Advanced Soil Mechanics
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Lecture-50
Modified Cam Clay

Welcome back all of you, in the last lecture we have started off with critical state model and the first one that we discussed was Cam Clay model. We know that this is an elastoplastic model, which utilize the concept of critical state. We have also seen the yield curve and we noted that it is a logarithmic spiral. Now there is some other developments in the critical state model and that led to modified Cam Clay.

So, in the last lecture we told that there are certain aspects related to plasticity behaviour of soil which also need to be incorporated. In the last lecture I purposely did not introduce those, because we first need to understand what a critical state model would look like. We have got a feel of it, we have defined the yield curve we have defined it mathematically. So, now we are conversant with the whole process.

So, in today's lecture we will see the modified Cam Clay model along with some basic aspects of plastic behaviour of the soil, I mean to say mechanical plastic response of the soil.

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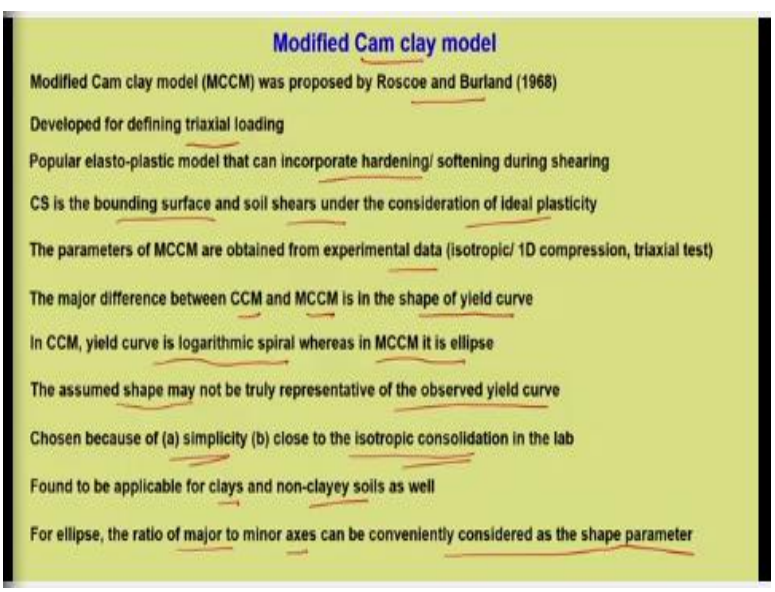

So, today's lecture we will start with modified Cam Clay model. So, this is what we will be dealing with in today's lecture. In fact Cam Clay and modified Cam Clay model, it was more or less developed during the same period. The initial one was proposed in Schofield and Wroth and later the modified Cam Clay model was proposed by Roscoe and Burland in 1968. And we know that these models were essentially developed for defining the triaxial loading or the triaxial test.

These are popular elastoplastic model, that can incorporate hardening, softening that happens during shearing. I mean to say it is not like elastic response and then failure, after yielding there is some other response that we have already noticed; it can be either hardening or softening. And when that gets incorporated we say it is an elastoplastic response. Critical state is the bounding surface, now what is so specific about Cam Clay and modified Cam Clay is it is entirely based on the critical state concept.

And hence the critical state forms the boundary of the soil, it is the bounding surface and the soil shears under the consideration of ideal plasticity. The parameters of modified Cam Clay model are obtained from experimental data and that is the beauty of Cam Clay model, rather we can get the parameters needed in a simplified manner. For example λ it comes from consolidation test, either it comes from isotropic or one dimensional consolidation.

The failure response in terms of M, it comes from triaxial testing. So, there is a way by which you can determine the parameters in a more simplified manner. The major difference between Cam Clay model and the modified Cam Clay model is in the shape of yield curve, I think we have discussed this before also. So, the major difference is in terms of the shape of the yield curve, the earlier one we have seen that it is a logarithmic spiral.

So, in Cam Clay model yield curve is logarithmic spiral whereas in MCCM it is ellipse. Now ellipse defines a more symmetrical and regular shape, it is not about symmetry, it is about the corners like in the logarithmic spiral we have seen that it comes and meets at a particular point. Now this may sometimes create issues in numerical instability as you solve the problem, because there is an issue of continuity of differential equation.

So, what happens is in the case of modified Cam Clay it becomes an ellipse, so it is more smooth shape we can say. It is not about symmetry, it is about more smoothness in the shape. Now the thing is the assumed shape may not be truly representative of the observed yield curve, if we actually want to determine the yield curve experimentally, we have seen that, it is a quite an extensive procedure.

You need to find out different ways by which the soil gets yielded and then you join all those points, it is not that easy. So, that is why the definition of the shape becomes important. Now what it says is that the shape which has been assumed, which is an ellipse may not be truly representative of the actual soil behaviour. So, it is an idealized condition, the actual behaviour may be something else some other shape**.**

But it is very close to it, a logarithmic spiral or ellipse can be considered close to the realistic yield curve. And it is essentially assumed because of it is simplicity in shape in defining the yield curve. And it is close to the isotropic consolidation in the lab which can be easily captured in this shape. It is found to be applicable for clays and non clay soils as well, even though there are deviations for some soils it may not behave properly.

But in most of the soils, in the absence of any other model this is found to be a fair approximation. Now the beauty of ellipse is the ratio of major to minor axis can be conveniently considered as the shape parameter. Now when I say ellipse, we also need to know how it changes depending upon the condition. Now these changes can be incorporated in terms of the shape parameter. And in this case the major and minor axis becomes a very important shape parameter, we will see how?

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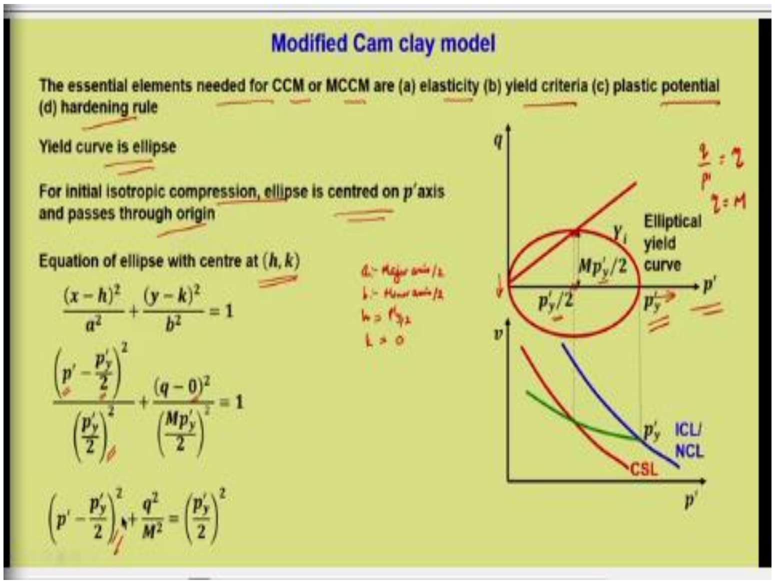

Now, let us start again with this statement. The essential elements needed for any elastoplastic models like CCM or MCCM are elasticity, yield criteria, plastic potential and hardening rule. It is very easy to understand what is elasticity because that is what we were exposed to most of the time during our undergraduate. And in some of the courses that we deal with in the post graduate as well, so it is pretty easy.

And the parameters are quite simple, if we make an extra assumption that it is linear, elastic, isotropic and homogeneous. So, this simplifies a lot of things and you are left with only 2 elastic parameters. You can define the elastic response of the soil using 2 elastic parameters, one is modulus and the other one is Poisson's ratio. And this modulus can be ends or Young's or bulk modulus that we have seen earlier, so that part is done.

Now for material like soils which exhibit plastic behaviour during shearing, we also need to know where it would yield. Now in this particular course we have extensively discussed on how soil is going to yield, at what point it would yield and we have discussed about yield curves. The various aspects related to yield curve, now when I say we have extensively discussed, it is still at an introductory level, please keep this in mind.

If you want to know more you have to read a lot further related to plasticity behaviour in geo mechanics. So, we have just understood how to learn and where to start with? So, we have discuss to that particular aspect, what is yielding and how do we define yield? And later, in the last case in Cam Clay model, we have also discussed about how it is going to yield and what is the boundary? That we have fixed.

So, we have clearly defined at some point the elasticity behaviour changes. Now yielding is not failure when it comes to elastoplastic behaviour. Elastic, perfectly plastic like models like Mohr Coulomb, we have seen that yielding itself is a failure condition. Now if you want to incorporate hardening, softening condition, this is not possible. So, that is where the elastoplastic models are important. Now yielding is the point where it undergo transition from elastic to plastic behaviour, but that is not failure.

The yield curve expands, and how it will expand? That part we do not know, we know that as the effective stress path go past beyond the yield curve, the yield curve start expanding. And when it meets the critical state line, I mean to say the effective stress path meets the critical straight line. That particular yield curve which is at the point of intersection of the effective stress path with the critical straight line that particular yield curve denotes the failure one, beyond which there is no further expansion.

Now how do we explain this many sequence beyond yielding? For that we need to know about the plastic behaviour of the soil as well. This particular aspect, we just told in Cam Clay model but we did not discuss this. Now putting everything in one shot is going to be difficult for you to understand, so we did not discuss this intentionally in the last lecture. But at the same time we need to know some basics of what happens to the soil during plastic behaviour beyond yielding?

So, this is the starting point where you can capitalize further and do further learning. So, this is important, please understand, whatever we are going to discuss now in terms of the third one that is the plastic potential and hardening rule which essentially state the plastic behaviour of the soil. This part is the beginning and this shows you what to learn and from where you have to start related to the plastic behaviour of soil?

And now you need to build on this for actually implementing this model into practice, so let us start. Now yield curve is ellipse in the case of MCCM, we will be discussing everything with respect to MCCM but that is remaining common for any other plastic behaviour as well. So, first of all elastic behaviour of the soil, then the yielding, then the plastic response, then the failure, so there are four stages in which we need to conceive a soil behaviour when it is loaded.

Any material will exhibit some amount of elastic behaviour, now it may be at a very small stress range or it may be at a higher stress range depending upon the stiffness of the material. Now there is a point at which the yielding happens and that is defined by the yield curve. Up to that stress strain response can be defined by elastic behaviour or elastic models. Then you need to fix a point where it will yield? That is yield criteria.

So, that also we have seen to some extent. Now beyond yielding it can undergo hardening or softening behaviour. Now there are different models to account for this behaviour and that is included in the elastoplastic models, various types of elastoplastic models. In that one particular model is Cam Clay model or modified Cam Clay model. Now how this is going to proceed beyond yield curve? That is defined by plastic potential or hardening rule.

That states the stress strain response under plastic condition. So, once the plasticity sets in how will you define the stress strain behaviour of the soil, how it will proceed? So, that is given by this, and finally the state boundary which is defined by the critical state defines the failure. So, these are the four steps in which you need to understand the soil when it is subjected to shearing, so yield curve is ellipse.

Now for initial isotropic compression, ellipse is centered on p’ axis and it passes through origin. Now this particular definition is with respect to MCCM, how the ellipse would look like in q p’ space? So, let us first define in q p’ space, so what it tells is now yielding as I told it need not mean failure. It can happen in isotropic compression, it can happen in one dimensional compression or it can happen in a triaxial loading.

So, in initial isotropic compression, ellipse is centered on p' axis. So, the center of the ellipse is on p' axis, so it is on this axis. And it passes through origin, the one side of the ellipse it passes through origin, let us see how? So, this is how we define the ellipse, so you can see that the center is on the p' axis and it passes through the origin. So, this is an elliptical yield curve in MCCM, let us call it as Y_i.

And the yield curve is defined by it is p'_y, that is the yield point. So, this also we know, we have already seen this. Now this on the ICL, NCL, p'_y is transferred to this particular point. Now we need to see that this is another kind of very important aspect that the midpoint of the major axis is here, so this is $p'_y/2$, this particular point. And that is the point through which the critical straight line goes, and how does that look in vp' plot, so this point.

That is the point where the unloading-reloading line passes or it crisscrosses the critical state. It does not mean that this meeting, but this is the location, so up to here what it means is that up to here this particular point it is elastic response and then it goes beyond. So, here in particular the elastic response of the soil and this is the point, so this is very specific to MCCM model that is the yield curve, please note these are very important aspect which you need to keep in mind.

That is when the yield curve touches the critical state line that corresponds to this loading, unloading line which is unloaded from p'_y, now $p'_y$is the extreme point of this ellipse. Now this is true for any bigger ellipse as the effective stress path moves ahead. Definitely this particular point will be M into $p'_y/2$ provided if that is the failure point. Now the stress ratio that is q upon p', if you call it as $\eta = \frac{q}{p}$'.

Anyway we are going to discuss this later, q by p' if it is η; $\frac{q}{p}$' is the stress ratio. Now this η will be equal to M, when the yield is corresponding to the failure condition, till that it is η. Now this let us consider it to be a failure condition, in that case q is given by $\frac{Mp'_y}{2}$ because this is p'_y, and the slope of the critical straight line is M. So, at this particular point where it touches at the extreme top point of the ellipse it is $\frac{Mp'_y}{2}$.

And this distance is half of the major axis of the ellipse which is $\frac{p'_y}{2}$. Now it is very easy since the elliptical yield curve is defined, we just need to define the equation of the ellipse, and that comes from our basic geometry. Now equation of ellipse with center at (h, k), now this is the point h, km this is the center. You can write it as

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where a and b are half of major and minor axis.

So, a is major axis /2, and b is minor axis /2, because as per the equation of ellipse, and h and k are the center. Now here h is for p', that is $p'_y/2$, and k it is on the p' axis, so it will be 0. So, this if you substitute,

$$\frac{\left(p' - \frac{p'_y}{2}\right)^2}{\left(\frac{p'_y}{2}\right)^2} + \frac{(q-0)^2}{\left(\frac{Mp'_y}{2}\right)^2} = 1$$

Now what we have done is, we have incorporated some parameters which will define the yield, and that is in terms yield curve, that is in terms of p'_y and M. So, p'_y and M will take care of the shape of the ellipse. If you further simplify you can write

$$\left(p' - \frac{p'_y}{2}\right)^2 + \frac{q^2}{M^2} = \left(\frac{p'_y}{2}\right)^2$$

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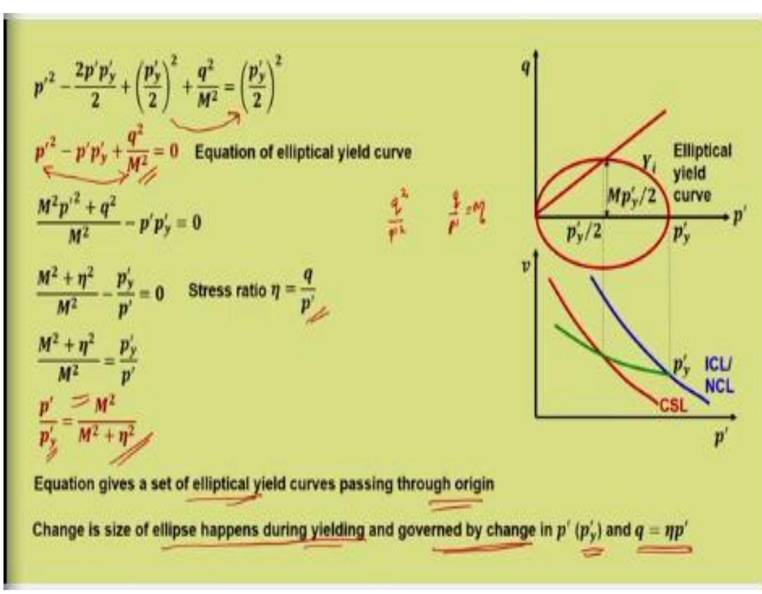

If you expand the previous one. So, then we will get

$$p'^2 - \frac{2p'p'_y}{2} + \left(\frac{p'_y}{2}\right)^2 + \frac{q^2}{M^2} = \left(\frac{p'_y}{2}\right)^2$$

So, we are left with the ellipse equation which is equal to

$$p'^2 - p'p'_y + \frac{q^2}{M^2} = 0$$

So, this comes from this particular expression, so this is the equation for elliptical yield curve. And that defines the boundary where the soil is going to yield. So, if you further rearrange this particular expression, we can write

$$\frac{M^2p'^2 + q^2}{M^2} - p'p'_y = 0$$

Now if we divide it by p' square, so we can write

$$\frac{M^2 + \eta^2}{M^2} - \frac{p'_y}{p'} = 0$$

And the stress ratio $\eta = \frac{q}{p'}$. Further we can write the another form of elliptical equation as

$$\frac{M^2 + \eta^2}{M^2} = \frac{p'_y}{p'}$$

Why I am discussing all different forms is that you will find these type of forms when you refer to the standard textbooks. So, the equation can be rewritten as

$$\frac{p'}{p'_y} = \frac{M^2}{M^2 + \eta^2}$$

Equation gives a set of elliptical yield curves passing through origin. And what is the kind of the size of the elliptical yield curve is defined by p' and M. A change in size of the ellipse happens during yielding, because how do you consider the increased size of the ellipse.

And that increase in size is completely governed by p'_y, whereas when you define p' in terms of p'_y, and $q = \eta p'$. Now $\eta p'$ is important, because the yield curve goes on increasing and that can go on till failure. Now when it is at failure, M gets activated, so that is the final bound but the concept of yielding keeps on happening.

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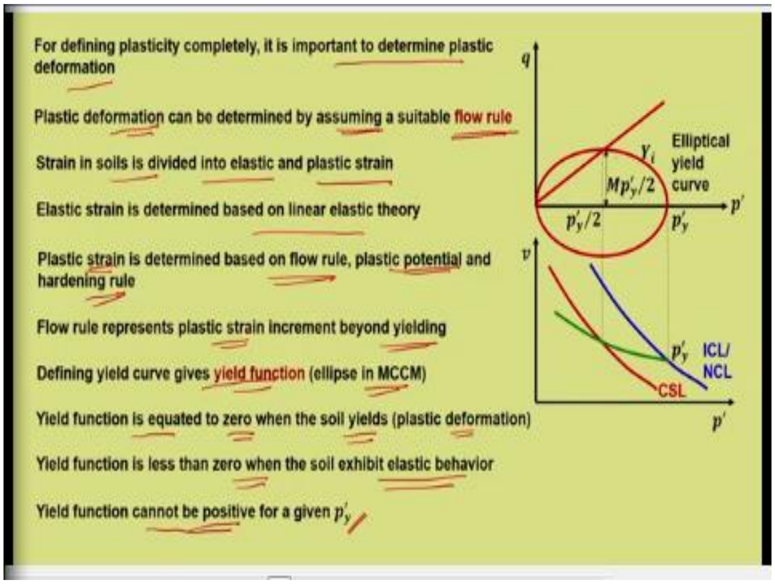

And that will depend upon whether it is hardening or softening. Now for defining plasticity completely, it is important to determine the plastic deformation. Now whatever we have done for Cam Clay model that is defining of the yield curve, we have done it for MCCM as well. Now this much we are comfortable with, now we will try to add a bit on what happens beyond yielding at till failure.

So, that is what we will see in this, and that is to determine the plastic deformation. Now for determining plastic deformation, we need to assume something known as flow rule, how does? It is more like a plastic flow. What happens to the soil behaviour during shearing, what kind of flow takes place once the yielding happens? So, that is dictated by some flow rule which we need to assume, how much it is close to the realistic behaviour of the soil that is a different aspect.

Because for engineers we need to define it mathematically and for that we need to make certain assumptions, so flow rule is one such kind of an assumption. Strain in soils, now where does this comes from? Why do we need to assume? Why do we need to account for flow rule? It is because strain in soils beyond yielding is divided into elastic and plastic strain. Now elastic strain is determined based on the concept of linear elastic theory.

Plastic strain is determined, now the next aspect is how do we determine the plastic strain? Now this can be determined based on flow rule what is known as plastic potential and then what is known as hardening rule. So, we need to have some basics on these aspects and that is what we will be doing in the further slides. Flow rule represents the plastic strain increment beyond yielding, so once it yields how the plastic strain increment happens which is defined by flow rule.

Defining yield curve gives yield function, and that is an ellipse in MCCM and such an yield function is a logarithmic spiral in Cam Clay model. Yield function is equated to 0 when soil yields and when the plastic deformation happens. So, once it is made equal to 0 that means that it is close to it has yielded and it is close to failure. Now yield function less than 0 is what it corresponds to elastic behaviour that is within this yield curve.

Whatever is the state of the soil within the yield curve, we know that that is elastic condition. And hence the yield curve function or the function of the yield curve less than 0 is supposed to be elastic behaviour. And yield function can never be positive, that is it is up to 0 where it yields, now beyond that it is not valid. So, greater than 0 is not possible, greater than 0 means either that particular yield curve is no longer valid.

It goes to the next yield curve and next yield point or if you are talking in terms of failure what it means is that, there is no soil state beyond that. So, that is why there is no question of yield function greater than 0, it cannot be positive for a given p'_y. So, that is why it is important, for that particular extreme yield point or pre-consolidation pressure the yield function cannot be greater than 0.

But it can expand, and when it expands the earlier yield curve is no longer valid, now it is a redefined yield curve. And if it is at a failure state if the effective stress path is touching the failure line or the critical state line, then there is no question of soil beyond that, so that forms the boundary.

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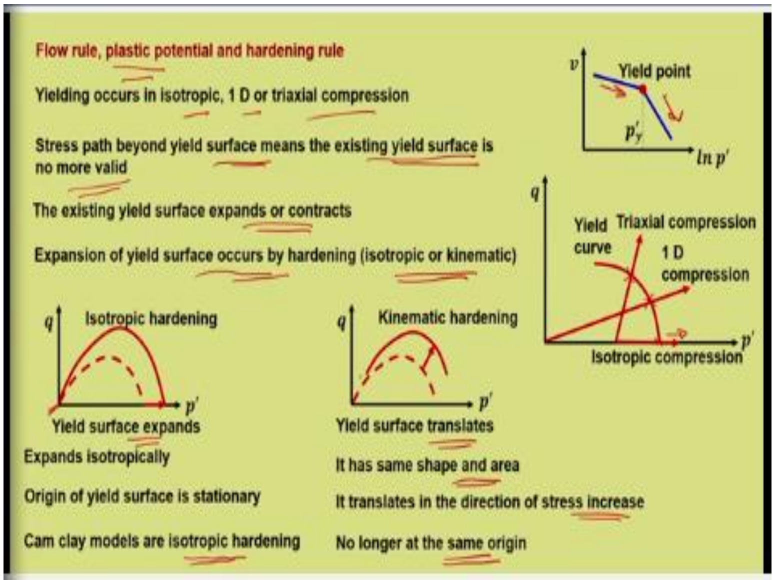

So, now let us see some basics of flow rule, plastic potential and hardening rule. Yielding can occur in isotropic one dimensional or triaxial compression. So, this notion should be very clear to you that yielding is not meaning failure. So, it can happen in one dimensional compression, isotropic compression where there is no shear stress developed, there also yielding can happen and it can also happen during shearing, so yielding is different from failure.

Now this is a general representation of yield point, so this is v versus ln p'. So, this is the point at which it yields and there is a substantial change in the v. This we know, this aspect we have already seen. Now in the case of qp' the yield curve, so you can see that this is the way in which the isotropic compression yielding happens, so this is the point. The one dimensional compression also can yield and triaxial compression in qp' that is shearing can also yield.

Now stress path beyond yield surface. So, it means that the stress path can still move forward, that also we have already seen in our previous lectures. When it moves beyond yield surface, means the existing yield surface is no more valid. So, once it crosses then it is for the new yield curve, then it keeps on extending till it fails. The existing yield surface expands or contracts depending upon whether it is a softening or a hardening behaviour.

Expansion of yield occurs by hardening or now this hardening behaviour is expressed in terms of isotropic or kinematic hardening. Now what we understand is that, after yielding if it is expansion of yield curve that is because of hardening. Now there are 2 ways in which the hardening is defined, that means expansion of the yield curve happens. Now that hardening is defined in terms of either isotropic or kinematic hardening.

Now this is our yield curve, the initial yield curve in general, so it need not be a critical state or modified Cam Clay or Cam Clay, it is a general yield curve. Now in the case of isotropic hardening, what do you mean by isotropic hardening? It is simply an increase in the size of the yield curve, the one which we have seen till now. And that is what we will be using because the Cam Clay model and the modified Cam Clay model uses the concept of isotropic hardening.

Now what do you mean by isotropic hardening? It is simply the increase in size of the yield curve as the effective stress path crosses beyond the existing yield curve. So, yield surface expands, expands isotropically. Origin of the yield surface is stationary; this point is not going to change, Cam Clay models that is both Cam Clay and modified Cam Clay models they are following isotropic hardening.

Now what is meant by kinematic hardening? This is the initial yield curve, and in kinematic hardening what happens is the whole of the yield curve gets shifted. You can see that it no longer follows the initial origin, it is getting translated from it is earlier position. So, that is what is meant by kinematic hardening. That is yield surface translates from this particular location to that particular location.

It has the same shape and area, that is initial shape and it is area is not going to change, it simply it is relocated. It translates in the direction of stress increase or how these stress path moves. No longer at the same origin, so that is essentially the difference between isotropic hardening and kinematic hardening. Now what is discussed in this particular course and what is not discussed in this particular course?

We understand what is hardening, we understand what is isotropic hardening and kinematic hardening. But we are not going to translate this into an exclusive mathematics for this particular model. So, that is not what is dealt in this particular lecture or in this particular course, and that will be the extension of your learning from taking this as the basics. So, that is about hardening like what happens to the yield curve beyond it yields. So, now let us see, we also need to define the plastic strain, now how do we account for plastic strain?

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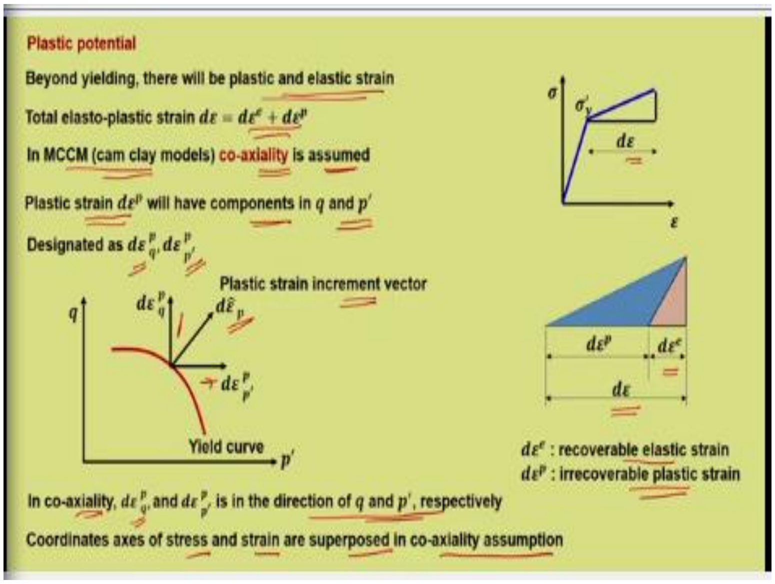

And for that we need to understand what is meant by plastic potential? Now beyond yielding there will be plastic as well as elastic strain that is why it is called elastoplastic response. So, elastic strain will also be there, and there will be quite a bit of plastic strain as well. Now we need define or we need to account or we need to determine these strains, then only our model is complete. And that is what we have not doing in this particular course, we are yet to define the strain completely.

But we will discuss like what we need to have further, and that is both elastic and plastic strain. So, let us take a typical σ versus ϵ response, this is elastic response, then it yields and there is a bilinear response which is shown here, σ'_y is the yield point. And this is a typical hardening behaviour, and this is $d\epsilon$, that is beyond the yield what is the total strain that is happening.

Now if I expand this and show here, this is the total strain that is happening beyond yield, this much portion is elastic strain and that is represented by $d\epsilon^e$. And the plastic strain is represented by $d\epsilon^p$, so total gives the $d\epsilon$ which is the total strain, $d\epsilon^e$ is the recoverable elastic strain, $d\epsilon^p$ is irrecoverable plastic strain. And the total elasto-plastic strain $d\epsilon = d\epsilon^e + d\epsilon^p$ Now to define this further I mean to say the strain, we need to make now further assumptions.

In modified Cam Clay or rather in Cam Clay models co-axiality is assumed. Now this is another assumption that we need to have in order to determine the strain. That means this particular aspect, the plastic strain that is $d\epsilon^p$ it will have components in q and p'. That means the plastic strain due to volumetric behaviour or volumetric changes and plastic strain due to deviatoric changes and that is represented.

And that plastic strain is decomposed into the component due to q and the component due to p'. And this is designated as $d\epsilon_q^p$ and $d\epsilon_{p'}^p$. Now this is due to the plastic strain due to q, and this is the plastic strain due to p'. So, this is the typical yield curve. Now this is what it represents $d\epsilon_q^p$ and $d\epsilon_{p'}^p$. So, this is corresponding to p' and this is corresponding to q, now who has told that this particular increment is in this particular direction?

That means it is in the direction of p', this is in the direction of q, who has told this? Now this is represented by how the strain increment vector happens, that is the plastic strain increment vector which is represented by $d\hat{\epsilon}_p$ with a crown. So, this is the plastic strain increment vector, now this plastic strain increment vector or it is knowledge is quite essential for defining the plastic strain.

So, plastic potential followed by plastic strain increment vector, these are 2 essential elements. But the question remains who has told that this is in this direction? Now in co-axiality, that is the assumption of co-axiality, we assume that $d\epsilon_q^p$ and $d\epsilon_{p'}^p$ is in the direction of q and p' respectively. Now that is the importance of co-axiality assumption. That means whatever is the strain that happens in the direction of stress, so that is what is mean by co-axiality. Coordinate axes of stress and strain are superposed in co-axiality assumption.

You can see that this particular axis and this particular axis which denotes the plastic strain is superposed with q and p' axis respectively, so this assumption is what is known as co-axiality. So, when you start learning plastic behaviour of soil or if you want to learn more mathematically, you will come across these terminologies. So, it will be very easy for you to understand further when you know these terminologies.

So, the assumption of co-axiality is valid for MCCM model, so for us we are discussing this with respect to critical state models. So, let us further go ahead, so now we have defined yield curve, we have defined the co-axiality assumption. And then we have seen the hardening rule that is isotropic or kinematic, now these are some of the elements. So, now we need to move further.

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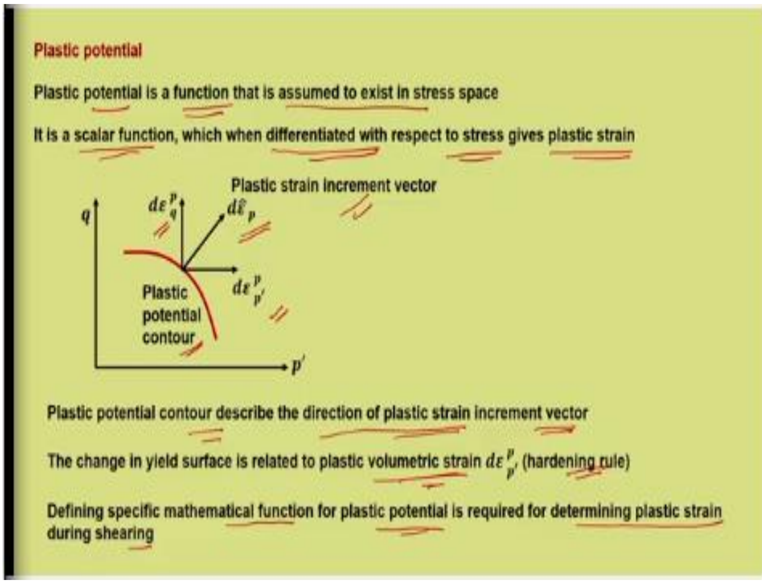

Plastic potential is a function that is assumed to exist in stress space. Now we need to define plastic strain increment vector. Now for that we also need to define what is known as plastic potential, now what is plastic potential? It is a function that is assumed to exist in stress space. Now what is the duty of this function? It is a scalar function which when differentiated with respect to that particular stress gives plastic strain.

Now you understand how do we get plastic strain, it is very essential to define plastic potential. Because it is a scalar function and when you differentiate that particular scalar function with respect to the corresponding stress or with respect to a specified stress, it gives plastic strain corresponding to that particular stress condition. So, define plastic potential and get plastic strain. So, let us say this is a plastic potential contour, this we have already defined, so $d\epsilon_q^p$ and $d\epsilon_{p'}^p$ and this is the plastic strain increment vector.

Plastic potential contour describe the direction of plastic strain increment vector, so very essential to define the plastic potential, so that this is obtained and this is also defined. The change in yield surface is related to plastic volumetric strain, and that is called hardening rule. That we have seen the expansion of yield curve is in terms of p'_y and that comes from the hardening behaviour, and that comes from the plastic volumetric strain.

Defining specific mathematical function for plastic potential is required for determining plastic strain during shearing. So, we need to define or we need to allocate certain function for this plastic potential, then only we will be able to determine plastic strain. Now this opens up another concept which is very essential in plastic behaviour of soil.

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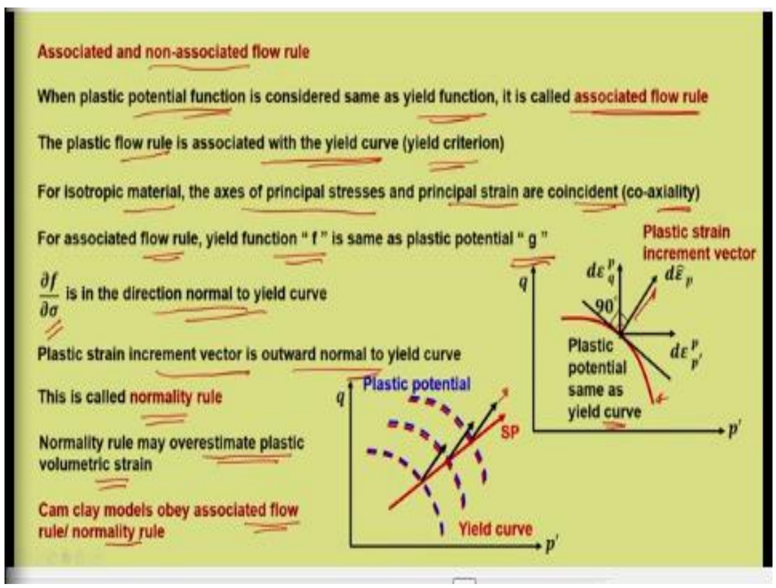

And that is what is known as associated and known associated flow rule. How do you define plastic potential? So, there it has something to do with this particular concept which is known as associated and non associated flow rule. When plastic potential function is considered same as yield function, it is called associated flow rule, very easy. Like what we are considering is we have already defined the yield function which is ellipse or logarithmic spiral or any other shape.

What we define is that the plastic potential function is same as the yield function, when you make such an assumption this is known as associated flow rule. The plastic flow rule is associated with the yield curve. Now it makes our life very simple, like we need to work with only one function and remaining things are known like how to evolve the plastic strain. Of course, I am not discussing that in this particular course, but definition of plastic potential is important.

So, we have already done that in the case of associated flow rule. For isotropic material, the axis of principle stresses and principle strain are coincident, that is co-axiality, we have already discussed. For associated flow rule, yield function F is same as plastic potential g. And $\frac{\partial f}{\partial \sigma}$ is in the direction normal to yield curve, so that is also very important. So, that is how it looks like q versus p' plastic potential which is same as the yield curve in the associated flow rule.

So, this is yield curve as well as plastic potential. And we have the plastic strain increment vector and that is normal to the yield curve. So, this is 90 degrees, so that is normal to yield curve. Now plastic strain incremental vector is outward normal to yield curve. So, this is outward normal to yield curve, so such an assumption is called normality rule, what it states? The plastic strain increment vector, this direction is perpendicular to the tangent at this particular point.

So, such an increment that is at 90 degrees it is outward normal to yield curve, this is called normality rule. So, that is also need to be satisfied and hence yield curve and plastic potential both are same, as you can see. And this is the stress path, and the plastic strain increment vector is given in this manner. Now when you define this mathematically that leads to the determination of plastic strain.

So, normality rule may sometimes overestimate the plastic volumetric strain, may not be realistic that is what it means. But these are this makes our life simple, that means the mathematical complexities reduced by such assumption. But always keep in mind there is a possibility that it may overestimate plastic volumetric strain. Now Cam Clay models both Cam Clay and modified Cam Clay obey associated flow rule and normality rule.

So, both associated flow rule and normality rule is applicable for these models and hence our discussion is mostly associated with that.

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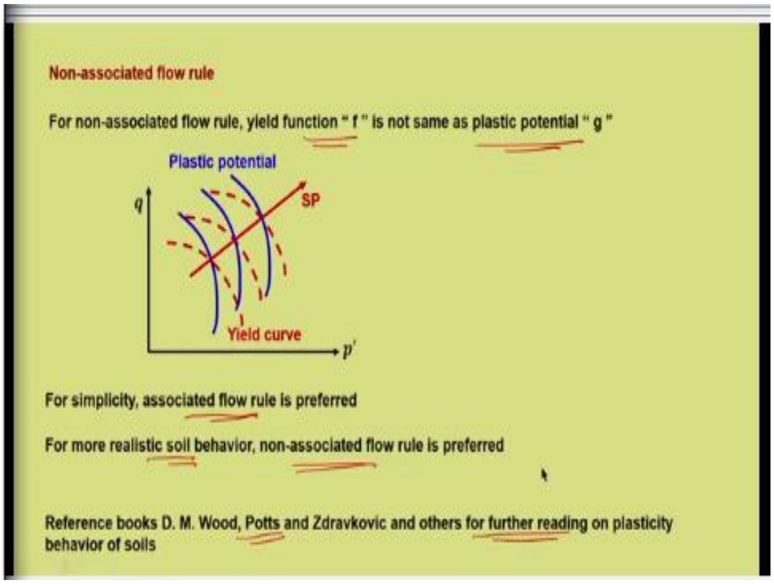

What is meant by non-associated flow rule? When we consider yield function is different from plastic potential or rather plastic potential g is different from F. So, then we say it is a non-associated flow rule. So, yield curve and the plastic potential they are different. For simplicity, associated flow rule is mostly preferred. For more realistic behaviour a non-associated flow rule is preferred.

Now further reading on this, now I cannot go beyond this because for this particular course this is mostly an introductory aspect of plasticity behaviour. There has to be an exclusive course on plastic behaviour of soil and for that the readers or the participants are advised to go through this particular reference book and any other reading material on plasticity behaviour of soils.

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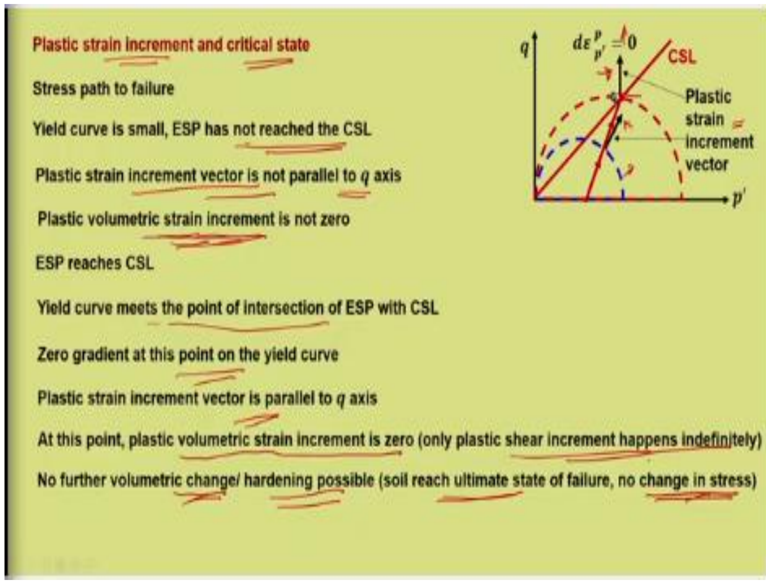

Now some important aspect related to plastic strain increment and with respect to the critical state. Let us say q p', now stress path to failure is defined. Now let us say this is the starting point of shearing, this is the critical state line and we can define the yield curve. The yield curve is small; ESP has not reached the critical state line. Now at this particular point if we try to mark the plastic potential or plastic strain increment vector, how it will look like?

Now the plastic strain increment vector is not parallel to q axis, that means the plastic volumetric strain increment is not 0, I will explain what actually is this? Now you can see that this is the plastic strain increment vector, the direction, at this particular point where the effective stress path is touching the yield curve. Now this has not failed, now at this particular point the direction of this that is the plastic strain increment vector, this is not parallel to q axis.

Now if it is not parallel to q axis, it means that plastic volumetric strain increment is not 0. Now what is the implication of this statement plastic volumetric strain increment? We will see what is the implication of this statement? This is the plastic strain increment vector. Now let us say the ESP reaches critical state line, this is the condition and that is defined by the yield curve or the elliptical yield curve.

Now at this particular point what happens is you cannot further increase the yield curve. Now we have seen in the previous slide, that increment of the yield curve is dictated by the plastic volumetric strain increment that is this one. Now once it has reached the critical state line soil has failed, there is no more question of existence of the soil beyond that. That means the plastic volumetric strain increment cannot happen beyond this.

From here it is possible, the plastic volumetric strain increment happens as the effective stress path moves further. But, once it reaches this there is no more, it has failed, there is no more soil beyond that particular point. So, the plastic volumetric strain increment becomes 0 at this particular point. And when that becomes 0 that is $d\epsilon_{p'}^{p}$ that becomes 0 means it has to be parallel.

That means this plastic strain increment vector it has to be parallel, that is yield curve means the point of intersection that is what I told and 0 gradient at this point on the yield curve. What is meant by 0 gradient? At this particular point it is 0 gradient that is at the point where it, so here the slope is 0 and the plastic strain increment vector is parallel to q axis. Now this is the situation where only $d\epsilon_q^p$ exist, $d\epsilon_{p'}^{p}$ it does not exist.

That means plastic volumetric strain increment is 0, only plastic shear increment happens indefinitely, now this is indefinite. Now we kept on telling this right from the day one when we started discussing critical state, at critical state the soil shears indefinitely. Now you will clearly appreciate what is meant by the statement in this particular slide? Like this plastic strain increment vector in the q direction means it is shearing indefinitely is infinite.

Whereas , $d\epsilon_{p'}^{p}$ which shows the plastic volumetric strain increment, that becomes 0 and there is no further change in stress because soil does not exist beyond this point. So, 2 conditions are satisfied when the effective stress path reaches the critical state line and for the yield curve which is defined by this. So, here, $d\epsilon_{p'}^{p} = 0$ and that is exactly what is meant by this condition.

Now for this particular plastic strain increment vector, there is , $d\epsilon_{p'}^{p}$, that is possible, that is why it is expanding. But at this point it stops, it becomes 0, and that is the condition where this plastic strain increment vector is parallel to q. No further volumetric change, so there are 2 things which are satisfied here. And no hardening possible, so I will reach ultimate state of failure and no change in stress.

So, no volumetric change and no change in stress but shears indefinitely. So, this particular figure shows exactly what is happening at critical state. Now if you want to understand this, we need to understand the basics of plasticity behaviour what we just discussed. So, that is all about MCCM and some basic elements of plastic behaviour which is very important. And now let us try to summarize what we have learned in this lecture.

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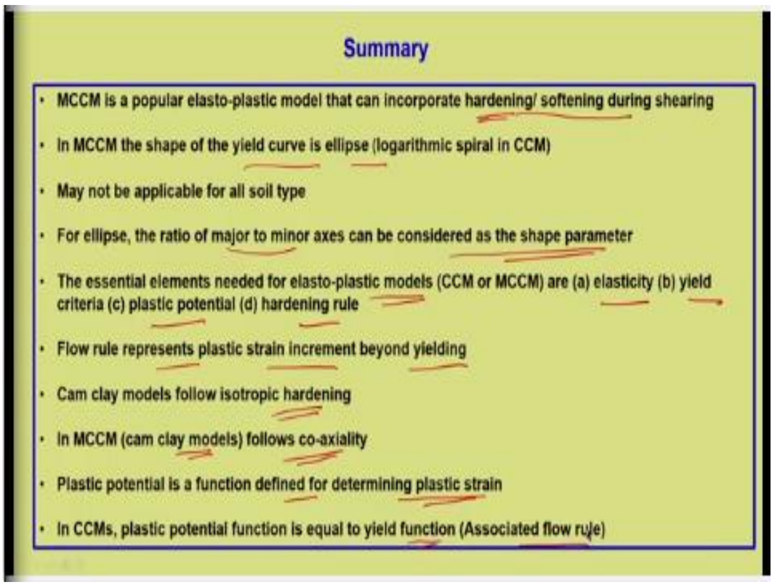

MCCM is a popular elasto-plastic model that can incorporate hardening, softening during shearing. In MCCM the shape of the yield curve is ellipse, but we have talked about hardening only, it is basically an isotropic hardening model. So, in MCCM the shape of the yield curve is ellipse and it is logarithmic spiral in CCM. It may not be applicable for all soil type, as I told when we do a mathematical modelling or mathematical definition we cannot make it generalized.

So, there will be some cases where it may not apply. For ellipse the ratio of major to minor axis is considered as the shape parameter, how? It accounts for further expansion. The essential elements needed for elasto-plastic models are elasticity, yield criteria, plastic potential, hardening rule, all of them the basic definition and what it actually means? We have discussed. Flow rule represents plastic strain increment beyond yielding.

Cam clay models follow isotropic hardening so we have these are basically hardening models. In MCCM or Cam Clay models it follows coaxiality which is important for defining the plastic strain increment vector. Plastic potential is a function defined for determining plastic strain. In critical state models plastic potential function is taken equal to yield function which is known as associated flow rule.

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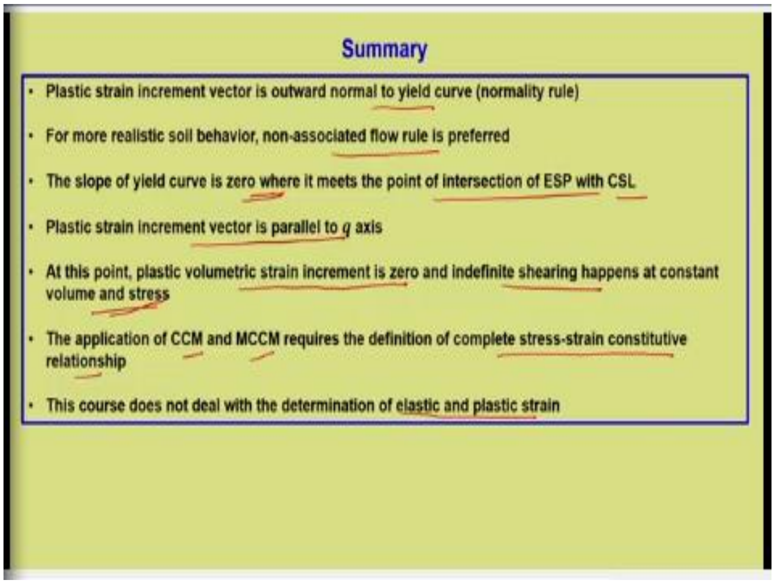

So, plastic strain increment vector is outward normal to yield curve, which is normality rule. For more realistic soil behaviour a non associated flow rule is preferred. The slope of yield curve is 0; please understand under what condition, where it meets the point of intersection of ESP with critical state line. And that is why at that particular point the plastic strain increment vector is in the upward direction.

Plastic strain increment vector is parallel to q axis at this particular point. At this point plastic volumetric strain increment is 0, indefinite shearing happens at constant volume and stress. The application of CCM and MCCM requires the definition of complete stress strain constitutive relationship. Now this course does not deal with the determination of elastic and plastic strain. We have defined the basic elements of plasticity behaviour but the complete constitutive relationship beyond the yielding we have not defined.

That means the plastic strain we are not actually determining in this particular course**.** So, that is all for the critical state models. Now we will see a bit about prediction of critical state behaviour in the next lecture, so that is all for now, thank you.