Advanced Soil Mechanics
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Lecture-44
Introduction-Critical State Soil Mechanics

Welcome back all of you, in the last lecture, we have completed module 3 on stress path. We have discussed various aspects of stress path which will become handy for module 4, which is the final module, which will be starting today. And the module is all about critical state soil mechanics. Now, the terminology critical state is not like you have already gone through, it is not new. So, are we going to study altogether a different concept in this module 4?

Not necessarily, what we will be doing is, we will be using the information that we have studied in our last few lectures basically related to module 2 and module 3, we will be placing it at appropriate locations and that is all about module 4. Of course, there will be some new terminologies that come into picture for understanding this framework better. So, with this, let us start the module 4 on critical state soil mechanics.

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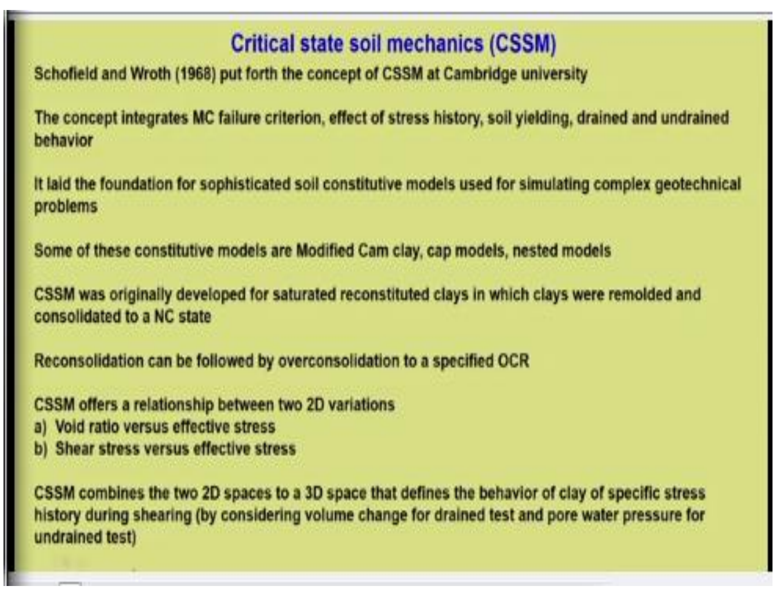

So, this concept, please remember this is a concept and which is put forth by Schofield and Wroth in 1968 at Cambridge University. It is about critical state soil mechanics, short form CSSM. Now, why this concept was evolved? The concept essentially integrates whatever we have learned that is the essential or the essence of soil mechanical behaviour, how do we define it is Mohr Coulomb failure criterion, the effect of stress history, the concept of soil yielding, drained and undrained behaviour.

So, it integrates, it tried to integrate these aspects together. This particular framework or the concept, it laid the foundation for sophisticated soil constitutive models used for simulating complex geotechnical problems. This we have already discussed a few of these models, I mean, we did not discuss into the depth, but conceptual understanding of these models we have done in our first module.

Now, there are a lot of models related to soil, which got developed to essentially cater to specific requirements or specific complex geotechnical conditions. In fact, we will not be discussing those in this; dedicated cause on geomechanics would help you to understand those, starting from the linear model to very complicated nonlinear models. You can very well go through the books by Potts, which is there already in the reference Haltz and Kovacs and many other books which discuss about these models.

So, critical state soil mechanics offers some foundation for understanding these models better. Some of these constitutive models include modified Cam Clay, cap models, nested models. This concept of CSSM was originally developed for saturated reconstituted clays in which clays were remolded and consolidated to a normally consolidated state. Essentially it was developed to understand triaxial behaviour of clays essentially remolded ones.

So, reconsolidation that is going to an NC state can be followed by over consolidation to a specified OCR. How do we generate over consolidated state by unloading it? CSSM offers a relationship between two 2-dimensional variations. In fact, there are 2 different variations which CSSM accounts and further it integrates these 2 different variations, what are these two 2D variations? The first one is void ratio versus effective stress.

Which again is something which we have already learned during consolidation, not in this particular course but then we have already have this basics from our undergraduate soil mechanics. A void ratio versus effective stress, it represents the consolidation condition. The second variation is shear stress versus effective stress, so effective stress is common here. So, this critical state soil mechanics combines these two 2D spaces to a single 3D space that defines the behaviour of clay of specific stress history during shearing.

So, whatever be, so there will be a particular stress history attached to that particular soil. So, critical state framework it helps to understand the soil behaviour during shearing in a three dimensional space which is basically obtained by integrating the two 2D variations. Now, for this, it is considered the volume change for drained test and pore water pressure for undrained interest. So, everything is accounted into this model be it the influence of stress history, be it the influence of drainage conditions all into one. And we try to understand the soil behaviour in a unique manner. So, that is the whole purpose of this development.

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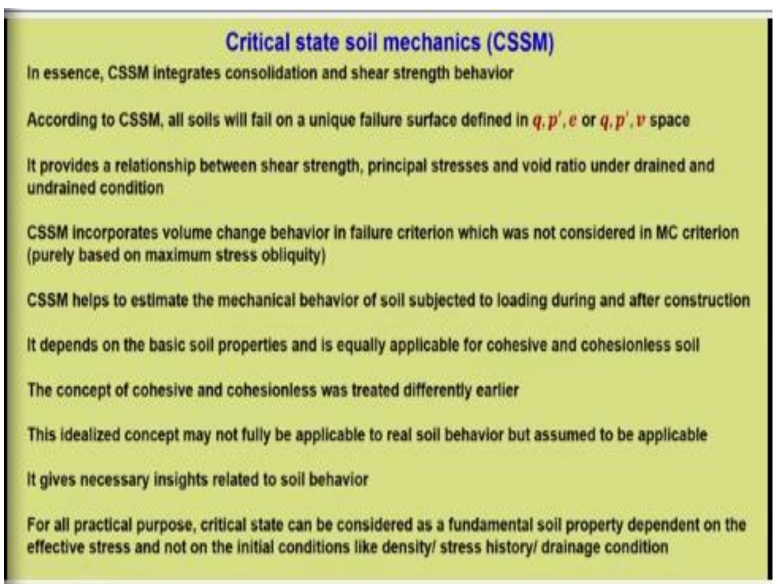

So, in essence CSSM integrates consolidation and shear strength behaviour. According to this concept, all soils will fail on a unique failure surface which is defined in q, p’ e or q, p’ v space. Now, q is deviatoric stress which represents the shear stress, p’ is the volumetric stress which represents the normal or effective stress condition and e the void ratio or v the specific volume that indicates the kind of pore structure or what is the kind of deformation behaviour.

So, the change in e helps to understand what is the kind of volume change that is happening within the soil. So, what we are trying to do, initially we talked in the stress part if you remember we have talked only in terms of two stress parameters. In fact, all our failure criterion is basically based on stress parameters only τ versus σ in Mohr Coulomb. We do not discuss about the volume change it has undergone during any kind of shearing.

So, critical state framework helps to bring in this new element. Now, in some textbooks you will find the volume change is accounted in terms of e and in some others it will be in terms of specific volume v, whatever be the concept remains the same, there will be some change in the parameters that helps to define the critical state framework that is all. So, in fact, we will try to discuss both side by side even though the prominent one is in terms of q, p' and v.

It provides a relationship between shear strength, principal stresses and void ratio under drained and undrained condition, as is visible in this q, p' e or q, p' v. $_{CS}$SM incorporates volume change behaviour in failure criterion, which was not considered in Mohr Coulomb failure criterion which is the most prominent model for soils, we do not talk about e or v at failure. So, that was purely based on maximum stress obliquity, I hope you understand this term σ'_1 / σ'_3 at failure.

$_{CS}$SM helps to estimate the mechanical behaviour of soil subjected to loading during and after construction. Later, you will see that you can predict based on this framework, what will be the kind of behaviour in drained, undrained or with respect to stress history. It essentially depends on basic soil properties and it is equally applicable for cohesive and cohesionless soil. If you remember in the earlier lectures, the concept of cohesive and cohesionless was treated differently.

When we discussed shear strength in module 2, we discussed separately for sands and clays, dense sand, loose sand against normally consolidated or over consolidated clays. Now this framework it does not actually need to see into the nitty gritties of these, we can all of them have been encapsulated into one particular model. This idealized concept, please remember this is an idealized concept.

Whenever we learn certain things related to mechanical behaviour of soils, somehow we tend to expect that soils should always behave the way we have studied. This may not happen and that is where we need to expand our understanding in the sense that whatever quantifications we are studying from various textbooks, these are for engineering applications for us to design, for us to understand the behaviour of soils.

So, all of them involves some sorts of assumption and idealization, here in the case of critical state soil mechani$_{cs}$ also, this is an idealized concept. So, if soils behave in a more ideal manner, this is how it will behave, but there will be always departure from this concept. Now that we also need to keep in mind, so this is a well idealized concept which is time tested and we understand that in general soil behaviour can be explained using this.

So, this idealized concept may not fully be applicable to real soil behaviour, but assumed to be applicable. Because this assumption is needed for us to predict the behaviour better the data when we monitor the behaviour, sometimes it may be different, but prior to any testing program, if we want to study the various scenarios this framework helps us to predict the behaviour better.

So, please do not take into account that, this is the final. So, soil is highly complex material and in nature this material behaves the in a most uncertain manner. So, every aspect may not go as per our technical understanding, it gives necessary insights related to soil behaviour. So, that is what is more important, for an engineer when he is planning for any sort of analysis or design certain insights related to soil behaviour is important and for that we need a framework.

And that framework is offered by $_{CS}$SM. For all practical purpose, critical state can be considered as a fundamental soil property dependent on the effective stress and not on the initial conditions like density, stress history, drainage conditions. So, this is one of the most powerful aspect of critical state soil mechani$_{cs}$. Earlier when we discussed Mohr Coulomb failure envelope, we discussed about friction angle and cohesion, but we could not associate it to be a fundamental soil property, because depending on the conditions these may change.

Now, here in this framework, it offers some solution partial solution to this problem by stating that this is more or less a fundamental property of soil which is independent of the initial conditions such as initial density, the stress history and what drainage condition does it account for. Finally, the state of the soil reaches a particular state called critical state, so that can be considered as fundamental behaviour.

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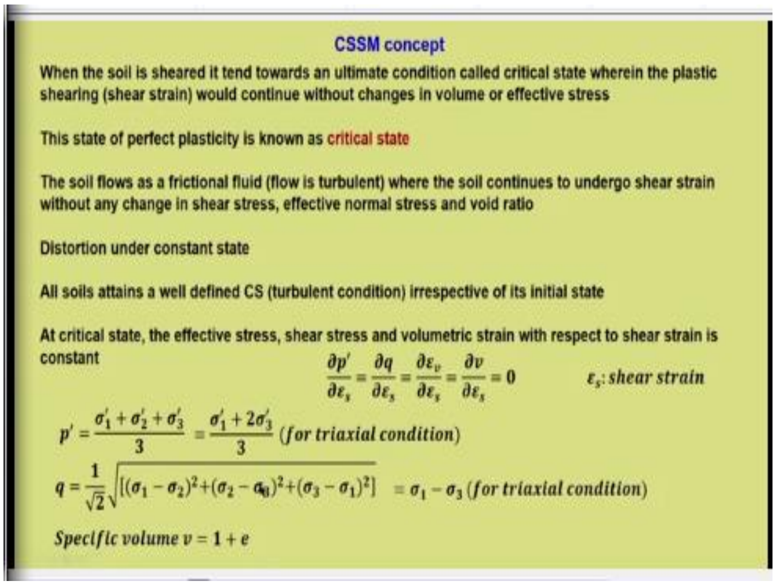

So, when the soil is sheared, it tends towards an ultimate condition called critical state which is a concept which we already know. So, that is what I told like we know these concepts only thing is we will be putting these in the right shelf, which will be easy for us to understand. Wherein the plastic shearing where we call it as shear strain would continue without changes in volume or effective stress.

So, any soil which has sheared it reaches a final state what is known as critical state, wherein the shear stress would continue but at constant effective stress and constant volume. So, the final state has constant volume, so shearing happens or the plastic shearing happens at constant volume when the soil reaches this critical state. This state of perfect plasticity is known as the critical state.

The soil flows as a frictional fluid and the flow is considered to be turbulent, where the soil continues to undergo shear strain without any change in shear stress, effective normal stress and void ratio. Please remember, we are just repeating the same concept again and again, so it does not matter that we are repeating, what is more important is that concept goes into our head very clearly. So, that is why even though it is getting repeated, every step we are adding some new information.

So, it is like this, at this particular critical state soil behaves more like a frictional fluid. Now, it is very difficult to understand like how it looks like a frictional fluid, why it is called frictional? Because there are solids which are involved, wherein the moment is happening and the friction is getting mobilized So, possibly that is the reason why the terminology frictional fluid, and at the critical state it is assumed that the flow is more like a turbulent manner.

So, it is not in an orderly manner, it is in a turbulent manner where the soil continues to undergo shear strain at constant shear stress, effective normal stress and void ratio. So, it is more than a distortion under constant state. All soils attain a well defined critical state that is a turbulent condition irrespective of it is initial state. Again this is a reputation but still it is important to repeat these statements.

At critical state, the effective stress, shear stress and volumetric strain, that is ε_v with respect to shear strain is constant. That means, $\partial p' / \partial \varepsilon_s$ is equal to this is p' is the effective stress or the volumetric stress equal to $\partial q / \partial \varepsilon_s$, where q represents the shear stress or the deviatoric stress, $\partial \varepsilon_v$ upon $\partial \varepsilon_s$ that is volumetric strain which is equal to ∂v upon $\partial \varepsilon_s$ where v is the volumetric component of the soil that is specific volume and that is equal to s.

So, all of these with respect to differentiated with respect to shear strain $\varepsilon_s = 0$, and where p' just for completeness I am adding, we know this, we have studied this $\sigma'_1 + \sigma'_2 + \sigma'_3 / 3$. And triaxial condition it is $\sigma'_1 + 2\sigma'_3 / 3$. q, the general expression for deviatoric stress is this that is equal to $\sigma_1 - \sigma_3$ for triaxial condition.

And specific volume v = 1 + e, so it is the volume of solids that contains unit volume of solids,. So, it is a specific volume is defined as the volume of soil containing unit volume of solids. So, 1 + c, if you remember the unit solid volume model, 1 + c is the total volume, in which the soil volume is considered to be 1. So, that is why it is 1 + e is the total volume of soil that constitutes unit solid volume that is 1.

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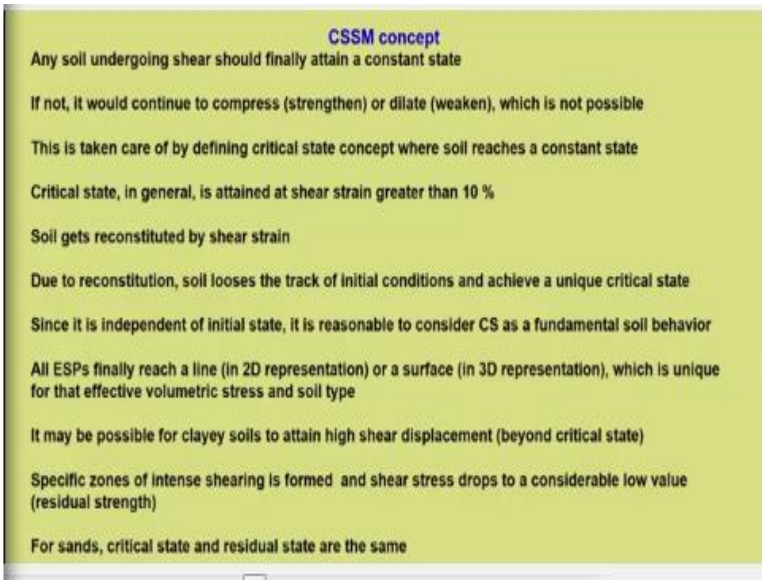

So, any soil undergoing shear should finally attain a constant state, why because if not, it would continue to compress that is strengthen or dilate that is weakening, which is not possible. This we have seen these two actions are common whether it is cohesionless or cohesive when the soil is getting sheared, it will either undergo compression or it will undergo dilation. Now can this be a continuous process? No, it has to stop somewhere and that is where the critical state offers that bound at which it can stop.

So, this is taken care of by defining critical state concept where soil reaches a constant state, which is neither compressing nor dilating, it reaches a constant volume state. So, critical state in general is attained at shear strain greater than 10%. Soil gets reconstituted by shear strain; due to this reconstitution soil loses the track of initial conditions and achieves a unique critical state. So, that is what is more important, why it is not dependent on?

The concept says that the reconstitution that happens during shearing enables soil to lose it is memory from where it has come. So, it is more like it is forgetting it is initial state of conditions what is the stress history to which it was subjected to, finally it reaches to this surface or line which is called critical state. Since it is independent of initial state, it is reasonable to consider critical state as a fundamental soil behaviour.

All effective stress paths finally reach a line in a 2D representation or a surface in 3D representation, which is unique for that effective volumetric stress and soil type. So, what is more important is soil type and the p' that we consider. So, what is the p' that we are starting from that is important, initial state means we tend to focus on what is the influence of stress history or what is the influence from the drainage condition or what is the influence of density.

Here p' is something to do with the confinement and we know that confinement do influence the shearing behaviour. So, here, it is basically dependent on what confinement or the initial state p' it starts and also what is the soil type. It may be possible for clay soils to attain higher shear displacement beyond critical state. Now this is one concept that can induce some sort of what you say doubt or confusion.

When we discussed our earlier lectures in shear strength, we also discussed about something known as residual strength. Now to alleviate those confusion, we are discussing this particular point in fact, it is not really relevant in this module. But still we would like to discuss just because to avoid that confusion. Now there are certain cases where soil can undergo very large displacement, but we need to understand that the soil has already been in a state of failure.

Still it has not completely failed, so there will be some sort of shear strength that is residual in that soil, that is what we discussed about residual strength as we discussed in module 2. Now this strength is when it is surpasses the critical state condition and it is still able to sustain without failure. So, there the constant shear strength condition is achieved at residual state, but critical state is a common terminology even if it surpasses.

You can consider the failure to have happened at critical state, but when you want to actually discuss slip lines of slope or when we consider the slip circle failure in the case of slope, there will be some amount of movement that happens. That means it is the soil is subjected to very large displacement before failure. To account for a very large displacement, the concept of residual strength is also there.

So, that is the specific zones of intense shearing is formed, and shear stress drops to a considerable low value, and that is what is known as residual strength. For materials like sands, we can notice that the critical state and the residual state are the same, for them there is no that such excessive movement that is possible. It is only valid for some certain kind of clays under certain situations, in that case the residual strength get invoked.

So, there is some difference between the critical state as well as the residual strength that we discussed. So, just to make to understand this concept better, let us try to understand this using a figure.

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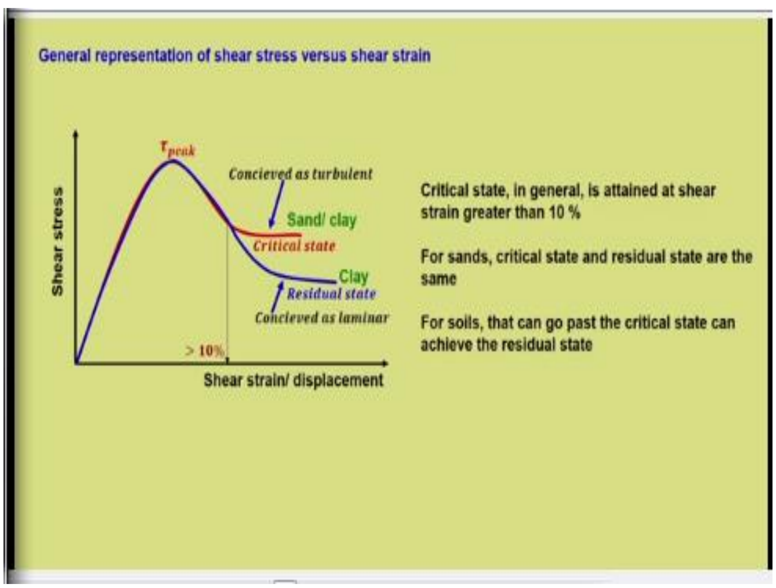

So, this is what we have already studied, shear stress, τ peak and then reaching a constant critical state, so this is equally valid for sand and clay. Now at this particular critical state, it is conceived there will be at kind of turbulent flow. And this critical state in general is attained at shear strain greater than 10% in general, so this what is marked here. For sands critical state and residual state are the same at this point. For soils that can go past the critical state can achieve the residual state, so that is what is shown by this blue line.

So, this is applicable only for clays where we call this as residual state at very large displacement. And here the flow is considered to be laminar because things have subsided there more or less, and particles or the clay particles are oriented in a more orderly manner. And hence, there will be the laminar kind of movement at residual state. Remember, whatever we are telling these are all concepts. So, now let us go into the first step of understanding critical state.

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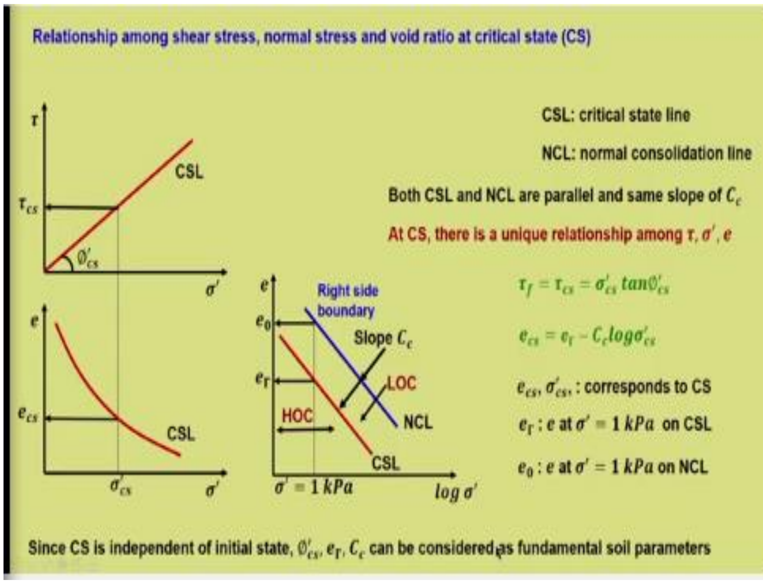

That is relationship among shear stress, normal stress and void ratio. So, we are considering all the three elements which are needed for critical state. So, first let us understand in terms of τ and σ', this is very well known to us is τ was a σ' and the failure criterion in terms of critical state friction angle, nothing to explain here. So, this is the critical state line, earlier we called it as failure line.

Here, now specifically we can call it as CSL critical state line. Then the other responses e versus σ', you can see that σ' which is the effective stress or the effective normal stress remains common for both the figures, so e represents the volume change. So, e versus σ', we can plot a line where in the locus denotes all the failure state or the e and σ' corresponding to the critical state.

So, that line is critical state line in e versus σ', when you start loading, we can find a particular point where the soil would fail, and that particular point what is the e and σ that is the e_{cs} and σ'

$_{cs}$ corresponding to critical state. So, the locus of all that point is what is denoted by CSL in e σ’. So, for example this is σ'_{cs} which is relevant to both the plots where on this particular plot that is e versus σ’.

We have e_{cs} and here we have τ_{cs}, e_{cs}, σ'_{cs} these three are connected. So, if we plot e versus σ’ on e log σ’ this is on base 10. We have critical state line which can be represented as a linear variation. And we can also represent normally consolidated line wherein it represents the e and σ’ variation of normally consolidated state. So, NCL is known as normal consolidation line.

So, we are now introducing certain important terminologies which are important for this framework, one is critical state line, the other one is normal consolidation line. Both CSL and NCL are parallel and with same slope of compression index C_c. Because now this is nothing but e versus log σ’ which we have seen during consolidation. And we know that the slope of this is going to be compression index C_c.

Now, when we discuss the next one that is q, p’ and v then the slope will change. So, that is the only difference but the concept remains the same. So, slope is C_c for both, so for both CSL and NCL, the slope will be C_c. And this normal consolidation line forms the right side boundary, why right side boundary? There is no soil state which is possible beyond this line, why? For any virgin soil that you consider in the field, it will undergo a consolidation along the normally consolidated line.

We cannot generate a state beyond this line, so it is an impossible state which we will see later. So for now we will understand that normal consolidation line forms the right side boundary. Whereas on the left side of NCL will be over consolidated state, it can be lightly or heavily over consolidate, why because all over consolidated state is created by unloading from the normally consolidated. So, unloading means it will be on the left side of normally consolidated line.

And critical state line falls in between, and how do we interpret this? This we will understand as we move further. So, lightly over consolidated will be somewhere around this, we will discuss more about it later. At critical state there is a unique relationship among τ, σ’ and e, that is at this particular line or this particular line there is a unique relationship in among all the three parameters that is τ, σ' and e.

For that, let us consider an initial point irrespective of any; let us start with a very low stress condition, which is more or less corresponding to let us say zero condition because log of 1 is 0. So, instead we take $\sigma' = 1$ kPa, we have some reference point on critical state line as well as on NCL line. Let us call the void ratio at 1 kPa, $\sigma' = 1$ kPa on the critical state line to be e_γ and void ratio on NCL to be e_0.

If that is the condition, then we can always write, τ failure which is same as τ critical state is equal to $\sigma'_{cs} \tan \varphi'_{cs}$ from this particular figure. And this is very familiar to us, which is very similar to what we call it as Mohr Coulomb failure envelope. And this is an new expression which takes into account the volume change aspect of soil. And where we can write e critical state, we are referring to void ratio along this critical state line, which is equal to which is a starting point at 1 kPa, it is e_γ.

So, $e_\gamma - C_c \log \sigma'_{cs}$ that is σ' along critical state line. So, these two are very important expressions, where e_{cs} σ'_{cs} corresponds to critical state, e_γ we have explained that is e at $\sigma' = 1$ kPa on CSL, e_0 is e at $\sigma' = 1$ kPa on NCL. So, since critical state is independent of initial state, φ'_{cs}, e_γ and C_c can be considered as a fundamental soil parameters, this is what we need to understand.

So, if we are dealing critical state framework in terms of τ, σ and e, then we need to understand that φ'_{cs}, e_γ and C_c are considered as fundamental soil parameters because these parameters are irrespective of the initial condition. Now we have discussed in terms of τ, σ' and e, but in most of the cases you will see that critical state framework is discussed in terms of q, p' and specific volume.

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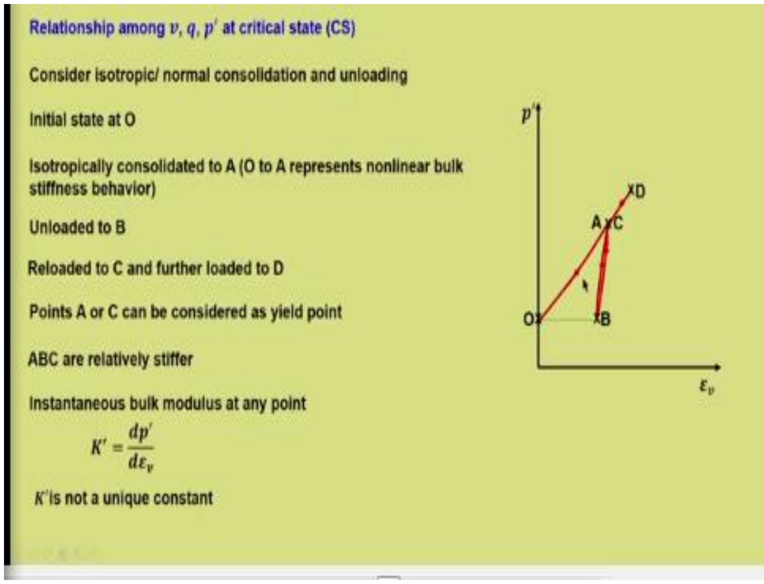

So, let us try to understand in that. Consider isotropic or normal consolidation and unloading. So, that is represented by p' and ε_v. So, initial state at O, so let us consider it O, isotropically consolidated to A or O to A represents nonlinear bulk stiffness behaviour. So, O A is the isotropical consolidation, then from A it is unloaded to B, as you can see it is unloaded to B. So, this O and B are at the same point, so it is at the same level of same p' level.

Then from B it is reloaded to C, where A and C is almost same and then further loaded to D. Points A or C, this particular point can be considered as the yield point for this particular behaviour. This is the point where it has yielded, why because this length C B is there, now this is the unloading reloading path, so this represents the yield point. ABC will be relatively stiffer because it is on the unloading path of the over consolidation part.

So, instantaneous bulk modulus at any point can be given as K', there is the bulk modulus is equal to dp' / d ε_v, that is the slope of this particular curve. And which you can see that it is keep on changing with the stress level. And hence it cannot be considered to be constant, it is not a unique constant, it changes with different stress level.

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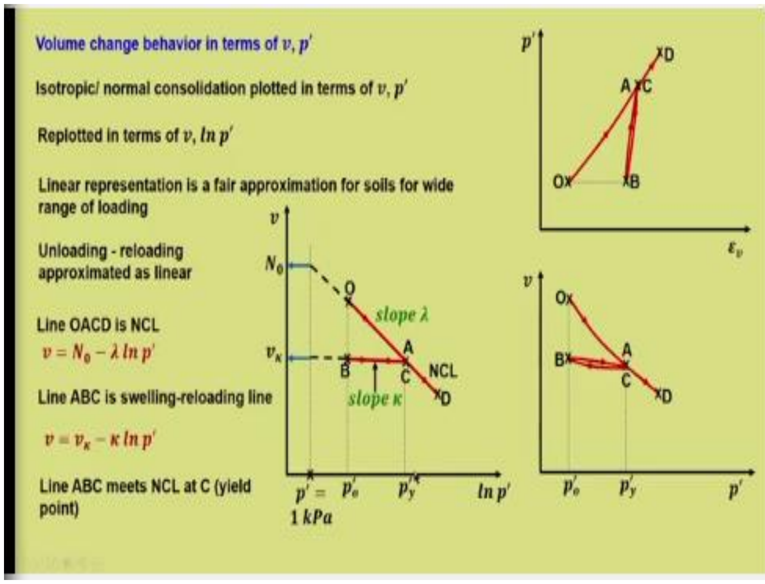

So, let us further understand volume change behaviour in terms of v and p', again isotropic normal consolidation, which is plotted in terms of v and p'. So, here this is the reputation of the figure from the previous slide, if this is translated to v, p', now here this p' ε_v we are plotting in terms of v p' which has the normal way in which these parameters are plotted for critical state framework.

So, let us start with the point O, it undergoes normal consolidation from O to A, then unloading-reloading that is to B. And then to C, reloaded to C and then again loaded to B, whatever we have discussed here, we have redrawn in v p' plot. Let this B p'_0, this is the yield point or the yield stress p'_y which we call it as pre consolidation pressure, replotted in terms of v ln p'.

Now this is in terms of v and p', what we want is a more general representation in terms of v ln p', whenever we use log or ln it is to account for a wide variation. So, here also v versus ln p', it is replotted and when it is replotted we know that it will become a straight line. So, both unloading-reloading is represented by a single straight line, average straight line A, B, C and further D, so this is what has been replotted.

So, this is normally consolidation line, and here you can see when it is represented in terms of v ln p', the slope is considered to be λ, what was our slope? Initially when we considered in terms of the other parameter in terms of void ratio it is e versus log σ', and that this slope is C_c, here it is slope λ. And the slope of unloading-reloading line or the over consolidation portion is κ is in terms of κ, this K represents Kappa.

So, and p'_y is the yield point p'_0 is the point the stress level at O or B. Again here also we form a reference point, that is p' = 1 kPa, the way we have done earlier. And if you extend these lines you will get what is the v corresponding to 1 kPa. And for normal consolidation line this is represented as N_0 and for over consolidation point, that is this slope κ, the initial point is considered to be $v_κ$.

Linear representation is a fair approximation for soils for wide range of loading. Unloading-reloading is approximated as linear, line OACD is NCL, where you can represent this line NCL by $v = N_0 - λ \ln p'$, same thing we have done earlier. Line ABC is swelling-reloading line, for which $v = v_κ - κ \ln p'$, that is to represent this particular line. Line ABC that is A, B, C meets NCL at C that is the yield point.

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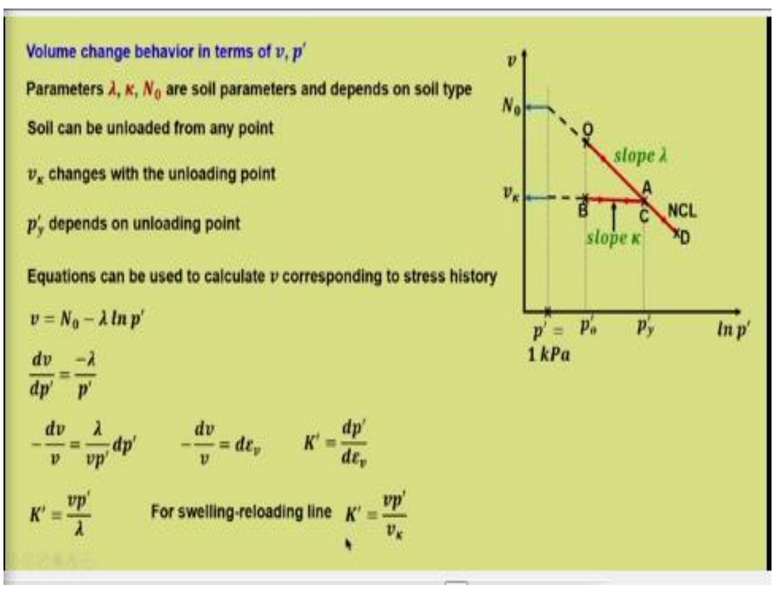

Now parameters λ, κ and N_0 are soil parameters and depends only on soil type. So, these parameters N_0, λ and κ this can be considered as a fundamental soil parameter. Now soil can be unloaded from any point, it can be unloaded from here; accordingly the yield stress would change. So, v_k changes with the unloading point. Now since it has been unloaded from point A, v_k is this, but if it is unloaded from here v_k would change.

So, v_k cannot be considered as a fundamental parameter, even p'_y will depend on the unloading point. So, if it is unloaded from here, this becomes the yield point or the yield stress. So, equations can be used to calculate v corresponding to stress history, by knowing the previous equations we can calculate what is the specific volume corresponding to different stress history whether it is here or on this.

$v = N_0 - \lambda \ln p'$, whatever we have written earlier, if you differentiate this with respect to p' we will get $dv / dp' = -\lambda / p'$, if you differentiate ln p' it is 1 / p'. Now if you write -dv / v, what is -dv / v? This is nothing but volumetric strain which is equal to $\lambda / vp'\ dp'$. We are just adding v here and bringing dp' on the side. Now let $- dv / v = d\varepsilon_v$ which is the volumetric strain.

And K', we know that we have already written the bulk stiffness $K' = dp' / d\varepsilon_v$. So, dp' / d, so this is replaced / $d\varepsilon_v$, so $dp' / d\varepsilon_v$ will be K' and this will go to the side. So, this will be $v p' / \lambda$. So, hence K' this is the bulk stiffness can be written as vp' / λ. Now this is the scenario of bulk modulus on normal consolidation line because it is λ.

But for swelling-reloading line $K' = vp' / v_\kappa$, that is this particular line. So, you can see that K' is not a constant parameter rather it gets changed depending upon whether it is λ or whether it is v_κ or with different position of v and p'.

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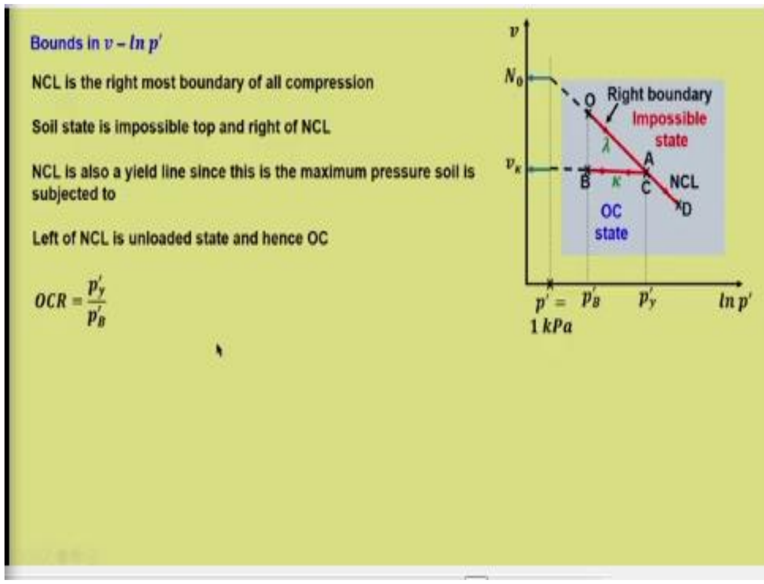

Now let us discuss about we have already discussed about the right most boundary that is the normally consolidated line we will try to add a bit more on this, so bounds in v ln p' plot. NCL is the right most boundary for all compression, so any compression that you do it gets terminated on a normal consolidation line, this is the right boundary. Soil state is impossible top and right of NCL.

So, any point soil related point on this part is not possible, so that is why it is marked as impossible state. NCL is also yield line, since this is the maximum pressure soil is subjected to, so it is otherwise also we understand because this is the maximum point to which the soil is subjected to. Left of NCL is unloaded and hence over consolidation, so here this particular portion represents the over consolidated state.

Where OCR is defined by p'_y / p'_B, where this is the yield pressure or the preconsolidation pressure p'_y / p'_B. If this is the point of consideration, if it is this then appropriate present stress has to be considered. So, here OCR is for B, point B is p'_y / p'_B.

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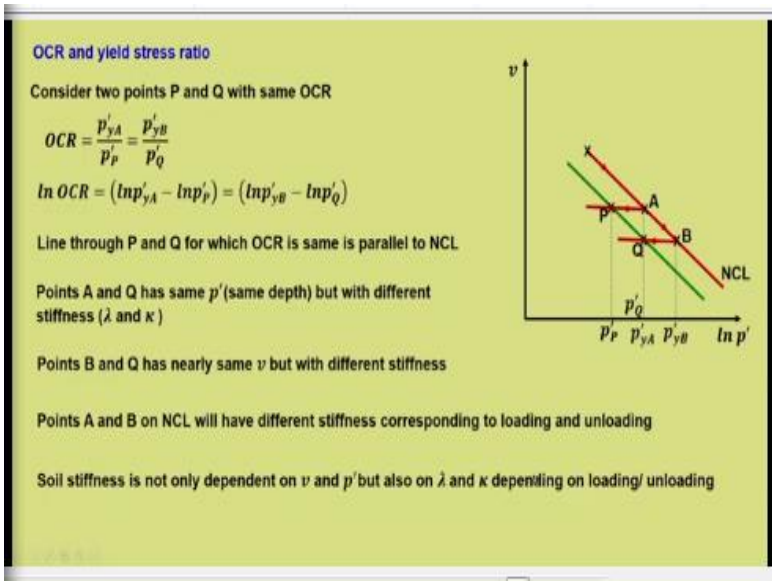

Now let us try to understand another terminology which is known as yield stress ratio and understand how yield stress ratio will be a better parameter than over consolidation ratio. For that let us consider 2 points P and Q with same OCR. So, here it is represented by a normally consolidation line and we have 2 unloading lines, one at A and the other one at B. Now consider point P and consider point Q, now if you refer to P and Q they have the same OCR.

That is $p'_{y A} / p'_{p} = p'_{y B} / p'_{Q}$. So, the OCR is same for the points P and Q, so that is what is written here. That is in terms of ln OCR we can rewrite this expression, line through P and Q for which OCR is same is parallel to NCL. So, if you draw a line through P and Q that will be parallel to NCL because these are of the same OCR, so which is shown here, this is the line.

Points A and Q has same p’ that is A and Q they are vertically below, so hence it has got the same p’ but with different stiffness λ and κ, that is what an important aspect even for the same initial pressure, you can see that if you consider point A, it is falling on NCL and hence the stiffness of A can be considered as λ whereas Q is on the loading-unloading line and hence the stiffness is κ.

Points B and Q has nearly same specific volume, why because it is on the unloading line, so the amount of regain of v will be marginal. So, B and Q is practically are nearly same specific volume but they can be considered of different stiffness, why? B is on NCL, so it will have a stiffness of λ whereas Q has a stiffness of κ. Now points A and B on NCL will have different stiffness corresponding to loading or unloading.

Now let us say that A and B are on NCL there is no doubt on that, if you consider a normal consolidation then A and B refers to a stiffness of λ. But it is already on a loading-unloading line, so at this point it can also come from the unloading line as well. So, unloading-reloading line as well, if that is a case A can also assume a value of κ. So, these points are very important as we move further when we try interpret the results, these informations are quite handy.

So, please keep this in your mind. Soil stiffness is not only dependent on v and p', I mean to say bulk stiffness but it also depends on λ and κ depending on whether it is on loading or unloading line.

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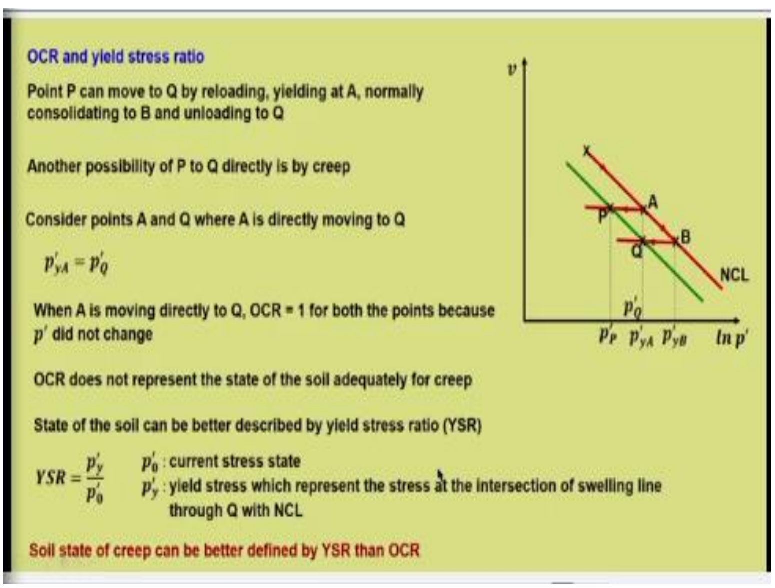

Now, let us come to yield stress ratio. Point P can move to Q by reloading, yielding at A, normally consolidating to B and unloading to Q. I mean to say if point P has to reach Q, what is the normal route it follows? It has to reload to A, so that is what is written here and further it is again yielding at A and then it moves towards B. And on unloading it reaches the condition of Q, but there is another possibility of P moving to Q and that is by creep.

Now here it is not actually creep because p' is changing but if we consider a point directly below then it represents creep. So, for understanding that better let us take the reference point via A and Q. Now whether from A to Q it can move from A to B and B to Q, now if it moves directly that is what is known as creep that is at constant stress condition. Consider a points A and Q, A and Q which are vertically below where A is directly moving to Q and this is possible when there is creep behaviour.

So, $p'_{yA} = p'_{Q}$ that is already marked, when A is moving directly to Q, that is A is moving directly to Q, we have OCR = 1 because stress has not changed. When A is moving to Q, OCR remains 1 for both A as well as Q for both the points because p' did not change, why? Because this is the yield stress we are not accounting for the yield stress at this point even though it is on the unloading line of B Q.

But here with reference to A to Q directly we will take OCR = 1, therefore OCR does not represent the state of the soil adequately for creep, because it is ignorant of this particular yield stress. State of soil can be better described by yield stress ratio which gives more clarity to the soil state, where yield stress ratio is always defined in terms of yield stress. That is p'_{y} upon the present stress that is p'_0.

So, here B is the yield stress for point Q, so p'_0 is the current stress state, p'_{y} yield stress which represents the stress at the intersection of swelling line through Q with NCL. So, this is the required p'. So, soil state of creep can be better defined by YSR than OCR, for example in this particular case both A and Q had an OCR of 1. If we are not bothered about p'_{y} but if you are defining in terms of yield stress ratio which is always with respect to p'_{y}.

For example if you are referring to point Q which is below A, you are referring to a swelling-reloading line with a slope of κ which meets NCL at B. So, for this the reference yield stress will be p'_{y}. so, p'_{yB} / p'_{Q} will be the yield stress ratio at Q which is always greater than p'_{yA}. Because the yield stress at A is p'_{yA} / p'_{A} which is equal to 1 which is a normally consolidated condition.

But if YSR at A will be 1 and YSR at Q will be more than 1 because p'_{yB} has been accounted. So, here the yield stress ratio helps us to interpret this condition in a more better manner and that is more relevant to conditions like creep. So, that is all for today's lecture like we have introduced critical state soil mechanics, a few more aspect also need to come in the introduction part which will see in the next lecture, that is all for now, thank you.