Advanced Soil Mechanics
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Lecture – 04
Stress Acting on a Plane

In the last lecture, we were discussing about stress acting at a point and we have specified Cauchy's stress tensor Cauchy's formula, wherein, the traction vector which is acting at a point is related to Cauchy's stress and the normal vector we have discussed this at length. At the end, we discussed about one of its application of the stress tensor in mechanics is defining the equilibrium equation. Following that, we will see today like what is the application of Cauchy's stress tensor with respect to stress acting on a plane? We will start.

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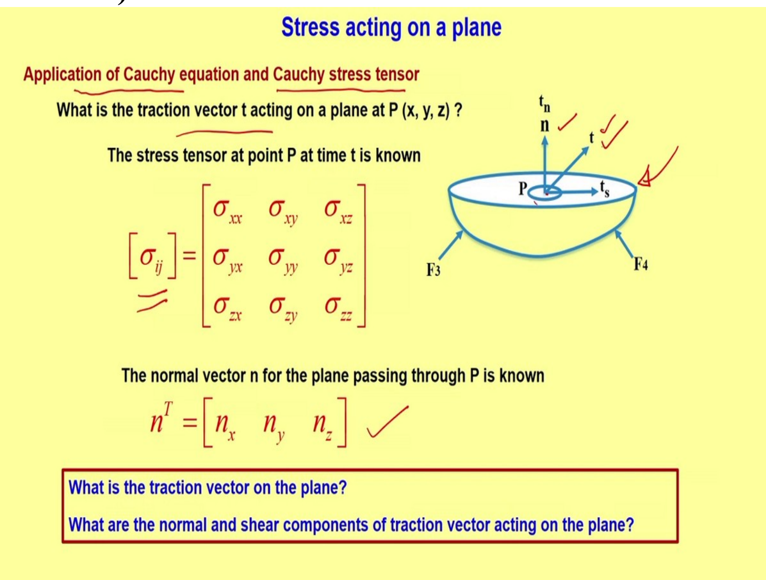

So, the discussion for today is stress acting on a plane we have discussed about stress acting at a point, but it is some effect of the whole body. Now, if you cut the body into 2 equal halves or 2 halves by a plane by a cutting plane, we are interested to find out what are the stresses acting on that plane? So, this is one typical application of Cauchy's equation and Cauchy's stress tensor I am not repeating Cauchy stress tensor, it is a 3 by 3 matrix form, what is the traction vector t acting on a plane at point P, point P is denoted by x, y and z.

This is the same plane which we discussed in the last lecture, where we evolved the Cauchy's stress tensor you can see here, this is the traction vector t. Now, stress tensor at a point P for a given time t, why time is important, because the stress tensor corresponds to a given t.

deformation of the body. So, stress tensor at point P at time t is known, so, that is known to us. So, this is represented by σ_{ij} and the components are given this we have already seen.

The normal vector for the plane P passing through point P is known. Now, this is very important, why because, the no any plane is always identified by its normal. So, normal vector n is known. So, for a given plane which is tangent plane at point p and what is the normal corresponding to that particular small elementary plane that gives the normal n? So, n is known and the direction cosine or the normal vector n for the plane is known and that is given as n vector.

Now, the question is what is the traction vector on the plane? So, I mean to say what is the traction vector t on the plane? What are the normal and shear components traction vector acting on the plane? So, these are the two questions which we have to discuss today. And for that what are the minimal information needed as explained?

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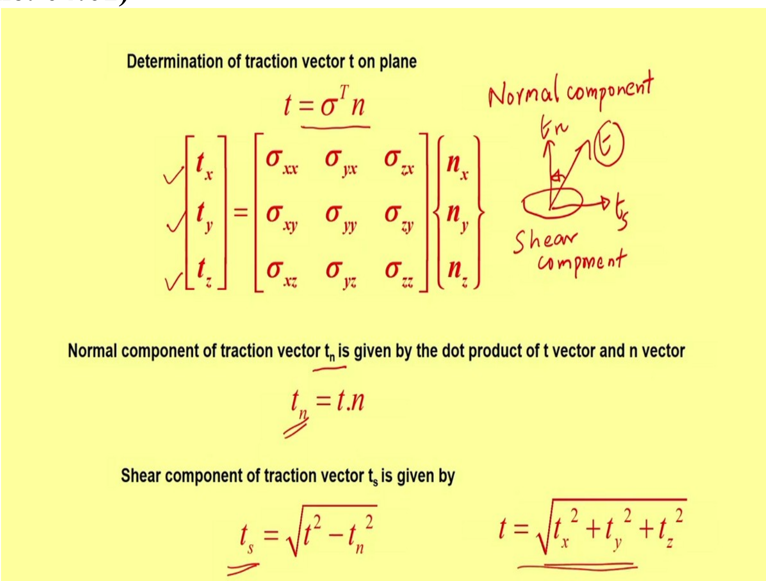

So, we will refer back to Cauchy's equation where determination of traction vector t on a plane is given by $t = \sigma^T n$. So, this is the Cauchy's equation. So, these are the elements of or components of traction vector t_x, t_y and t_z these are the elements which we need to determine now, once we determine t_x, t_y, t_z it is understood that the traction vector t is defined. now, that can be written based on Cauchy's equation as $t = \sigma^T n$, normal component of traction vector t_n.

Now, as we have discussed in the previous slide t is any traction and then we have normal component of traction vector now, t_n is the normal component. So, normal component t_n of the traction vector. similarly, we have the shear component of the traction vector t_s. So, normal component of traction vector t_n is obtained by the dot product of t vector and n vector why because t this component is known.

So, this is known. So, when you take dot product, it is the same protection law that we already know this is projected onto the normal. So, t vector or the traction vector and normal vector the dot product gives you the normal component of traction vector. So, similarly, the shear component which I have already shown here, the shear component of traction vector t_s is given by so, once we know t_n.

So, t_n is known now, we know that t_s because these are orthogonal, we already know that it is

$$\sqrt{t^2 - t_n^2}$$

that means, the magnitude of t_s that is the shear component of the traction vector t_s is given by

$$\sqrt{t^2 - t_n^2}$$

Now, this determination of stress is acting on a plane when I say stress is acting on a plane it means normal stress or the shear stress or if we tell more specifically it is the normal component of traction vector and the shear component of traction vector can be determined.

So, t_n and t_s are the required components. So, here what is t? t is the magnitude of the traction vector which is obtained as

$$\sqrt{t_x^2 + t_y^2 + t_z^2}$$

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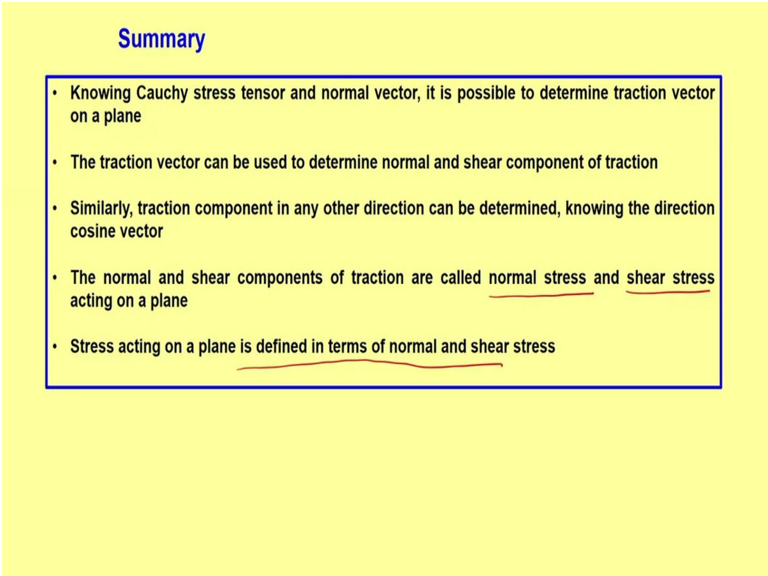

So, to summarize knowing Cauchy's stress tensor and normal vector, it is possible to determine traction vector on a plane the traction vector can be used to determine normal and shear component of traction. Similarly, traction component in any other direction can be determined knowing the direction cosine vector. So, if the direction cosine is known, then what is the component of traction vector in that direction can also be determined?

As of now, we have determined for normal direction, the normal and shear components of traction are called normal stress and shear stress acting on a plane. So, whenever we say stress acting on a plane, we define it in terms of normal and shear stress or the normal component and shear components of the traction vector itself is considered as normal stress and shear stress acting on a plane.

The stress acting on a plane is defined in terms of normal and shear stress. So, this is all about stress acting on a plane and it is a typical example of applying Cauchy's stress tensor and Cauchy's formula for finding out stress acting on a plane.