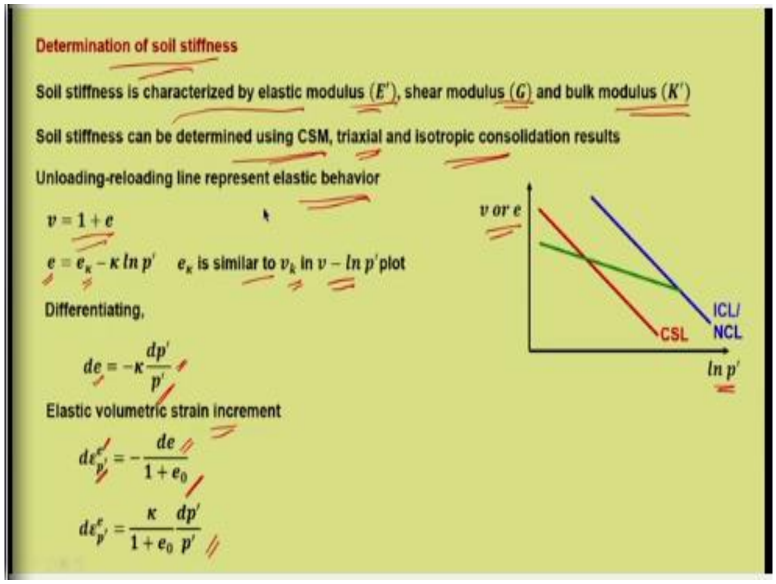

Now that is all related to the prediction and another aspect that we can predict based on critical state framework or very specifically MCCM is the soil stiffness. So, the next discussion is determination of soil stiffness. Soil stiffness is characterized by elastic modulus E', shear modulus G and bulk modulus K'. K' and E' is shown to show that this is relevant only for effective stress condition when it comes to soil behaviour.

We also have u and K_u which comes from the undrained test, but for very specifically the mechanical properties we will just consider E' and K', and G does not need any prime because it is a shear modulus. Now all of these are elastic parameters. Now in the critical state framework can we use MCCM or critical state framework for predicting the values of these stiffness. So, we need to confine only to the elastic response of the soil.

Now probably you would get a hint here, like we will be referring only to the unloading, reloading line, not the ICL. Because ICL exhibits elastoplastic behaviour, we are concerned only about the elastic stiffness. So, for prediction we need to now account for the unloading, reloading line of critical state framework. Soil stiffness can be determined using critical state model, triaxial testing and isotropic consolidation results.

We need for determining the critical state parameter, isotropic consolidation and or the triaxial test results, yes. So, how to do this? Let us see, so unloading-reloading line represent elastic behaviour. Now I think none of few would be having any doubt related to this, that it is unloading-reloading line that represents the elastic behaviour. Now when we talk about stiffness, it is always better that we discuss it in terms of void ratio, because that is what we are use to.

But the same, everything remains same only thing is the axis changes by 1 + e in the case of v. So, either I discuss in terms of e or in terms of v the responses are same, but the slope would change it is λ and CC, CC in e plot, λ in v plot. So, there is only difference in the slope values, but all the concept remains same, it is v or e versus ln p'. So, here instead of writing v = vκ - κln p' I am writing

$$e = e_k - \kappa \ln p'$$

This e_κ is same as corresponding to one unit pressure that is 1 kilo pascal and instead of v_κ, it is e_κ. So, e_κ is similar to $e_k \; is \; similar \; to \; v_k \ln v - \ln p'$. Now if you differentiate this we will get $de = -\frac{\kappa dp'}{p'}$. And the elastic volumetric strain increment that is given as

$$de_{p'}^{e} = -\frac{de}{1 + e_o}$$

Please note that here the p' corresponds to volumetric stress, and e corresponds to elastic.

The same thing we have seen $de_{p'}^{p}$ means it corresponds to plastic. So, here it is elastic volumetric strain increment, you can write it as -de that is change in void ratio upon original volume that is 1 + e_o. So, initial volume is 1 + e_o, change in volume to it is original volume that is 1 + e_o that will give the elastic volumetric strain increment. So, we know what is de from this expression, if you substitute this you can get

$$de_{p'}^{e} = \frac{\kappa}{1 + e_o} \frac{dp'}{p'}$$

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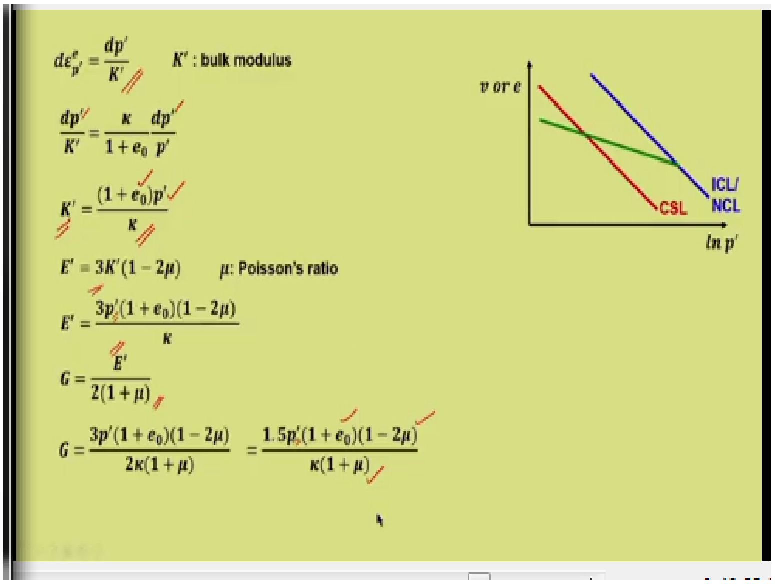

But we also know

$$d\epsilon_{p'}^{e} = \frac{dp'}{K'} \quad K': bulk\ modulus$$

Now substituting this,

$$\frac{dp'}{K'} = \frac{\kappa}{1+e_o}\frac{dp'}{p'}$$

One can write the expression for bulk modulus

$$K' = \frac{(1+e_o)p'}{\kappa}$$

You can see that the bulk modulus can be determined by knowing the value of κ and e_o and the value of p’. Now we already know the relationship,

$$E' = 2K'(1-2\mu) \quad \mu: Poisson's\ ratio$$

Substituting for K’ we will get the value for E’ that is

$$E' = \frac{3p'(1+e_o)(1-2\mu)}{\kappa}$$

And G we know it is

$$G = \frac{E'}{2(1+\mu)}$$

substituting in the value of E’

$$G = \frac{3p'(1+e_o)(1-2\mu)}{2(1+\mu)} = \frac{1.5\,p'(1+e_o)(1-2\mu)}{\kappa(1+\mu)}$$

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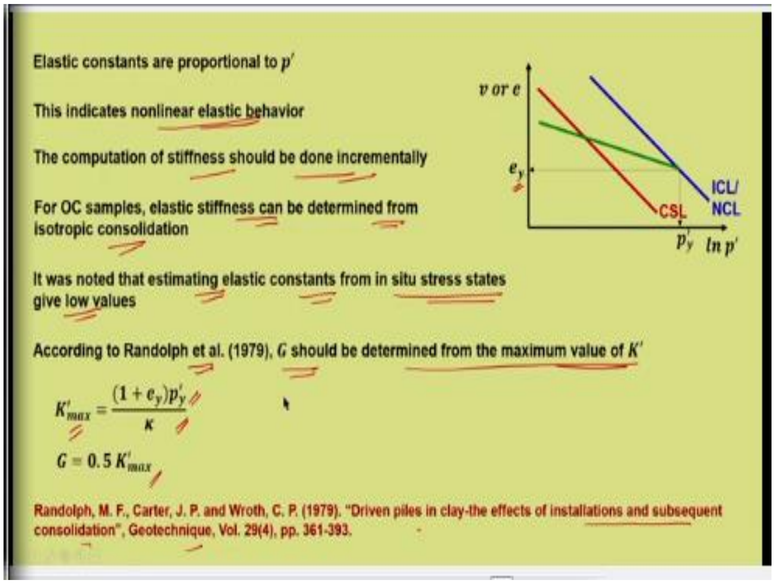

So, elastic constants we can see whatever we have discussed here, it is all proportional to p’. You can see in the expression K’, E’ and for G it is a function of p’. So, elastic constants are proportional to p’, this indicates the non-linear elastic behaviour. So, it is not that we simply substitute and get the result; it should be in an incremental manner. The computation of stiffness should be done incrementally.

For OC samples the elastic stiffness can be determined from isotropic consolidation, because we know it would follow the loading-unloading line. And why should we have the elastic modulus for normally consolidated because that is going to exhibit elastoplastic behaviour. When we say elastic response, we know that it is mostly relevant for OC samples. So, it was noted that estimating elastic constants from in situ stress states it gives low values.

That means whatever we determine based on the in situ stress state will give low values of elastic constants. So, according to Randolph, in 1979 he suggested that G should be determined from the maximum value of K’. Now what is meant by maximum value of K’? Maximum value of K’ means, it should correspond to maximum p’. And what is the maximum p’? That is the yield stress for that particular yield curve that is the extreme point of the yield curve which is p'_y.

So, K'_{max} should be

$$K'_{max} = \frac{(1 + e_y)p'_y}{\kappa}$$

So, that is according to the recommendation given by Randolph and others. So, this p'_y it is referring to and the corresponding void ratio is e_y, and G is taken as 0.5 of K'_{max}according to the suggestion. So, this is reported in this particular paper which is driven piles in clay-the effects of installations and subsequent consolidation, published in Geotechnique**.** So, one can refer to this for further reading.

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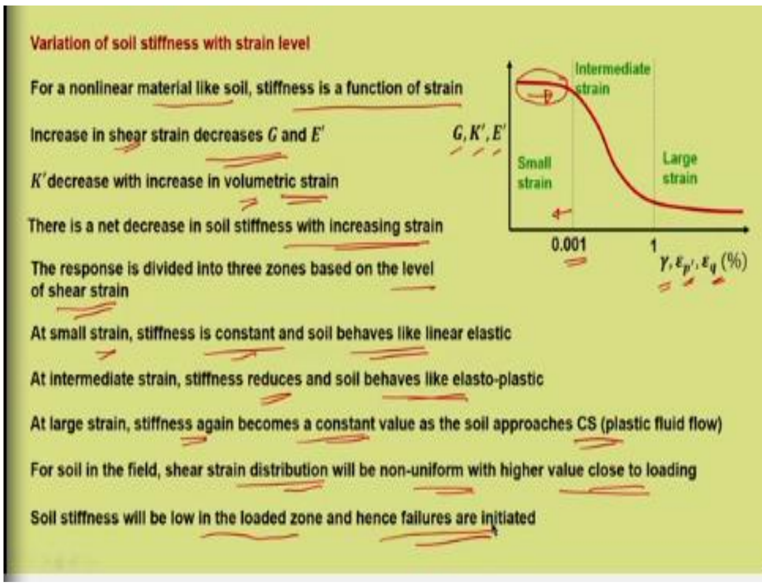

A variation of soil stiffness with strain level, we can note that for a nonlinear material like soil stiffness is a function of strain, it keeps changing. So, at a low strain we know it is a linear response and then it becomes a nonlinear response. Now increase in shear strain decreases G and E'. K' decreases with increase in volumetric strain, because each of this is associated with shear response and volumetric change.

So, there is a net decrease in soil stiffness with increasing strain. The response is divided into three zones based on the level of shear strain, which is given as how G, K', E' varies with shear strain or volumetric strain or deviatoric strain, so how does it vary? So, it varies something like this, is an s shaped curve. There are some indications of the values which is 0.001%, 1%. So, the strain which is less than 0.001 is considered as small strain 0.001 to 1% is intermediate strain, and greater than 1 is large strain.

So, you can see that even a very small percentage of strain itself is considered as a large strain, in general discussion it is considered like that. At small strain, stiffness is constant and soil behaves like linear elastic. So, here this is a linear elastic case where the response is stiff and we have very less volume change, so or whatever is the amount of strain that is happening is quite less and hence it remains more or less constant.

At intermediate strain, stiffness reduces soil behaves like elastoplastic and there is a significant reduction that happens. At large strain, stiffness again becomes a constant value as the soil approaches the critical state. So, as it reaches critical state then you can see that it reaches more or less the constant value again. For soil in the field shear strain distribution will be non uniform with higher value close to loading.

So, when some loading happens in the field what we can see that, the strain will be high close to the loading location. And hence whatever is the stiffness, in that particular zone that drastically reduces as per the given discussion. So, as the strain level increases, the stiffness starts reducing, so closer to the loaded points you can see that the stiffness will be less as compared to other part of the soil. And that is how the failure zones or the failure points starts generating at from a given zone. So, soil stiffness will be low in the loaded zone and hence the failures are initiated.

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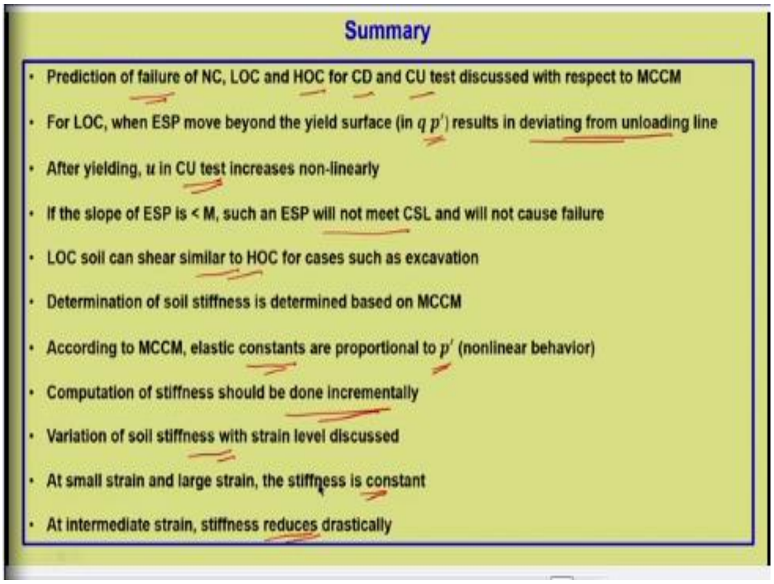

So, that is all about a prediction based on MCCM. Let us summarize today's lecture prediction of failure of NC, LOC and HOC for CD and CU test discussed with respect to MCCM. For LOC, when ESP move beyond the yield surface that is in q p’ plot, it results in deviating from the unloading line. So, we have shown that relationship with respect to yielding and the yield curve. So, from unloading reloading line it deviates towards the critical state line.

And that information we get because we have integrated yield curve along with ESP. After yielding u in CU test increases nonlinearly. The slope of ESP is less than M, such an ESP will not meet critical state line and it will not cause failure. LOC soil can shear similar to HOC for cases such as excavation. Determination of soil stiffness is determined based on MCCM. According to MCCM elastic constants are proportional to p’.

Computation of stiffness should be done incrementally even though there is an equation, it does not mean that it gives a particular constant value for the range of p’, it will be different. Variation of soil stiffness with strain level discussed. At small strain and large strain the stiffness approaches a constant value. At intermediate strain stiffness reduces drastically. So, that is all for this lecture, in the next lecture we will see prediction of strains from the critical state concept**.** So, that is all for today, thank you.