Advanced Soil Mechanics

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Lecture-48
Soil Yielding

Welcome back all of you, in the last lecture we have seen peak state. We have discussed various aspects related to peak state and which is relevant for critical state soil mechanics in terms of the boundary. Now we have discussed shear strength in module 2, stress path in module 3, and our essential focus is on the mechanism of failure and to define a more unique failure concept. And that is where we reached the fourth module where we talked about critical state soil mechanics.

And we have already understood the various aspects of critical state soil mechanics, the introduction part. And we have also discussed the various boundaries the right most normally consolidated or isotropic consolidated case at the top peak and to the left tension cut off zone we have discussed. Now you will see that, adding one more very essential information to these will enhance our understanding of soil in shearing, and that additional understanding is soil yielding.

There are certain aspects which remain hidden in module 2 as well as in module 3. Now a complete explanation is possible when we define what is known as soil yielding and the limiting surface or the limiting curve in two dimensional space given by the yielding. So, that is what we will be seeing in today's lecture.

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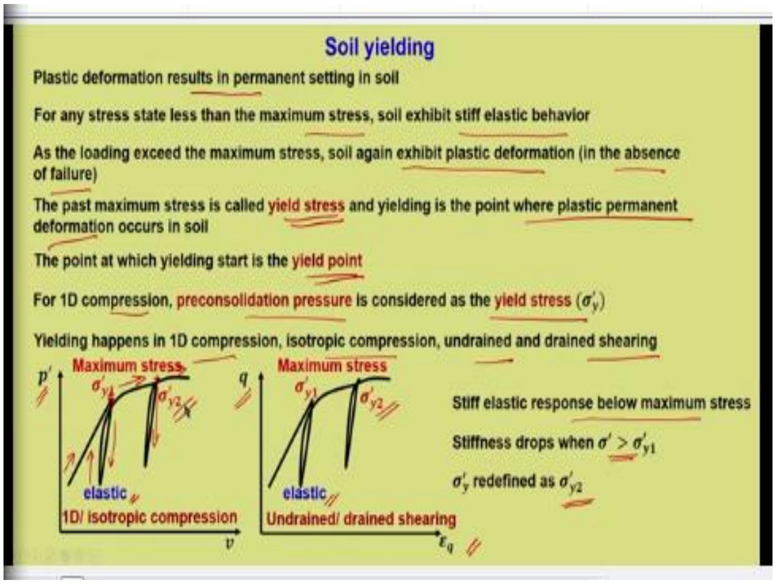

So, we will start with soil yielding. The plastic deformation results in permanent setting in soil, all of us know this; this has been discussed in this particular course as well. Like after elastic the soil yields and then it undergoes softening or hardening behaviour depending upon the stress history and then it undergoes some plastic setting or permanent setting because of plastic deformation.

For any stress state which is less than the maximum stress, soil exhibits stiff elastic behaviour, this also we have seen and that is where the concept of stress history comes into picture. A soil is already subjected to a maximum stress in the past. And any stress below that maximum stress soil will behave in a stiff elastic manner. As the loading exceed the maximum stress soil will again exhibit plastic deformation in the absence of failure.

Like when we start loading this is not a continuous phenomena, it will fail at some point of time depending upon the nature of stress application. So, what it means is that it keeps yielding when it crosses it is maximum past stress, and it continues till the point of failure. So, the yielding happens if the soil has not failed. The past maximum stress is called yield stress, now this is what is of importance in today's lecture.

And yielding is the point where plastic permanent deformation occurs in soil, well this is not very difficult to understand. The point at which the yielding start is called the yield point, so we are just understanding specific terminologies yield stress, plastic permanent deformation then yield point. For one dimensional compression which is basically coming from the oedometer response, the preconsolidation pressure is considered as yield stress, σ'_y.

All these are repetition of information which we have already seen in the past lectures. Yielding happens in one dimensional compression, isotropic compression, undrained and drained shearing. So, there are 2 aspects which we need to distinguish here, the first one is yielding and the other one is failure. Yielding does not mean failure has occurred, that is the reason whether it is a one dimensional compression or isotropic compression yielding would happen.

But that may not result in failure of the soil mass, for failure we basically talk in terms of sharing. So, drained, undrained sharing also bring about failure, in all these cases soil will yield and that we have seen, it is a very good example is oedometer test result you load, unload, then again load at the pre-consolidation pressure it will again yield. So, that yielding means high deformation that is on the NC curve again, so that is what it means.

So, yielding is a general phenomena, but it need not mean that the soil has failed. Now let us try to understand this, this is a typical one dimensional or isotropic compression. So, which is talked in terms of volumetric stress p', so this is loading and from this point there is unloading, then again reloading. And after this particular point we can see that the plastic deformation is quite high, and v, the change in v from this point to this point is drastic again loading, unloading.

And once it crosses, now from here this point to this point what has happened? The maximum stress to which the soil has been subjected to is changing from this point to this point. And then for all the stress range which is less than this will be stiff elastic, and once it crosses due to again reloading once it crosses then we will see that again the plastic deformation happens. Now this is in terms of drained and undrained shearing where we talk about deviatoric stress versus shear strain.

And then is the same response as that of the p' versus v response. So, the yielding concept remains same, stiff elastic response below maximum stress, we have already seen that. Now in the first case, this is the maximum stress where we call it as the yield point σ' $_{y1}$ or σ' $_{y1}$ is the yield stress, now you can see that this limp corresponds to elastic behaviour. Stiffness drops once due to reloading σ' increases beyond σ' $_{y1}$.

In the next loop when it is reloaded and when it exceeds σ' $_{y1}$, then the stiffness of the soil or the elastic response drastically varies and it changes to plastic response. σ' y defined as σ' $_{y2}$, now it is the yield point σ' $_{y1}$ is redefined as σ' $_{y2}$, now this becomes the new yield point for the soil.

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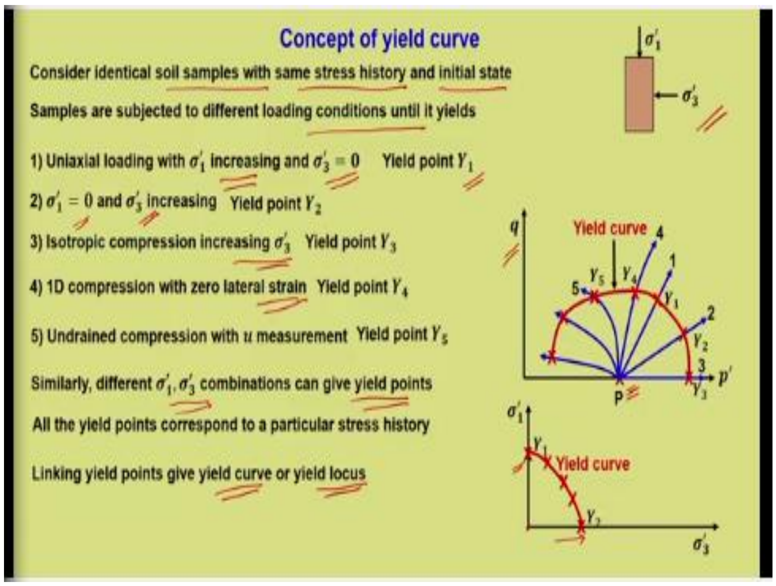

So, now let us see a bit on the concept of yield curve, because this yield curve concept is important for defining the critical state framework. Let us consider identical soil samples with same stress history and initial state. So, we have the same stress history and we have same initial state. Samples are subjected to different loading conditions until it yields. Let us consider a triaxial condition σ'$_1$and σ'$_3$.

Let us first consider a uniaxial loading wherein σ'$_1$ is increased, and σ'$_3$ is remaining constant, so it is a uniaxial loading. Let it yield at point y $_1$, now this can be represented in q p' plot, P is the point that is the identical point with same stress history and initial state, it is also shown on σ'$_1$ versus σ'$_3$ plot. Now here it is very easy to mark because σ'$_3$ is 0 that means this particular point.

And σ'_1 is increasing means it is going along the y axis and at some σ'_1 the soil would yield, so that is the yield point y_1. Now if you compute this for q and p' we will also see that it is corresponding to the first one and then it is y_1. So, y_1 is the point at which this particular soil yields. Now let us take another case, where $\sigma'_1 = 0$ and σ'_3 is increasing, so what will happen σ'_1 is 0.

So, here this point is 0 and σ'_3 is increasing towards this and yield point is y_2, so let this be y_2 on σ'_3 axis. The same thing you can see in q p' plot let this be y_2. Another case is isotropic compression increasing σ'_3, now one may always wonder like isotropic compression is not going to lead to any failure, but remember it can yield. So, isotropic compression increasing σ'_3 let us consider it to be yield point y_3.

So, this is isotropic compression, now please note that in isotropic compression there is no q, there is no shear stress developed, so that is why it is along the x axis or p' axis, and now let that be y_3. Let there be one dimensional compression with zero lateral strain, now this is a typical case of oedometer where there is no lateral strain because it is infinite rigidity offered by this steel ring, so it is with zero lateral strain, yield point y_4, now let that be y_4.

Then we have undrained compression with u measurement, yield point y_5, so this is y_5. Now we have discussed at least five cases, now there are different such cases where we can load the soil by giving different σ'_1, σ'_3 combinations, and all of them will yield at some point of time. So, that we can show it as different points, so these are different ways of stress path, applications based on different combinations of σ'_1, σ'_3 and it yields.

Now if we join all these yield points, all the yield points correspond to a particular stress history, we have to keep in mind it is the starting point is P and it corresponds to the same stress history and initial state. Linking yield points give yield curve or yield locus, so let us join all these points and that will give us the bound within which the soil mass remains elastic in nature, so this is the yield curve.

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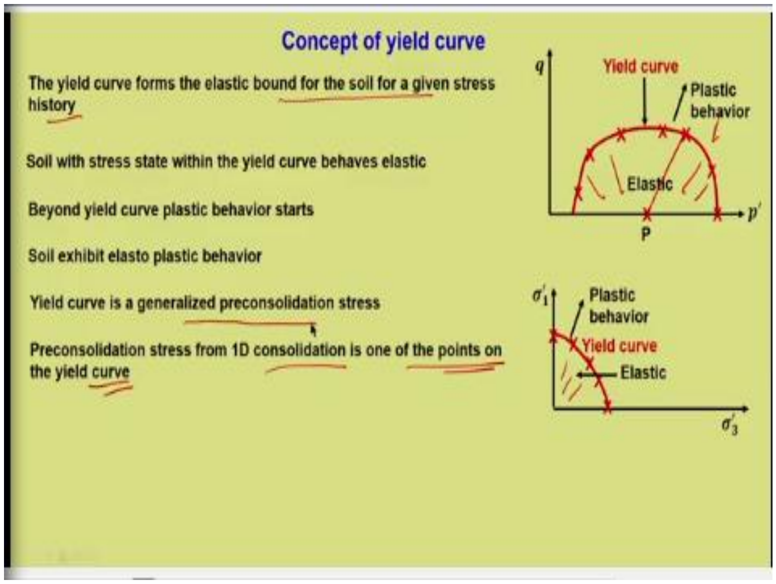

So, that is what the basic concept of yielding, yield point, yield curve is all about. Now it is in two dimensional spaces it is yield curve the same thing becomes yield surface when it is in three dimensional representations. The yield curve forms the elastic bound for the soil for a given stress history. Now this particular yield curve corresponds to a given stress history, now for that whatever is the domain which is shown here, now that corresponds to elastic domain.

Soil with stress state within the yield curve behaves elastic, so both are elastic. So, within this yield curve is a very important information that within the yield curve the soil state is completely elastic, and for both the domains it has been shown here. Beyond yield curve, now once the let us say the stress path reaches here and once it exceeds the yield curve then the plastic behaviour starts. Soil exhibit elastoplastic behaviour, so beyond that it is plastic behaviour.

Yield curve, it is a generalized preconsolidation stress, what does that mean because in one dimensional compression the extreme point corresponds to yield stress or preconsolidation stress. Now here the yield curve is more like a generalized preconsolidation stress, so whichever is it does not correspond to one dimensional consolidation alone but it encompasses different ways of yielding for the same soil and with same stress history.

Now preconsolidation stress from one dimensional consolidation is one of the points on the yield curve, so that is why it is called generalized preconsolidation.

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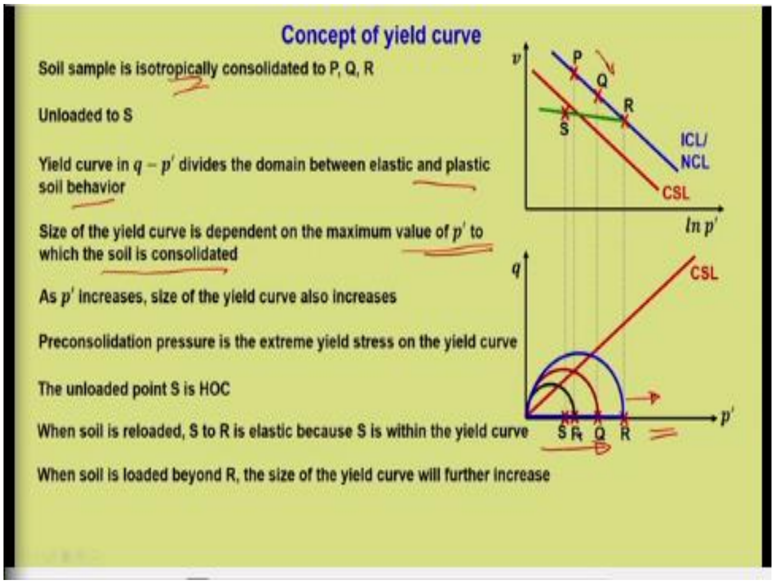

Now let us say that the soil sample is isotropically consolidated to P, Q and R, this is v ln p', this is the ICL, NCL line and CSL is also shown, let P, Q, R be isotropically consolidated points. From R the soil sample is unloaded to S, so this is unloaded to S. Now the same is shown P, Q, R on, so this is on the x axis because it is isotropically consolidated. So, all the points are there on the p' axis.

Now yield curve in q p' divides the domain between elastic and plastic soil behaviour, you may see that some of these facts are getting repeated but that is important. Now we are slowly getting into the concept of yield curve, and probably you will be able to appreciate why yield curve is needed for understanding the shear behaviour of the soil better. Now this for point P, for point P the maximum stress that it has been subjected to is σ' or p'_p.

So, this is the maximum point and hence the yield curve the extreme point is defined by the maximum stress. So, this is the yield curve which corresponds to point P, why because this should be the extreme point of the yield curve as well, for one dimensional compression. Now you may be wondering that how did I choose a kind of elliptical curve? Now there are different shapes and it need not be exactly elliptical.

Now you will see in the subsequent lectures later that the assumption has been made related to the shape of yield curve, it is more like an idealization. It is close to but may not be exact, so here we are just showing it as an example. So, here we are assuming it more or less close to an elliptical behaviour because it is easy to understand. Now as we load from p to q what is happening to yield curve?

As we load from p to q the yield curve increases or the yield curve gets expanded, so this is the fact which we have to keep in mind. And when we load it to R, it again increases, now the yield curve has changed from the black one to brown, and now the yield curve is now blue, because the maximum stress has now becomes R. Size of the yield curve is dependent on the maximum value of p' to which the soil is consolidated.

Obviously there is an additional condition which is needed here provided the soil has not failed, so beyond failure there is no concept of yielding. As p' increases, size of the yield curve also increases which is very well shown in this figure. Preconsolidation pressure is the extreme yield stress on the yield curve, so that particular aspect also we need to keep in mind. In all the cases the preconsolidation pressure is the extreme point of the yield curve.

The unloaded point S is heavily over consolidated, here this particular point, now from here the unloading has taken place. When soil is reloaded from S to R, from here if it is reloaded we know that from S to R is elastic; S is within the yield curve**.** So, from here to here, from S to R it is completely elastic. When soil is loaded beyond R the size of the yield curve will further increase. Now let us say that after unloading to S, reloading to R and further it is again increase then what is going to happen is yield curve further increases in it is size.

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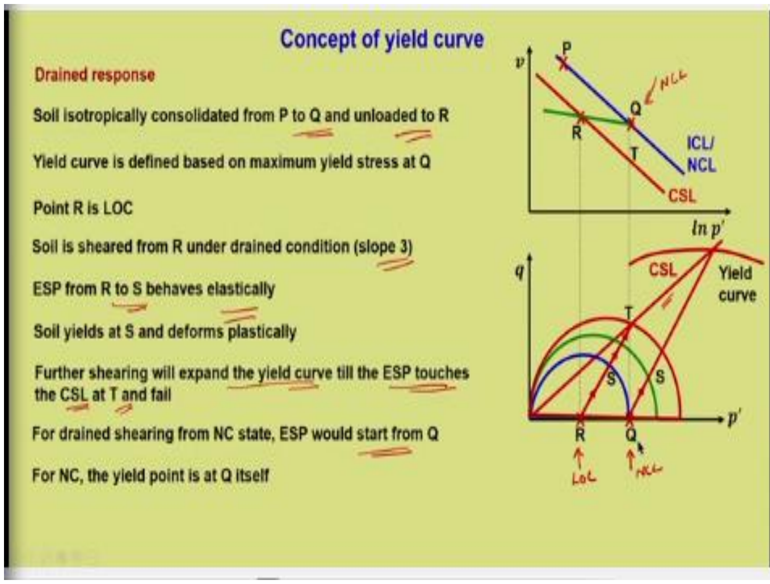

Now let us try to add the drained response and try to understand in with respect to the yield curve domain. Soil is first isotropically consolidated from p to q and then unloaded to R, let us see how it is, v ln p' plot, q p' plot from P it is isotropically consolidated to point q and then it is unloaded to point R. So, now this particular point R, do you have any kind of understanding like what will be the state of point R?

R will be close to a lightly over consolidated soil, because we have defined critical over consolidation ratio in the previous lecture. Above which or towards the right it is always lightly over consolidated, and we have also seen that the critical state line is in between COCR and the normally consolidated line. So, here point R we can confidently say that this is a lightly over consolidated point.

So, this is the extreme point of maximum stress application and hence the yield curve also corresponds to the maximum or the extreme point corresponds to p' q and R is the lightly over consolidated point. Now yield curve is defined based on the maximum yield stress at q, point R is lightly over consolidated. Now soil is sheared from R under drained condition, now remember in the previous slide we have not sheared it.

We have again followed the unloading, reloading line, so mostly in terms of one dimensional compression in oedometer. But here we are specifically shearing it under drained condition. And in q p’ plot, please refer back to stress path the slope of the effective stress path under drained condition is 3, so that is what is written here, just for reminding you. So, from point R, now the stress path will move at a slope of 3.

Now this is at inclination 3 with slope 3 and R S is the ESP. Now ESP from R to S it behaves elastically. So, please remember our discussion on stress path in our previous lectures, we never talked about yielding at that point of time. We know the starting point R, we know what is the ending point and the inclination or the slope is known at 3, so if you just plot a line it will go and meet the critical state line, it has to go and meet the critical state line.

Now in that case we never talked about yield, so yielding was more or less hidden in our explanation, even in the module 2 also we never talked about yielding. So, this lecture on yielding is an additional information in shear behaviour of soil. So, we need to understand that for any sort of failure, the first thing is the effective stress path has to touch the yield curve, please underline this fact because that is important in understanding the behaviour.

So, from R to S when the soil is sheared R to S exhibits elastic behaviour because now we know what is the yield curve, so till that point it will behave elastically. Now that was not told in our previous lecture. Now at this point S, soil yields and then it deforms plastically. Further shearing, what will happen if you share further? Now the plastic deformation has started and further shearing will expand the yield curve till the ESP touches the critical state line at T and fail.

So, let us see what happens on further shearing. On further shearing you can see that it is with the same slope but it start propagating towards the critical state line. And once it crosses this particular yield then the yield curve is also increasing. So, it is expansion of yield curve that is happening. Now finally it touches the critical state line at point T for which the yield curve is defined in red colour.

Now is there any case beyond this? No, because the soil has already failed, so T is the final point, so T is marked here. For drained shearing from normally consolidated state ESP would start from q. Now we have discussed here lightly over consolidated, so this point is LOC, now what will be the situation? If the soil is normally consolidated, there is no difference the only aspect is Q is the normally consolidated point this is NCL point, so this is NCL point.

So, instead of starting from R the stress path will start from Q, the drained stress path. So, that is what is shown here which goes and fails at critical state line, and the failure yield curve is shown in this figure, I have not completed the whole because that is not needed here. For NC the yield point is at Q itself, now in the case of R which is lightly over consolidated point you can see that it will yield in the process of shearing.

So, during shearing it yields, but for normally consolidated point that is the yield point itself or the yield limit itself, the point Q itself is the yield point.

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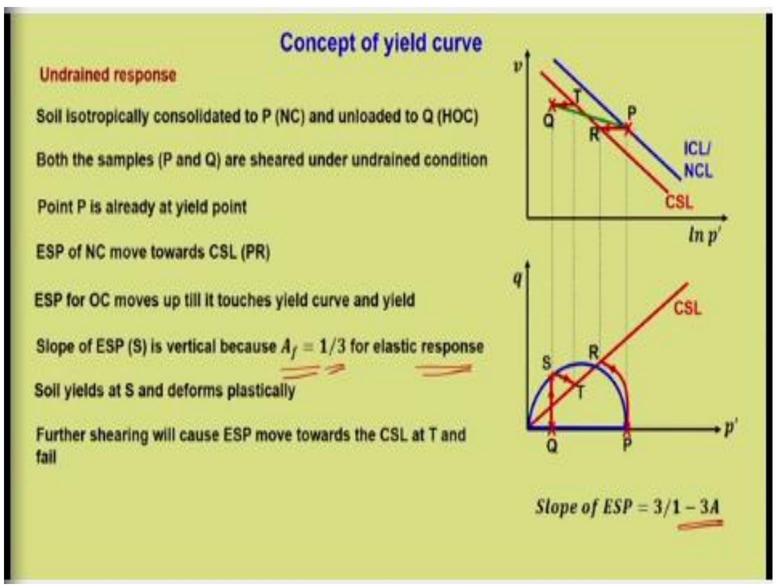

Now the same thing we will see for undrained response. Soil is isotropically consolidated to P that is normally consolidated, and then unloaded to Q which is a heavily over consolidated point. Let us say P and then it is unloaded to Q which is an HOC point, the same is shown in q p' plot and we know that this is the yield curve because that is the maximum pressure. Both the samples P and Q are sheared under undrained condition.

Now normally consolidated is also sheared, heavily over consolidated is also sheared, now let us see what happens. Point P is already at yield point, now this corresponds to yield stress, so this corresponds to the extreme yield point on the yield curve, P is already yielded, there is no need for further yielding. Now ESP of normally consolidated, we know that positive pore water pressure develops and hence it will move left towards the effective stress path of normally consolidated will move left wards for undrained shearing.

Please refer back if you have not followed this. We know that this is PR is the effective stress path and it goes and touches at the critical state line. Now that is what is shown in v ln p', this part we have already discussed, I do not want to spend more time, what is important for us is that the yield curve changes. Now ESP for OC, what happens for OC? It moves up till it touches the yield curve and yield, we are going to discuss this again.

But here for the point Q, what it means? Now this is not an yield point, it has to yield during the process of shearing. So, now the question is how we have to draw the effective stress path? Now before discussing let me ask you, like this particular point Q whether it is plastic or elastic? We all know from our previous discussion that it will be elastic in nature, now if it is elastic it is effective stress path; we know that it is governed by the slope in q p'; it is 3 divided / 1 - 3 A**.**

So, if it is 1 - 3A and the value of A = 1 / 3 for elastic response, we know now the effective stress path correspond to elastic because it has not yielded. So, A value is equal to 1 / 3 which is shown here, if you substitute that in the slope of ESP which is 3 divided / 1 - 3A, if you substitute 1 / 3 here it becomes infinity. So, that is, it means that it moves vertically upwards, so this is what is shown here Q S, now how long it will move vertical?

In the last lectures what we have seen this; we have seen that the stress path goes and touches the peak point. Now here there is a kind of understanding that the peak response and the portion defined by the yield curve is more or less the same. So, in the earlier lecture please refer back the effective stress path goes and touches. And we have told at that point of time that it is something relevant to yielding, the soil has to yield first, so that is what is shown here.

So, the soil has yielded at this point S, soil yields at S and deforms plastically. Further shearing will cause the ESP move towards critical state line at T and fail. Now in this case it will be negative pore water pressure because it is HOC and the final pore water pressure will be negative. And hence the effective stress path will move towards T, S T that is shown in v ln p' plot.

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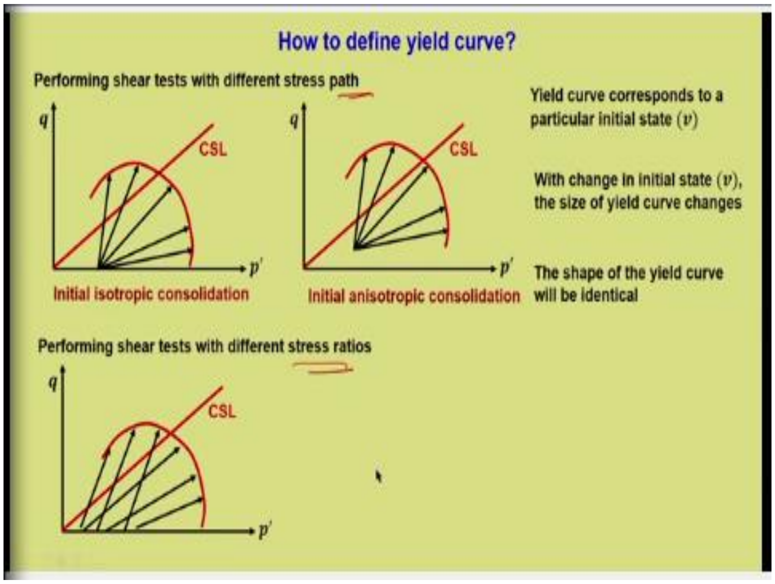

Now we have discussed more or less various aspects of yield curve, now how do we define this? We have a clue we have to conduct the test, and see how where the points are yielding and then join it but it is going to be a tough task always. So, if you want to determine because there are various ways by which we need to generate the yielding condition. Now that is why the mathematical framework is important even for yield curve.

So, let us see performing shear test with different stress path, that is initial isotropic consolidation condition, it remains same but we understand different stress path and then join the point of yielding. So, this is how normally it is done, the same thing can be done with initial anisotropic consolidation but the initial state remains same for all the stress path. So, this is another way by which but you can see that the shape of the yield curve will more or less remain same.

Yield curve corresponds to a particular initial state, please remember when we say yield curve it corresponds to a particular initial state or stress history. With change in initial state the size of the yield curve would change. For example if it is normally consolidated to point P, we will have a particular yield curve, now if it is further increased then the yield curve will increase. The shape of the yield curve will be identical.

Now there is another way of obtaining the yield curve that is performing shear tests with different stress ratio. Instead of different stress path different q / p' can be obtained, so if that is the case we will have again the yield curve. But you will see that whether it is determined using different stress ratios or with stress path, the shape of the yield curve will be identical.

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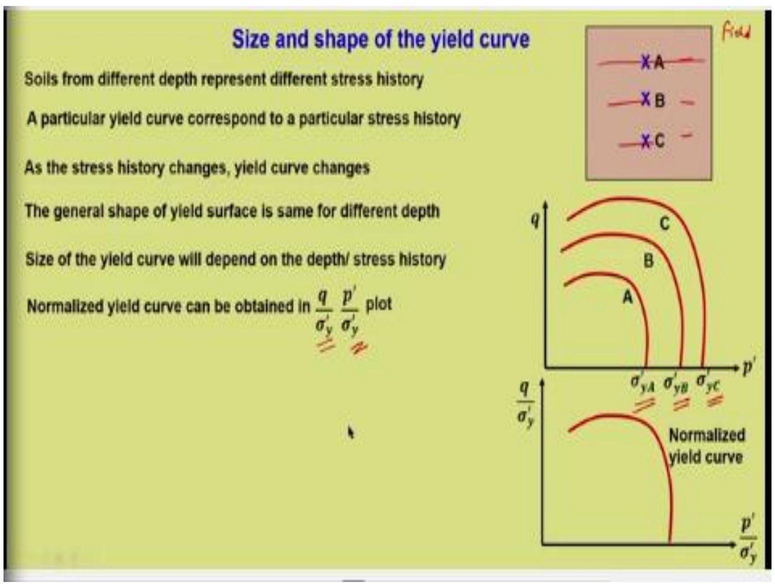

Now let us talk something about size and shape of the yield curve. Let us say soil from different depth represent different stress history. For example in the field let us say this is a field condition, you can see these are different depths from which the soil is collected. Now for this particular depth at A, at B and C soil will be of identical stress history but A, B and C it represents different stress history.

Now a particular yield curve corresponds to a particular stress history, now it is very clear to us. Like for example at point A we have a particular yield curve and the maximum stress point is $\sigma'_{y\,A}$ that is the yield curve at point A. As the stress history changes that means as the depth increases the yield curve changes. The general shape of yield surface is same for different depth B and C, this is how it goes.

Now size of the yield curve will depend on the depth or the stress history, now whatever we have discussed in the previous slide we are just understanding it in terms of what is happening in the field. Now it is very clear that this is the determining part of the size of the yield curve. Now if we try to normalize q p' plot with respect to σ'_y, what is going to happen? We can see that there will be a normalized unique yield curve which is obtained.

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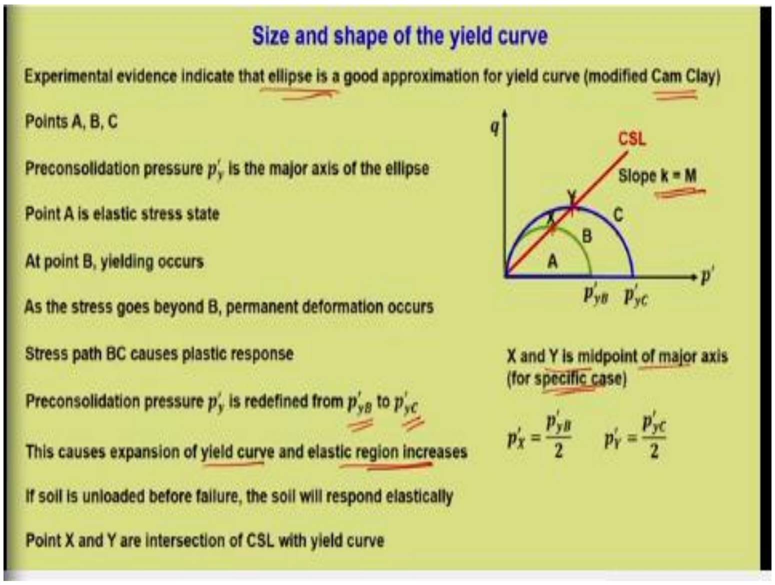

Now experimental evidence indicate that ellipse is a good approximation for yield curve, and that is very specifically defined in modified Cam Clay. Now Cam Clay is a critical state framework mathematical model, we will be discussing in brief Cam Clay and modified Cam Clay. Now it is apparent that there has to be an yield curve which is defined, now if it is a Cam Clay you will see that the yield curve is defined in terms of logarithmic spiral and for modified Cam Clay it is identified as an ellipse.

So, here I am just introducing that that ellipse is found to be a good approximation for the yield curve otherwise it becomes extremely difficult for everyone to determine the yield point separately. So, that is why it is quantified or it is idealized to be ellipse or any other shape. So, slope, let us say the slope we know that in q p' slope of CSL is M, but here I am generalizing it to be k = M.

Now let us say we have point A, B and C, now A is inside this yield curve, so preconsolidation pressure p'_y is the major axis of the ellipse. Since we have introduced ellipse here what it means is that p'_y is the major axis of the ellipse, so this is p'_y, p'_{yB} for the yield curve B, and this is p'_{yC} for yield curve C. Point A, now we are just revising whatever we have told in the previous slides point A is at elastic state.

At point B yielding occurs for the inner yield curve. As the stress goes beyond B, now once it departs then the permanent deformation occurs. Stress path BC causes plastic response, so preconsolidation pressure p'_y is redefined from p'_{yB} to p'_{yC}. Now all these aspects we have just now discussed. This causes expansion of yield curve and the elastic region increases. So, now we have redefined the yield curve, now all these position from here to here it becomes elastic.

If soil is unloaded before failure soil will respond elastically. Now this point x that is this particular point x and y are intersection of yield curve with critical state line. Now since we have introduced ellipse and it is the condition in modified Cam Clay, I would also like to add few of the aspect here where x and y is the midpoint of major axis. Now this is only for this particular specific case where in case of modified Cam Clay and ellipse.

So, $p'_x = p'_{yB} / 2$ and $p'_y = p'_{yC} / 2$, this information is handy as we move forward in our lecture and we will discuss certain prediction in terms of modified Cam Clay as of now we just need to understand this.

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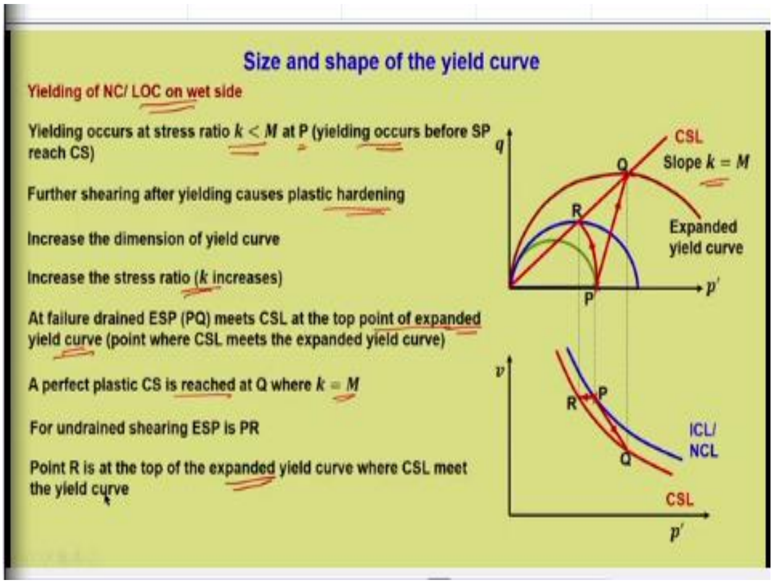

Now whatever we have learned previously we are just going to reiterate again in terms of normally consolidated and LOC which is on the wet side. Now here what we are trying to understand is we are just going to understand the concept of softening and hardening in terms of yielding. So, yielding occurs for NC and LOC, yielding occurs at stress ratio k less than M at P, that is yielding occurs before SP reach CS.

So, we need to understand that yielding will occur before the stress path reach the critical state. So, here what is the relevance of k less than M? Now let us say this is the point P, so in the case of normally consolidated or lightly over consolidated, we know that the value of k is less than M, that is the stress ratio, stress ratio is q / p’, it is less than M, M is the slope of CSL, now that P is shown on the NCL line. Further shearing after yielding causes plastic hardening, now we have already learned this.

Now this is the point which is normally consoled, this is normally consolidated. Now it is already yielded, there is no more yielding needed. So, once it is sheared it is directly proceeding towards the critical state line towards failure. Now what it happens, we know that it is subjected to failure by plastic hardening, and when it hardens what will happen to the slope k? So, definitely we know it will increase the dimension of the yield curve, now due to plastic hardening that is going to increase.

Now what is the net effect in the case of stress ratio, k also increases because we know the failure is going to happen only when it reaches k = M. So, earlier k is less than M for an NC soil now as for that point P and then it is going to increase and k finally when it reaches M it will fail. At failure drained ESP that is PQ meets critical state line, so this is what from P to Q, it meets the critical state line at top point of expanded yield curve.

Please remember top point of expanded yield curve is relevant only when you are considering the elliptical yield curve, where point where the critical state line meets the expanded yield curve, now this is the failure point PQ. Now this is the expanded yield curve, now this is at the midpoint of this outer ellipse. At a perfect plastic critical state is reached at Q where k = M, the stress ratio reaches M.

Here the stress ratio was less than M, and now when it reaches Q it has become k = M. For undrained shearing ESP is PR, now that is shown and this we have already seen and that is the yield curve for normally consolidated. Point R is at the top of the expanded yield curve, here please note it is expanded yield curve for NC and LOC where CSL meet the yield curve, this is R and this is the failure point.

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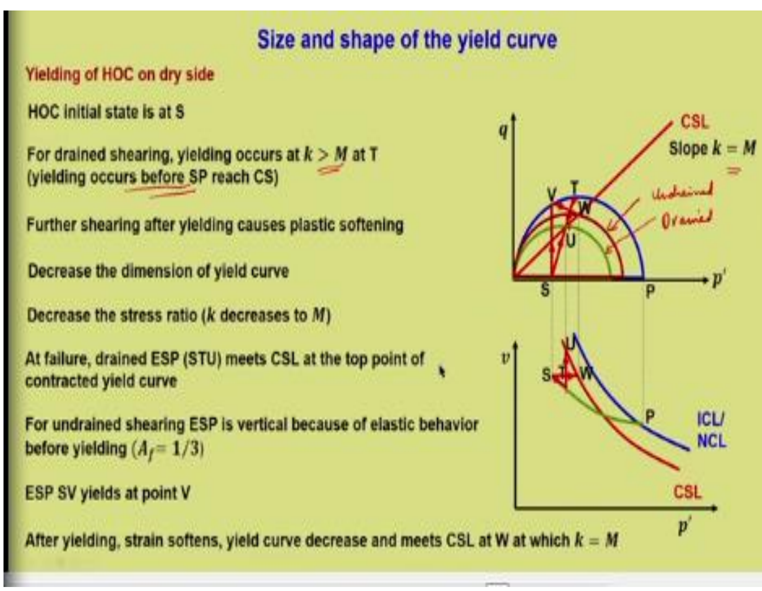

Now let us say for yielding of HOC on dry side. For that the heavily over consolidated point correspond to S which with it is initial state at S. So, this is point P, it has been loaded to point P and then unloaded to S, now that is shown here as S which is the starting point. Now for drained shearing yielding occurs at k greater than M at T. That is yielding has to occur before stress path reach critical state.

Now please remember this statement is a very important while understanding the shear behaviour. So, from S we know that there has to be a point T, now this is the point T where it has to yield, this we have seen in our previous lecture as well. Now if you consider the stress ratio at point T, the slope at T will be greater than that at M, so that is what is shown here k is greater than M at point T which corresponds to the yield curve**.**

So, this is the maximum stress to which, so this becomes the reference yield curve for us. And the stress path for drained it will be at slope 3, it will go and touches the yield curve and that is the point where it yields. Now further shearing after yielding causes plastic softening, so NC it is plastic hardening, now it is plastic softening. So, at from point T it is plastic softening and hence the stress comes down and fails at critical state, you can see that from T it reverse back and then meets the critical state line.

So, k greater than M as it is sheared it will meet the particular yield curve and then it gets back where k = M, so M, so the k value the stress ratio is reducing from a higher value to M. Whereas in the case of NC, k value has increased from an initial value to M. Decrease the dimension of yield curve, please remember we have discussed more or less the same thing in most of the cases but we are adding one by one in each of these slides.

In this particular slide what we are adding is about plastic softening and plastic hardening, even that you know but then in the context of yielding how does it translate to? So, decrease the dimension of yield curve in the case of NC it is increasing of yield curve, whereas in the case of OC it is decreasing of yield curve. So, this is the point where the final yield curve would lie, so that is what is shown here.

Now decrease the stress ratio k decreases to M from an initial value. At failure, drained ESP that is STU it meets the critical state at the top point of contracted yield curve. Now this is the contracted yield curve, remember this midpoint, top point it is relevant because we are discussing this in terms of ellipse for our convenience, it can be other shapes as well. Now that is what is shown in the case of critical state line.

Now we have now made it very, very precise, in the earlier case we defined it in terms of peak line it is now replaced by yield curve. Now you can see that from here this point it is along the unloading curve and then it moves towards U. Now if it is for undrained shearing from S, now this is for drained shearing, we know that for undrained shearing the slope is governed by 3 / 1 - 3A, till it yields it has to be the slope for which A = 1 / 3.

Because A = 1 / 3 is for the elastic condition and hence when we put it has to be vertical. So, we know that from S it has to be in the vertical direction. So, ESP SV it yields at point V, so now how it will be? This is the reference yield curve from S, it moves towards V and it yields at this point V, so up to here it is elastic behaviour. Then after yielding strain softens, now from here also it has to drop down and yield curve decreases and meet CSL at W at which k = M.

So, this is the point where it meets critical state at point W. Now at this point k = M, at point V, k is greater than M and that reduces to k = M, so that is shown in the v p' plot as well. So, this is the contracted yield curve for undrained, so this is for drained, and this is for the contracted yield curve for undrained.

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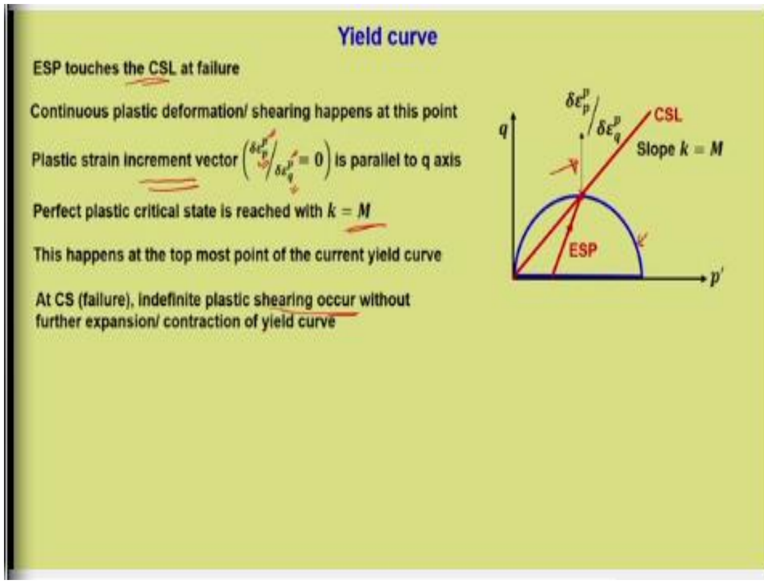

Now ESP touches the critical state at failure which we know. Continuous plastic deformation shearing happens at this point, which point at the point where the ESP touches the critical state line. Now this is what it means, it touches the critical state line at this point and this is the yield curve. Now at this particular point, the plastic strain increment vector that is $\delta\varepsilon_p$ of in plastic, this p means it is volumetric, here p it means it is plastic.

Here it is deviatoric and this p means it is plastic, and this ratio is known as plastic strain increment vector. Now that is equal to 0 and it is parallel to q axis, it is something like this, now I will not get into the details of plastic strain increment vector, because it is purely based on the plastic analysis. And we have not touched upon to that details in this course and we are not going to do that as well.

But this information that it is parallel to q axis may be handy for you when you work in this direction. So, that is the reason why I have just introduced this particular terminology. Perfectly plastic critical state is reached with k = M at this particular point and this happens at the topmost point of the current yield curve. Now this is the current yield curve, remember this is only for a specific case where we discuss in terms of elliptical yield curve for modified Cam Clay.

At critical state which is the failure state indefinite plastic shearing can occur and that is the reason why it is in this particular manner, without further expansion contraction of yield curve.

Once the yield curve meets or the effective stress path meets the critical state line and that particular yield curve this plastic increment vector is equal to 0 and it is in upwards. And at this particular condition what it means is that infinite plastic shearing can happen without any change in the yield curve.

That is there is no volume change, there is no other change that is happening and there is no plastic softening, plastic hardening that is happening, it is simply a plastic shearing case, an infinite or indefinite shearing that happens.

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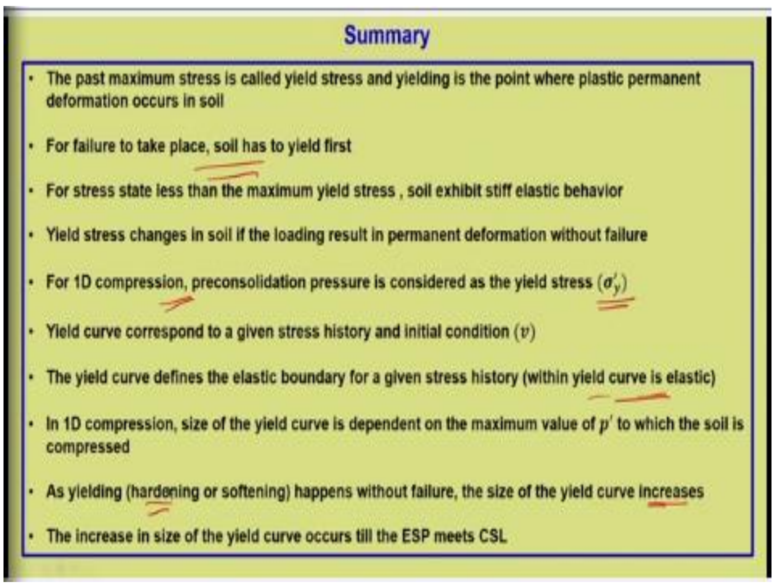

So, let us now try to summarize the points that we have learnt. So, we have today understood a very important aspect of soil yielding which need to be integrated with the shear strength as well as the stress path portions which we have studied earlier which adds a complete meaning to shear behaviour of soil and the failure mechanics of soil. The past maximum stress is called yield stress, and yielding is the point where plastic permanent deformation occurs in soil, so up to yielding it is elastic.

For failure to take place soil has to yield first, now this is a very important condition which is needed in critical state soil mechanics. For stress state less than the maximum yield stress soil exhibit stiff elastic behaviour. Yield stress changes in soil if the loading result in permanent deformation without failure. So, that is what a very good example is one dimensional compression.

For one dimensional compression preconsolidation pressure is the yield stress, σ'_y. Now this is specific only to one dimensional compression because this is the extreme most point. Yield curve corresponds to a given stress history, and initial condition or which is the specific volume. The yield curve defines the elastic boundary for a given stress history that is within the yield curve is elastic.

One dimensional compression, size of the yield curve is dependent on the maximum value of p' to which the soil is compressed as yielding happens without failure that is either hardening or softening the size of the yield curve increases or it may be decreasing as well. The increase in size of the yield curve occurs till the ESP meets, please remember increasing happens because of hardening and decrease happens because of softening.

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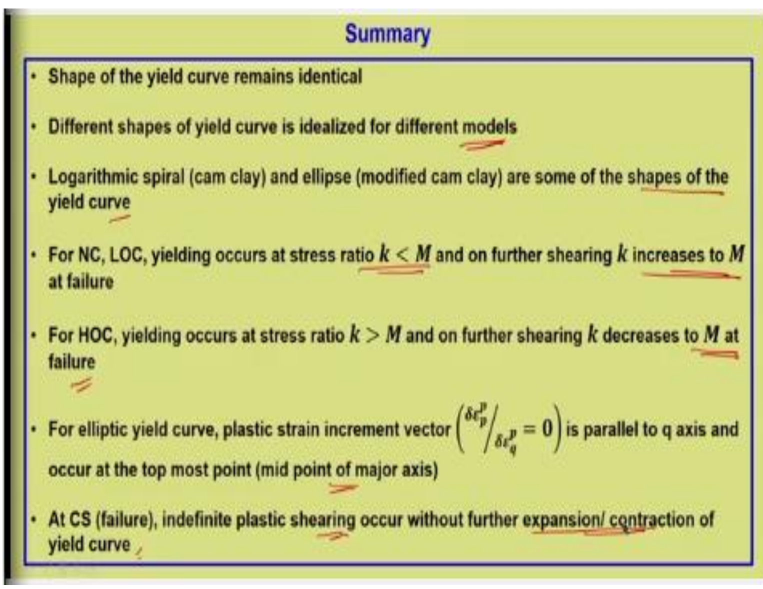

Shape of the yield curve remains identical, even though we are conducting we are finding out the yield curve at different stress history the shape of the yield curve is more or less identical. And this is understood from the experimental evidence which is already reported in the literature. Different shapes of the yield curve is idealized for different models that I have already told you for example in the case of modified Cam Clay, it is elliptical.

So, it is a idealization of the shape of the yield curve. The logarithmic spiral, it is used in Cam Clay and ellipse in modified Cam Clay are some of the shapes of the yield curve which has been used by previous researchers. For normally consolidated lightly over consolidated yielding occurs at stress ratio k less than M and on further shearing k increases to M at failure. For HOC yielding occurs at stress ratio k greater than M and on further shearing k decreases to M at failure.

For elliptic yield curve, plastic strain increment vector which is equal to 0, it is parallel to q axis and it occur at the top most point or midpoint of major axis. At critical state which is the failure state indefinite plastic shearing occur without further expansion, contraction of the yield curve. Further expansion, contraction means further hardening or softening which results in the size of the yield curve**.** So, that is all about soil yielding, we will see in the next lecture, thank you.