Advanced Soil Mechanics
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Lecture-51
Prediction of Soil Behaviour from MCCM 1

Welcome back, all of you. We were discussing about critical state models and we have finished Cam Clay model and modified Cam Clay model. And most of the numerical modelling adopts mostly modified cam clay. So, in the next part of the lecture will be how to predict the failure state of soil, based on the critical state model specific to modified Cam Clay and the procedure remains the same.

So, we will have 2 lectures in this, the first lecture will be a general discussion on how to predict. And the next lecture will be specific to most of the matter remains the same, but then we will discuss with respect to consolidated drain, consolidated undrained for NC and HOC soil. So, this is what we will be doing the next 2 lectures, so it is all about prediction of failure state from modified Cam Clay model.

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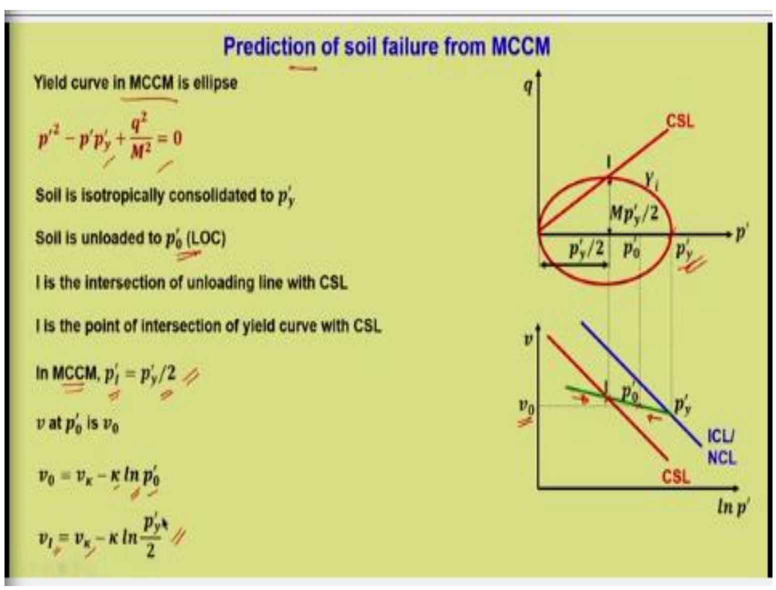

So, just to refresh, yield curve in modified Cam Clay model is ellipse. We will not go into the details of it, this has been discussed. We have q p' axis, the first yield curve is marked, the yield stress marked as p'_y, the point where the critical state meets this yield curve, that is marked as M $\frac{p'_y}{2}$ and we know that this is $\frac{p'_y}{2}$. The equation is given

$$p'^2 - p'p'_y + \frac{q^2}{M^2} = 0$$

Soil is isotropically consolidated to p'_y, this one, so this point. Same is shown here p' y from where the unloading can happen. Soil is unloaded to p_0' which is a lightly over consolidated state. So, somewhere here, so that it is slightly over consolidated, so p_0', this is the unloading line, so you can see that it unloads to p_0'. Now I is the intersection of unloading line with CSL, this is the point.

And that corresponds to this particular point, I is the point of intersection of yield curve with CSL, now all these information we have already discussed. And we also have the condition, so without knowing MCCM, this lecture will not be good for you to understand. So, what are the properties according to MCCM, we know that $p'_I = \frac{p'_y}{2}$, because these informations are needed for formulating the prediction

V at p_0' is v_0, so this is the starting point because we are starting with an lightly over consolidated state, so the v corresponding to p_0' is v_0, so this one. We can write this particular point of p_0'as

$$v_o = v_k - \kappa \ln p'_o$$

from the equation of unloading reloading line. Now this point v_I, we can represent in terms of unloading, reloading line as well as in terms of critical state line.

So, in terms of unloading, reloading line we can write

$$v_I = v_k - \kappa \ln \frac{p'_y}{2}$$

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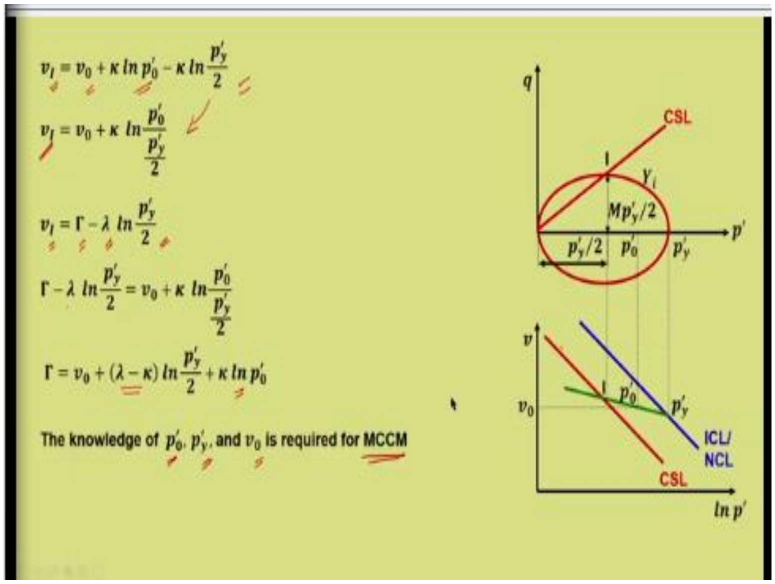

We have

$$v_O = v_k - \kappa \, ln \mathrm{p}'_\mathrm{o}$$

$$v_t = v_o + \kappa \ln \mathrm{p}'_\mathrm{o} - \kappa \, ln \frac{p'_y}{2}$$

from where it comes from we are substituting it for v_k. And v_I can be written as

$$v_t = v_o + \kappa \, ln \frac{\mathrm{p}'_\mathrm{o}}{\frac{p'_y}{2}}$$

Now based on critical state line we can also write

$$v_t = \Gamma - \lambda \, ln \frac{p'_y}{2}$$

the same point but from critical state line. We can substitute it for v_I, so when you substitute for v_I we can write

$$\Gamma - \lambda \, ln \frac{p'_y}{2} = v_o + \kappa \, ln \frac{\mathrm{p}'_\mathrm{o}}{\frac{p'_y}{2}}$$

So, we can write gamma as

$$\Gamma = v_o + (\lambda - \kappa) \ln \frac{p'_y}{2} + \kappa \, ln \; p'_o$$

Now the knowledge of p 0’, p’ y and v 0 is required for MCCM prediction.

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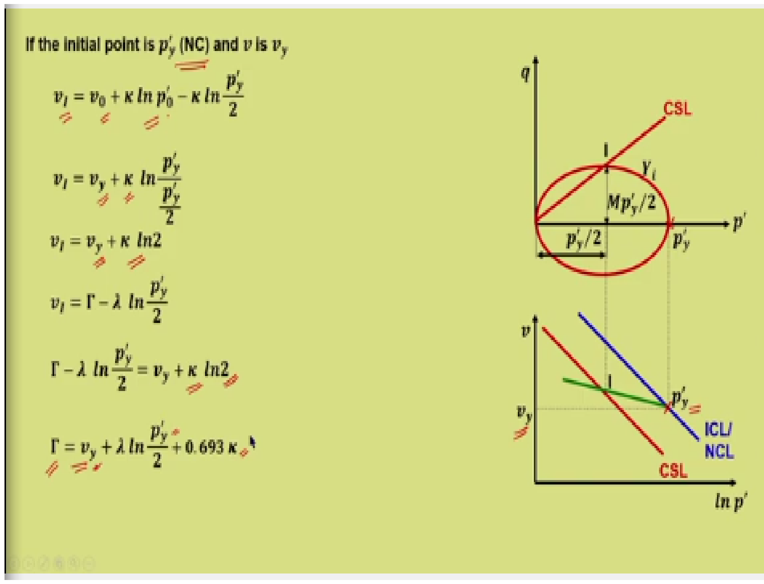

Now if the initial point is p'$_y$, so whatever we have discussed is for the lightly over consolidated point. Let us consider this particular point itself that is the normally consolidated point, so this is the point, this is normally consolidated. And v is v_y, so the initial condition is v_y, so if that is the case then v_I can be written as

$$v_I = v_o + \kappa \ln \mathrm{p}'_\mathrm{o} - \kappa\, ln \frac{p'_y}{2}$$

Now here p_0' it is written in general, now our starting point here is p_0' = p_y'.

So, here v_I can be written as instead of v_0 the starting point is v_y because it is a normally consolidated point. So, normally consolidated point it is v_y here,

$$v_I = v_o + \kappa\, ln \frac{\mathrm{p}'_\mathrm{o}}{\frac{p'_y}{2}}$$

So, we can write

$$v_t = v_o + \kappa\, ln2$$

Also

$$v_t = \Gamma - \lambda\, ln \frac{p'_y}{2}$$

If we substitute we can from the same procedure for LOC, we are doing it only the starting point is different. So,

$$\Gamma - \lambda\, ln\frac{p'_y}{2} = v_y + \kappa\, ln2$$

$$\Gamma = v_y + \lambda\, ln\frac{p'_y}{2} + 0.693\kappa$$

Now gamma can be predicted based on the given conditions. If we know v_y, p_y' and κ which are the critical state parameters gamma can be determined. Now once gamma is known then the particular critical state line is also defined.

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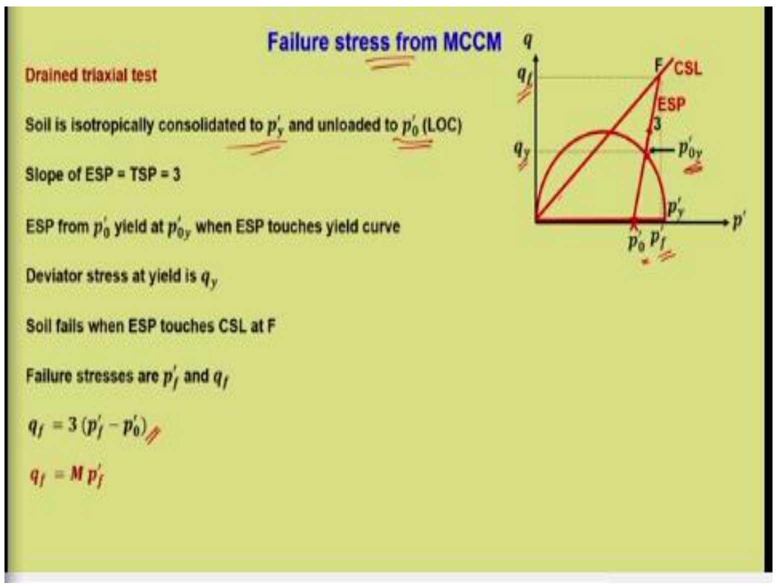

Now let us come to specific of drained triaxial test. Like what are the failure stresses from MCCM. Now we know what are the failure stresses? we are just trying to understand how we will determine the failure stresses based on the given MCCM model. Now where is the role of MCCM coming into picture, because the yield curve matters? So, with respect to any effective stress path yield curve matters, so that is defined in addition to what we have done in module 2, module 3 onwards.

So, here we are adding another component while defining the failure state of soil and that is yield curve. Again soil is isotropically consolidated to p_y'; unloaded to p_0', let us consider a case of LOC in general. So, we have p_y', we have defined the yield curve corresponding to p_y', and critical state line, and then p_0', up to here there is no need for discussion. Now whatever is needed we have discussed till our last lecture, now it is only summing up.

So, please follow the previous lectures very properly, so that we will be a bit fast in this lecture and hence you will be able to pick up further. Slope of ESP = TSP because it is a drained triaxial test. Now ESP from p_0' yield at p'_{oy}, when ESP touches the yield curve, so it is this particular point where it yields and the slope is 3, this information comes from our third module. Now this is the yield stress that is p'_{oy}, where it touches the yield curve.

Now remember the same discussion we have done in module 3. We do not have any idea about where it is going to yield; we just talked about the final state or the failure state where the ESP touches the critical state line. Now we are adding one more information that is the yield stress, now where the ESP touches the yield curve, that is p'_{oy}. Deviator stress at yield is q_y, so that q_y is given. Soil fails when ESP touches CSL at F, so this is the point F. Failure stresses are p'_f and q_f.

So, all these can be determined, like we can determine q_y, p'_f, p'_{oy} is written because the initial point is p'_o and the failure stress q_f, and p'_f, all can be determined. Now looking at the geometry of this figure; that is considered this triangle p'_o and p'_f. We can always write

$$q_f = 3(p'_f - p'_o)$$

So it is very conveniently we can write this. And we know

$$q_f = Mp'_f$$

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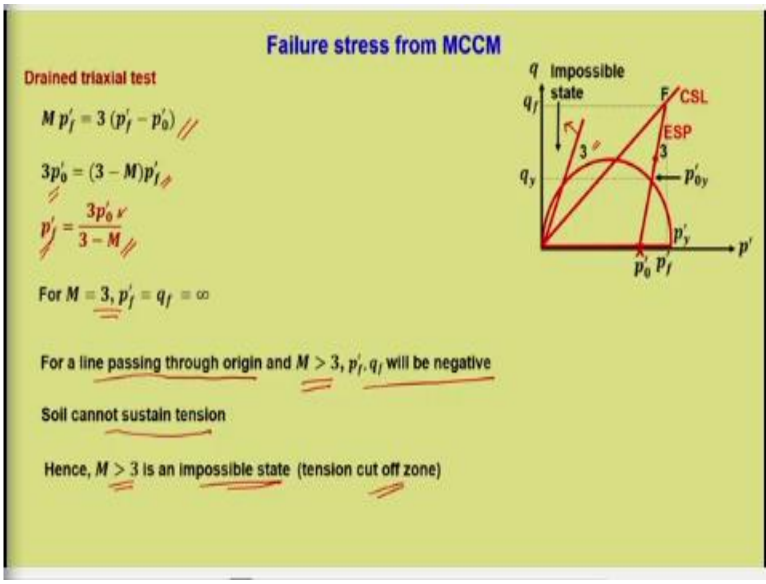

So, then this can be substituted and written

$$Mp'_f = 3(p'_f - p'_o)$$

So, then rearranging, we can write

$$3p'_o = (3 - M)p'_f$$

From which the failure stress can be determined that is

$$p'_f = \frac{3p'_o}{3 - M}$$

You can see this particular equation where p'_f is the required failure stress now that is a function of p'_o which is the initial condition and M which is a critical state parameter.

So, once we know p'_f , we can determine q, q_f. Now for one additional condition which we need to discuss here is for M = 3, p'_f = $q_f = \infty$. If you check here what will happen to p'_f when you put M = 3, so this will become infinity, so that is p'_f = $q_f = \infty$.. That means if at the particular point 3, if you remember our discussion earlier like we have discussed about tension cut off zone from the origin.

When we discussed about this earlier in our previous lecture, we told that this same thing will be discussed later how you are drawing a tension cut off zone at an inclination of 3 from the origin. Now what will happen if M is greater than 3? If M is greater than 3 you will have p'_f to be negative.

So, for a line passing through origin and M greater than 3 both p_f' and q_f will be negative, and that is the reason and the soil cannot sustain tension.

So, hence M greater than 3 is an impossible state, and that impossible state is at an inclination of 3 we have drawn. Now anything more than this is not possible because it will be negative and soil cannot sustain tension, and that is the reason we have defined it as a tension cut off zone. another state boundary on the left side. So, that is the boundary beyond which the soil will not exist.

Now I hope the explanation is very clear and it is justified how we have drawn an inclination of 3 at origin and marked it as a tension cutoff or an impossible state of the soil. So, that is otherwise known as tension cut off zone.

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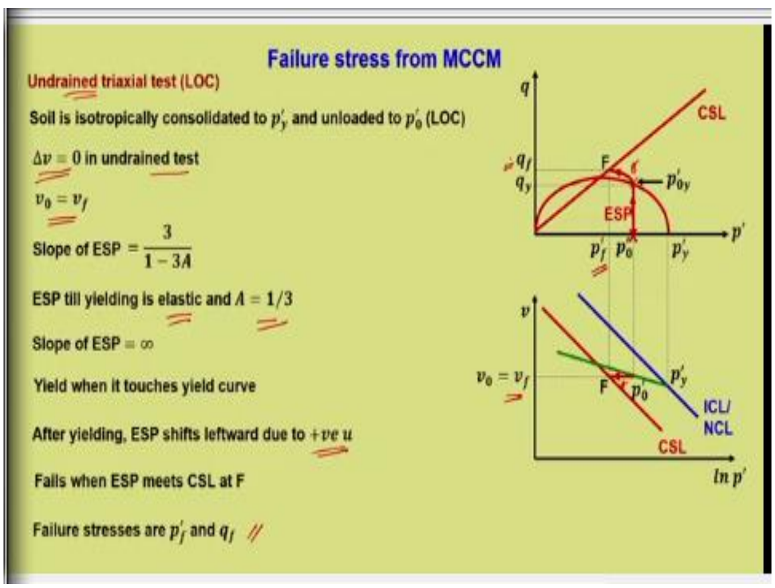

Now for the same case undrained triaxial test for lightly over consolidated state. Soil is again the same situation p_y' unloaded to p_o' lightly over consolidated state, so p_o'. The same is marked in the figure, so the initial condition is fixed. Now this is an undrained test, the very important fact that Δv or the specific volume there is no change in undrained test. When this is the condition we know that the initial specific volume and final specific volume they are same.

Now this makes a lot of difference in our mathematical computation of failure stress. So, here the failure happens when it meets the critical state line, and that F is the failure point. So, $v_0 = v_f$, now slope of $ESP = \frac{3}{1-3A}$ and we know that till yielding it is elastic $A = \frac{1}{3}$ in fact this particular aspect we have discussed several number of times. Now what is happening is some bit of information is getting added all the time.

So, it becomes almost very clear when we repeat these facts but make sure like we are pointing towards the same idea. Whatever we have discussed the same ideas we are discussing. But in this particular case we are adding yielding coming into picture, but all these facts remain same. So, now for if it is elastic behaviour, A has to be 1/ 3, when you substitute it, it will be infinity, so it will be vertically upwards, so ESP is vertically upwards.

Now once it yields this is an undrained condition, so the yield stress is p'_{oy} and q_y. Now after this it yields and then what happens, ESP shifts towards leftwards due to positive pore water pressure, this is an undrained test. So, here the effective stress path moves leftwards, so this is due to positive pore water pressure, fails when ESP meets critical straight line is F. So, we are not going to spend much time here because all these facts are known. Now failure stresses are p'_f and q_f which need to be determined. So, q_f and p'_f , so this is the failure stress.

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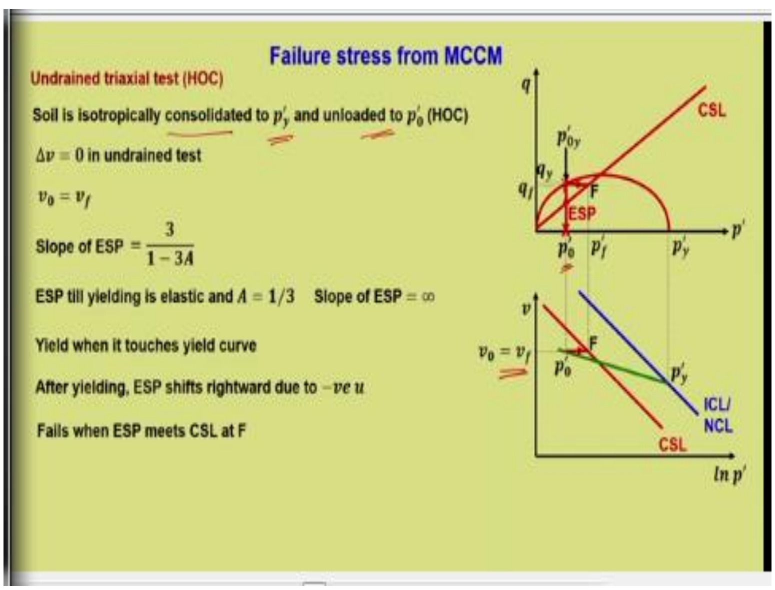

That is for undrained test for lightly over consolidated. Now let us see undrained traction tests for heavily over consolidated. So, soil is isotropically consolidated to p'_y and then unloaded to p'_o, the same procedure but the unloading is much higher. So, p'_y, yield curve, critical state line, now you can see that it is slightly towards the left end because it is heavily over consolidated.

The same is marked on v ln p' plot, you can see that this much is the over consolidation, that is happening $v_o = v_f$, Δv = 0 in undrained test, so $v_o = v_f$ slope of $ESP = \frac{3}{1-3A}$ ESP till yielding is elastic, the same information yield when it touches the yield curve. So, up to here it is all clear, so this is q_y and p'_{oy}, the yield stresses. After yielding ESP shifts rightwards due to negative pore water pressure, again it is a known fact fails when ESP meets critical state line at F. So, that is F it is undrained test, so it meets at F, and p'_f and q_f are the failure stresses.

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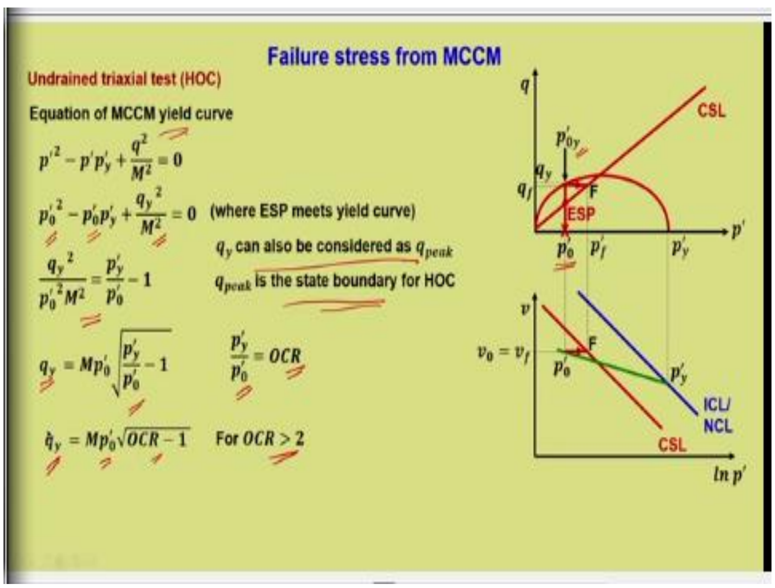

Now equation of MCCM yield curve is this. Now we know that this effective stress is meeting the yield curve. So, if you substitute this particular point,

$$p'^2 - p'p'_y + \frac{q^2}{M^2} = 0$$

$$p'^2_o - p'_o p'_y + \frac{q_y^2}{M^2} = 0 \text{ (where ESP meets the yield curve)}$$

In this particular case q_y is defined by the yield curve, can also be considered as q peak. Now keep q_{peak} is the state boundary for heavily over consolidated soil. You might be remembering this very well, like in our previous lectures we have discussed with respect to peak point of for a heavily over consolidated soil. So, what happens, the soil the stress strain response increases, it reaches the maximum that is the peak point and then strain softens.

Now what we have drawn is, we have drawn a peak line and then it touches. So, the maximum point to which the soil can reach as a heavily over consolidated soil can reach is the peak point. So, here in this particular case we are defining it by means of a yield curve. So, here if you consider yield curve to be same as the state boundary surface then you can always approximate yield point to be that of peak point, in fact that is depending upon the critical state model.

Now whether the soil actually yields at that peak point, this is not very clear, it can yield slightly before or maybe slightly after or maybe at peak, this is not very clear. But here we are assuming the yield point to be same as the peak point. Because both serves as the maximum point, state boundary point, so that is why. So, q_{peak} is the state boundary and hence q_y that is the yield point this particular point can be considered equal to q_{peak}.

If that is the case we can write

$$\frac{q_y^2}{{p_o'}^2 M^2} = \frac{p_y'}{p_o'} - 1$$

So, then q_y that is the yield deviatoric stress can be written as

$$q_y = Mp_o'\sqrt{\frac{p_y'}{p_o'} - 1}.$$

So, this is a quick way of computing q_y, and we know that $\frac{p_y'}{p_o'} = OCR$ with respect to the initial point. So, you can replace

$$q_y = Mp_o'\sqrt{OCR - 1} \;\; For\ OCR > 2$$

and this is mostly valid for heavily over consolidated soil where OCR is greater than 2, it is not mostly valid, it is valid for heavily over consolidated soil.

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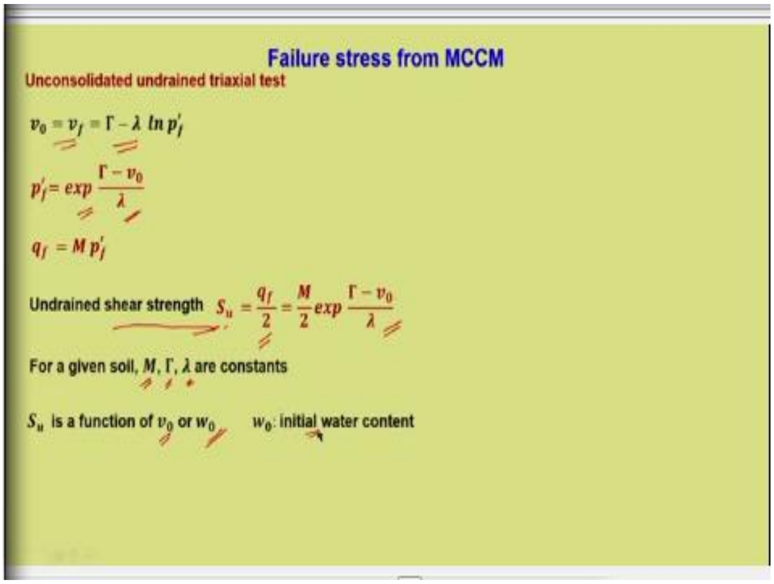

Now let us discuss about unconsolidated undrained triaxial test. We know

$$v_o = v_f = \Gamma - \lambda ln\, p'_f$$

because it is an undrained test. Now please refer back if you are finding it difficult, now we are not going to repeat it. So, p'_f can be written as

$$p'_f = \exp\frac{\Gamma - v_o}{\lambda} \text{ and } q_f = Mp'_f$$

Undrained shear strength

$$S_u = \frac{q_f}{2} = \frac{M}{2} exp\frac{\Gamma - v_o}{\lambda}$$

For a given soil M, Γ , λ are constants. So, what $S_u$is merely a function of v 0 that is the initial specific volume or we can write in terms of initial water content, w_o is the initial water content.

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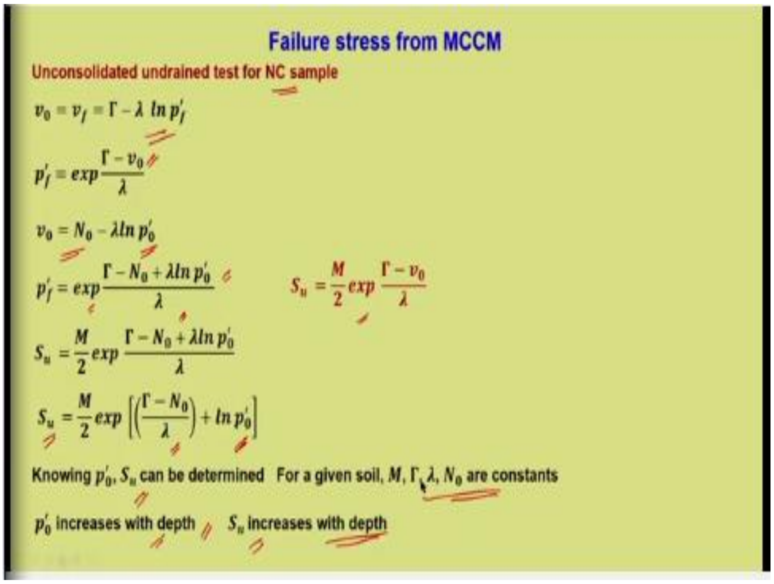

Now let us say unconsolidated undrained test for NC sample. If it is v_o it is the same equation we are considering,

$$v_o = v_f = \Gamma - \lambda ln\ p'_f$$

we have

$$p'_f = \exp\frac{\Gamma - v_o}{\lambda}$$

But we also have since it is an NC sample we can also write

$$v_o = N_o - \lambda \ln p'_o$$

Now if you substitute for v_o here,

$$p'_f = \exp\frac{\Gamma - N_o + \lambda \ln p'_o}{\lambda}$$

for p'_f can be written as

Now S_u is also equal to

$$S_u = \frac{M}{2} exp \frac{\Gamma - v_o}{\lambda}$$

So S_u can be written as

$$S_u = \frac{M}{2} exp \frac{\Gamma - N_o + \lambda \ln p'_o}{\lambda}$$

$$S_u = \frac{M}{2}\left[\left(\frac{\Gamma - N_o}{\lambda}\right) + \ln p_o'\right]$$

Knowing p_o' , S_u can be determined because all others are the part of critical state parameters. For a given soil, these are all constants. p_o' increases with depth, why?

Because p_o' is a mean vertical stress, so mean stress condition and with depth mean stress condition increases. So, since p_o' increases with depth the S_u for normally consolidated soil increases with depth. So, if you want to prove that S_u of NC will always increase with depth, this is the proof and p_o' increases with depth. And according to this equation S_u is merely a function of p_o' because all others are constants.

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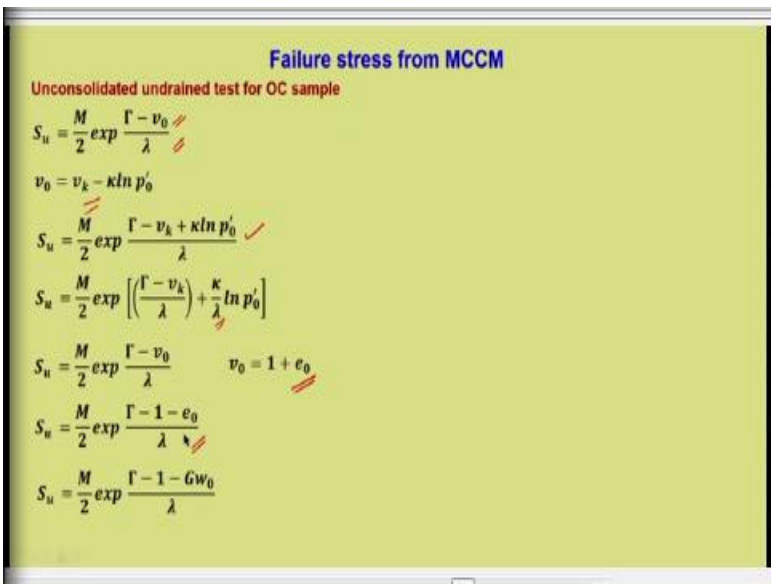

Now the same exercise for OC sample

$$S_u = \frac{M}{2} exp \frac{\Gamma - v_o}{\lambda}$$

$$v_o = v_k - \kappa \ln p_o'$$

Because this is an unloading-reloading case. Substitute for v_o

$$S_u = \frac{M}{2} exp \frac{\Gamma - v_k - \kappa \ln p_o'}{\lambda}$$

Again like previous case we can write

$$S_u = \frac{M}{2}\left[\left(\frac{\Gamma - N_o}{\lambda}\right) + \frac{\kappa}{\lambda}\ln p_o'\right]$$

Also we have

$$S_u = \frac{M}{2} exp \frac{\Gamma - v_o}{\lambda}$$

we can substitute for v_o that is equal to

$$S_u = \frac{M}{2} exp \frac{\Gamma - 1 - e_o}{\lambda}$$

what is e_o? e_o can be substituted as Gw_o because S_re = G w. So, based on that

$$S_u = \frac{M}{2} exp \frac{\Gamma - 1 - Gw_o}{\lambda}$$

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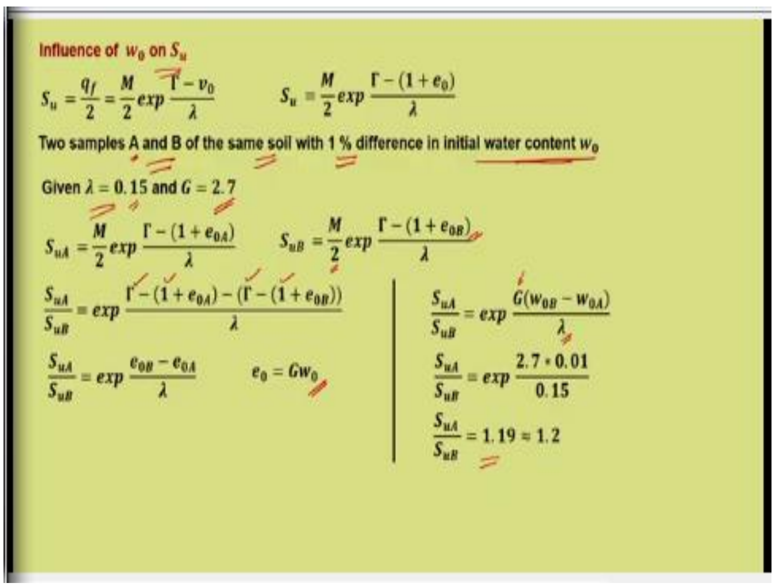

So, let us try to understand why we have done the previous expression is to understand what is the influence. How much is the influence of initial water content on undrained shear strength? Just to prove that,

$$S_u = \frac{q_f}{2} = \frac{M}{2} exp \frac{\Gamma - v_o}{\lambda} \qquad S_u = \frac{M}{2} exp \frac{\Gamma - (1 + e_o)}{\lambda}$$

Let us consider 2 samples A and B of the same soil with 1% difference in initial water content w_o. So, we are just trying to see this 1% difference in initial water content will translate to how much difference in S u. So, we are considering the critical state parameters or in the critical state parameters λ = 0.15 and specific gravity as 2.7.

$$S_{uA} = \frac{M}{2} exp \frac{\Gamma - (1 + e_{oA})}{\lambda}$$

And S_{uB} is e_{oB} only, the initial void ratio there is a difference.

$$S_{uB} = \frac{M}{2} exp \frac{\Gamma - (1 + e_{oB})}{\lambda}$$

Now if you divide $\frac{S_{uA}}{S_{uB}}$, we can write exp,

$$\frac{S_{uA}}{S_{uB}} = exp \frac{\Gamma - (1 + e_{oA}) - (\Gamma - (1 + e_{oB}))}{\lambda}$$

So, here you have

$$\frac{S_{uA}}{S_{uB}} = \exp \frac{e_{oB -} e_{oA}}{\lambda}$$

So, substituting for $e_o = Gw_o$ we have

$$\frac{S_{uA}}{S_{uB}} = \exp \frac{G(w_{oB} - w_{oA})}{\lambda}$$

Substitute for G that is 2.7, the difference in water content is 1%,

$$\frac{S_{uA}}{S_{uB}} = \frac{2.7 - 0.01}{0.15}$$

$$\frac{S_{uA}}{S_{uB}} = 1.19 \approx 1.2$$

So, 1% difference in w_o results in 20% difference in S_u, so that is the conclusion. So, a very small difference in w_o will be a glaring difference in S_u. So, natural water content in the samples should be preserved as far as possible.

When we are doing sampling from the field, it is very important that is why it is told that it has to be packed properly. And hence we can preserve what is the natural moisture content or field moisture content as it is otherwise it is going to influence the strength which is determined. Now here it is influence on undrained strength is quite high. So, if we do not take care of initial water content, then S_u would considerably vary and we will not be getting the condition that is existing in the field.

So, that is all for the initial part of prediction. So, we have tried to understand the various equations that can be used for finding out the failure stresses.

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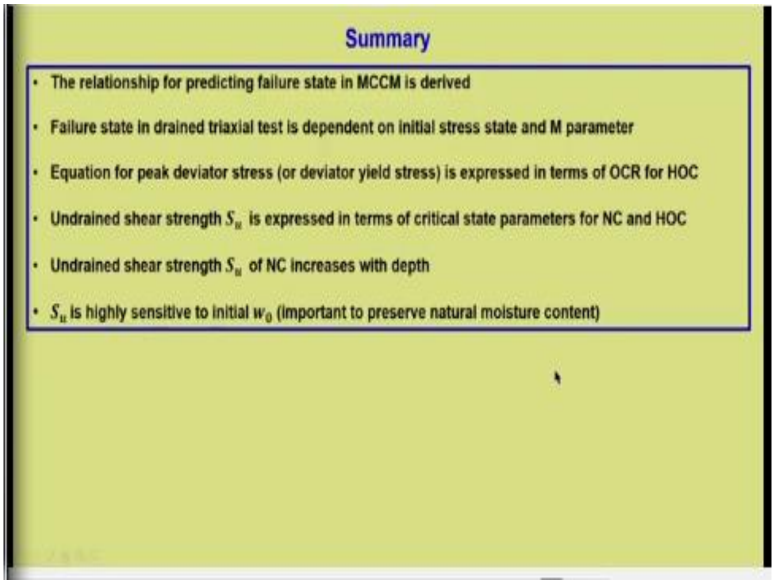

So, let us try to summarize today's lecture. The relationship for predicting failure state in MCCM is derived. Failure state in drain triaxial test is dependent on initial stress state and M parameter. Equation for peak deviatoric stress or deviator yield stress is expressed in terms of OCR for heavily over consolidated soil. So, we have also found out that q_y and p'_yalso can be determined for any given type of situation.

Undrained shear strength S_u is expressed in terms of critical state parameters for NC and HOC. Undrained shear strength S u of NC increases with depth that we have shown. S_u is highly sensitive to initial water content, so that is important to preserve natural moisture content. So, that is all for today's lecture, we will be continuing the same in the next lecture with more details added to it.

All the discussions remains the same but we will also try to see the prediction of $C_u \ and \ C_d$ for NC and OC material or soils and with respect to the volume change or the pore water pressure also added to it. So, we will see this in the next lecture, that is all for now, thank you.