Advanced Soil Mechanics
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Lecture-49
Cam Clay

Welcome back all of you, in the last few lectures we were dealing with critical state soil mechanics. If you have noticed we have been adding certain important information one after the other. And lastly we discussed about soil yielding, now where are we heading to? We are heading towards defining a state boundary surface for the soil. And when I say boundary surface, it means the domain within which the soil would exist, soil would exist mechanically, mechanical means because of it is mechanical response.

Otherwise soil is always there, now whether soil would exist as a load bearing medium when it is subjected to shearing, that is what it means. So, when we say boundary surface, we are trying to understand the limit within which the soil would exist during shearing as a load bearing member. Now for that we need to define a limit within which the soil may not exist. And one possibility was in terms of soil yielding.

So, yielding is a good indication, that, yes, soil is going to be in severe distress that is plastic deformation starts setting in. And we have seen in the last lecture that as the yield curve progressively increases it reaches a point where the yield curve meets the effective stress path and the critical state. So, though that particular yield curve which passes through the point of intersection of effective stress path with the critical state line, that particular yield curve happens to be the final boundary.

During the expansion of yield curve that defines the final boundary. Now we have understood it physically, now for any engineering application we need to define it mathematically. So, there need to be certain models based on which the yield curve is defined. So, in today's and the upcoming lecture we will see how yield curve is defined mathematically based on 2 popular models which is based on critical state framework.

So, the first model is Cam Clay model which we will see in today's lecture. I would also like to mention here that in this particular course you will not be studying all the aspects of such a model or critical state framework. We are basically understanding the concepts in stress parameters in terms of stress variations rather mostly applicable for stress path. We are not in a position to gain more knowledge from the strain point of view.

I mean to say when I say a strain elastic strain and followed by plastic strain, a bit of it we will be discussing in the next module, next lecture I mean. So, here what we are understanding would be the basic understanding which is needed further for understanding the plastic response. So, with that let us try to understand what is a Cam Clay model today and with that understanding we will move on to the next model. And possibly you will be able to appreciate why we will not be able to deal the entire aspects in this particular lecture.

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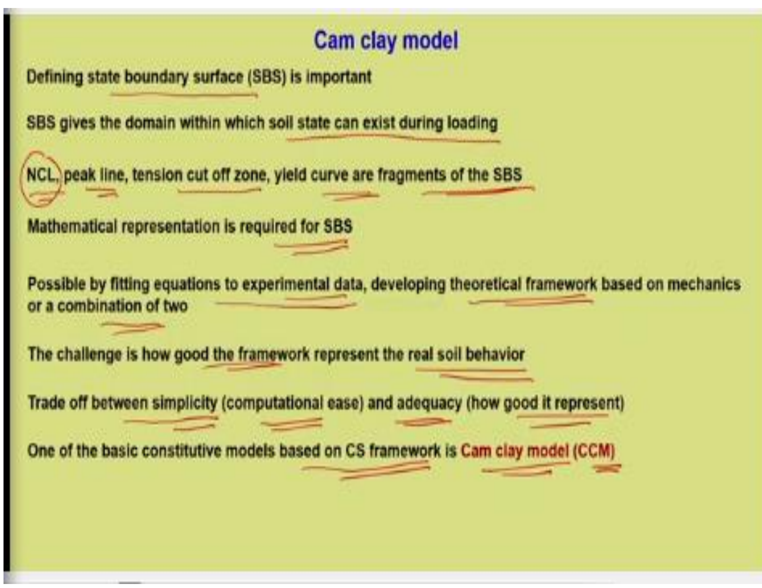

So, the first model is the Cam Clay model. Now the name suggests that it is a kind of clay or something like that, it is not so. I would strongly recommend you reading Schofield and Wroth’s book wherein it is stated that cam is the name of the river which flows besides or close to Cambridge University. Now clay is here clay in the sense it is not meaning a kind of particular clay may be which is obtained from there, it is not like that.

Cam clay is a generic name for that particular model, it has nothing to do with the clay response, it has something to do with the theoretical model which they have developed. And they wanted to make it a more generic name in nature, so that is why such a name which is Cam Clay. Now as I already told defining state boundary surface is important, we are heading towards that. Now there are different ways by which you can define the state boundary.

Some of them we have already seen, we have seen a peak line that is also a state boundary that is a part of state boundary. We have seen the yield kind of curve in the last lecture; we have seen the tension cut off zone, so these are all fragments of state boundary. Now there can be a specific shape and model based on which we define this, that is also state boundary. So, there are different ways by which the state boundary surface is defined.

This state boundary surface gives the domain within which soil state can exist during loading. So, this later part of the sentence is important, we are discussing the existence of soil state during loading otherwise soil is very much there. We are not bothered about that existence; whether the soil will be able to bear it is load without failure. The normally consolidated line, the peak line, we will see this normally consolidated why it becomes a kind of fragment of SBS? That we will see later.

Peak line we have understood clearly, tension cut off zone that also we have understood, we know that nothing is going to be there towards the left end. Of course in normally consolidated in vp' plot we have already seen that it is the right most boundary, so in that sense that is also a state boundary surface. And yield curve, specific yield curve, when I say yield curve not all the yield curves because yield curve can expand in it is size, is the final yield curve where I told already which passes through the point of intersection of effective stress path meeting with the critical state line.

These are all fragments of SBS. Now we will see, we will put all these facts together and finally understand what is a kind of state boundary surface in 2D as well as in 3D variation or the framework? Mathematical representation is needed for SBS then only it will be useful for engineers. This is possible, now what is possible? Mathematical representation is possible by fitting equations to experimental data developing purely a theoretical framework based on mechanics or the combination of 2.

Now the biggest challenge is how good the framework represent the real soil behavior? Now this is asking for more, like we are trying to mathematically quantify something which is beyond our imagination and that soil behaviour. We have to admit the fact that in spite of all our knowledge and development still there is a gap in understanding the actual soil behaviour. So, a particular model which may work very well for a specific type of soil and situations may not work for another type of soil in the same situation.

Or for a different situation it may not work for the same soil, so there are different possibilities. So, we cannot generalize a particular model, but how close we are not looking for a 100% matching, how close the given model is with respect to the actual or the real soil behaviour? That is what the biggest challenge is. So, there is always a tradeoff between simplicity which is essentially needed for computational ease and adequacy, how could it represent?

So, there is always a we need to balance this, if we want to clearly model every bit of aspect of the soil that is going to be computationally highly intensive. And that may not be really advantageous from a design perspective where there are equally good amount of uncertainties otherwise also. So, we need to strike a balance between how simple our model is vis-a-vis how good it represents? So, that is why simplicity and adequacy.

One of the basic constitutive models based on critical state framework is the Cam Clay model in short CCM. So, we will now try to understand the critical state model, Cam Clay model which is the primary development, this is the original Cam Clay model. And then it was modified a bit and that is known as modified Cam Clay which we will see in the next lecture.

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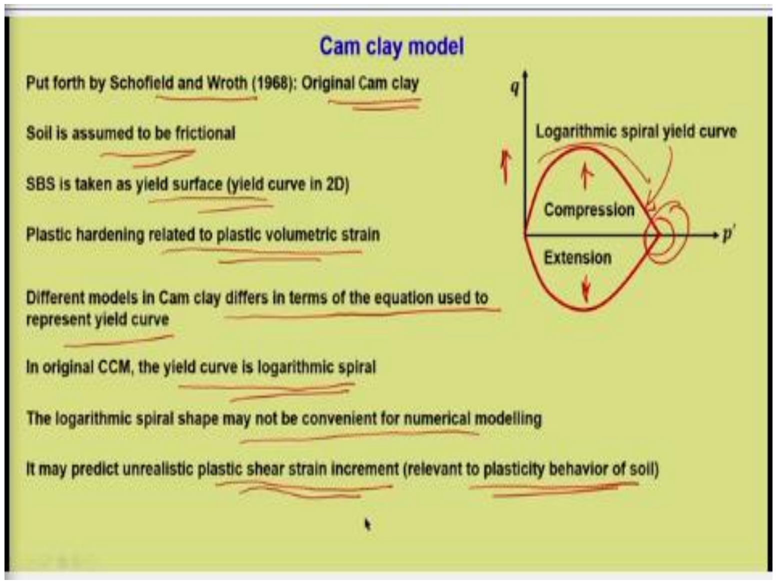

So, let us start with Cam Clay, understanding Cam Clay model all the understanding which we have till now related to 2D representation of critical state and soil yielding becomes handy in understanding the Cam Clay model. Without knowing that it becomes difficult to appreciate what is Cam Clay model. So, that is why we have grown or we have dealt the lectures in steps, adding one information one after the other.

So, I would strongly recommend that you have to go through the previous lectures before starting understanding this Cam Clay model. This was put forth by Schofield and Wroth in 1968 which is the original Cam Clay. Soil is assumed to be frictional, essentially it is assumed to be frictional. Soil state boundary surface is taken as the yield surface or the yield curve in two dimension, it is very easy for us to understand this particular concept.

It is a bound which we have seen and that can be used for defining the boundary. Plastic hardening related to plastic volumetric strain. Now I would be a bit cautious in using these terminologies in this particular course. The very reason is we are not discussing anything important or in depth about the plasticity behaviour of soil. So, here plastic hardening whatever the expansion of the yield curve is basically related to the plastic volumetric strain.

You remember the v versus p' plot where the normally consolidated line it moves along the normally consolidated line. So, every time it moves we have seen in the last lecture the size of the yield curve would increase. So, basically the plastic hardening is related to plastic volumetric strain. Now there is another kind of strain that is going to take place that is shear strain during loading, so that part is also there, unfortunately we will not be able to discuss those in detail.

Different models in Cam Clay differs in terms of the equation used to represent the yield curve. So, basically the difference between different Cam Clay models in the same Cam Clay family is the kind of yield curve that is used. Otherwise the kind of critical state concepts that we use in all these models are pretty same. In original Cam Clay, the yield curve is logarithmic spiral because of the closeness in whatever we have seen the kind of boundary that we have seen logarithmic spiral happens to be closer.

And that is why for Cam Clay model that shape is adopted as the yield curve. So, it looks something like this, it is in q p' plot the yield curve is shown in this manner, so this is the yield curve. Now this is the logarithmic spiral yield curve, now for completeness we have shown for compression as well as for extension. But in our subsequent lectures we will be basically concentrating on the compression portion.

So, that is one half of the yield curve, in fact the yield curve is mirror image on both sides in compression as well as in extension. So, we will be basically dealing with positive value of q. The logarithmic spiral shape may not be convenient for numerical modelling; you can see this particular quotient wherein there is a pointed edge here. Now for all numerical modelling there has we need to ensure the continuity of the differential equation which is used.

So, basically here this particular location it creates problem in some of the numerical models and in the stability of the algorithm. So, for that is the reason why the modification has been given, but to understand the basics of Cam Clay we need to understand this particular model, original Cam Clay also. So, it may predict unrealistic plastic shear strain increment, now again I would like to use this particular word with caution.

Because we are not discussing this particular term the plastic shear strain increment, it is basically relevant to plasticity behaviour of soil. So, there is an issue that it can predict because of this particular model can predict unrealistic plastic shear strain increment.

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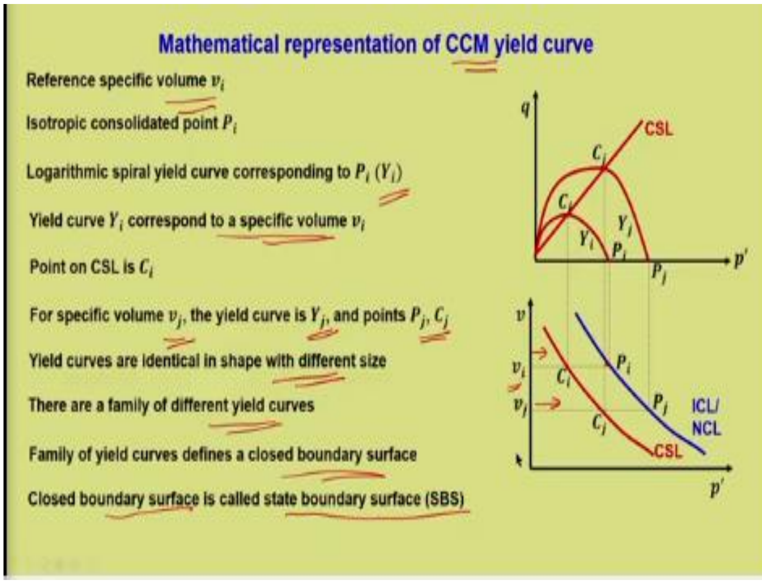

So, let us understand what is the mathematical representation of Cam Clay model yield curve. Let us consider a specific volume of v_i, so this is q p' plot and this is vp' plot. Now let us consider the reference v_i, so this is the v_i corresponding to which there is an initial consolidation point which is P_i on the ICL, NCL. Now this can be either isotropically or it is one dimensional consolidated point P_i for which the specific volume is v_i.

Now let us try to understand it in q p' plot, now this is the initial point Pi, logarithmic spiral curve corresponding to this particular point. Now we have already seen in the yielding lecture that the extreme point is given by this preconsolidation pressure. Now for this particular case it is P_i, so the extreme point of the yield curve Y_i will be Pi. So, we need to fit a logarithmic spiral Y_i which is the yield curve corresponding to P_i.

Now yield curve Y_i correspond to a specific volume vi, now we need to understand that this particular yield curve is specific to this particular specific volume vi. Now as the specific volume changes which means P is going to change, so let us say if it is above or below the kind of P_i will also change. Now when P_i changed that means that the yield curve also changes, so now for the time being we need to understand that a particular yield curve is associated with a given specific volume.

Or we can say that this particular yield curve represents a kind of undrained behaviour, what is undrained behaviour? Where v is not changing, so it is both ways true, if I say that yield curve corresponds to a given specific volume or this particular yield curve represents a kind of undrained behaviour, both explanation is same. Point on critical state line is C_i, so let us say for this particular v_i the point on the critical state line is C_i.

Now when you transfer this how will it look like? Now this part is very important, now when we transfer this particular point it looks like this. That is the point where the critical state line meets the top portion of the yield curve, top portion means the extreme top portion, so that is C_i. So, this relationship one should keep in mind. Let us consider another specific volume of v_j which is lesser than v_i, so this is v_j.

Now the yield curve corresponding to this will be y j and the points will be P_j and C_j, so this is P_j that is taken to the q p' plot and this is C_j. So, that C_j will lie on the critical state line and the kind of yield curve will be like this. Now yield curves are identical in shape with different size. The shape of these 2 yield curves are very close to each other but the shape differs depending upon whether it is v_i or v_j.

So, it is apparent that there will be a family of different yield curves. Family of yield curves defines a closed boundary surface. Now this how does a closed boundary surface look like, I am not showing it here we will discuss when we discuss about the state boundary surface. We need to imagine that there will be different family of such yield curves leading to a complete boundary surface in a given three dimensional space.

This closed boundary surface is called the state boundary surface. Again we will see about state boundary surface bit later. Only when we understand this Cam Clay and the prediction clearly we will understand the state boundary surface.

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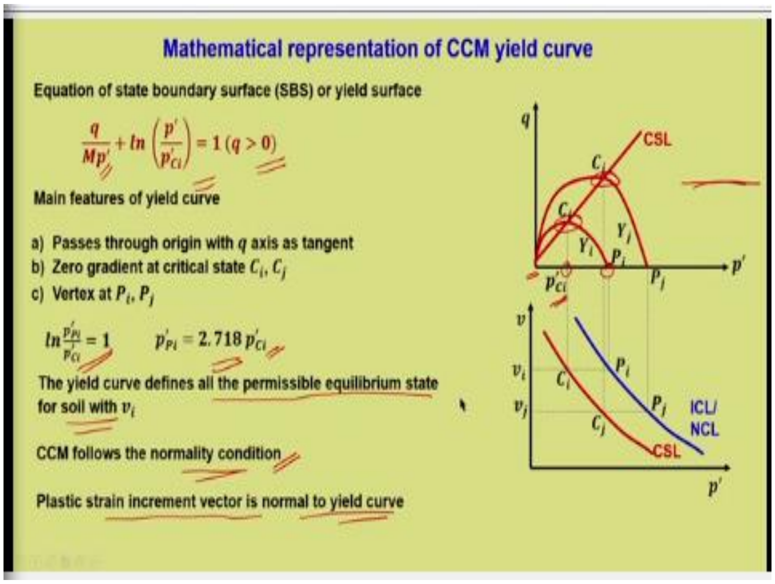

So, we have understood the basic framework and now the equation of state boundary surface or yield surface, that is how do we represent this particular yield curve. The logarithmic spiral yield curve is given by this expression which is

$$\frac{q}{M_p'} + \ln\left(\frac{p'}{p'_{Ci}}\right) = 1\ (q > 1)$$

And here we are taking the condition q greater than 0 that means on the compression. Now q we know, M, M is the slope of critical state line. Which is again a critical state parameter, fundamental parameter, p' we know, ln p', p'_{ci}, what is p'_{ci}? P'_{ci} is the effective pressure corresponding to this particular point c_i, so it will be somewhere here. So, that is also known, so whatever is needed is known from this particular equation equal to 1 and for the condition q greater than 0, so this is p'_{ci}.

Now the main features of yield curve that it should pass through origin where q axis is the tangent at that particular point. So, that is clearly visible here, you remember when we discussed about the last few lectures, we discussed that it is better that the failure envelope passes through the origin because every time we cannot ensure the existence of cohesion. So, that is why in this particular framework we have already seen that it is purely taken as a frictional material.

So, it is passes through the origin with q axis as tangent, zero gradient at critical state. Now at this particular location C_i and C_j what it means is that there is a zero gradient. If I draw a tangent at the top most portion, it is horizontal, so it is zero gradient at critical states at this point C_i and C_j. Vertex is at P_i and P_j and that happens to be the extreme point of the yield curve on the right side.

It also follows this important relationship that is

$$\ln \frac{p'_{pi}}{p'_{Ci}} = 1$$

That is the relationship between this point the extreme point and this point of intersection, that is the mean stress. At this particular point and this particular point is related and what is the relationship? It is $\ln \frac{p'_{pi}}{p'_{Ci}} = 1$ or if you take the anti log it is $p'_{pi} = 2.718\, p'_{Ci}$. So, this is one of the attributes of this particular yield curve.

Similarly if it is p'_j it will $\ln \frac{p'_{pj}}{p'_{Cj}} = 1$, so that relationship also holds. Now how this relationship is coming? We will see in the subsequent slide. The yield curve defines all the permissible equilibrium state for soil with v_i, so all the possible condition is governed or it is taken care of by this yield curve. Now CCM follows the normality condition, the critical state model follows the normality condition, and what is normality condition?

Again this has to come from the discussion of plasticity behaviour of soil. Now if somebody is interested to know this more, I strongly advise you to go through the plasticity behaviour of soil in this specific textbook; especially you can refer to the textbook by Potts, so what is meant by this? It means that the plastic strain incremental vector is normal to yield curve, so in the last lecture also we have seen that it is perpendicular, it means that. Now if I start discussing this our designated portions is going to suffer, so I cannot get into the details of plastic behaviour.

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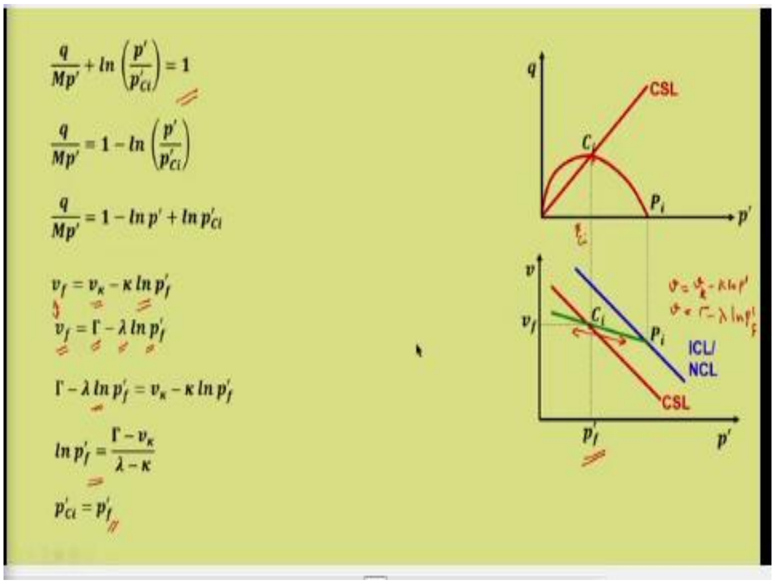

Now let us again start with I am referring to the same equation here $\left(\frac{q}{M'_p} + \ln\left(\frac{p'}{p'_{Ci}}\right) = 1\ (q > 1)\right)$

Let us start with this particular equation $\frac{q}{M'_p} + \ln\left(\frac{p'}{p'_{Ci}}\right) = 1$ So, we will start from here this is the yield curve and this is C_i P_i and this is marked. Now we have understood one part, now we are slowly adding another part here, what is that part? That is the unloading line or the elastic component line, where does it fall with respect to whatever we have discussed.

In the last slide we have not introduced this unloading line. Now you can see that P_i is here, this particular point C_i is nothing but the point of crossing where this unloading line is crossing the critical state line, I would not say that it is touching. It is only notional in this particular plot v p', it is the point where it is crossing, so this particular point corresponds to C_i, so what is the relationship? That is what we are trying to understand.

So, when we understand Cam Clay model we need to keep this these aspects in mind, so then only it will work. So, C_i is the point where the yield curve intersects the critical state line and that is where what it is shown here. Now here you need to also see that this particular line the unloading line represents elasticity, the yield curve also defines the domain of elasticity. So, that is the point here the crossing point is C_i.

Now,

$$\frac{q}{M_p'} = 1 - \ln\left(\frac{p'}{p_{Ci}'}\right),$$

$$\frac{q}{M_p'} = 1 - \ln p' + \ln p_{Ci}'$$

So, here you can also designate this particular point as v_f, why? Because that is the point where the effective stress path would meet and this any point on the critical state line can be considered as the failure point or VCS both are same. If we consider v_f, we understand that it is the failure point and that corresponds to VCS as well.

So, now this is the point that C_i is a point on the critical state line, so designating it as v_f holds good. It also mean it is p'f, now p' f is same as p'cs. Now from this we can always write

$$v_f = v_k - \kappa \, ln \, p_f'$$

This is the point and that point lies on this particular line as well here. Now how do we represent this particular loading, unloading line again refer back to our critical state discussion wherein we can write this particular line as v.

In general any point on this loading, unloading line can be written as $v_f = v_k - \kappa \, ln \, p_f'$. We can also write the same point because it is lying on the critical state line, now how do we represent critical state line. Now here we know that it is always p_f' because it is lying on the critical state line. So, similarly for this particular point can also be designated as

$$v_f = \Gamma - \kappa \, ln \, p_f'$$

Now we can see this both represents v f, we can equate it.

$$\Gamma - \kappa \, ln \, p_f' = v_k - \kappa \, ln \, p_f'$$

Now if you rearrange it,

$$\ln p_f' = \frac{\Gamma - v_k}{\lambda - k}$$

So, we got an expression for ln p_f'. Now we know that $p'_{Ci} = p'_f$ the same point.

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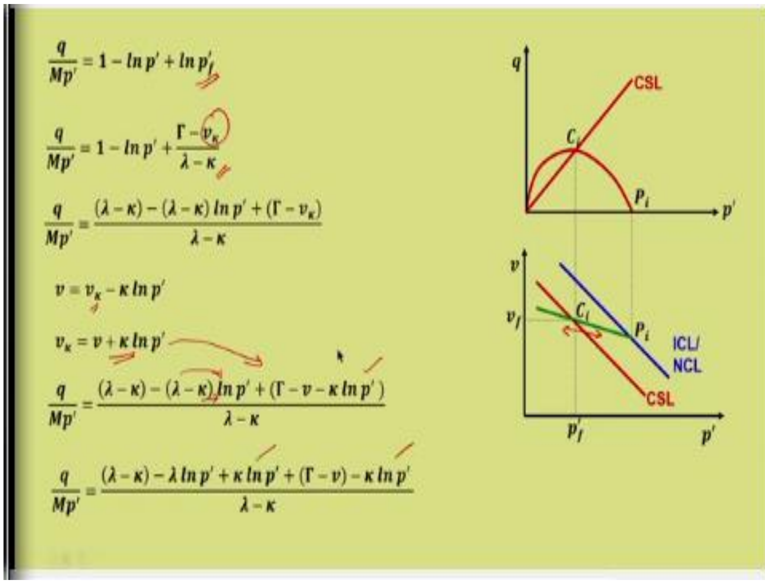

So, now we have got an expression for $ln\, p'_f$, substituting that here we can write

$$\frac{q}{M_p'} = 1 - \ln p' + \ln p_f'$$

Now what are we doing? We know one expression which represents the yield curve; we are just trying to represent the same equation in a more general form involving the critical state parameters.

$$\frac{q}{M_p'} = 1 - \ln p' + \frac{(\Gamma - v_k)}{\lambda - k}$$

Now v_k again we know v kappa is not a fixed parameter, it depends upon where the unloading has been done, again we have discussed this point. So, we need to replace v_k, so how do we replace v kappa? Now we know that is

$$\frac{q}{M_p'} = \frac{(\lambda - k) - (\lambda - k)lnp' + (\Gamma - v_k)}{\lambda - k}$$

Now we need to replace this $v = v_k - kln\, p'$ again the equation for this particular loading-unloading line.

From which we can write $v_k = v + kln\, p'$. So, we can substitute for v_k .

$$\frac{q}{M_p'} = \frac{(\lambda - k) - (\lambda - k)lnp' + (\Gamma - v - kln\, p')}{\lambda - k}$$

So, we are substituting this to here. So, then if you expand it,

$$\frac{q}{M_p'} = \frac{(\lambda - k) - \lambda lnp' + kln\,p' + (\Gamma - v) - kln\,p'}{\lambda - k}$$

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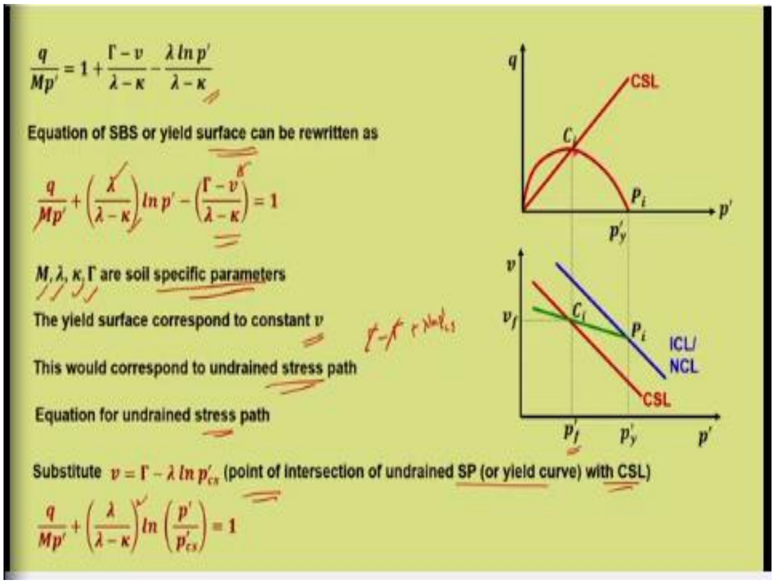

So, after simplifying we can write

$$\frac{q}{M_p'} = 1 + \frac{(\Gamma - v_k)}{\lambda - k} + \frac{\lambda \ln p'}{\lambda - k}$$

So, equation of state boundary surface in terms of all the critical state parameters can be rewritten as

$$\boldsymbol{\frac{q}{M_p'} + \left(\frac{\lambda}{\lambda - k}\right) ln\,p' - \frac{\Gamma - v_k}{\lambda - k} = 1}$$

So, this is the equation for the yield curve which is represented by original Cam Clay model.

And you can see that lambda, kappa, M these are all the critical state fundamental parameters of soil, so what we have done? We have defined the yield curve in terms of fundamental soil parameters which is coming from the critical state framework. So, this is one part of the story, and in this particular course we will be discussing only this part of the story, what is the other part of the story?

The other part of the story deals with the plastics straining, where we need to define how the plastics straining happens and how it propagates. So, that particular aspect we are not discussing in this course. So, here $M, \lambda, \kappa\ and\ \Gamma$ are soil specific parameters. The yield surface correspond to constant specific volume, we have already discussed that. This would correspond to undrained stress path, why?

Because it is corresponding to a constant specific volume, so it is as good as telling it corresponds to a undrained stress path. Now equation for undrained stress path, we can also derive substitute for $v = \Gamma - \lambda \ln p'_{CS}$ which is the point of intersection of undrained stress path or yield curve with critical state line. Now if this is the particular point, now if that is considered as $p'_f\ or\ p'_{cs}$.

And for this particular v, we substitute this equation $v = \Gamma - \lambda \ln p'_{CS}$, what we will get? So, we get

$$\frac{q}{M'_p} + \left(\frac{\lambda}{\lambda - k}\right) \ln\left(\frac{p'}{p'_{CS}}\right) = 1$$

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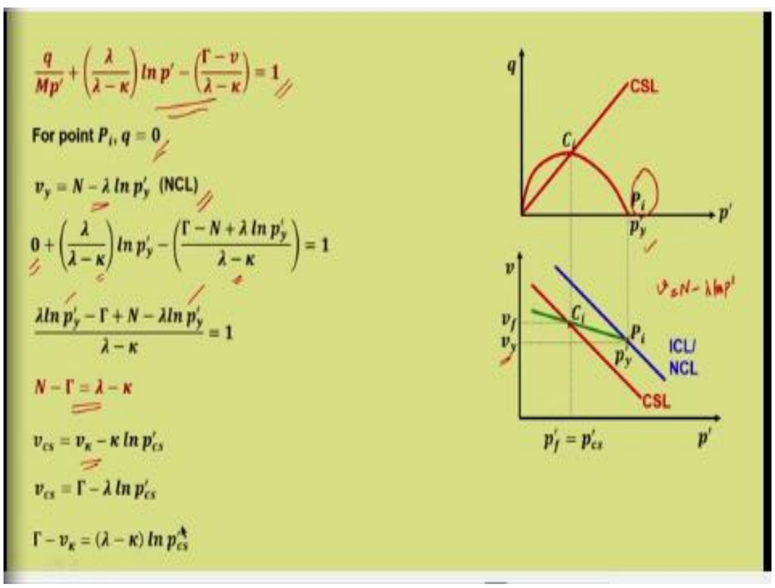

Now considering this to be yield stress (p'_y), we will start from the same equation

$$\frac{q}{M'_p} + \left(\frac{\lambda}{\lambda - k}\right) ln\, p' - \frac{\Gamma - v_k}{\lambda - k} = 1$$

Because we need to find out the relationship between $p'_y\ and\ p'_{Ci}$. So, for point P_i, so this particular point P_i q = 0, at this particular point q = 0, also if you see here this is the same point if you consider here p'_y, this is considered to be v_y corresponding to that particular yield.

We can write

$$v_y = N - \lambda \ln p_y' \ (NCL)$$

what is N? N is the point on the normally consolidated line corresponding to unit pressure which we have already seen. So, specifically for v_y we can write so

$$v = N - \lambda \ln p'$$

Now for this particular point we can write $N - \lambda \ln p_y'$, so that is what is written here from the normally consolidated line. Substituting this in the state boundary surface equation or the yield curve equation q is 0,

$$0 + \left(\frac{\lambda}{\lambda - k}\right) ln\ p' - \frac{\Gamma - N - \lambda \ln p_y'}{\lambda - k} = 1$$

Now rearranging, the denominators are same, so we can write

$$\frac{\lambda \ln p_y' - \Gamma + N - \lambda \ln p_y'}{\lambda - k} = 1$$

We can write

$$N - \Gamma = \lambda - k$$

So this relationship comes from here.

Now for the critical state point, this particular point we can write

$$v_{CS} = v_k - k \ln p_{CS}'$$

point from the loading, unloading line. From the critical state line the same point can be represented as

$$v_{CS} = \Gamma - \lambda \ln p_{CS}'$$

Now substituting these two,

$$\Gamma - v_k = (\lambda - k)\ ln\ p_{CS}$$

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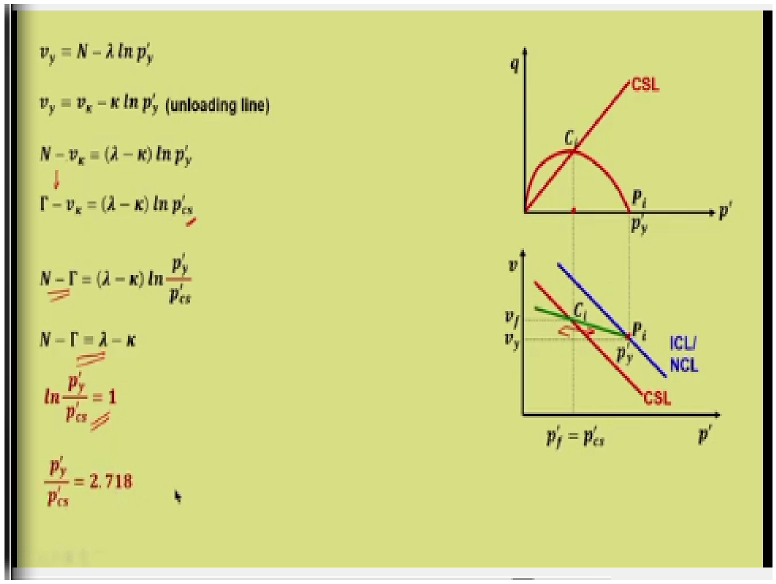

So,

$$v_y = N - \lambda \ln p'_y$$

equation from the normally consolidated line, v_y can also be written from this particular loading, unloading line.

$$v_y = N - k \ln p'_y \ \ (unloading\ line)$$

Substituting these 2 we can write

$$N - v_k = (\lambda - k) \ln p'_{CS}$$

Now from the previous slide considering the critical state point we can write

$$\Gamma - v_k = (\lambda - k) \ln p'_{CS}$$

Now $(N - v_k) - (\Gamma - v_k)$ will give

$$N - \Gamma = (\lambda - k) \ ln \ \frac{p'}{p'_{CS}}.$$

But we already have derived this particular relationship where $N - \Gamma = \lambda - k$ substituting that we can write

$$ln \ \frac{p'}{p'_{CS}} = 1$$

this is where we started off with that this particular relation holds good. So, that we have just now proved that this particular point and this particular point on the p' axis is related by this expression $ln \frac{p'}{p'_{CS}}$, that is the attribute of this particular yield curve and

$$\frac{p'}{p'_{CS}} = 2.718$$

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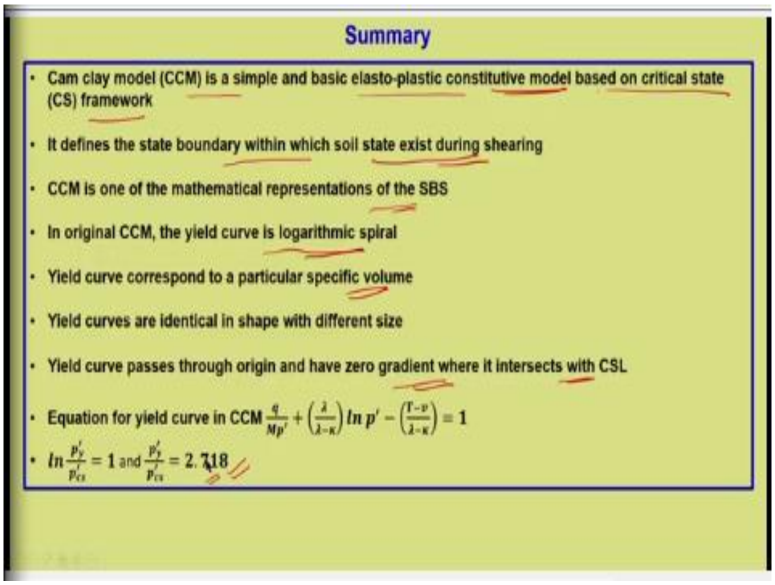

So, that is all some very basic information related to critical state framework in Cam Clay model**.** So, the one part is what we have discussed now, let us try to summarize this particular portion. We need to understand that this type of understanding we need to develop in steps. So, what we are exactly doing is that adding one by one. So, here for this particular course we need to stop here our level of understanding.

Now the next level of understanding for you will be in terms of plasticity behaviour of soil, where both the complete constitutive relationship becomes important. And for which the stress strain in both elastic and plastic framework need to be understood. So, let us summarize today's lecture. The Cam Clay model CCM is a simple and basic elastoplastic constitutive model based on critical state framework.

Now again I would like to use a caution while telling elastoplastic constitutive model, we have not completed the entire constitutive model. We can say it is constitutive model when there is a proper relationship between stress and strain**.** So, we have discussed the partial framework right now. It defines the state boundary, now here our entire effort is in understanding the state boundary surface.

And that is fully justified in this type of discussion, and that you will see that whatever we have learned just now we will understand that in terms of three dimensional state boundary surface. So, that part is completely dealt in this particular course in which the soil exists during shearing. CCM is one of the mathematical representations of the state boundary surface; there are different ways by which it is done now this Cam Clay model is one of such representations.

In original Cam Clay the yield curve is represented by logarithmic spiral, yield curve correspond to a particular specific volume when the specific volume change the size of the yield curve also changes but the shape essentially remains the same. So, yield curves are identical in shape but with different size, size changes with specific volume. Yield curve passes through origin and have zero gradient where it intersects with the critical state line and that is at the top most portion.

The equation of yield curve in Cam Clay model is

$$\frac{q}{M_p'} + \left(\frac{\lambda}{\lambda - k}\right) ln\, p' - \frac{\Gamma - v_k}{\lambda - k} = 1$$

And it also holds this particular relationship,

$$ln\, \frac{p'}{p_{cs}'} = 1, \qquad \frac{p'}{p_{cs}'} = 2.718$$

So, that is all for this particular lecture, we will see the modification of this original Cam Clay in terms of modified Cam Clay framework in the next lecture, thank you.