Advanced Soil Mechanics
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Lecture-42
Stress Path Problems

Welcome back all of you, in the last lecture we have finished this module 3. And we have discussed various aspects of stress path. In this lecture we will work out some representative problems related to stress path.

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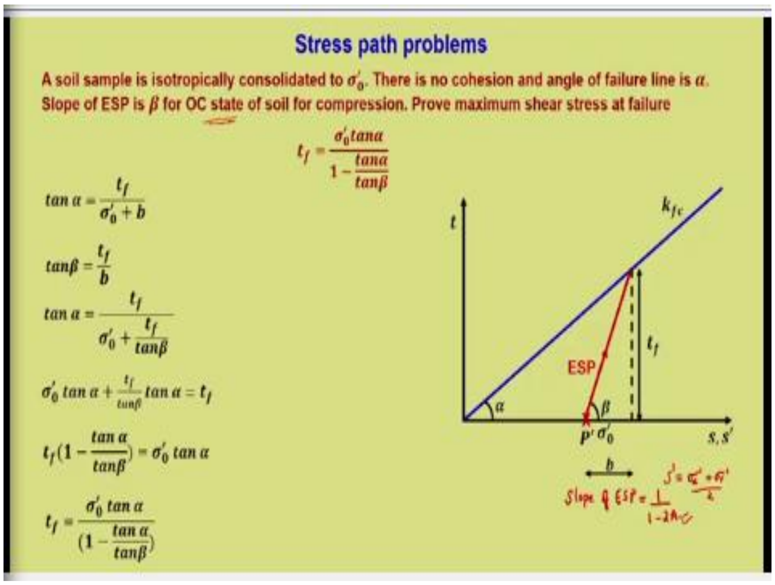

So, the first question is a soil sample is isotropically consolidated to σ'_0, there is no cohesion and angle of failure line is α, slope of effective stress path is β for over consolidated state of the soil and that is in compression. Prove maximum shear stress at failure

$$t_f = \frac{\sigma_0' tan\alpha}{1 - \frac{tan\alpha}{tan\beta}}$$

So, this is a representative problem which helps us to interpret the question and draw the stress path.

There are a few conditions which are given and the most importantly it is over consolidated, alpha means it gives a clue that it should be in t- s, s' plot, there is no cohesion. So, A' is not going to come into picture, so first it is isotropically consolidated that means we know where is the starting point. And we need to first find out what is the slope of ESP**,** so effective stress path, so it has to be in a more general manner, had it been total stress path we know like there are clear specific leads 1 is to 1 is the slope.

But since it is effective stress path, we also need to take care of other aspects as well, now let us try to solve this problem. So, it is clear that it is on t-s, s' and the slope of the failure line is α, so that is marked here. Next is the initial point, soil sample is isotropically consolidated to σ'_0. So, it gives a clear indication that it has to be on the isotropic line and which is the isotropic line? So, this is the isotropic line.

So, the initial point that is p' which corresponds to σ'_0 will be here, now how does this point become σ'_0. Because we know that $s' = \sigma'_0 + \sigma'_0 /2$, why is it so? We know that $s' = \sigma'_0 + \sigma'_r /2$, so since it is isotropic $\sigma'_0 + \sigma'_0 /2$ that will be, so s'_0 that is the initial point $s'_0 = \sigma'_0$.

Now how did I draw the effective stress path with such an inclination? Now nothing is told only thing is it is told that the effective stress path is at an inclination of β. Now this β can be in this direction as well. So, how did I plan to draw ESP in this direction? ESP in this direction, now we have to look back to the slope of ESP in t-s, s' plot, that is 1 /1 - 2A. Now for over consolidated state the A value at failure if that is to be considered as constant.

Let us take A_f to be constant and we know that for heavily over consolidated soil A_f will be negative. So, if you substitute for A equal to a negative value whatever be the value, this whole of the slope will become positive. So, we are shear that for OC state this ESP is going to be positive and that is what I have drawn with a positive slope and that is equal to β. And what is of maximum shear stress at failure? This is the maximum shear stress at failure.

Because where effective stress path touches the failure line, now this is t_f. And now the problem is very simple, the only aspect we need to keep in mind while solving stress path problem is that, we should be able to plot it correctly. Once we get the geometry then it is very easy to solve the problem. So, what is important is to determine the slope? The certain basic knowledge that failure is always brought about / effective stress path.

So, these bare minimal conditions we should keep in mind and that will help to solve the problem**.** Now the problem is simple, it is only geometry now. Let us consider this distance to be b, so tan α considering this particular triangle tan α = t_f divided / this is σ'_0. So, this distance is σ'_0 and this distance is marked as b, so that is σ'_0 + b, tanβ, this triangle, we have t_f / b, so that is what is written here.

Then I can replace b, so tan α = t_f /(σ'_0 + (t_f /tanβ)). Now further rearranging we have

$$\sigma_0' tan\alpha + \frac{t_f}{tan\beta} tan\alpha = t_f$$

$$t_f \left(1 - \frac{tan\alpha}{tan\beta}\right) = \sigma_0' tan\alpha$$

$$t_f = \frac{\sigma_0' tan\alpha}{1 - \frac{tan\alpha}{tan\beta}}$$

So, hence it is proved**.** This expression you can use it for solving further problems as well**,** so that is all about the first problem.

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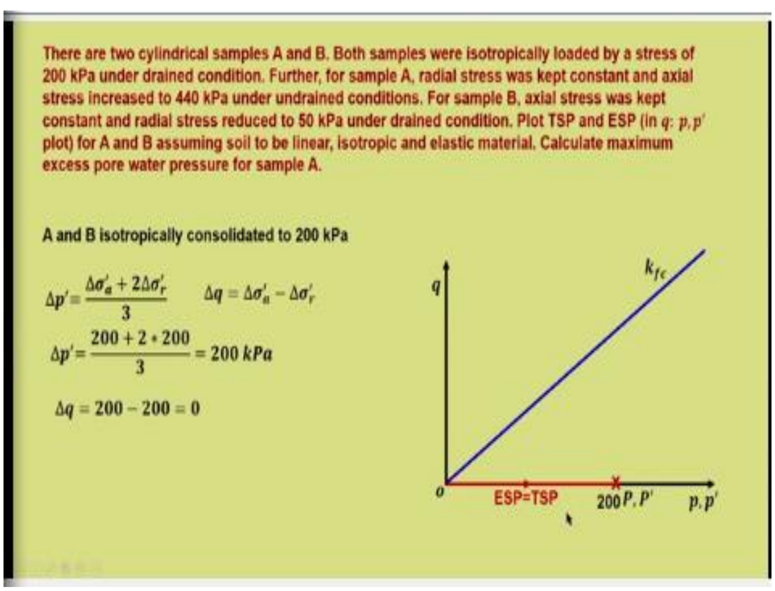

Now let us move on. So, the next problem is as follows, there are 2 cylindrical samples A and B, now both the samples were isotropically loaded / a stress of 200 kPa under drained condition. So, there are 2 samples A and B, let us consider it to be a triaxial sample, both of them are isotropically consolidated. So, isotropically consolidated, it is very clear that it is under drained condition otherwise it will not consolidate.

And the initial isotropic consolidation pressure is given that is 200 kPa. Further, let us take the case of sample A, for sample A radial stress was kept constant, radial stress means σ_r is kept constant, and axial stress is increased to 440 kPa under undrained conditions. Now you remember that it is a straightforward case of compression. So, radial stress was kept constant and actual stress increased / 440 kPa.

And now we have to keep in mind that it is under undrained condition. For sample B axial stress was kept constant and radial stress reduced 250 kPa under drained condition. So, this is also compression because axial stress was kept constant and you are releasing or reducing the radial stress. Now earlier radial stress was 200 it is reduced to 50 kPa under drained condition, now that also we have to keep in mind.

Now we need to plot total stress path and effective stress path in q-p, p' plot, for A and B assuming soil to be linear, isotropic and elastic material. Calculate maximum excess pore water pressure for sample A. So, it is a straightforward problem, we are not asked to find out any failure conditions rather we just need to find in what manner the total stress path and effective stress path would be in this particular problem.

So, let us say A and B is isotropically consolidated to 200 kPa, and before that I think we also need to take the value of A param. That is the pore water pressure param also, so we will do that or we will assume that in the due course**.** So, A and B is isotropically consolidated to 200 kPa, so p' is, we need to find out what is the value of p' and q', because that q because that is where we have to plot it.

And Δ q = $\Delta\sigma'_a$ - $\Delta\sigma'_r$, so this is known to us. So, the initial condition is 200 + 2 into 200 / 3 this 200 and Δ q is 0. Let us plot this initial condition, so we are starting from 0, we are consolidating it isotropically, here you can see q is same. So, Δ q is 0 and Δ p' keeps increasing to 200 kPa, so this is p, p' because it is isotropically consolidated, there is no pore water pressure existing in the beginning**.** So, that is 200 and this initial point is p, p'. Here effective stress path is equal to total stress path, and these things are very clear to all of us.

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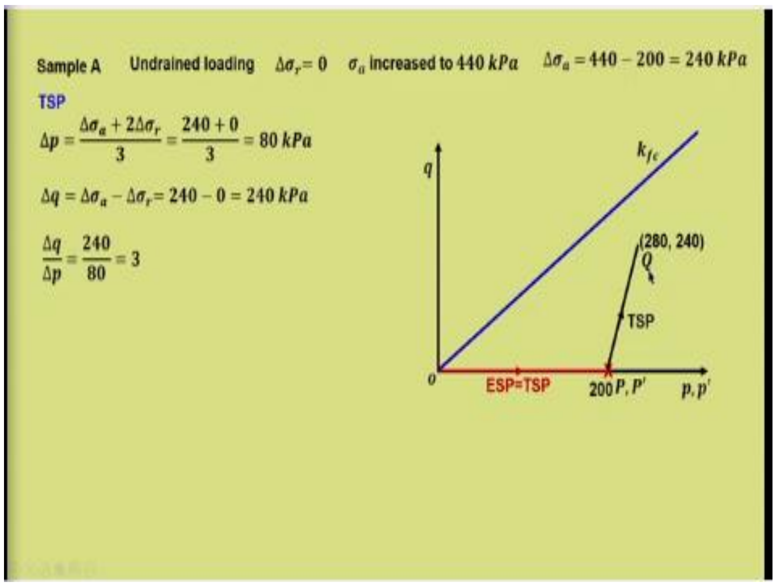

Now specifically for sample A, let us start it is an undrained condition. Undrained loading, we know that it is at constant radial stress, so Δ σ_r = 0. So, interpreting the question is very important, Δ a is then increased to 440 kPa, so Δ σ_a will be 440 - 200. Because earlier there is an isotropic stress condition further which it is increased up to 440 kPa. So, do not misinterpret that 440 is in addition to 200, it is increased to the axial loading is increased to 440.

So, the total axial stress increment will be 440 - 200 which is 240 kPa, so these data is known. Now for total stress path it is pretty straight forward Δ p = Δ σ_a + 2 Δ σ_r / 3, Δ σ_r is 0, that is substituted Δ σ_a is 240 and hence Δ p = 80 kPa. Where you can go wrong in this question? If you interpret that it is 440 is over and above there is a possibility of misinterpretation, in fact it is loaded to the final value.

So, those terminologies and English usage you have to keep in mind Δ q = $\Delta\sigma_a$- $\Delta\sigma_r$ which is 240 - 0 which is 240 kPa, Δ q upon Δ p is 240 / 80 which is equal to 3. Now this slope we already know, in q-p, p’ we have already seen for drained condition that is total stress path equal to effective stress path, the slope is equal to 3. Now in the undrained case for total stress path, this slope will always remain same.

So, that is what we have just found out and that is equal to 3. So, this is P Q is the total stress path at an inclination of 3 or a slope of 3. So, here this value will be 280 and 240, now the final value will be because this is Δ p, so increment is 80. So, up to here it is 200 + 80, so that is 280 on x axis and y axis which is represented / Δ q which starts from 0. So, whatever it is 240 that appear as 240, so this is the point q.

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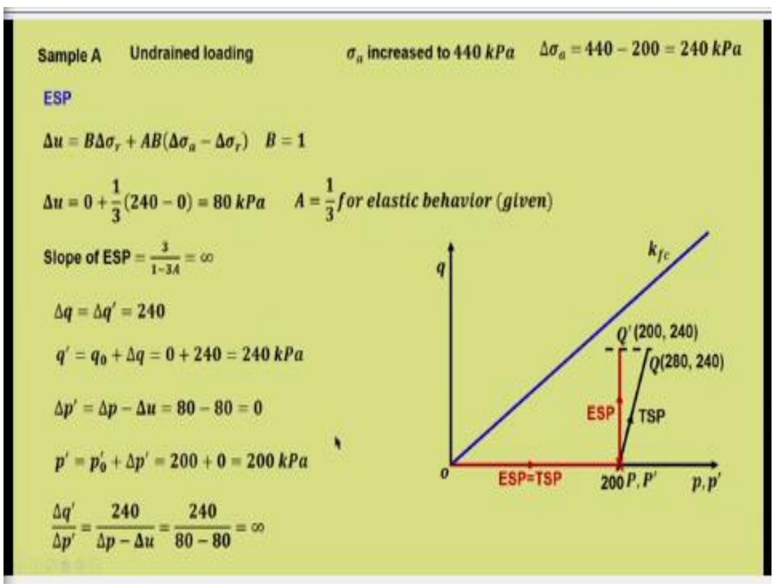

Now, we also now need to find out what is ESP? So, the moment for undrained case we need to plot ESP, we also need to determine what is the variation in pore water pressure? So, Δ u is the excess pore water pressure which is B * $\Delta\sigma_r$ + AB ($\Delta\sigma_a$ - $\Delta\sigma_r$), B = 1. And Δ u = 0 + (1 / 3) (240 – 0) which is equal to 80 kPa. Now what is the catch here? It is already given in the question; assume the soil to be isotropic and elastic.

Now if Skempton's pore water pressure equation for triaxial condition is not clear to us, it is difficult to assume the value of A. Here for elastic condition we know that the value of A = 1 / 3, and that is what we have done here A has been substituted to 1 / 3, so Δ u we will get to be 80 kPa. So, this is the condition that we have used, so slope of effective stress path is equal to, this also we know 3 / 1 - 3A.

Now A, the value is 1 / 3, so here it becomes 0 and hence it will be infinity. So, that means it grows vertically upwards because it is Δ q upon Δ p. So, Δ p there is no change in p, so it has to go vertically upwards, Δ q = Δ q’ = 240 which we have already seen in the previous slide**.** So, maximum value for this particular case is 240 and so q’, I have just taken q’ just to make sure that it is for effective stress path.

But we already know q’ and q are the same, so q’ = q_0 + Δ q that is 0 + 240, so it will be Δ p’ = Δ p - Δ u which is 80 - 80 which is equal to 0, so that is also now known. So, there is no change, so p’ = p'_0 + Δ p’ that is 200 is the initial value + 0 which is 200 kPa. And hence Δ q’ / Δ p’ = infinity, and this information we already have here, is just for your understanding we have worked it out.

So, here is the effective stress path, so this we have understood. Now this is Q’ 200 and 240, now what is this distance? It is already there Δ u which is equal to 80, so 280 - 80 that gives 200, q value is 240, as we have determined here. So, that is all about sample A for undrained loading.

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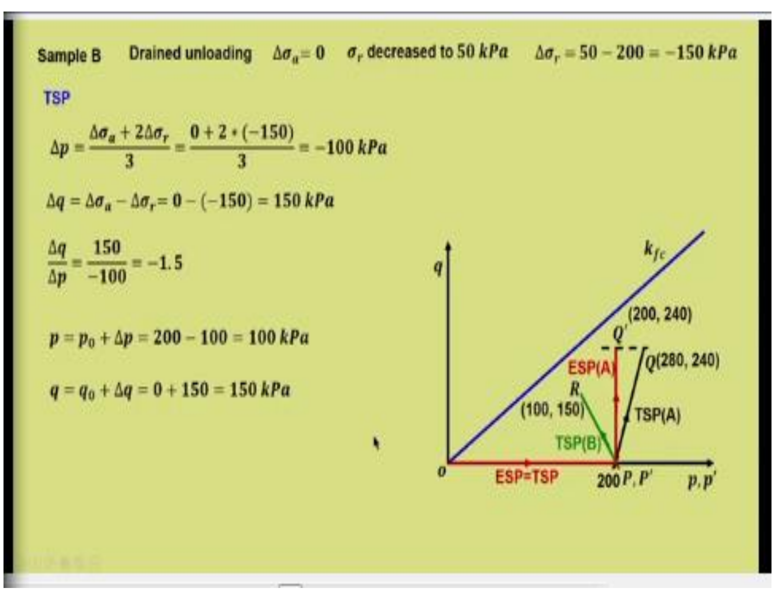

Now sample B, so we know that it is a drained unloading, so here it is release of radial stress, so it is unloading. So, drained unloading $\Delta\sigma_a$ is 0, axial stress kept constant, σ_r decreased to 50 kPa. So, there was an initial value of 200 from there it is decrease to 50 kPa. So, $\Delta\sigma_r$ = 50 – 200, that is - 150, all these sequence of loading we have clearly discussed while discussing the stress path.

So, you can cross refer to those for understanding this better. So, TSP, now TSP and ESP both are same because it is drained unloading. So, Δ p = $\Delta\sigma_a$+ 2 $\Delta\sigma_r$ / 3, $\Delta\sigma_a$ given 0 + 2 into - 150 / 3, it is -100 kPa, Δ q = $\Delta\sigma_a$- Δ 0 minus of minus it is +150. So, Δ q upon Δ p = 150 / - 100 which is equal to -1.5, so it is very clear that the slope is negative.

p = p_0 + Δ p, so p_0 is 200 and Δ p = -100, substituting that we know that it is 100 kPa but in the left ward direction. And q = q_0 + Δ q q_0, initial value is 0 + 150 Δ q is +150 that will give you 150. So, 150 in this direction 100 it has to go in this direction, so that is the slope it is negative slope. So, TSP for B, so this is the P R is the required TSP for B and all other interpretation it comes from this calculation.

Because it is minus, so we know that it is moving in this particular direction, and apart from that the slope is also minus, so we know that it has to be in the left ward direction. Now should it be in which direction that it should it be in this or in this, so that information you will get from the actual values of P and Q**.** Here it is looking positive but please remember that it is going in the reverse direction. Earlier value was 200; since it is negative it is decreasing. So, we know that in which direction it will be moving, so that is 100 and 150.

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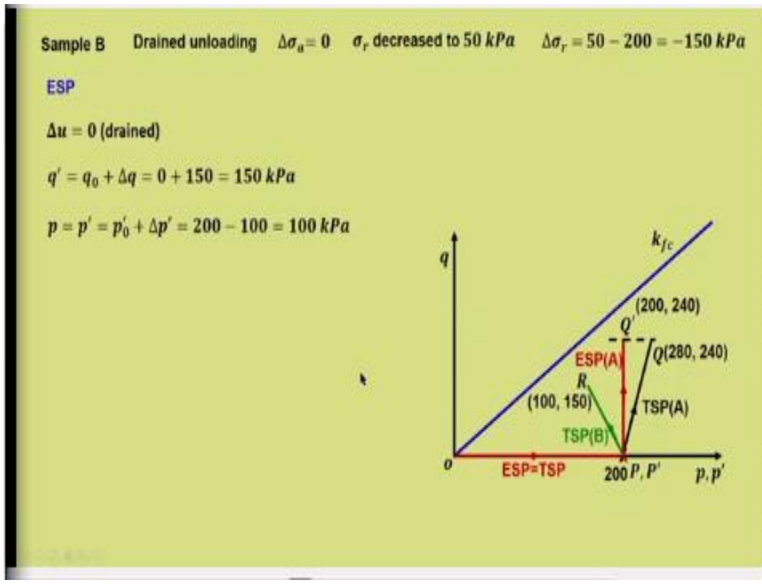

So, ESP of this Δ u = 0 drained, so q' = q_0 + del is 150, p is 100 kPa and hence it is the same. So, there is no difference for ESP and TSP, so since it is drained we do not have to explain it further. So, that is all about the second question we will move on to the next one.

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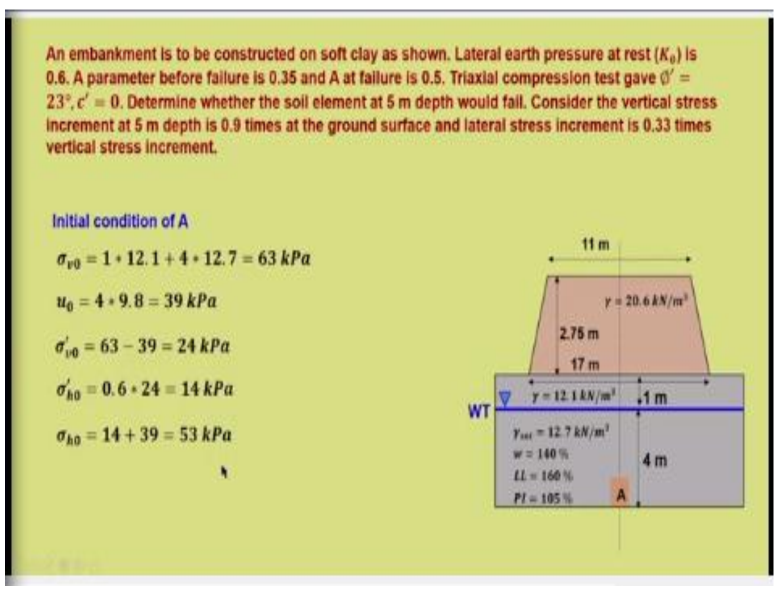

So, the next question is, an embankment is to be constructed on soft clay as shown. So, this is the embankment the top width is 11 m, bottom width is given it is 17 m, the height is 2.75 m and the bulk unit weight of the material is 20.6 KN/m^3. And this is the soft clay below with the given properties, there is a water table at 1 m depth, above the water table gamma or the bulk unit weight is 12.1 KN/m^3.

Below the water table the saturated unit weight is 12.7 KN/m^3, initial water content is 140% below, liquid limit is 160 and plasticity index is 105% and the lateral earth pressure at rest K_0 is 0.6, that is given. A parameter before failure is 0.35, and A parameter that is A_f at failure is 0.5, so Skempton's A parameter is given, triaxial compression test in the lab you can add gave the effective friction angle = 23 degrees and c’ = 0.

Determine whether the soil element at depth 5 m would fail or not? So, this is the reference soil element which is at 5 m below the ground surface. Now we need to understand based on stress path whether this particular element would fail or not. Consider the vertical stress increment at 5 m depth is 0.9 times at the ground surface and lateral stress increment is 0.33 times of vertical stress increment. What is the importance of last statement?

So, there are 2 cases, first is the initial condition which is due to self weight, now under self weight it is not going to fail. Now when you are placing the embankment load whether the element would fail or not, that is the question. Now when you add the embankment there will be some amount of stress that gets added up at the element A. Now if you studied the stress distribution we know that there is a way / which we can calculate it.

Here we are adopting this particular information for finding out what will be the stress increment at point A, that is consider it is at0.9 times, now if you consider what is the stress because of this embankment at the ground surface then at point A it will be 0.9 times of what is the stress due to embankment at ground surface. So, first we need to find out what is the stress increment due to embankment at the ground surface.

It is 0.9 times will be the stress at the element A, and the lateral stress increment is 0.33 times of vertical stress increment at A. So, those information are given, so let us see how to solve this problem. So, initial condition of A, it is because of the sulfate, so σ_{v0} = 12.1 * 1 + 4* 12.7, so that is the total vertical stress which is equal to 63 kPa and that is the initial condition.

u_0, if you consider a piezometer here it will rise up to 4 m, so 4 into 9.8 which is 39 kPa. So, σ'_{v0} = 63 - 39 which is 24 kPa, so σ'_{h0} = 0.6 because lateral earth pressure at rest is given to be 0.6, so

0.6 into 24, that will give us 14 kPa. So, σ_{h0} = 14 + 39, that is a total horizontal stress 14 + 39 which is equal to 53 kPa.

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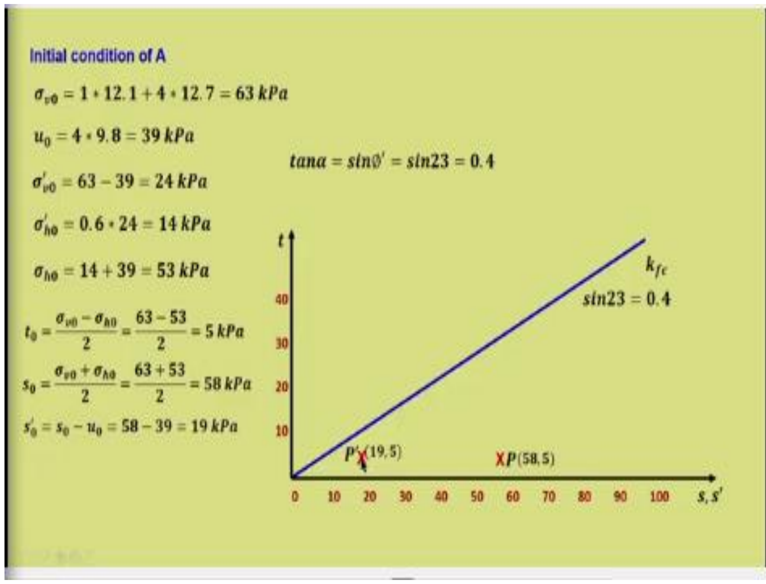

So, initial condition is known. So, t_0 we have plotting this in t-s, s’ plot, so we can find out what is t_0? Which is σ_{v0} - σ_{h0} / 2 all the values are given 63 - 53 / 2 which is 5 kPa, s_0 the same + 63 + 53 / 2 which is 58 kPa. So, the initial condition is now known. So, $s'_0$is s_0 - u_0 which is equal to 58 - 39 which is equal to 19 kPa, because u_0 is 39. So, all these initial conditions are known now.

So, t-s, s’, k_{fc} is the failure line and you can easily denote what is k_{fc} because φ’is known. tan α = sine φ’, φ’ is 23, so we get0.4, so sin 23 = 0.4 is the slope of the failure line without this, this problem cannot be solved. Now the starting point is the point P, we know that it is t_0 and s_0 which is 5 and 58. So, here it is somewhere 58 is s_0 and t_0 is 5, so 58 and 5, we have s'_0 which is s_0 - u_0 that is p’ is around 19 kPa and t_0 remains same.

So, at 5 height we have 19, so it is close to here 19, so p’ that is 19 5. So, you can see that the initial point itself is very close to the failure line. So, you can make some judgment right at this point itself, like there are not much of elements for failure. Like it is very close and factor of safety is substantially low. So, factor of safety how do you be in the last lectures we have seen that when it is close to the failure line the factor of safety is less. So, accordingly we can easily make out the factor of safety or the safety margin is quite less in this particular case, this is 39**.**

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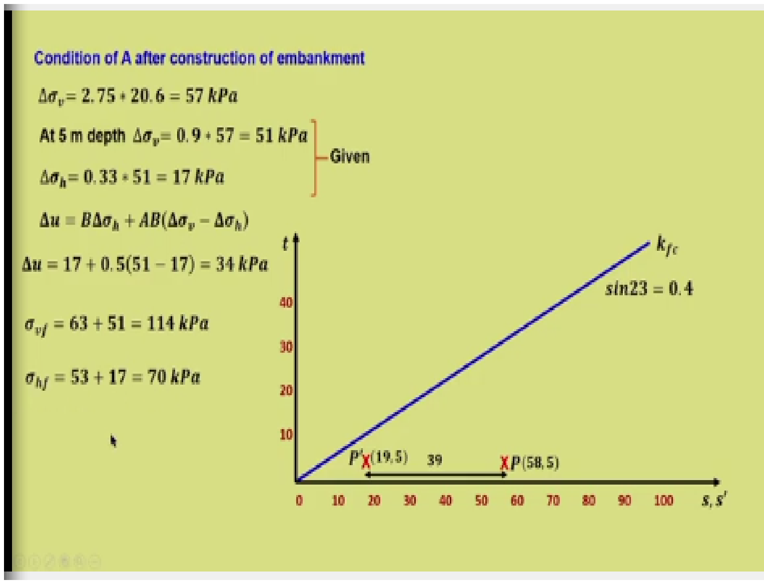

Now after the construction of embankment what will happen? Whether it was going to fail or not? So, the incremental stress is $\Delta \sigma_v$ which is equal to 2.75 which is the height of the embankment into 20.6 is the unit weight given for the embankment material which is 57 kPa at 5 m depth, the entire stress is not going to be delegated it is 0.9 times according to what is given in the question.

$\Delta \sigma_h$ = 0.33 into 51 which is equal to 17 kPa, this is given and this is as per what is given in the question. Now Δ u what will be the change in pore water pressure because of embankment construction which is B* $\Delta \sigma_h$ + AB* ($\Delta \sigma_v$ - $\Delta \sigma_h$). Substituting this we have all the information with us, we have Δ u = 34 kPa. So, σ_{vf} is 63 + 51, the incremental load that is 114 kPa, σ_{hf} = 23 + 17 which is 70 kPa.

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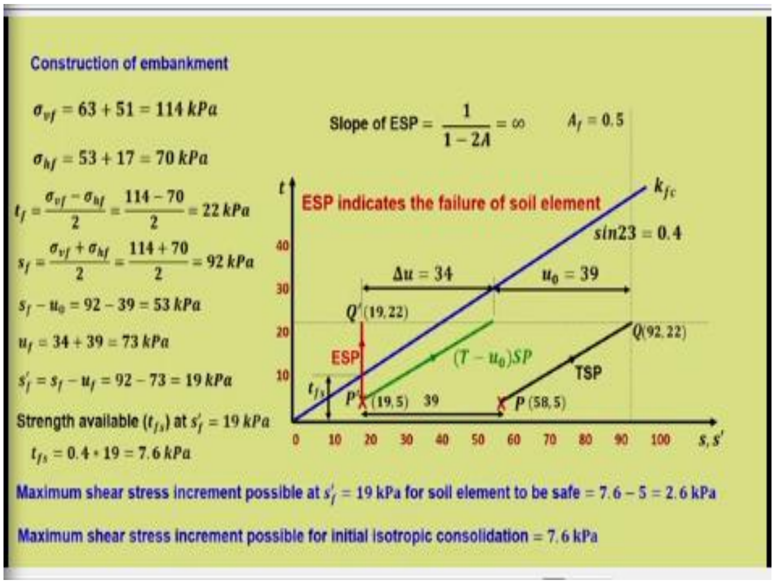

Now with this we can find out what will be the final condition of total stress that is $t_f = \sigma_{vf} - \sigma_{hf} / 2$ which is 114 - 70 / 2, which is 22 kPa. So, here it is 22, I am drawing a dashed line here, what is s_f? $s_f = \sigma_{vf} + \sigma_{hf} / 2$ 114 + 70 / 2 which is 92 kPa, so somewhere here. So, I am drawn a vertical dashed line $s_f - u_0$, that is 92 - 39 which is equal to 53, so we already know that it is 39 which is 53 kPa.

So, u_f = 34 + 39 because in the previous slide we have seen that Δu because of embankment construction is 34. So, initial is 39, so total pore water pressure at final is 73 kPa, why did I use this particular $s_f - u_0$? This is to just show that the total stress path here is translated to a total stress path here that is TSP - u_0 stress path. So, t - u_0 stress path, we have total stress path TSP and from the total stress path whatever is the initial porter pressure we minus that.

So, that we draw the total stress path at this particular point, so that is another way of representing total stress path, t - u_0 stress path. That also we have discussed in the last lecture, you may please refer back. So, $s'_f = s_f – u_f$ that is 92 – 73, from here you need to minus the final pore water pressure 73, that is 19 kPa. So, it is at the same point here s'_f is at this point, so this is the total stress path P Q that is 92, 22.

So, this is 92 and here it is 22, that is due to the construction of embankment. Now when the embankment is getting constructed the total stress path is progressing in this direction, and that is

92, 22, that is total stress path. Now this is t - u_0 stress path, it is parallel to stress path but it is drawn at this initial point. And that is initial pore water pressure 39 is minus from this and this is the t - u_0 stress path.

Now the effective stress path is the s'_f is 19, so it is at this point, and what is t_f? It is 22. So, it moves in this direction and touches here, why it is moving in the vertical direction? It is already given in the question that the final pore water pressure at failure will be the A parameter corresponding to this will be 0.5. And we know that the total effective stress path of t-s, s' plot is 1 / 1 - 2A**.**

According to that, that is slope is 1 / 1 - 2A, so A_f is given as 0.5. So, if you substitute A_f = 0.5, this will be infinity, so that is why it is moving vertically upwards, so that is what is shown here A_f = 0.5 and that gives infinite. So, it is in the vertical direction and this is 19, 22. Now we do not need to discuss further to say whether it will fail or not, yes the stress state according to the final construction of embankment is well above the failure line which is an impossible state.

So, the soil would have failed by the time it reached here, it cannot cross the failure line in any circumstances. So, the embankment construction would fail the soil element which is at 5 m depth. So, ESP indicates the failure of soil elements, so how beautifully the effective stress path helps us to understand a lot of facts whether it will fail, at what time it will fail, and how it is going to fail?

All these information we can derive / plotting stress path. So, this is a very good example which clearly demonstrates for a field problem how we can interpret various situations based on stress path plotting. So, now whatever we are asked to find out, it is already done, we have found out what is the excess pore water pressure, we have found out whether the embankment construction would fail the soil element or not, all these things are known.

Now we are curious to find out what is the possible strength available if the condition is to be safe. There is a strength available that is t fs corresponding to s dash f, because this is a starting point is equal to 19 kPa, that is s'_f = 19, that is this particular point what is the available strength?

That is t_{fs}, so this is what is shown as t_{fs}. Now t_{fs} is equal to we know that the inclination is 0.4, 0.4 into 19 this distance, that is 7.6 kPa is the strength available here.

So, maximum shear stress increment which is possible due to the construction of embankment if the soil has to be safe is equal to 7.6 - 5 which is equal to 2.6 kPa. So, 2.6 kPa is the only permitted increment, why it is 2.6? Because we know that A_f is 0.5, and it is moving in the vertical direction. Now if it is moving in the vertical direction the only possible increment available with us is 2.6 kPa.

So, we can construct only that much of the embankment that will give an incremental stress of 2.6, beyond that it is going to fail. So, we have understood like where it is going to and how it is going to fail. Now here already the initial condition is at 5 kPa above now let us say that if it is isotropically consolidated. Then we would get an allowance of 7.6 kPa before failure that is this point will come down.

So, total possible increment would be 7.6, it is only an added information which we can get from the stress path plot. For example if the starting point is again towards left, we will have even less allowance or if this point is slightly here then we have this much of strength available. So, that much amount of embankment construction is possible. So, this particular plot help us for various possibilities and understanding related to a given field condition.

This is what we have discussed related to embankment. Now this can be any problem, it can be a foundation, it can be retaining walls like the one we have discussed in the last lecture, so that is all. So, now we have almost completed module 3 and some representative problems we have done.

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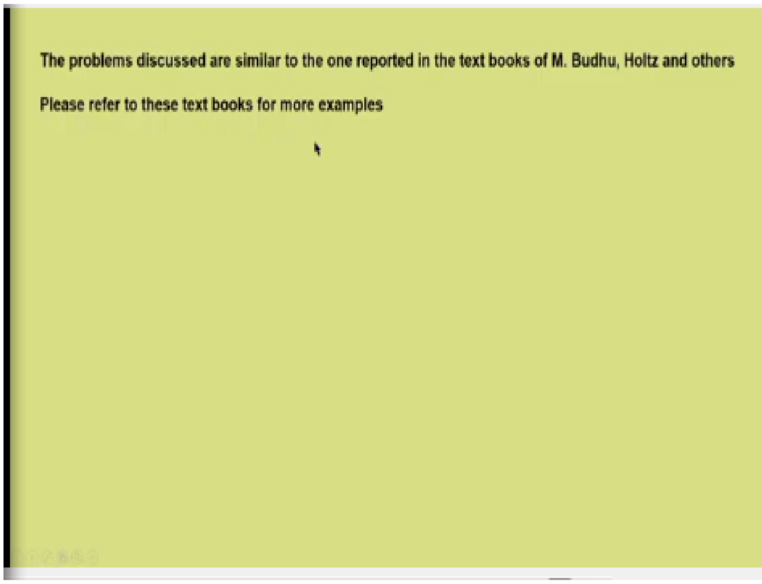

The problems which are discussed in this lecture, these are very similar to the one reported in the textbooks of M. Budhu, Holtz and others**.** So, please refer to these textbooks for more examples. So, that is all for now in this lecture, the last lecture in module 3 is summarizing the whole of module 3 what we have learned, so that is all for now, thank you.